**Real Analysis - I**
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**Lecture - 1.3**
**Square Root of 2**

In the last module we had used the fact that, the $\sqrt{2}$ is an irrational number. Let us prove that. Before I begin this proof let me remark that the proof I am about to give is considered one of the most elegant proofs in mathematics. It is very straightforward and simple at the same time very beautiful. First, let me make a very precise statement as to what I want to prove.

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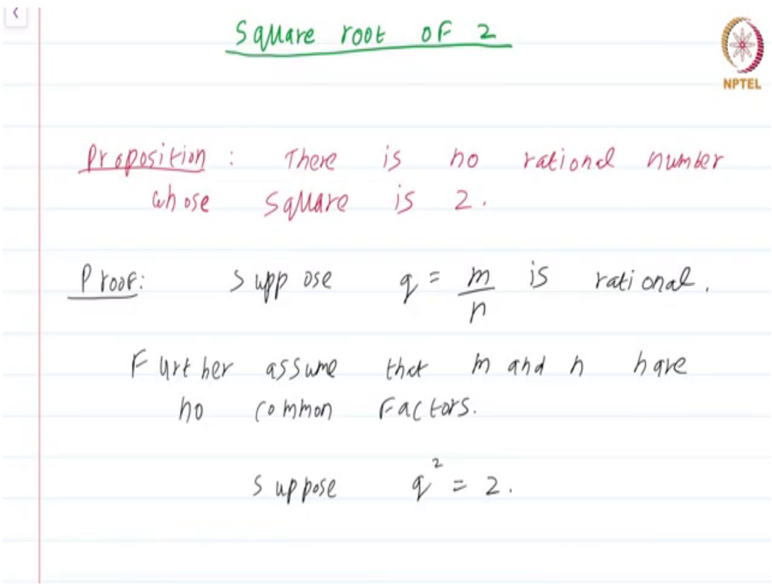


Proposition: There is no rational number whose square is 2.

So, I am not stating that $\sqrt{2}$ is irrational, we do not even know what irrational numbers are so far. What I am stating is, if you take a rational number and square it you cannot possibly get the number 2. Let us see the proof.

Suppose, $q$ equal to $\frac{m}{n}$ is rational, ok.

Further assume that $m$ and $n$ have no common factors, well, we can assume that simply, because if there is a common factor between $m$ and $n$, I just cancel it out, I can get rid of all the common factors and put $q$ in its lowest form. Now, suppose $q^2 = 2$. So, this is going to be a proof by contradiction. I am going to assume that the result is false and that there is a rational number whose square is 2 and somehow arrive at a contradiction.

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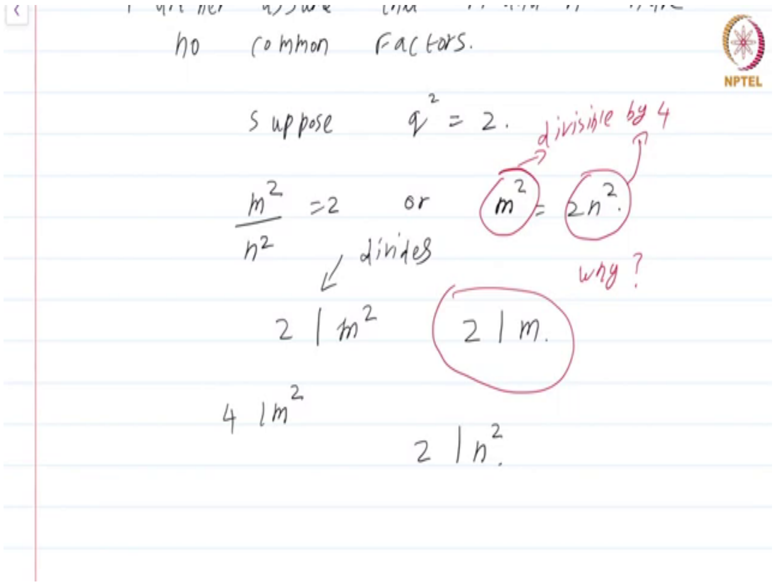


This just means that $\frac{m^2}{n^2} = 2$ or in other words $m^2 = 2n^2$, ok. Now, what does $m^2 = 2n^2$ tell us? It tells us that $2$ divides $m^2$. This vertical line just means divides.

Because the right hand side is $2n^2$, 2 must divide the left hand side which is $m^2$, but if you think about it since 2 divides $m^2$, it must be the case that 2 divides m. Think about why this is true. If 2 were to divide $m^2$ there is no choice but for 2 to divide $m$, that means 4 divides $m^2$ ok.

4 must divide $m^2$, why is this the case because 2 divides m therefore, 4 must divide $m$. That means the LHS here is divisible by 4; that means, the RHS must also be divisible by 4. Now, there is already a 2 coming from $2n^2$ squared putting all this together we get 2 must divide n square ok. Because 2 divides $n^2$, by the same logic 2 divides $n$; and hence 4 divides $n^2$.

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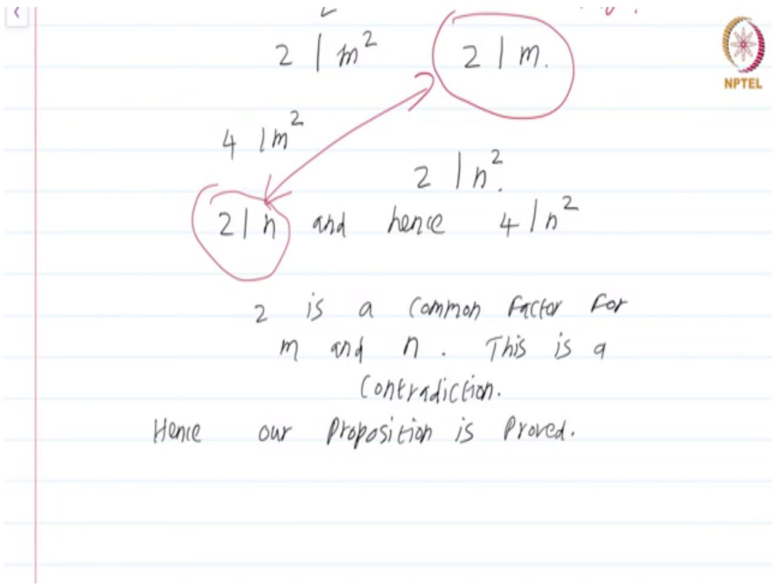


This is actually not going to be useful in the proof all I need is that 2 divides $n$, ok. But I am just making that remark, but that means 2 divides $m$ and 2 divides $n$ as well. What is the upshot of all this? 2 is a common factor; 2 is a common factor for $m$ and $n$. This is a contradiction.

Why is this a contradiction? Because we have assumed that $m$ and $n$ have no common factors. Hence, our proposition is proved. This is a course on real analysis and you have just watched the module on the square root of 2.