Introduction to Probability and Statistics
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Lecture – 21
Additional Examples

In this lecture we look at some Additional Examples in probability. This will be the concluding lecture in this course on Introduction to Probability and Statistics. We look at some new examples and we try to explain some interesting ideas from problems that are well known, and we are going to look at 5 to 6 different examples in this lecture. Some of these have already been covered in earlier lectures. And I will also try to give some additional explanation or an alternative way of solving the problem.

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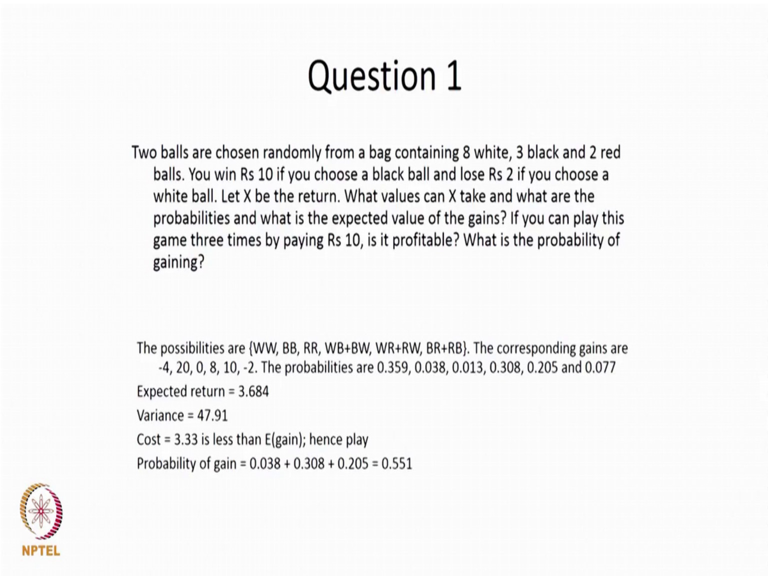

So, we begin with this problem, 2 balls are chosen randomly from a bag containing 8 white 3 black and 2 red balls. You win rupees 10 if you choose a black ball and lose rupees 2 if you choose a white ball. Let X be the return. What values can X take? What are the probabilities? What is the expected value? And if you can play this game 3 times by paying rupees 10 is it profitable what is the probability of winning. Since 2 balls are chosen there are 8 plus 3 11 plus 2 13 balls. So, the 2 balls could be chosen as white and white black and black red and red. It could be white black, black white, it could be white red red white, it could be black red and red black.

Now, the corresponding gains are given. So, minus 4, because you lose 2 rupees if you choose a white ball. So, if we do a W, W then it is minus 4. So, the gains are written here minus 4, 20, 0, 8, 10 and minus 2. Probabilities are also given alongside. So, probability of W B and B W both are added and they are given here. You could also check that the probabilities add up to 1.

So, the expected return would be the value of the gain multiplied by the probability; that is, minus 4 into 0.359 plus 20 into 0.038 and so on, which is 3.684 with the variance of 47.91. Now the cost is to play 3 times we can play by paying rupees 10. So, the cost of 3.33, the expected return is 3.684. Therefore, it is profitable and the probability of gain is the sum of the probabilities where the return is positive, and therefore, we get 0.551. So, more than half the time there is a chance of making some money in these trials.

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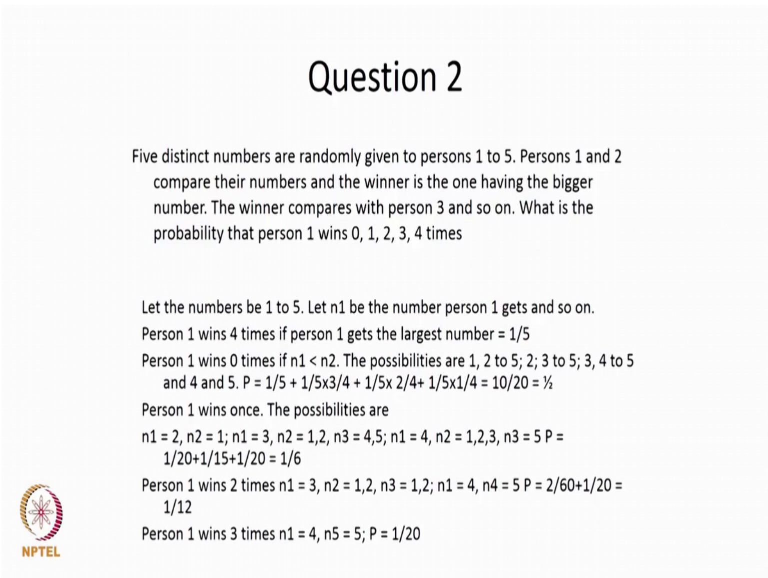

We come to the next question, 5 distinct numbers are randomly given to 5 persons. So, persons 1 and 2 compare their numbers, and the winner is the one having a bigger number. The winner then compares it with person 3 and so on. What is the probability that person 1 wins 0 times, 1 time, 2 times 3 times and 4 times.

So, the solution is let the numbers be 1 2 5 and let n 1 be the number the person 1 gets and so on. So, the numbers are one to find the persons are also called 1 2 5. So, person 1 will win 4 times, if person 1 gets the largest number. Because irrespective of who person 1 compares and starts comparing with 2 and then wins and then compares with 3 and then wins and so on; that can happen when person 1 gets the largest number which is 1 by 5.

Person 1 will win 0 times if n 1 is less than n to; which means the number that person 1 gets is less than the number that person 2 gets. So, this can happen in many ways. Person 1 can get number, one while person 2 can get anything from 2 3 4 and 5. Person 1 may get number 2, and person 2 may get numbers anything between 3 and 5. Person 1 may get number 3 and person 2 may get 4 or 5, and person 1 may get number 4 and person 2 gets number 5. So, the probabilities are person 1 getting number 1 is 1 by 5 , which means person 2 will get anything from 2 or 3 or 4 or 5 person 1 getting number 2 is 1 by 5.

Now, that is multiplied by person 2 getting numbers 3 or 4 or 5, 3 out of the 4 remaining numbers so, 3 by 4. Now person 1 getting number 3 is 1 by 5 and person 2 getting 4 or 5 out of the remaining 4 numbers is 2 by 4. And person 1 getting number 4 and person 2 getting number 5 is 1 by 5 into 1 by 4. So, we multiply and add to get 10 by 20 which is half. Person 1 wins only once and these possibilities are person 1 gets number 2 and person 2 gets number 1. Person 1 gets number 3, person 2 gets numbers 1 or 2. Person 1 gets numbers 1 or 2, and person 2 gets 4 or 5.

Person 1 gets number 4, person 2 gets 1, 2, 3 person 1 gets number 5 and so on. So, let me repeat, person 1 wins only once. So, this can happen, when person 1 gets the number 2, and person 2 gets the number one so that person 1 will win. And next time the person will lose, because person 3 would have got a number higher. Second cases person 1 gets the number 3, person 2 gets numbers 1 or 2 so that person 1 would win once. And person 3 should get 4 or 5 so that person 1 now having won the first round with person 2.

Now, faces person 3 who has a higher number and losses and therefore, person 1 wins only once. The other cases person 1 will have number 4, person 2 will have numbers 1, 2, 3 and person 3 has number 5. Therefore, again person 1 will win only once. And then face person 3 who has 5 and therefore, will lose so. The probabilities are 1 by 20 plus 1 by 15 plus 1 by 20, which are calculated in the usual way to get 1 by 6.

The case where person number one wins 2 times, can happen when person 1 gets number 3 person 2 gets numbers 1 or 2 person 3 also gets numbers 1 or 2 so that when person 1 meets person 4, that would have had a higher number and therefore, person number 1 will win only 2 times. The other cases person number one gets number 4, and person number 4 gets number 5, then person number one would have won only 2 times. So, there are only 2 cases, and the probabilities are 2 by 60 plus 1 by 20 which is 1 by 2. Person 1 wins 3 times means person 1 gets value 4 and the fifth person gets the number 5 so that finally, in the 4th comparison person 1 will lose and therefore, would have one only 3 times.

There is only one case and therefore, the probability is 1 by 20. So, this is a very interesting question which kind of makes us think and look at all the possibilities. And then list out the number of ways by which these things can happen. And once we are able to get that then assigning the probabilities and multiplying them, and adding them becomes relatively simpler and we can solve such problems.

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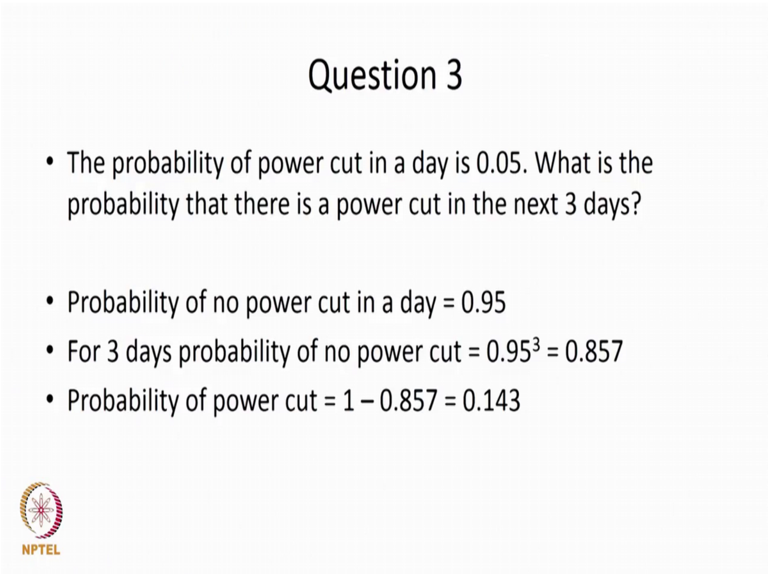

Now, we look at question number 3 which is some a question that we have seen earlier. Probability of power cut in a day is 0.5 what is the probability that there is a power cut in the next 3 days. So, this we have seen this slide earlier probability of no power cut in a day is 0.95. So, for 3 days probability of no power cut is 0.95 cubed which is 0.857. And therefore, probability of power cut at least one power cut in the next 3 days is 1 minus 0.857 which is 0.143.

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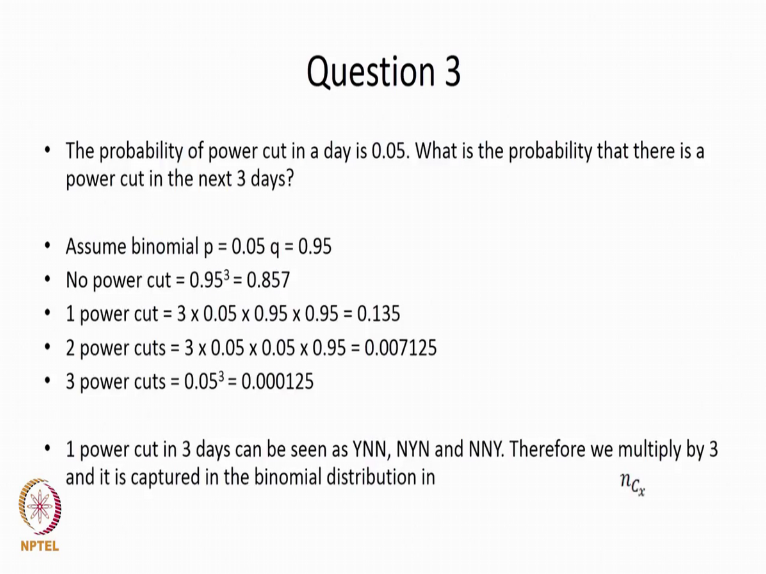

Now, we can do the same problem using the binomial distribution, and that is the reason why we are revisiting this again. So, assume binomial with p equal to 0.5 which is the probability of a power cut and q is equal to 0.95. So, in this case success is like saying there is a power cut, and failure is cell there is no power cut. So, no power cut will be straightaway 0.95 cubed which we have already seen. So, this is n C x p power x q power n minus x.

So, n C 0 p power 0 q to the power n minus 0 which is q to the power 3; which is 0.95 to the power 3 which is 0.857. There is one power cut is n C 1 p to the power 1, q to the power 2 which is 3 into 0.05 and 2.95, 2.95 which is 0.135 to power cuts is 3 C 2 p to the power 2 q to the power 1, you see 3 C 2 is 3 p to the power 2, 2 times multiplied q to the power 1 0.007125. And 3 power cuts is 0.05 cubed this 0.000125. So, one power cut in 3 days can be seen as yes, no, no, no, yes, no and no, no, yes which happens 3 times. So, therefore, we multiply by 3 which comes here this 3 which is captured in the n C x which is in the binomial distribution.

So, finally, the answer that we got here is 1 minus 0.857 which is 0.143. So, probability of there is one power get in the next 3 days is 1 power cut plus 2 power cuts plus 3 power cuts which is equal to 1 minus probability of 0 power cut. So, 1 minus 0.857, which is 0.143.

So, the same problem we now looked at using the binomial distribution and the idea of 2 outcomes and the Bernoulli trial leads us to the binomial distribution. Now we look at the next question.

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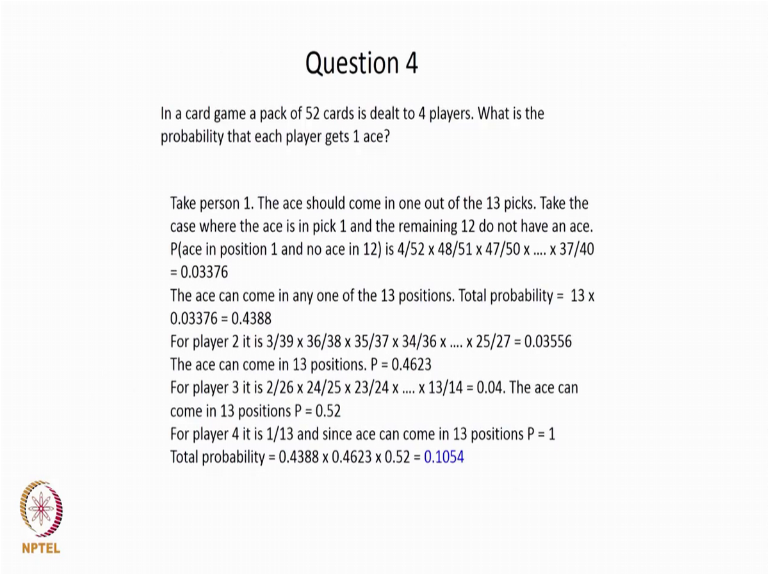

Again this question we have seen earlier. We have seen this slide earlier; we will now see another way of working out the same problem. So, in a card game a pack of 52 cards is dealt randomly to 4 players. What is the (Refer Time: 13:00) probability that each player gets exactly one ace? So, we have seen this description earlier therefore, I will go slightly faster on this description. So, if we take the first person, the air should come in one out of the 13 picks.

So, take the case where ace is in pick one, and the remaining 12 do not have a niece. So, ace in position 1, and no ace in remaining 12 positions is 4 by 52 into 48 by 51 etcetera, etcetera, which is 0.033764 by 52 comes. Because there are 4 aces in 52 cards, and then one card has been picked up. So, remaining 51, and then this one should not be an ace. So, there are 48 non ace cards and therefore, 48 by 51 and it keeps reducing for 13 picks and you get 0.03376.now this is can come in any one of the 13 positions. Therefore, 13 (Refer Time: 14:00) into 0.03376 is 0.4388.

Now, for player 2 it is 3 by 39 because there are fail player 1 13 cards have been given. So, there are 39 cards remaining and there are 3 aces. So, 3 by 39 into 36 by 38, because the second position, should not have an ace and so on. So, once again it can come in 13 positions 0.4623. For player 3, it is 2 by 26, because players 1 and 2 we have already used up 26 out of the 52 cards, and remaining 26 cards are available out of which there are 2 aces. So, 2 by 26 into 24 by 25 and so on and this comes in 0.5. For player 4 it is 1 by 15 because player 4 does not have a choice, all the remaining 13 come out of these 13 there should be one s. So, 1 by 13 and this can come in 13 positions. So, we multiplied all of them to get 0.1054.

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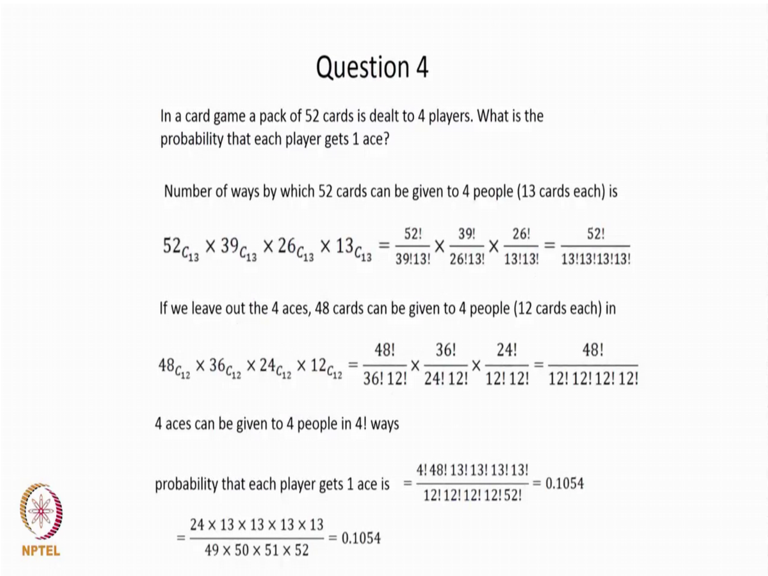

Now, let us look at the permutation combination way of doing this. So, number of ways by which 52 cards can be given to 4 people, 13 cards each is 52 to see 13 into 39 see 13 into 26 see 13 into 13 see 13; which is 52 factorial by 39 factorial into 13 factorial. This is 39 factorial by 26 factorial into 13 factorial. This 26 factorial by 13 factorial into 13 factorial, the last one is one because the remaining 13 cards goes to the 4th person. And we see something interesting happening, this 39 factorial is getting cancelled, 26 factorial is getting cancelled and so on. And therefore, we get 52 factorial divided by 13 factorial 4 times.

If we leave out the 4 aces, 48 cards can be given to 4 people 12 cards each, in a similar manner 48 see 12 into 36 see 12 into 24 see 12 in to 12 see 12; which on simplification will give 48 factorial by 12 factorial into 12 factorial into 12 factorial into 12 factorial. The 4 remaining aces as a 13th card can be given to the 4 people in 4 factorial ways. And therefore, each person getting an ace is 4 factorial multiplied by 48 factorial divided by 4 times 12 factorial.

Divided by 52 factorial which comes here and the 4 13 factorials go to the numerator, and we get 0.105 for the simplification is also shown here. So, the same problem can be a can be viewed or approached in 2 different ways. I mean one can critically look at both these and say that they are actually the same, but they are certainly 2 different ways of working out the same answer for problems of this kind.

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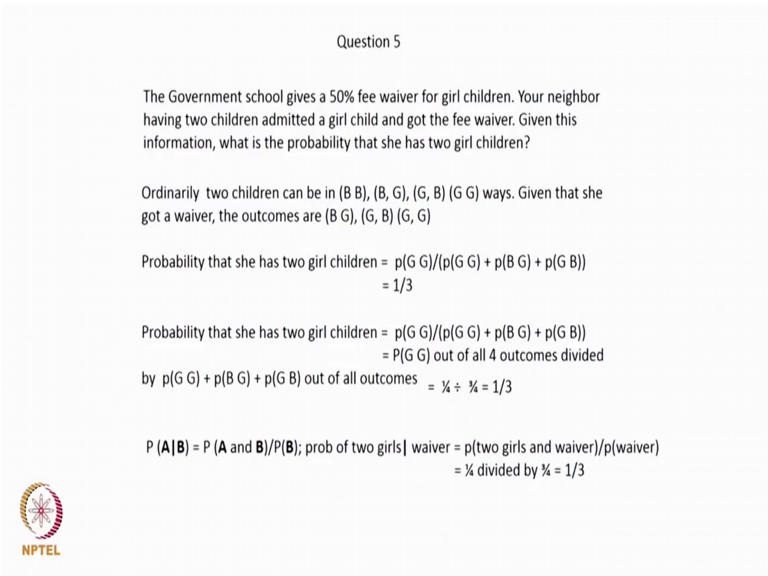

So, we look at a fifth question. A government school gives a 50 percent fee waiver for girl children. So, your neighbor has 2 children and has admitted a girl child and got the fee waiver. Given this information what is the probability that she has 2 girl children? So, this is a very interesting question and first let us solve this and then try to understand a couple of more things about it. So, ordinarily 2 children can be in 4 ways boy, boy, boy, girl, girl, boy and girl, girl.

Assuming that the child is either a boy or a girl, given that she got a waiver, the favorable outcomes are boy girl and girl boy and girl girl. We have not spoken about first child and second child and therefore, you still have boy, girl, girl, boy and girl, girl. Now probability that she has 2 girl children is probability that of gold girl divided by girl girl plus boy girl plus girl boy which is 1 by 3. The other way to do is girl girl comes as 1 by

4; girl girl girl boy boy girl is 3 by 4. So, 1 by 4 divided by 3 by 4 is 1 by 3. You can also use the base theorem.

So, p f or the conditional probability theorem. So, P of A given be equal to P of A and B by P of B. So, probability of 2 girls given a waiver is probability of 2 girls and waiver divided by probability of waiver. So, 1 by 4 divided by 3 by 4 which is 1 by 3.

So, we find many problems like this, but then we may approach these kind of problems this way. At times the boy girl problems also have different solutions where the order and other things are involved. So, being an introductory course we would restrict ourselves to this explanation, and then follow this idea of solving this problem even though other interpretations and other solutions exist for what are typically called the classes of boy girl problems in probability.

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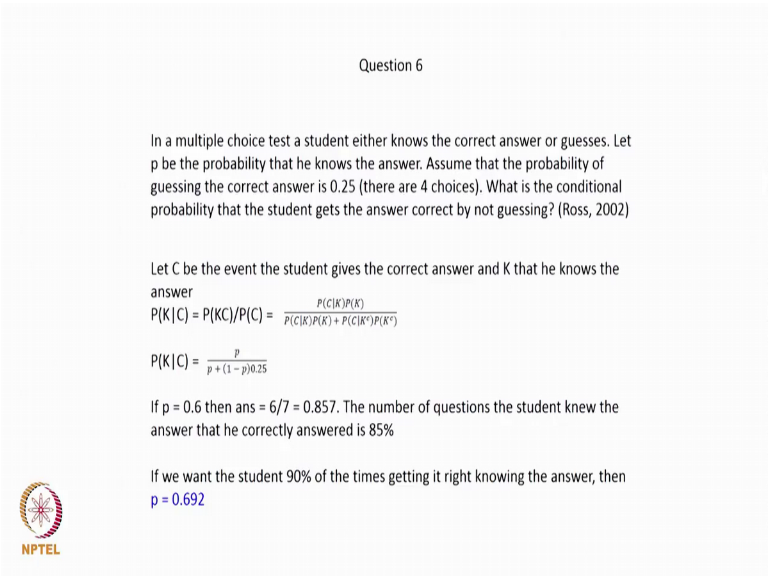

We look at one more question, in a multiple choice test, a student either knows the correct answer or guesses the answer. Let P be the probability that the student knows the answer. Assuming that the probability of guessing is 0.25, because there are 4 choices what is the conditional probability that the student gets the answer correct by not guessing. So, this has been discussed in r and have given the reference for that.

So, let us see P the event that the student gives the correct answer, and K the event that the student knows the answer. So, probability that the student knows the answer given that the student has actually given the correct answer is the probability that he probability of knowing the answer into giving the correct answer divided by probability of giving the correct answer; which is P of C given K into P of by P of c given K into P of K plus P of c given K transpose plus into P of K transpose.

So, probability p of C given K into P of K is P divided by P plus 1 minus P into 0.25. 0.25 is the probability of guessing and not knowing. So, 1 minus P is 0.25 is the probability of guessing 1 minus p is the probability of not knowing. So, probability of not knowing guessing and getting the correct answer is 1 minus P into 0.25. Probability of knowing the answer writing it and getting it as P therefore, P by P plus 1 minus P into 0.25. So, the numerator is about knowing the correct answer, writing it and getting it with the probability of P.

So, you could get the right answer either by knowing the right answer and giving it or by not knowing the right answer and guessing it. So, knowing the right answer and giving it is p into 1, knowing the right answer in guessing it is P into 1 not knowing the right answer is 1 minus P. And guessing it and getting it right is multiplied by 0.25 therefore, P by P plus 1 minus P into 0.25. If P equal to 0.6, then the probability is 6 by 7 so, 0.857.

So, the number of questions that the student knew the answer and correctly answered is about 85 percent, if the student knows the probability of the answer correct answer is 0.6. If we want the student 90 percent of the times getting it right knowing the answer, then we back calculate and find out P is equal to 0.692 which is roughly 0.7. So, if the student knows the correct answer with the probability of 0.7. And thus the remaining by guess then the person can actually get about 90 percent of the correct answers.

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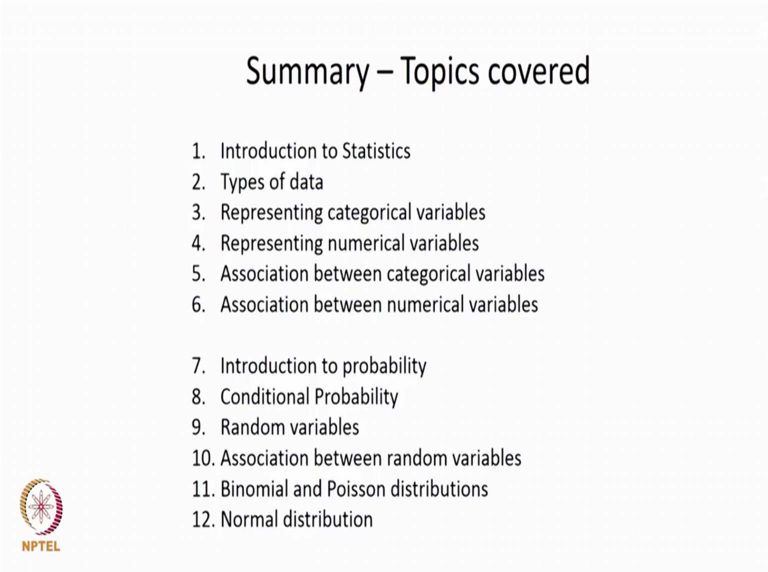

So, to conclude we take a look at the topics that we have covered in this introductory course. So, we broadly divided the course into the statistics component and the probability component. So, we started with introduction to statistics, what is statistics? Types of data, the 4 broad classifications of data and then we looked at representing categorical variables; using pie charts and bar charts and then representing numerical variables using histograms stem and leaf and so on.

Then we looked at association between categorical variables in the form of chi-square and crime as v and then association between numerical variables in terms of variance standard deviation and coefficient of variation. Earlier, when we represented numerical variables, we also an categorical variables, we looked at measures of central tendency; with mode for the nominal variable, median, the ordinal and interval variables, and arithmetic mean median and mode for the ratio level variables.

Then we went or studied probability we define the axioms of probability and the theorems. And we worked out some problems in probability. And then we looked at conditional probability and base theorem. We introduced random variables their expected value and the variance. And then we looked at association between random variables trying to find out the correlation coefficient covariance and so on. And then we studied the binomial and poisons distributions discrete distributions and then we get a little bit of normal distribution in this course.

So, with this we formally wind up this course. I hope you have enjoyed looking at these videos, and hope that you are able to get introduced to the basics of probability and statistics; which would help you do advanced courses in the future.

Thank you.