Mathematical Methods 1
Prof. Auditya Sharma
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Linear Algebra
Lecture - 01
Vectors

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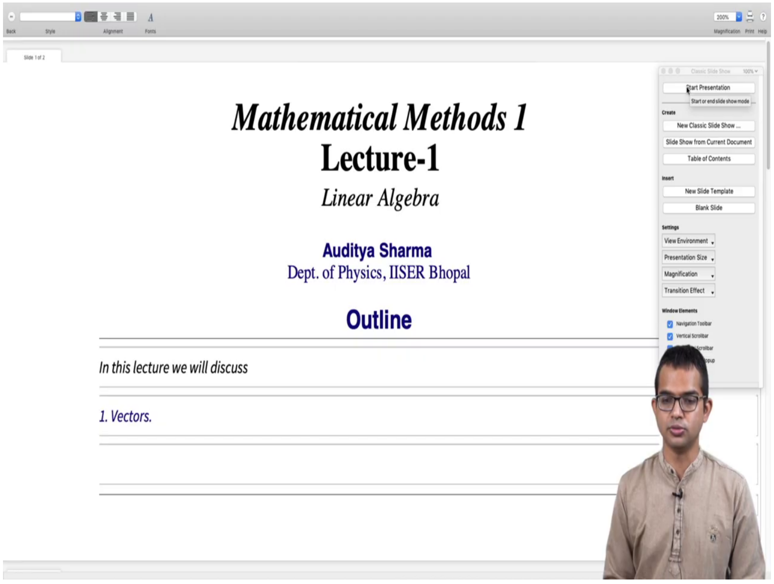

So, let us get started. So, this is going to be a series of lectures on Linear Algebra, right. So, for organizational purposes I am dividing the content into many sub units. So, we will have you know lots of these small pieces. But all of them I am going to just call them lectures, so lecture 1, lecture 2, so on, but there will be each of them will be tiny lectures.

So, let us start with a, I guess it is more in the nature of recall, right. Most of us should be familiar with vectors. So, let us get started. So, this lecture is about vectors that we are familiar from high school, ok.

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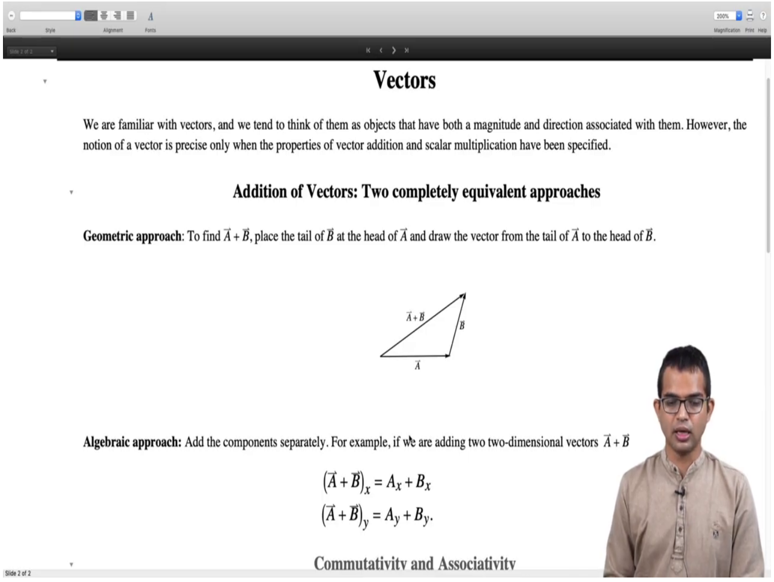

So, we tend to think of vectors as objects that have a magnitude and a direction associated with them, right. So, however, one has to be a little more careful, not everything which has both a magnitude and direction is going to be a vector.

So, for a vector to be a vector it has to satisfy certain properties. The key, one key property is how two vectors add, right. So, if you add two vectors you must get another vector and which has a very precise form, right. So, we are also familiar with this. So, let us just recall how we thought of addition of vectors, right.

So, there is one way of thinking of this is the geometric approach. You can think of you know a vector A and a vector B, and if you want to add these two you place the tail of vector B at the head of vector A and then you draw a vector from the tail of vector A to the head of vector B, right. So, that is going to be A plus B, right. So, there is an algebraic approach which one can come up with which is to add the components separately, right.

So, suppose, so I am here for the purpose of illustration I am thinking of these as two-dimensional vectors, so in general you can have you know whatever dimension you want. So, then you have all these components associated with these vectors, you add these component the vectors component wise. So, the x component of the vector A plus B is simply A x plus B x and the y component of A plus B is A y plus B y, right.

$(\vec{A} + \vec{B})_x = A_x + B_x \qquad (\vec{A} + \vec{B})_y = A_y + B_y$

So, this is another way of thinking about addition of vectors, right.

So, a key aspect of this operation which we have called addition is that it is commutative and it is also associative, right. So, this is an essential aspect of what this operation called addition means, right.

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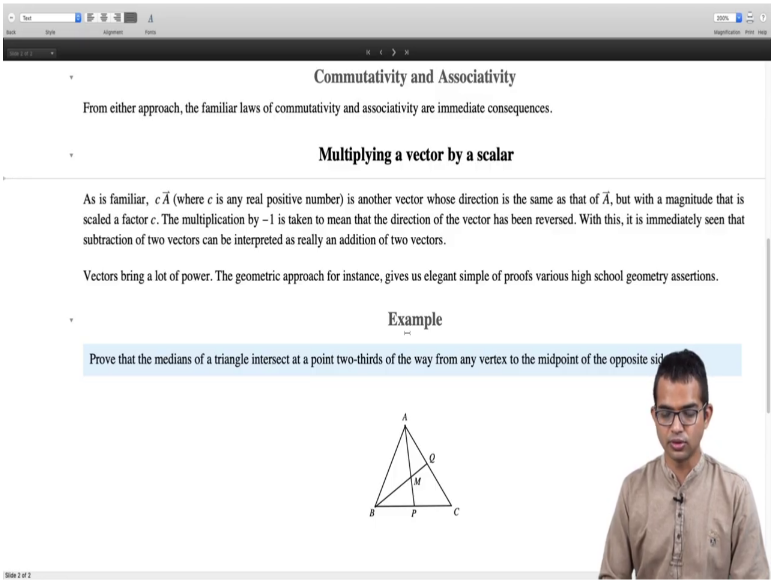

So, the reason why we are going into these apparently very simple operations is because we want to look at vectors in a much more abstract way, which we will start doing from the next lecture onwards. But the point is that we will look at a familiar object, you know the vectors and try to pull out the key properties, and then we will just work with your you know more abstract quantities which have these properties, right.

So, we are looking at vectors and we observe that addition is commutative and also it is associative. What does commutativity mean? It simply means that it does not matter whether you add A to B or B to A, so the result is the same, right. We could have, in the geometric approach we could have you know started with the vector B and you know place the tail of vector A at the head of vector B and then add it would still give us the same answer, right.

So, A plus B is the same as B plus A and it is evidently clear also with the algebraic approach. And associativity means if you are adding three vectors A plus B plus C, now it does not matter where you put the brackets, right.

So, you could put the brackets as A plus you know B plus C being added first and then you add A to it or you could think of it as the addition of A and B first you know you put a brackets you know corresponding to A and B, A plus B brackets plus C. So, it does not matter in which order you do this operation you are going to get the same answer, right. So, that is what associativity means, right.

So, we can you know look at the way we have defined addition and immediately verify that these two properties hold, right. So, we were going to make this more general and then, but we will try and we will preserve these properties. That is why it is important to observe this property.

So, the other you know aspect of a vector is multiplication with a scalar. If you take a vector and multiply by a scalar, right. So, for the moment let us think of just real numbers, if you take a real number and multiply it with some vector, its direction is going to remain unchanged in this for these familiar two-dimensional or three-dimensional vectors, but its magnitude is going to get magnified or diminished by this factor c, right. It is all very familiar. We have thought about this in high school.

And the multiplication with minus 1 is going to just change the direction, but keep its magnitude unchanged, right. So, we are going to come back to this later on. So in fact, we will say that multiplication by an arbitrary complex number is possible, a scalar could be taken from a complex field, we are going to generalize this later on, right. And the notion of a vector itself is going to be generalized, but subject to these properties, that is why it is important to identify these properties, ok.

So, but before we go on to thinking about abstract vectors, let us very quickly point out an example where the use of vectors already gives you a lot of power, right. So, from high school we geometry there are all these theorems which we proved and many of these had very complicated proofs. So, let us just look at one example where simply the use of vectors as we have defined it so far itself can give you very elegant and instructive proofs.

So, one statement that we are familiar from high school is that if you take a triangle and then draw the medians of the triangle they are all going to intersect at the same point first of all, and they are going to intersect at a point which is two-thirds of any of these medians. So, let me show you a diagram. So, I have this triangle ABC, and we are asked to prove that if you drop these medians AP and BQ, right, they are going to intersect at some point M, for sure they are going to intersect.

And so, the assertion is that this AM, the length AM is two-thirds of the length AP and likewise the length BM is two-thirds of the length BQ. And if you have drawn the third one which I am not drawing here that too would be the same. So, CM is going to be two-thirds of the median from C to AB to the side AB, ok. Let us prove this using vectors and we will appreciate the power of you know this simple abstraction itself.

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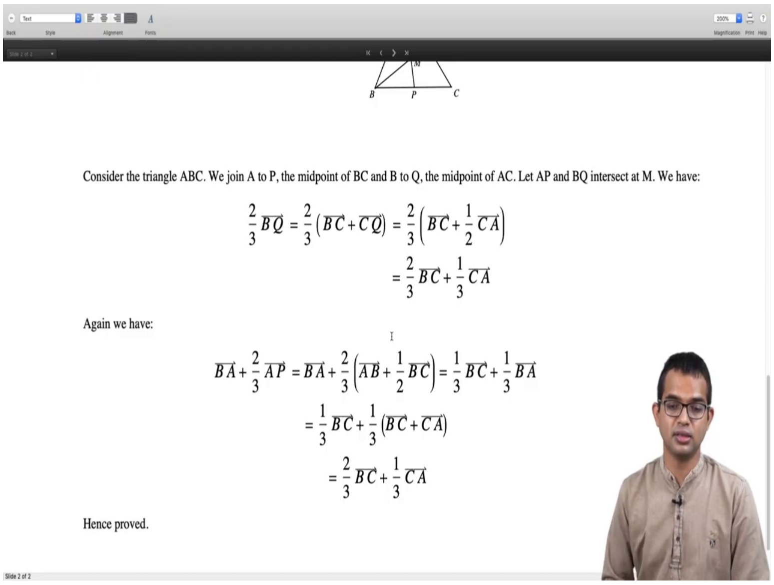

So, consider this triangle, ok. So, I have joined AP and I have joined BQ. So, let them intersect at M, right. So, all of this is by construction we have just drawn these things and we have named them. So, now let us look at this vector BQ, right. So, I want to look at two-thirds of the vector BQ. So, it starts at B and goes all the way up to Q, so let me see what is two-thirds of BQ.

But what is BQ? BQ is from this parallelogram law of addition BQ is nothing, but BC plus CQ, right. It is important to get the direction right. So, BC plus CQ is equal to BQ. So, I have two-thirds of BQ is equal to two-thirds of BC plus CQ, but which is the same as two-thirds of BC plus half of CA, right because Q is the midpoint of AC, right. So, two-thirds of BC plus half CA which in turn is nothing, but two-thirds BC plus I am just multiplying this out. So, I have two-thirds BC plus one-third CA. So, let us keep this as it is.

Now, let us compute the same two-thirds BQ in a different way, right. So, we have BA plus two-thirds AP, right. So, let us compute BM. So, I am claiming that if I go from B to A, and then I come down two-thirds of this direction along AP of this length AP, then I am going to get to the same point, right, I have to show this.

So, I am you know in principle it could have been some other point, but we will show that in fact, it is going to be the same as the other vector BQ. So, if I do BA plus two-thirds of AP that is the same as saying its BA plus two-thirds of AP is nothing, but AB plus half BC, right. So, think of it as I have this vector AP, AP is I could come down from A to B and then from B to P, but B to P is nothing, but half BC.

So, I write it as AB plus half BC. So, I have AP is AB plus half BC, so BA plus two-thirds of AB plus half BC, right. And, so then I just expand this out I have one-thirds, one-third BC plus one-third BA, right. So, we have to realize that the vector BA is the same as minus vector AB, right.

So, taking this into account I get one-third BC plus one-third BA. But what is BA? BA is BC plus CA, right. So, I can write this as one-third BC plus one-third of BC plus CA which is if I collect all these terms together I get two-thirds BC plus one-third CA.

So, what I managed to show is whether I directly go along this vector BQ, if I with a magnitude which is just two-thirds of this length BQ, I am going to get the same vector as BA plus two-thirds of AP, right. I could equally well have done it for the third side, and I will end up at the same point, right.

So, first of all it shows that all these three lines by symmetry. So, all these three lines intersect and they intersect at a point which is two-thirds of each of these medians, right. So, it is a clever elegant proof. And so, in fact, you can use vectors in a similar manner to prove many of your high school theorems, right. So, I will probably send out a homework around this.

But this was just to quickly illustrate the power of vectors with such a minimal level of abstraction, right. So, we are going to make them more abstract and come up with the notion of what is called a vector space in the next lecture.

Thank you.