Introduction to Probability and Statistics
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Lecture – 14
Rules of Probability
(Continued)

We continue the discussion on probability and concepts and we also look at a couple of examples in this lecture to understand these concepts. As, has been the practice, we also have some simple discussion questions, which we will try to solve in this lecture.

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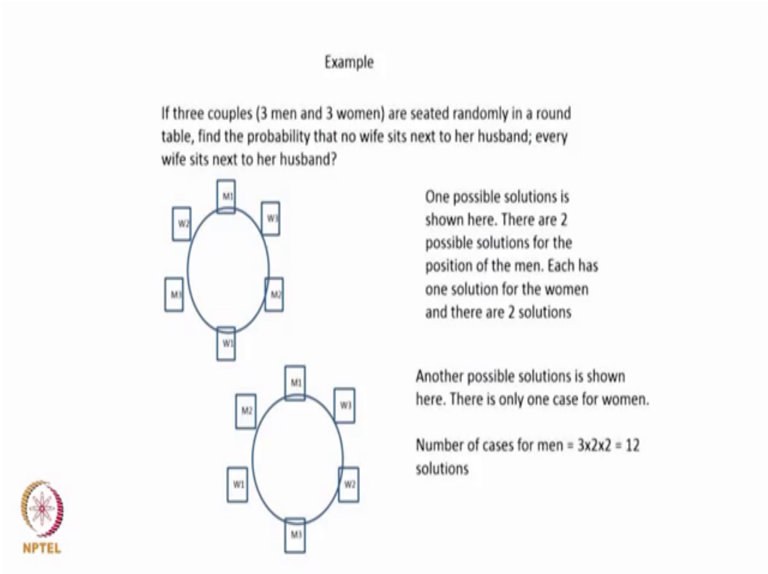

So, let us look at one more example. So, three couples, 3 men and women are seated randomly in a round table. Now, find the probability that no wife sits next to her husband and find the probability that every wife sits next to her husband. So, there are two questions, the first question talks about no wife sits next to her husband. So, one way to look at it is the first picture that we show here this picture, where we have a round table and without loss of generality, we call the 3 men as M 1 M 2 and M 3 and their wives as W 1 W 2 and W 3.

So, one possible way of sitting is shown in the first picture. So, here there are two possible solutions for the position of the men and each has one solution for the women. For example, we could have M 1 there and we have shown M 2 and M 3 here, and since the wives are not, not even on pair is setting close to each other. So, if person M 1 is sitting here, then W 2 and W 3 have to sit like this or they can interchange.

They cannot interchange that easily, because M 3 is here, when they interchange then M 3 and W 3 come next, to each other. Therefore, if we have M 3 M 2 and M 3 in this manner, we can have only one configuration of the wives, but W 1 W 2 and W 3. So, for each position; that is if person M 1 sitting here and then M 2 and M 3 are here, there is only one case with W 2, W 3 and W 1. But another case is possible where person M 1 is here, M 2 comes here and M 3 can come here, in which case the W 2 and W 3 will interchange and therefore, we have two solutions. So, what is written here is there are two possible solutions for the position of men. So, it is either M 1 M 2 M 3 or M 1 M 3 here and M 2 here, but for a given M 1 M 2 M 3, there is only one position for the women.

So, there are only 2 solutions that are possible if the men are seated this way, another way that the men could be seated are M 1 and M 2 are sitting, are sitting next to each other this way. And therefore, we can have only, we can have only W 3 here we could have W 1 here, M 3 is here and once we have M 1 M 2 M 3 this way, M 1 M 2 M 3 this way. So, W 1 W 2 will have to sit like this and W 3 will have to do this. So, another possible solution is shown here for this M 1 M 2 M 3 kind of sitting of men, there is only one case for the women, but the men themselves can be seated in 3 into 2 into 2 12 ways therefore, there are 12 solutions in this.

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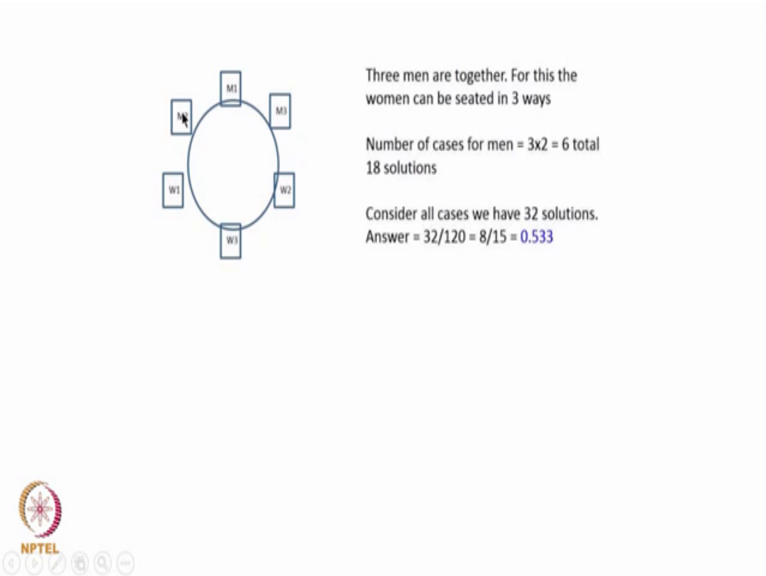

And then there is a third, when all the 3 men actually sit, next to each other and then if they sit next to each other this way then M 2, W 1 has to be here and the women can be seated in 3 ways. So, we you could have W 1 W 2 W 3 we could have them in three different manner, because against M 2 we could have W 1 as well as W 3 and so on. So, the women can be seated in 3 ways, the number of cases for the men is 6 therefore, there are 18 solutions. So, totally we have 32 solutions out of a possible 120, because it is a circle and since there are 6 people, it is not 6 factorial. It is 5 factorial, because it is a circle and therefore, there and 120 possible ways; 32 of them meet our requirement that no wife sits next to her husband. So, the probability is 0.533.

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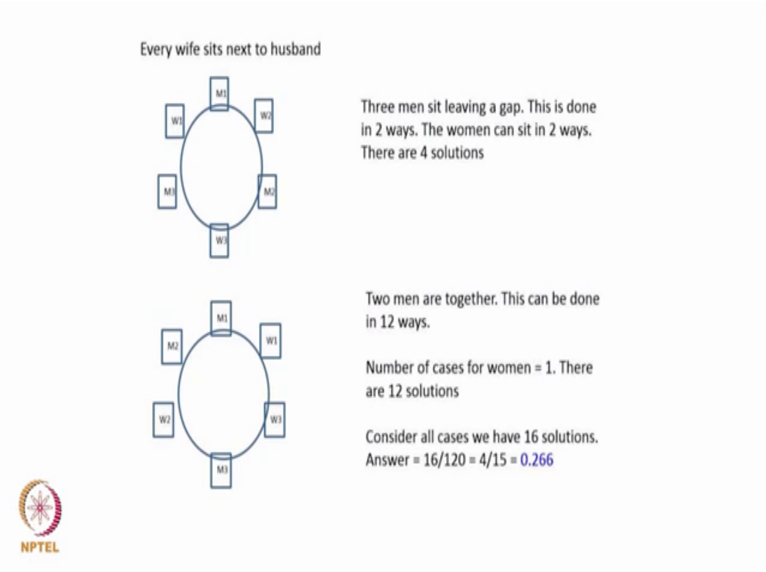

Now, we look at the next case when every wife sits next to the husband. So, if we have the men configuration is M 1 M 2 M 3. So, men sit leaving the gaps and the women can sit in 2 ways. So, there are 4 solutions for each of the men configuration, there are 2 configurations for the women and there are 4 solutions. The next case 2 men are together and this can be done in 12 ways and in each case there is only 1. So, there are 12 solutions. So, totally there are 16 solutions out of 120 and 0.266 is the probability that every wife sits next to her husband.

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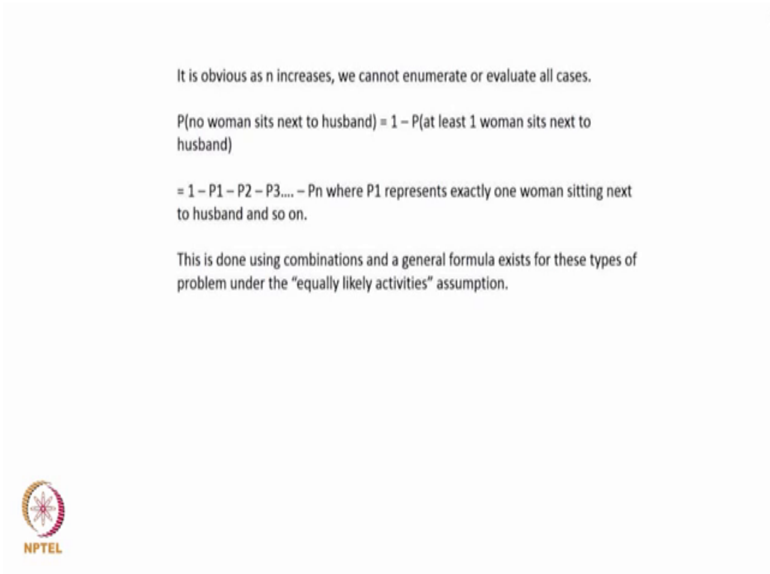

So, obviously, has n increases, we cannot enumerate or evaluate all these cases. So, we could go back and try to do probability that no woman sits next to the husband is 1 minus probability of at least 1 woman sits next to the husband and so on and then we could kind of extend. This whole thing for a more generalized expression, but by and large we learn a few thing about seating in particular and when they are seated in the form of a circle; it is not n factorial, it is n minus 1 factorial ways of doing it.

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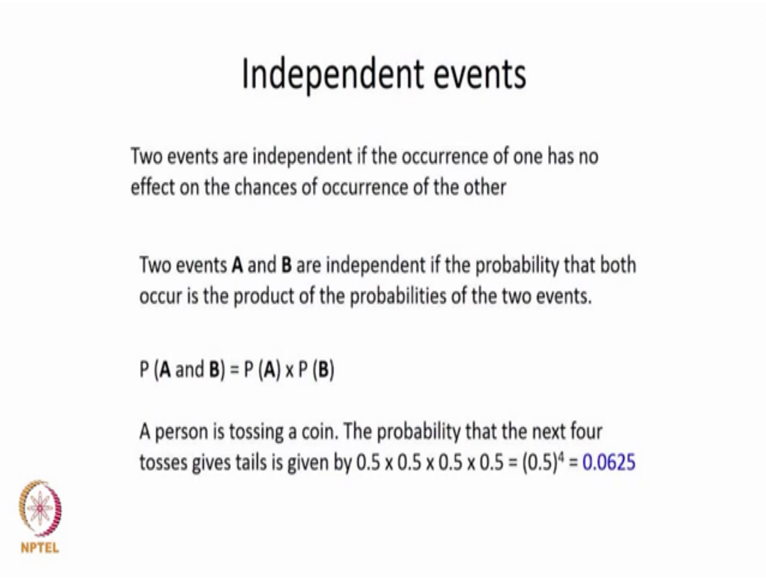

We then look at what are called independent events. So, two events are independent, if the occurrence of one has no effect on the chance of occurrence of another. So, two events A and B are independent, if the probability that both occur is the product of the probabilities of the two events. So, P of A and B is equal to P of A into P of B simplest example is tossing a coin person is tossing, a coin probability that the next 4 tosses gives tails is given by 0.5 into 0.5 into 0.5 into 0.5, which is 0. 0625 most of the times when we do this tossing, the coin example; we have independent events, it has no bearing on the previous one.

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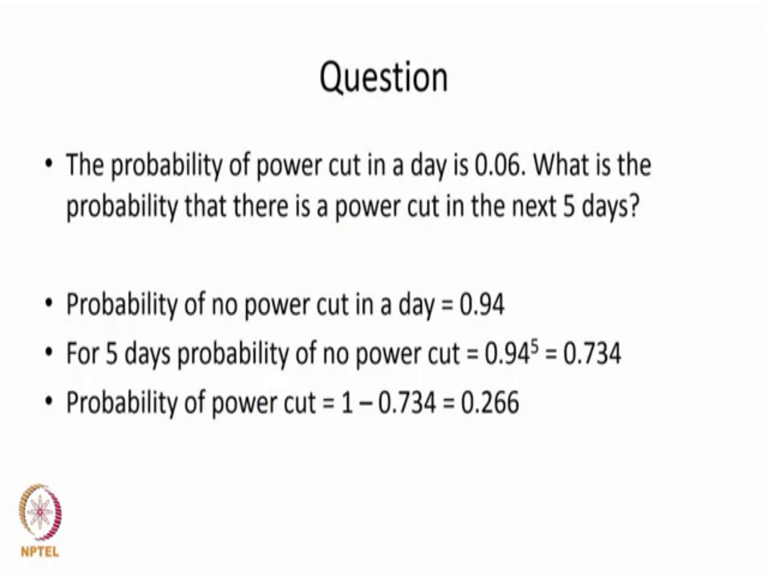

Now, probability of a power cut in a day is 0.06, what is the probability that there is a power cut in the next 5 days. So, probability of no power cut in a day is 0.94. So, for 5 days probability of no power cut is 0.94 to the power 5 which is 0.734. Therefore, probability of power at least 1 power cut in the next 5 days is 1 minus 0.734, which is 0.266. Many times we also work on problems like this, where we actually have a binary kind of a thing probability of power cut in a days 0.06.

So, there is at least 1 power cut in the next 5 days is 1 minus probability of no power cut in the next 5 days. Otherwise we have to calculate probability of exactly power cut in 1 of the 5 days 2 of the 5 days 3 4 and all 5 days and then we have to add and so on, instead we do 1 minus probability of the other.

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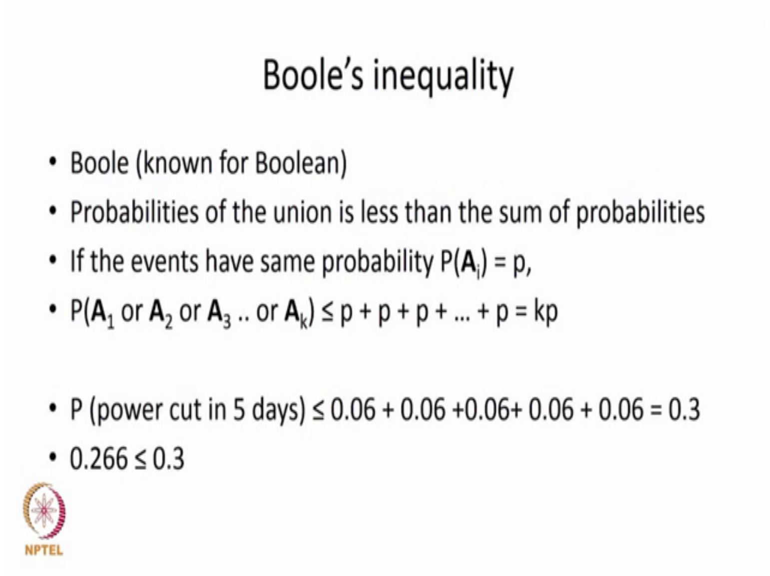

The last one, among the concepts is, what is called Boole's inequality? Probability of the union is less than the sum of the probabilities, if the events have the same probability P then P of A 1 or A 2 or A 3 or etcetera is less than or equal to p plus p plus p, which is kp, which we can verify in the power cut example. So, power cut in at least 1 power cut in 5 days is individual power cut if 0.06. So, 5 times if we add, we get 0.3, but the actual answer turned out to be 0.266 at least 1 day at least 1 power cut in the next 5 days is 0.266, which is less than or equal 0.3. It is a general way of understanding that it actually becomes less than the sum of the probabilities.

Now, we continue, we have a discussion with some simple questions on all the concepts that we have learnt in probability till now, we have already explained most of these concepts through examples. And, we take further examples to explain them and two kind of recap or refresh, what we have learnt under probability till now.

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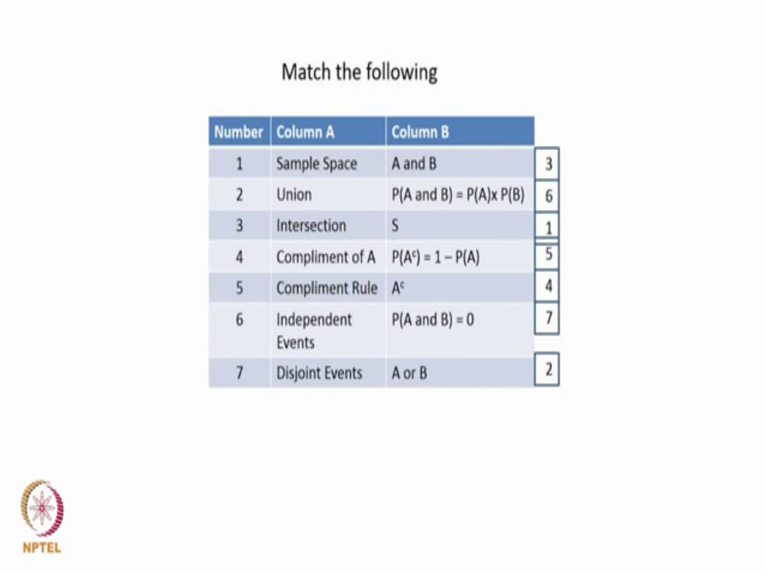

Number	Column A	Column B	
1	Sample Space	A and B	3
2	Union	P(A and B) = P(A)x P(B)	6
3	Intersection	S	1
4	Compliment of A	$P(A^c) = 1 - P(A)$	5
5	Compliment Rule	A^c	4
6	Independent Events	P(A and B) = 0	7
7	Disjoint Events	A or B	2

So, we will first do a simple match, the following there are 7 things given in column A as well as column B. So, those in column A are sample union intersection complement of A, complement rule independent events and disjoint events, some of these we can easily do. So, let us take this. So, intersection sample space is given by S. So, the 1 here indicates that S is the answer for the first one in column A. So, sample space is given by S union of 2 A or B. So, we have two events A and A B. So, the union is A or B, the third one is intersection which is A and B, it is fairly obvious.

So, both happening A and B, the fourth one complement of A is simple. So, A compliment the way it is defined that is the answer first one is the compliment rule. So, the compliment rule is 1 minus P of A, probability of a compliment is 1 minus probability of A, which is the compliment rule, independent events P of A and B is equal to P of A into P of B, which we saw now, there is no bearing on that. And, the last one disjoint events probability of A and B is 0, because they are mutually exclusive.

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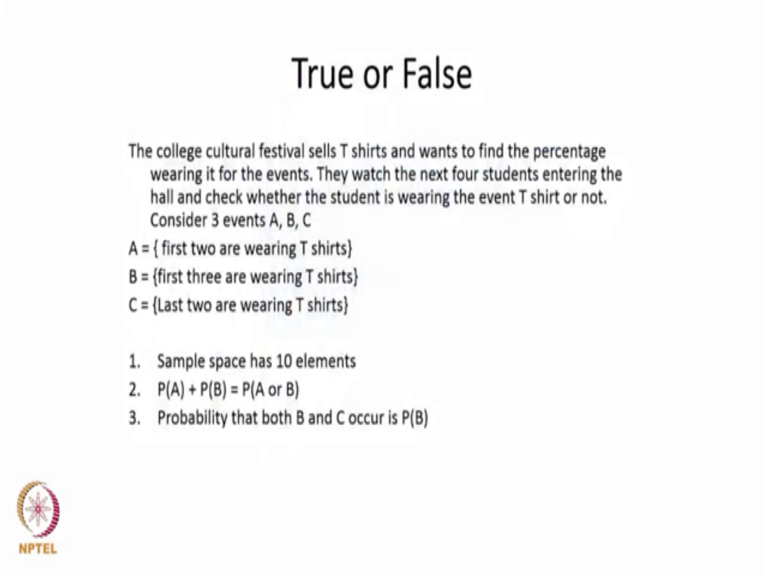

Now, let us look at some true or false kind of questions. The college cultural festival sells T shirts and wants to find out the percentage of people wearing the T shirt for the events. They watch the next four students entering the hall and check whether the student is wearing T shirt or not. So, look at 3 events A first two are wearing T shirts, B first three are wearing T shirts and C the last two are wearing T shirt. So, the sample space has 10 elements, the sample space, there are four students. So, we could have 4, all 4 wearing, any 3 out of 4, wearing any 3 out of 4 wearing and 1 person wearing. So, we have 10 elements in the sample space.

So, P of A plus P of B is equal to P of A or B, is it true? So, probability of A first two are wearing T shirts, probability of B first three are wearing T shirts. So, P of A plus P of B is P A or B is not true; probability that both B and C occur is P of B. So, first three wearing T shirts, last two wearing T shirts again, it does not happen. So, probability that both B and C, occur is not P of B in this case.

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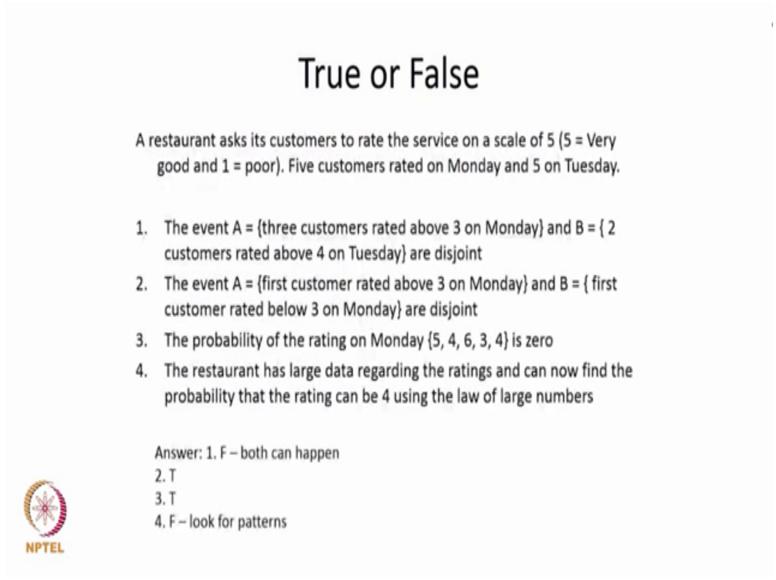

Now, let us look at this. A restaurant asks its customer to rate the service on a scale of 5, 5 being very good and 1 being poor. 5 customers rated on Monday and five customers rated on Tuesday. Now, question 1 the event A, where 3 customers rated above 3 1 Monday and B 2 customers rated above 4 on Tuesday are disjoint the answer is well both can happen. And therefore, they need not be disjoing disjoint implies only one can question number 2, the event a first customer rated above 3 on Monday and B the first customer rated below 3 on Monday are disjoint the answer is yes, because only one of them can happen first customer rated above 3 on Monday and first customer rated below 3 on Monday.

Let us assume when we say rated above 3, let us assume, it is greater than or equal to 3 and therefore, they are disjoint even otherwise they are disjoint, if we say that the first customer rated either 4 or 5 and while the first customer rated 1 or 3 probability of the rating 5 4 6 3 4 is 0, the answer is true, because the person can rate a maximum of 5 therefore, we assume that nobody as rated 6 and therefore, the probability is 0 is true.

The restaurant has a large amount of data regarding the ratings and now, can find the probability that the rating can be 4 using the law of large numbers, this is slightly involved question, the answer is not necessarily true, what is more important is the organization the, the, the patterns that exist at times can distort the probability or proportions that we have. And therefore, one has to look for patterns before making this decision or before saying that the answer is true.

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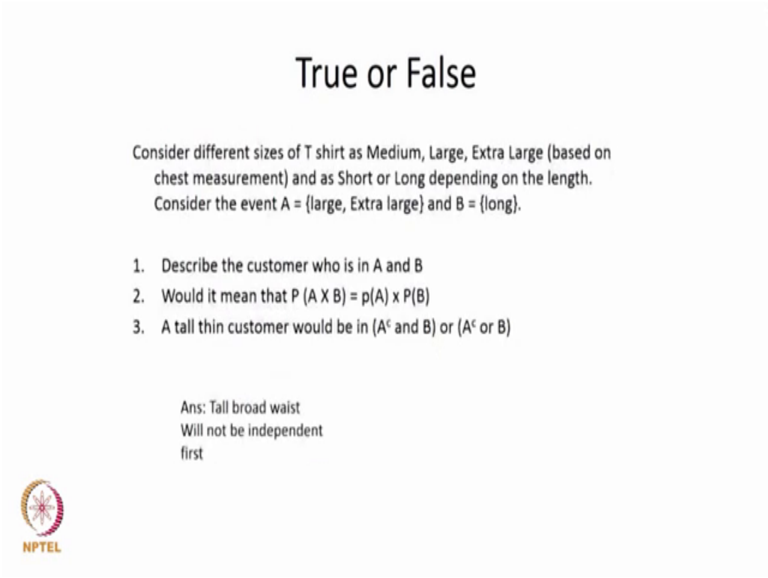

Now, considered different sizes of T shirts as medium, large and extra large based on chest measurement and short or long depending on the length. So, consider the event, A is large, extra large and B is long describe the customer, who is in A and in B. So, the customer who is in A and in B can be assume to be a person with their slightly broader chest and tall person. So, that the person belongs to large or extra large T shirt category as well as long as T shirt category based on height, would it mean that P of A into B is equal to P of A into P of B will not be independent.

So, that could be dependent so on. For example, a person, a short person can have a larger waist. So, on a tall thin customer, a tall customer would look for long and a thin customer, would look for medium, a tall and a thin customer. So, thin customer implies the waist size is small and therefore, would look for medium therefore, who the person would be in a compliment and the person would be in long. So, a tall thin customer would be in a compliment and B and will not be in a compliment or B. So, a tall thin customer will be in A complement and B and not in A complement or B.

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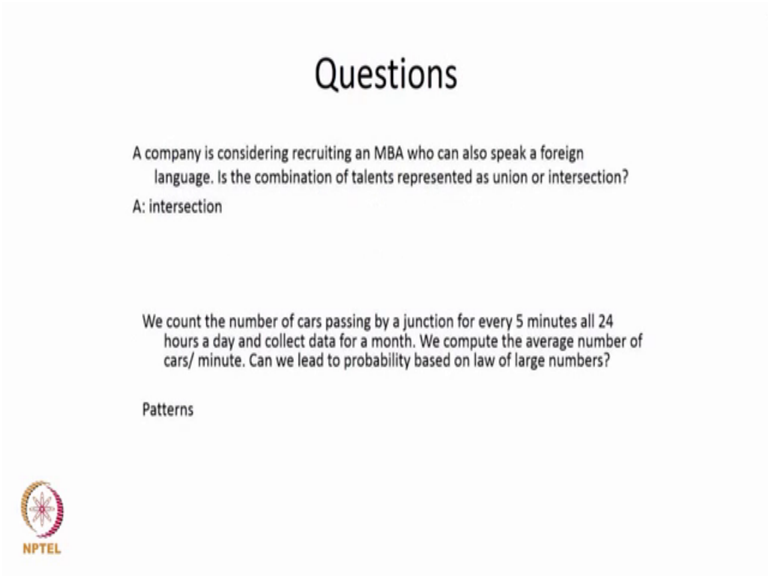

Now, let us look at some more questions. A company is considering recruiting an MBA student who can also speak a foreign language. Is the combination of talent represented as a union or as an intersection? The answer is intersection, because we want a student with an MBA degree and a student who can speak a foreign language. So, it is an intersection of two things for example, if we have a set of people, who have an MBA degree and we have another set of people, who can speak a foreign language, we would like to find out is there a common name in both the lists and that is the person we are looking at, and therefore, it is intersection. We count the number of cars passing by a junction for every 5 minutes in all 24 hours a day and collect data for a month.

We now, computer the average number of cars per minute. Can this lead to probability based on law of large numbers? Same answer patterns exist, because that could be holidays or other days where the number of cars would be large and so on. So, if it is a high way then one could think of holidays, having a slightly larger proportion whereas, if it is busy intersection in a city, we could have the working days and peak hours and there are so many patterns that come therefore, we have to take into account all of these patterns before we approximate these by using the law of large numbers.

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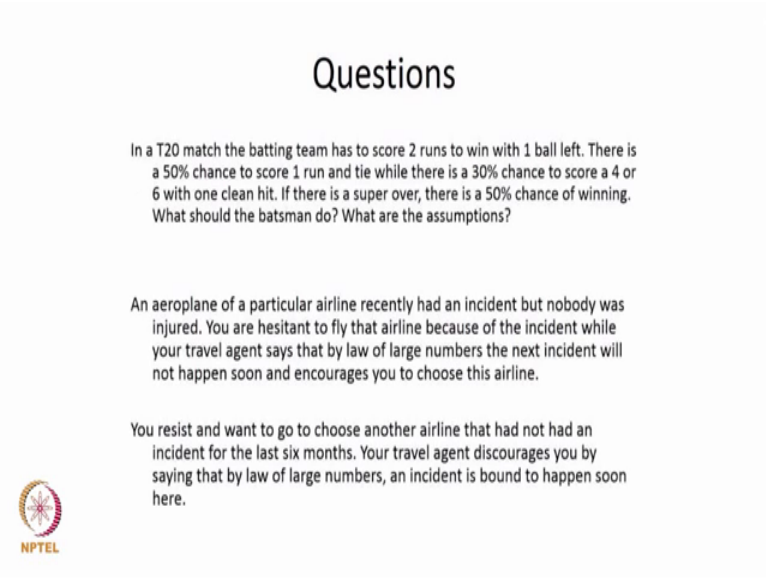

Next question in a T 20 match the batting team has to score 2 runs to win with 1 ball left. There is a 59 percent chance to score 1 run and tie the match while there is a 30 percent chance to score a 4 or a 6 with 1 clean hit. If there is a tie and there is a super over, there is a 50 percent chance of winning. So, what should the batsman do and what are the assumptions? So, if the person decides to go for a clean hit then the probability of winning is 30 percent or 0.3 if the person decides to take 1 run and tie the match, probability is higher it is 0.5, but then the team goes into a super over with a 59 percent chance of winning.

So, the probability of winning in that case becomes 0.5 into 0.5 which is 0.25 and therefore, the batsman will try to go for a clean hit and try to score a 4 or a 6 to win, because the probability is higher assumptions. We do the multiplication, have to get, 0.5 into 0.5 which is 0.25 so on. That is independent. Next question, an aeroplane of a particular airline recently had an accident, but nobody was injured. You are hesitant to fly that airline, because of the incident while your travel agent says that by law of large numbers, the next incident will not happen soon and therefore, encourages you to choose this airline.

Now, we have to understand the independent events and just, because an incident happened, it is quite likely that it may not happen today, is also wrong. There is an equal probability therefore; one cannot say by the law of large numbers that the next incident will not happen very soon. You resist and want to choose another airline that had not, had an incident for the last 6 months; your travel agent discourages you by saying that by the law of large numbers that is bound to happen.

So, once again, it is not true based on the law of large numbers, whatever it is the probability remains the same and though we might be tempted to argue and defend our decisions based on these kind of interpretations. One has to understand that these are not entirely true and whatever is the probability of an incident or an accident happening, the probability remains the same whether it happened yesterday or whether it did not happen yesterday.

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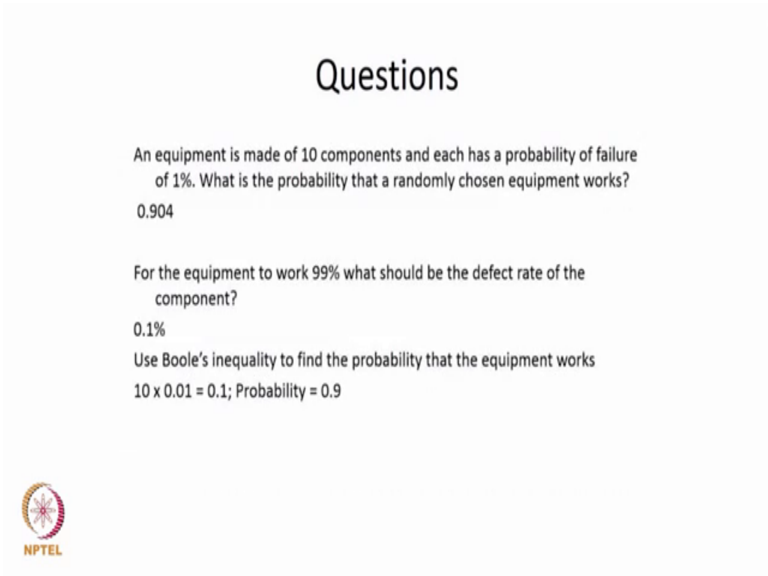

Next question, an equipment is made of 10 components, each has a probability of failure a, 1 percent. What is the probability that a randomly chosen equipment works? So, the probability of 1 component working is 0.99 and for the equipment to work, all 10 components have to work. So, it is 0.99 to the power 10 which is 0.904, which is the answer given.

Now, for the equipment to work 99 percent what should be the defect rate of the component? Then what happens is, let the defect rate be P or whatever 1 minus P to the power 10 is equal to 0.99 and then we realize that P is 0.1 percent. Please note that it was 1 percent in the earlier case it gave us 0.9 and to make it 0.99, we need to have a 0.1 percent. Now, Boole's, use Boole's inequality to find the probability that the equipment works. So, 10 into 0.01 is 0.1 and the probability is actually, becomes 0.9 and then you see here it is 0.904.

So, 1 minus 0.904 will be slightly smaller than this 0.1 or the other way that if we have this 0.1 here and then 1 percent is 0.01. So, 10 into 1 percent is 0.1; so, that gives as the probability of 0.9. We randomly chosen equipment works for 0.904, but then we have to compare the other way; so, 1 minus 0.904 will be less than 0.1.

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Now, you live in the fourth floor of an 8 story building with people living in floors 1 to 8 and ground floor is used for parking. You are waiting for the elevator in the ground floor when a person with a ladder comes and waits for the elevator. Would you let the other person enter first? Now, what happens you are going to the fourth floor while the, this person at the moment we can assume can go to any floor from 1 to 8 and the person is carrying a ladder.

So, if the person is going from 4 to 8 then what you can do is let the other person, if the other person enters first and you enter later, you can come out of the elevator without inconveniencing the person with the ladder. So, that probability is 5 by 8, because the person going to any floor including 4 to 8 is 5 by 8 whereas, if the person with the ladder is going to floors 1 2 and 3 and you let that person go first, then you have to come out of the elevator and you are inconvenienced.

So, because the probability is 5 by 8, you might as well as the person with a ladder to go inside and then you join the person in the elevator; so, that you can come out first.

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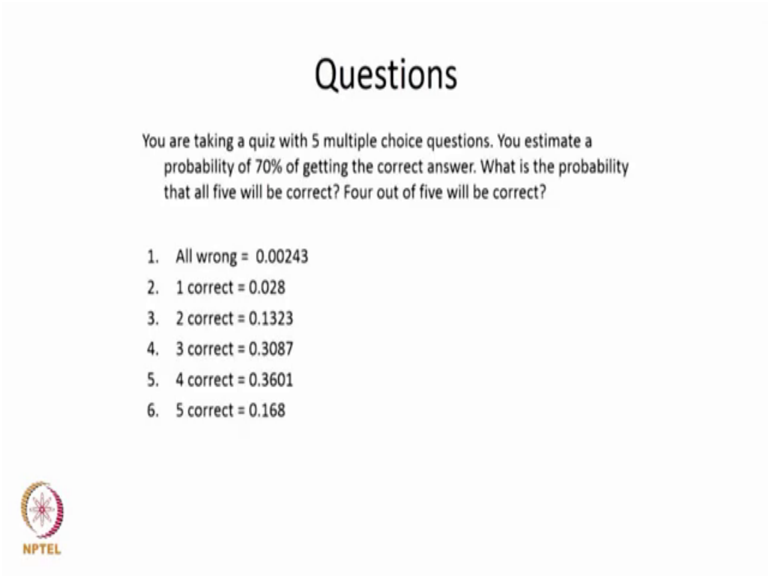

Another question is you are taking a quiz with 5 multiple choice questions. You estimate a probability of 70 percent of getting the correct answer. What is the probability that all 5 will be correct and 5 4 out of 5 will be correct. So, probability that all wrong is 0.00243, we can, you can do this comfortably, because probability of getting it right is 0.7, probability of getting into wrong is 0.3. So, 0.3 into 0.35 times and so on. So, we have given all the answers 1 characters 0.0282 characters, 0.1323 and so on and they add up to 1

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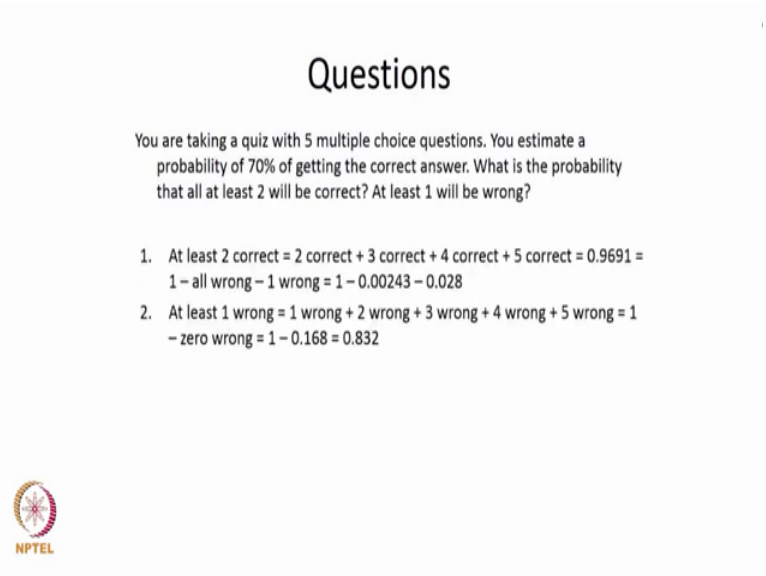

You are taking a quiz with 5 multiple choice questions; probability of 70 percent of getting the correct answer. What is the probability that at least 2 will be correct and at least 1 will be wrong. So, at least 2 will be correct is 2 correct plus 3 correct plus 4 plus 5 correct, which is also equal to 1 minus all wrong minus 1 wrong and so on. And, at least 1 wrong will be 1 wrong plus 2 wrong plus 3 wrong 4 wrong plus 5 wrong, which is 1 minus 0 wrong and so on.

So, these are the ways by which we answer some of these questions. So, with this we come to the end of our discussion, on general concepts of probability. Now, we will be looking at the next topic, which is called conditional probability and we do that in the next lecture.