Mathematical Methods 1
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Linear Algebra
Lecture – 13
Rank of a matrix: Consequences

So, in the last couple of lectures, we looked at the operation of row reduction or the row echelon form or any matrix can be brought into this form and then, we also looked at the concept of a rank of a matrix right and so, here we will see that once we know the Rank of a matrix or a rank of two matrices corresponding to a set of equations right.

So, we are able to say whether this set of equations are consistent or if there is a unique solution, if there are lots of solutions right. So, these statements can be made directly from knowing the rank of the augmented matrix and the coefficient matrix ok.

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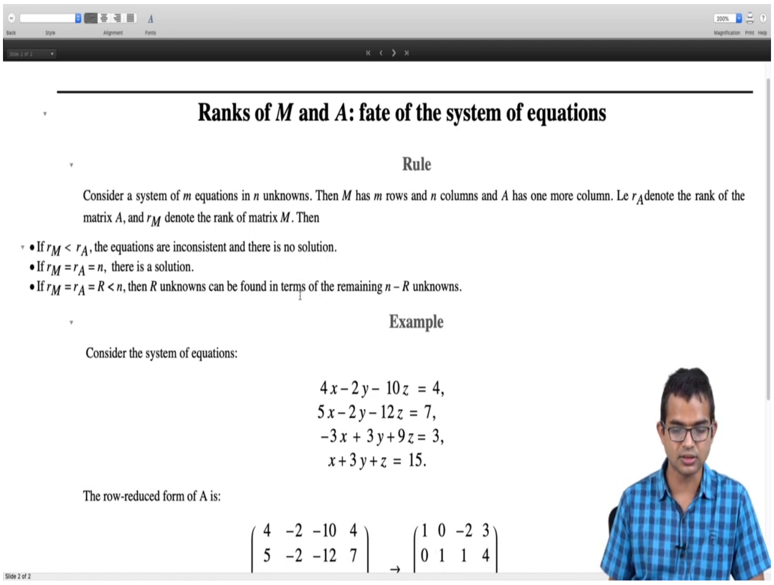

So, let us just give it out as a rule right. So, the rule is simply that if you have a system of m equations in n unknowns. So, you have m rows and n columns right. So, you write down all the equations as different rows and so, you have and A has one extra column right. So, these are the numbers which appear on the rightmost end.

So, let r A denote the rank of matrix A and r M denote the rank of matrix M. Then there are only three possibilities right; either r M is equal to r A or r M is not equal to r M r A right. If r M is equal to r A, you again have two possibilities; either r M is equal to r A is equal to n, n is the number of unknowns right.

So, you also have this number of unknowns n and in addition to the number of equations right. So, you could have lots of equations and the number of unknowns could be a different variable n right. So, the question is how are all these related and what can we say about the solution right depending upon all these variables.

So, if r M is not equal to r A, it turns out that r M the only possibility is for r M to be less than r A, r M can never be greater than r A right. You can see it you know when we discuss how the rank of a matrix can also be extracted from determinants right. So, the matrix A has, if at all, more information than the matrix M, it can never have less information than the matrix M right; so that is also clear intuitively.

Now, r M is if r M is less than r A, then it is a problem. So, for this system of equations, there will be no solution, there is an inconsistency right. Only if the rank of r M, rank of M is equal to rank of A only then are your system of equations consistent. Now, if r M is equal to r A and also equal to n the number of unknowns, then not only are they consistent, but there is a unique solution, there is just one precise solution.

But on the other hand, if r M is equal to r A is equal to some R which is less than n then R unknowns can be found in terms of the remaining n minus R unknowns, right. So, this is the theory right. So, now, it can happen that you may actually have a ton of equations right. So, I mean why would we want some very systematic approach like this, it is essential if you are trying to put them on a computer right.

You have like 100 variables let us say and I do not know 1000 equations, so, it may well be that there is a lot of redundancy in it, all these 1000 equations are not you know giving you one extra bit of information, but many of them can be just you know removed and the question is do they contain 100 pieces of information enough to extract the values or do they contain even lesser information or they even inconsistent right.

So, this can be you know extracted using the rank approach right and that has a very well defined, very systematic approach that we have already defined, it can be coded on a computer and you can you know get it to read out what the rank of the augmented matrix is, what the coefficient matrix rank is from which you can get you know get an answer to this question whether they are consistent or not ok.

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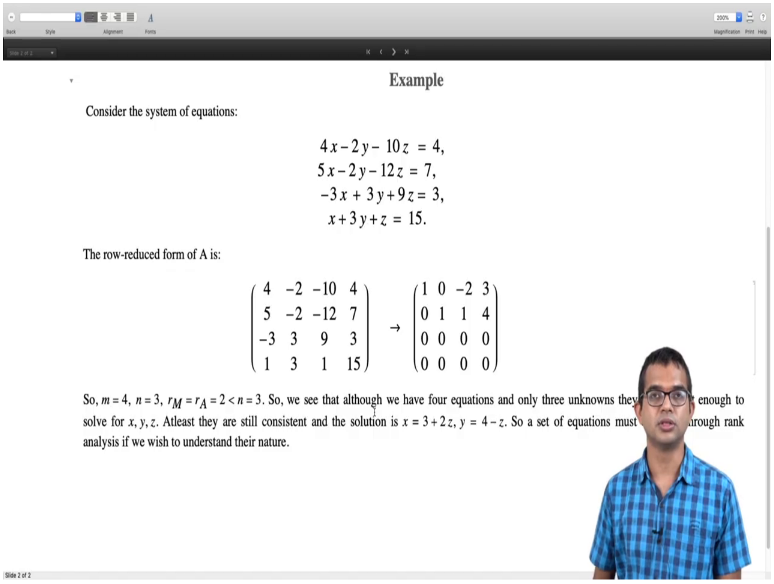

So, I have an example here. I have only three unknowns x, y and z, but I have four equations. So, we might think that surely this should be solvable right, but you know the test for such a case is of course, to go for the you must look at the rank of these whether two matrices involved only after that can you make a statement on whether they have enough information to solve for x, y and z or they do not have information or they have inconsistent information.

So, in this case, it turns out again I will not do the elaborate procedure, I have given you know the method and I expect you to work this out. You should show that the row reduced form of A in this case, you know results in this form which is just a rank 2 matrix, right.

So, the fact that both the augmented matrix and the coefficient matrix are both of the same rank, implies that they are consistent, these equations are consistent, there is no problem you can solve for you know various equations in terms of the other. But it is not; it is not enough to pin down the values of x, y and z because there are only two equations worth of information although there are four.

And so, in this case r M is equal to r A is equal to 2 which is less than n which is equal to 3, right. So, we started with m equal to 4 and n equal to 3. So, in this case, r M and r A are 2 less than n which is equal to 3 and therefore, you have actually infinitely many solutions in some sense. You can, you have the freedom to choose z to be whatever you want and then once you have been z, you can get x and y in terms of z right.

As you can see from the first equation and the second equation in the row reduced form. So, x equal to 3 plus 2z you can read off from the first row and y equal to 4 minus z you can read off from the second row, but that is it, there is nothing more you can do, z can take any value and then from which you can solve for x and y right.

So, the whole reason for this discussion about ranks is you know it is intimately connected to the notion of linear dependence and linear independence and so on which we will come back to, we will go back to this discussion you know related to abstract vector spaces and so on. But before that, we also have a discussion of determinants which is going to follow in the lectures ahead. That is all for this lecture.

Thank you.