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NPTEL ONLINE COURSE
Transform Techniques for Engineers
Solution of Heat and Laplace Equations by
Fourier Transform
With
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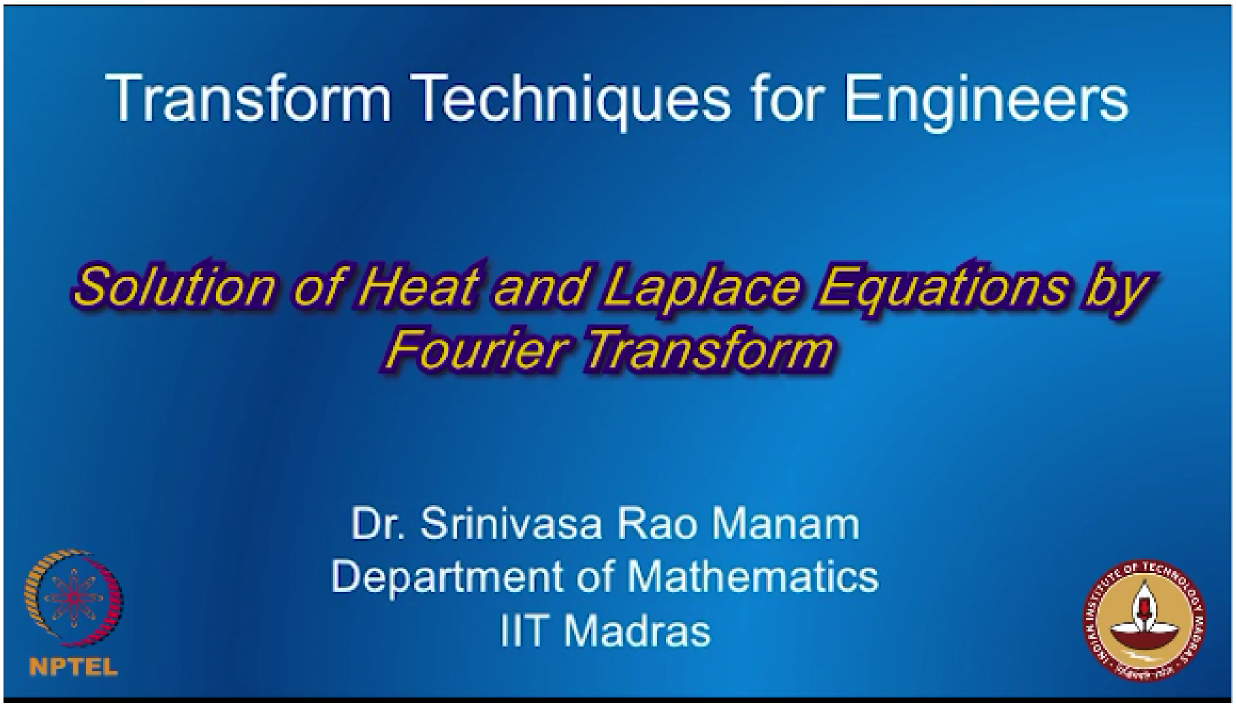
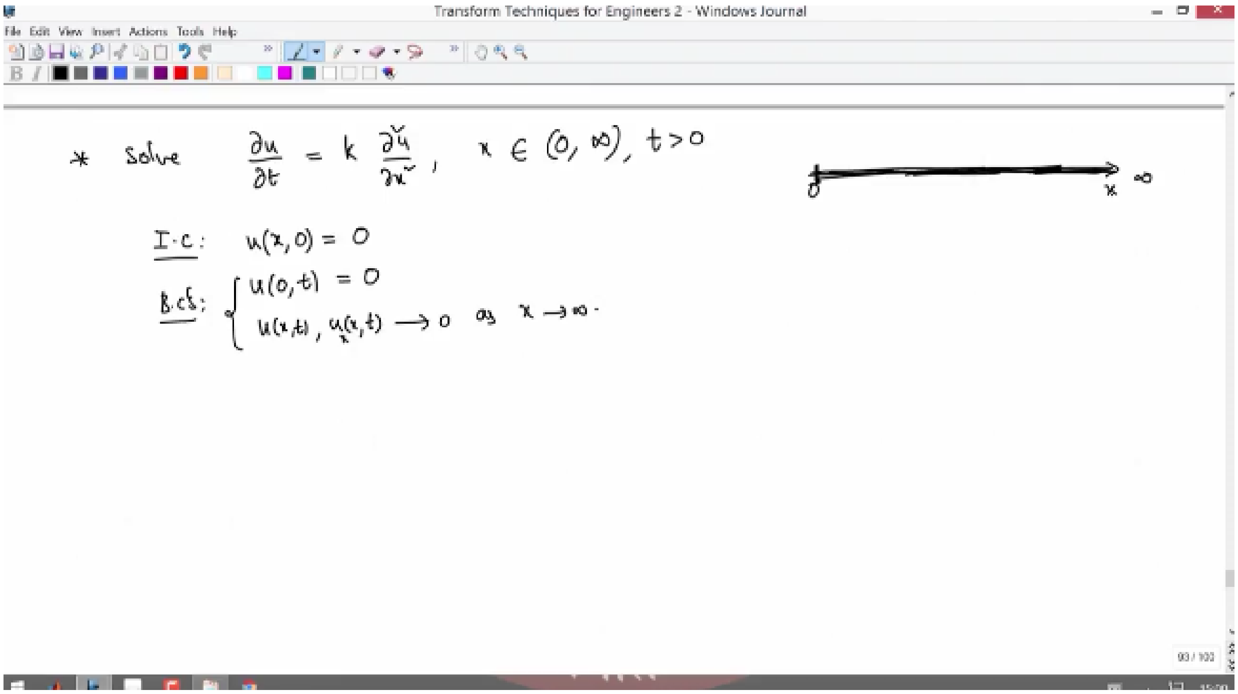


Welcome back, in the last video we have seen the application of Fourier transform to certain partial differential equations, so boundary value problems involving partial differential equations, typical linear equations we have solved with boundary conditions and initial conditions.

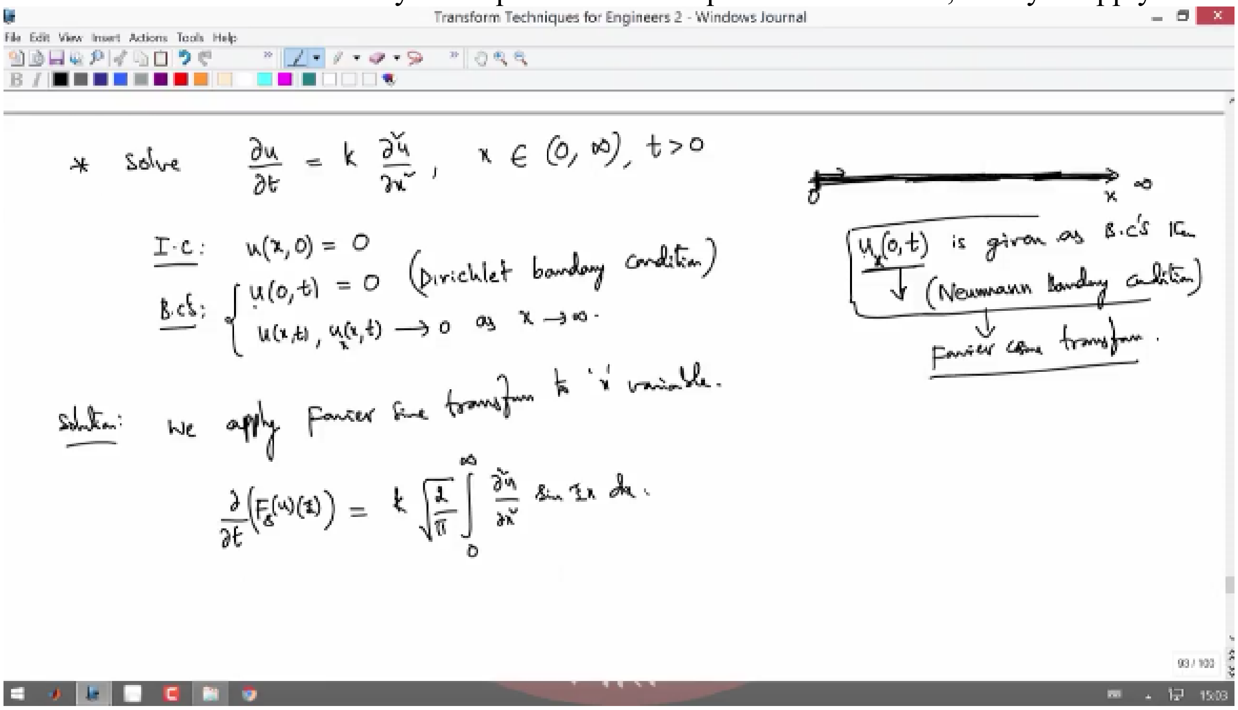
In this video we will see how to apply Fourier sine transform or cosine transform when the problem is a, boundary value problem is defined over domain that is a semi-infinite that is from 0 to infinity, okay.

So we'll just start with a problem like this solution, solve if you want to solve this equation this is a heat equation dou square U/dou T, dou U/dou T which is equal to that kappa specific heat constant, so you have dou square U/dou X square, and now X belongs to 0 to infinity so you have a semi-infinite rod which you have 0 to X that is infinity, same infinite rod you have its one-dimensional literally insulated so that all the propagation of heat is only one directions, and the T positive, so initially at T = 0 you should provide, because it's a one derivative T is our time derivative is first order so you have at U at X, T = 0 you provide let's say 0, so at 0 its temperature is 0 of this rod at T = 0 temperature of the rod is 0 and then you have this boundary, at this boundary that is at U at 0 for all times maintain 0 temperature, okay.

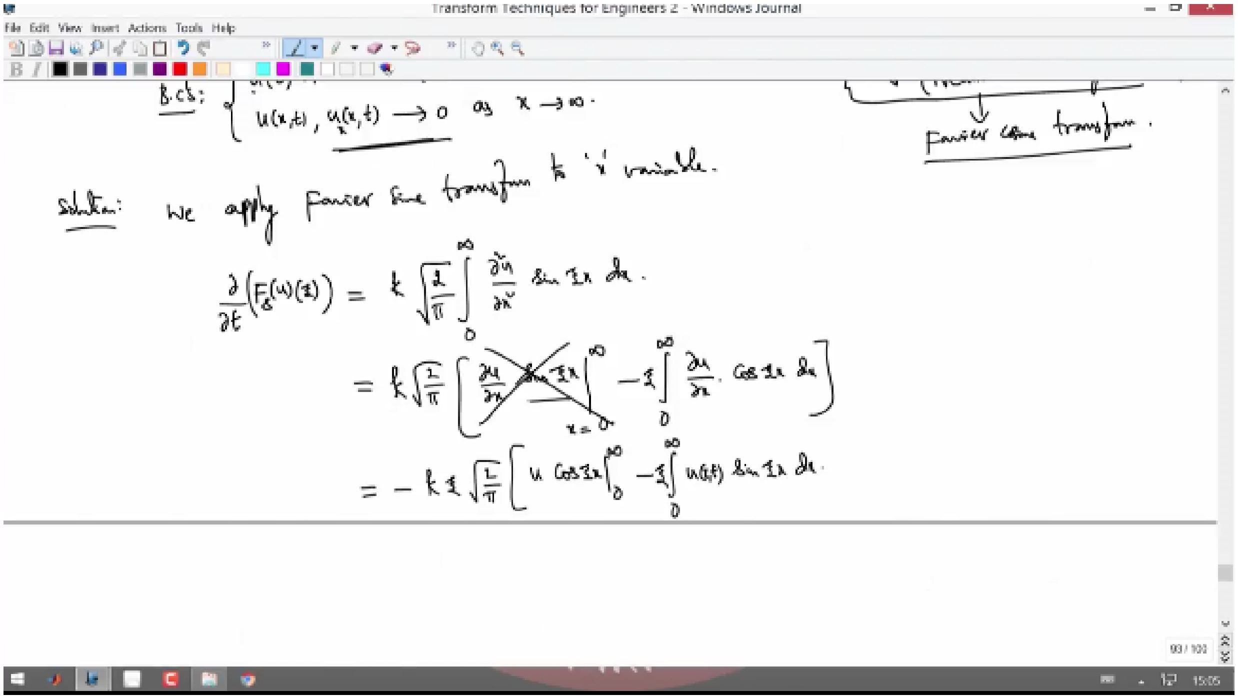


And then what else you have? So you have a boundary, if you want to apply the Fourier transform here, I need Fourier transform it has to be absolutely integrable function minimum so you expect, so you're going to apply since the domain is 0 to infinity you expect to apply Fourier cosine or sine transform for this U and UX, so you need another boundary condition so this is the actual initial condition and these are the boundary conditions, what are the boundary conditions? One is this, other one is simply U and U(x,t), UX(x,t) should go to 0 as X goes to infinity, so this is, so that means the temperature at infinity can expect in the rod that's going to 0, so this problem we can solve by Fourier, see once you see this boundary condition this boundary condition, the moment you see the boundary condition as Dirichlet condition, this is a Dirichlet condition when once you have this 3 type of boundary conditions, so you have a boundary at the X = 0, if U is provided that is called a Dirichlet boundary condition, okay, so if your boundary condition is flux condition that means heat is either going inside or going outside, going outside or going inside let us say for all time if you keep that as a boundary condition mathematically that is U, flux means it's going out so this is our derivative, normal derivative to this boundary at X = 0, at X = 0T this is provided, this is given if this is given as a boundary condition then you should use Fourier sine transform, so such a boundary condition this is called Neumann condition, first of all it's called Neumann boundary condition.
This is Neumann boundary condition in this case we use, if such is the boundary condition we can use Fourier sine transform, you will understand why this is so, naturally we are going to use this boundary condition if you use Fourier sine transform here I think in this case we use Fourier cosine, Fourier cosine transform in this case we use Fourier sine transform so that naturally we can make use of this boundary condition in the process.
So if you are given a Neumann boundary condition you use this Fourier cosine transform, if you use this Dirichlet condition you use Fourier sine transform, so let's solve this by Fourier sine transform, we apply Fourier sine transform to X variable, X variable is from 0 to infinity so if you do that you have dou U by, dou u let's use if you use this derivative dou/dou T of, if you apply for this left-hand side you get Fourier sine transform of U(xi) for this the time derivative

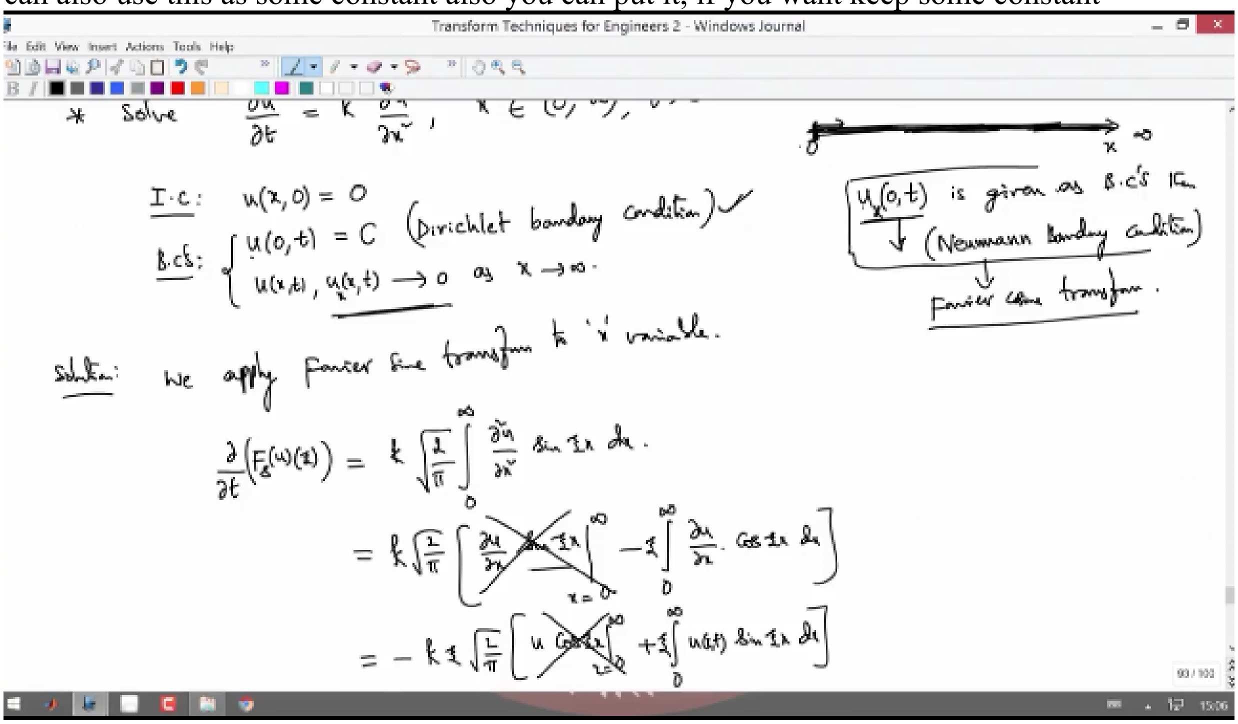
which is equal to kappa, now if you can apply Fourier sine transform so you have the definition of sine transform is 0 to infinity dou square U/dou X square sine xi X DX, so if you apply this



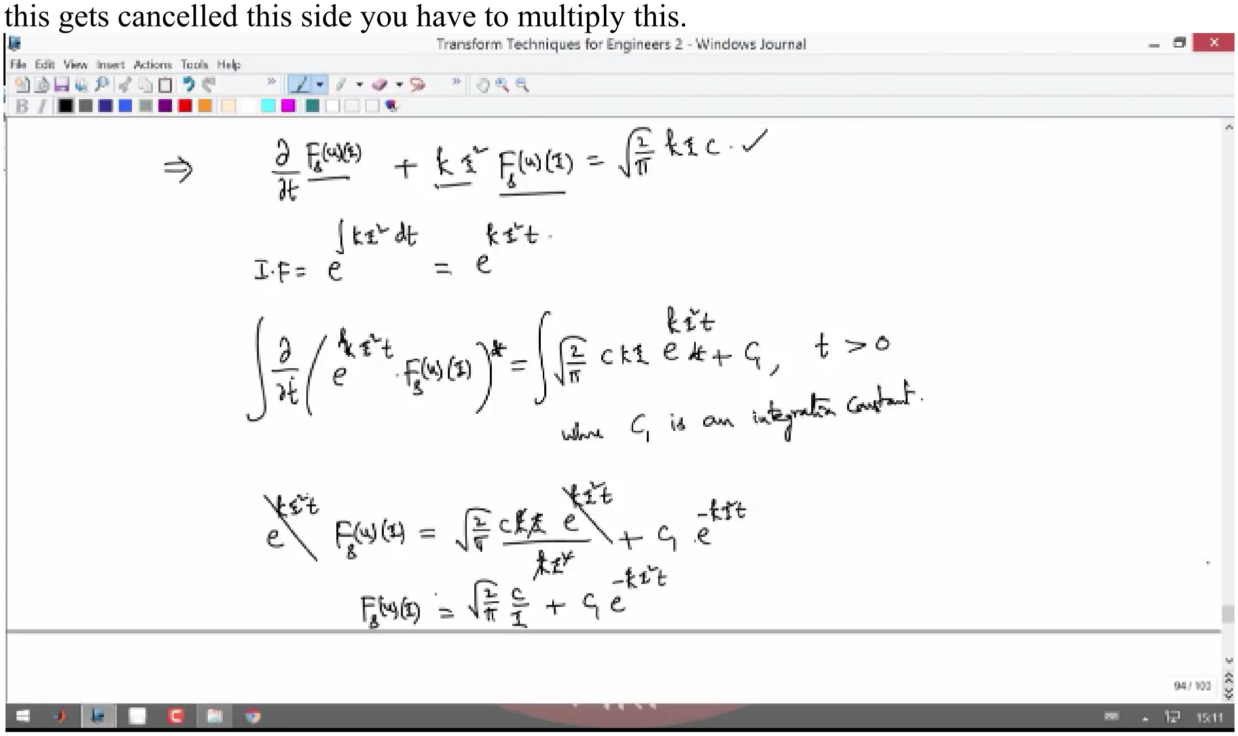
you can do the integration by parts kappa times root 2/pi and you have dou U/dou X sine xi X so you can easily see this, so this is the boundary terms you get for this integral when you do the integration by parts minus what you get is, when you get integral 0 to infinity dou U/dou X times derivative of sine xi X that xi comes out you have cos xi X DX, so you want to avoid this boundary condition you can make them nullified, you can nullify these boundary conditions if you use, if you use cosine transform you need to provide dou U/dou X okay, dou U/dou X at infinity or dou U/dou X at 0 okay, because of sine xi X at once you put X = 0 infinity this is naturally going to, you don't require this dou U/dou X, okay, so if you use this at infinity dou U/dou X that goes to 0, and if you use this condition this is 0 and at X = 0 sine xi X is 0 so this together it's nullified, so you end up getting K xi root 2/pi 0 to infinity, so again if you do one more time integration by parts here, so you get U cos xi X for this you might have this and -integral 0 to infinity, so again cos xi X you are going to differentiate that becomes sine xi X DX this is going to be U(x,t).



So cos xi X - the derivative, when you differentiate cos xi X this is going to be –sine xi X that minus the integration by parts minus is going to be plus, okay, so here now you can use this Dirichlet boundary condition to make this, and first of all this is going to be U(x,t) as X goes to infinity 0, when you put X = 0 cos xi X is 1, U(0,t) that is the Dirichlet boundary condition that makes it 0 here, so you end up getting finally -kappa xi square root 2/pi, root 2/pi together with this integral that is simply Fourier sine transform (u)(xi), so you need not take it this as 0, you can also use this as some constant also you can put it, if you want keep some constant

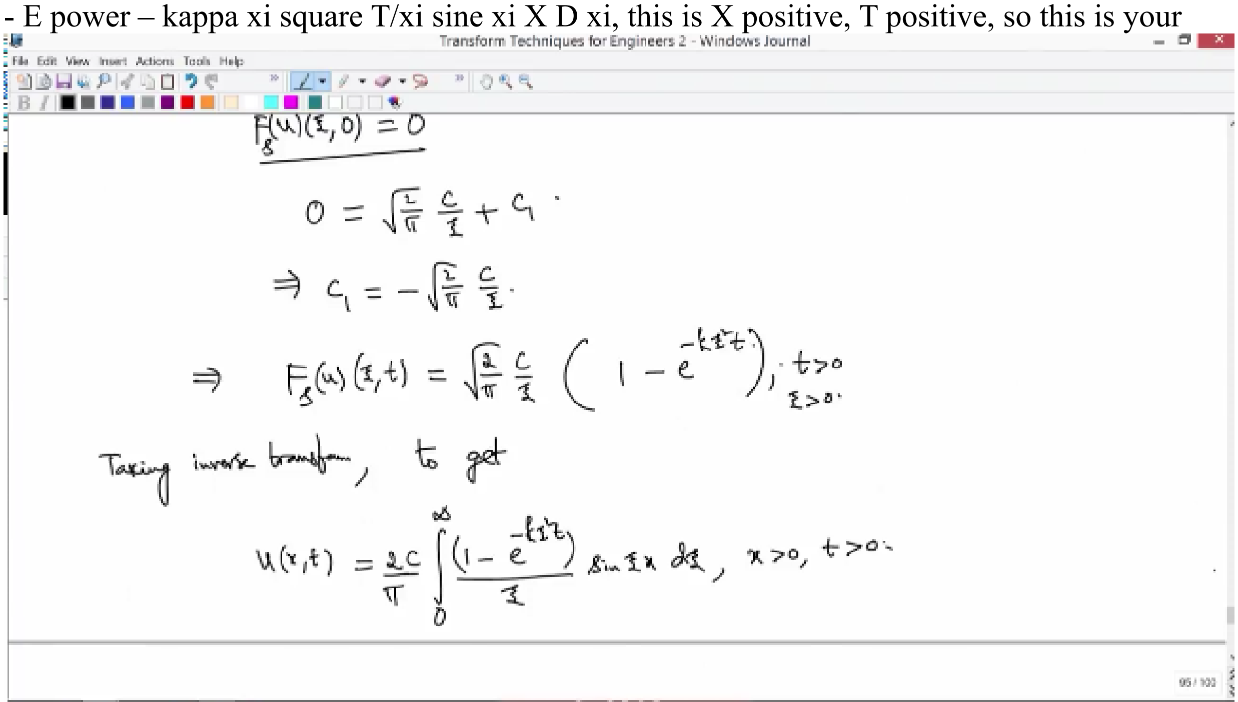


temperature here, so let's use some constant C so then you end up getting here this won't be 0 here so at infinity this is 0 this minus minus plus, what you end up is, what you end up is xi root 2/pi cos xi X, at X = 0 its 1, and U(0,t) that is C, so you have this is what you get, that C is Dirichlet boundary condition, you can do this one also.
So what you end up is this ordinary differential equations dou U/dou T of FS(u)(xi) + K root 2/pi, there's no root 2/pi right, so you have K xi square FS(u)(xi) = root 2/pi kappa xi C, okay, so this is your ordinary differential equation, it's a linear equation for this unknown this one, this is your unknown and if we find the general solution integrating factor you have to multiply this with integrating E power integral this is your P DY/DT + P(t) into Y(t) which is equal to some given function of, this is just a constant it doesn't involve any function of T, so it's just a constant, so integrating factor is if you calculate E power integral kappa xi square that is a constant DT, so that will give me E power kappa xi square T, so this you try to multiply to the equation so you see that left hand side will become dou/dou T of E power kappa of xi square T times FS(u) which of xi that is a Fourier sine transform of U which is function of xi, this is equal to root 2/pi CK xi times E power kappa xi square T, so if you integrate both sides you get, you end up getting, you integrate both sides, so if you do both sides integration, so what is your time? Time is the definition of domain is time derivative, so T which is positive we can integrate both sides actually, if you integrate both sides you end up getting some constant K, let us say some C1, that C1 is integration constant, this is your general solution so if you do this, this is with respect to DT so you end up getting E power kappa xi square T times Fourier sine transform = root 2/pi C kappa xi, and this is divided by K xi square E power kappa xi square T that is the integration of that, + C1, so if KK goes, xi xi goes you end up getting root 2/pi C/xi and if you simply multiply both sides you end up getting here E power -K xi square T, okay, this gets cancelled this side you have to multiply this.

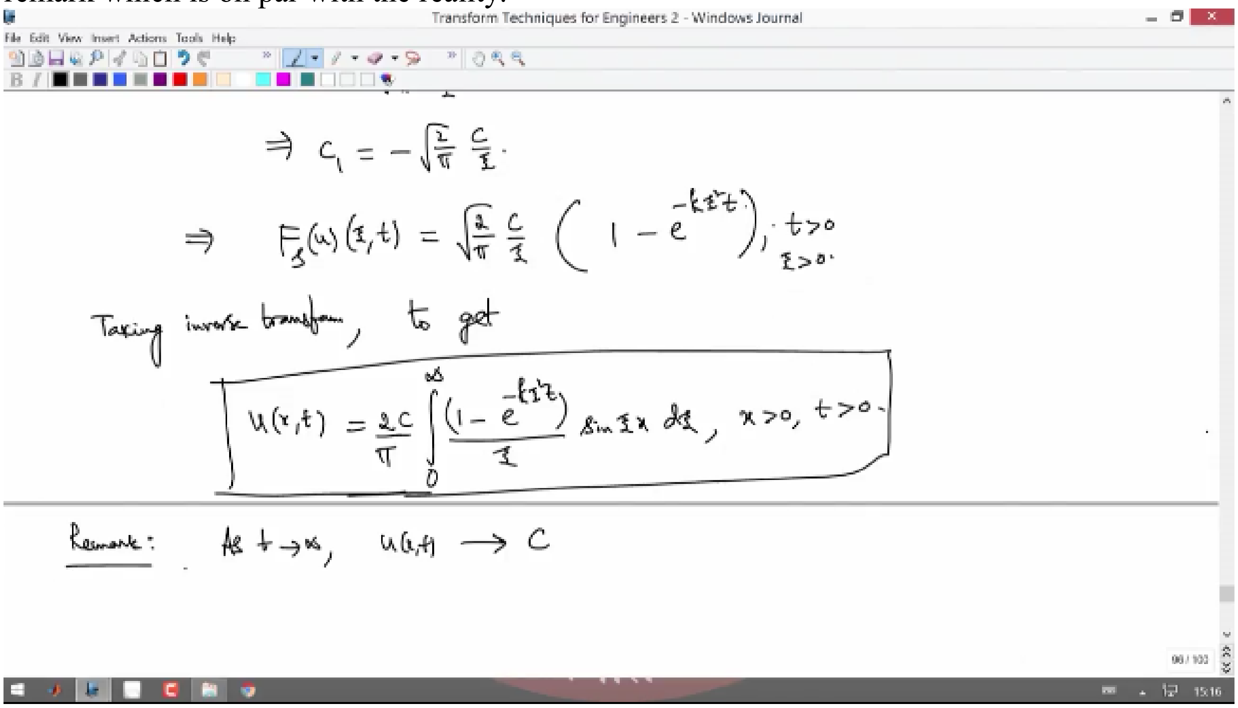


So this one times + C1 E power -kappa xi square T, and this is actually your Fourier sine transform of this. So far we have used only boundary conditions, boundary condition initial condition as well so, only boundary condition have used, we have not used this initial condition, so if you use that initial condition if you want to use that initial condition so you apply Fourier sine transform through this initial condition, so if you apply because this is a variable U at X = 0, T = 0, X belongs to 0 to infinity, if you apply, you can apply Fourier sine transform there so since initial condition U(x,0) = 0 we have by Fourier sine transform FS(u)(xi) is also 0, okay, so xi,t this is a function of both xi,t, so actually throughout when you apply this which is function of xi, so you also have T, okay, Fourier sine transform of(xi,t) both the variables, so we have a Fourier sine transform (xi,t) both, you end up getting xi,t, xi,t and you have this is 0, at T = 0, so because I'm taking the Fourier sine transform here, so since this is the case if you apply this here you end up getting the left hand side 0, this is equal to root 2/pi C/xi + C1, so because T = 0 exponential quantity is 1, so that makes it C1 = -root 2/pi C/xi, so you go and substitute there into your solution for U which is function of xi, which is also function of T, this is equal to root 2/pi C/xi, C is given constant + C1 is -root 2/pi C/xi times E power -kappa xi square T, so if you remove this, take this common root 2/pi or C/xi this is going to be 1- of this, so this is exactly your, so this is T positive, xi is positive, because this Fourier sine transform.

So what is your solution? You just take the inverse transform, taking inverse transform, taking inverse transform, inverse sine transform we get or to get U(x,t) this is equal to you end up getting finally a root 2 pi, root 2 pi so you have 2/pi C/xi which is a constant, you integrate from 0 to infinity, you have root 2 pi, root 2 pi is here that's become 2/pi so I write it outside and C is a constant I write it outside, so you have 2/pi 2C/pi, integral 0 to infinity this quantity 1 - E power – kappa xi square T/xi sine xi X D xi, this is X positive, T positive, so this is your

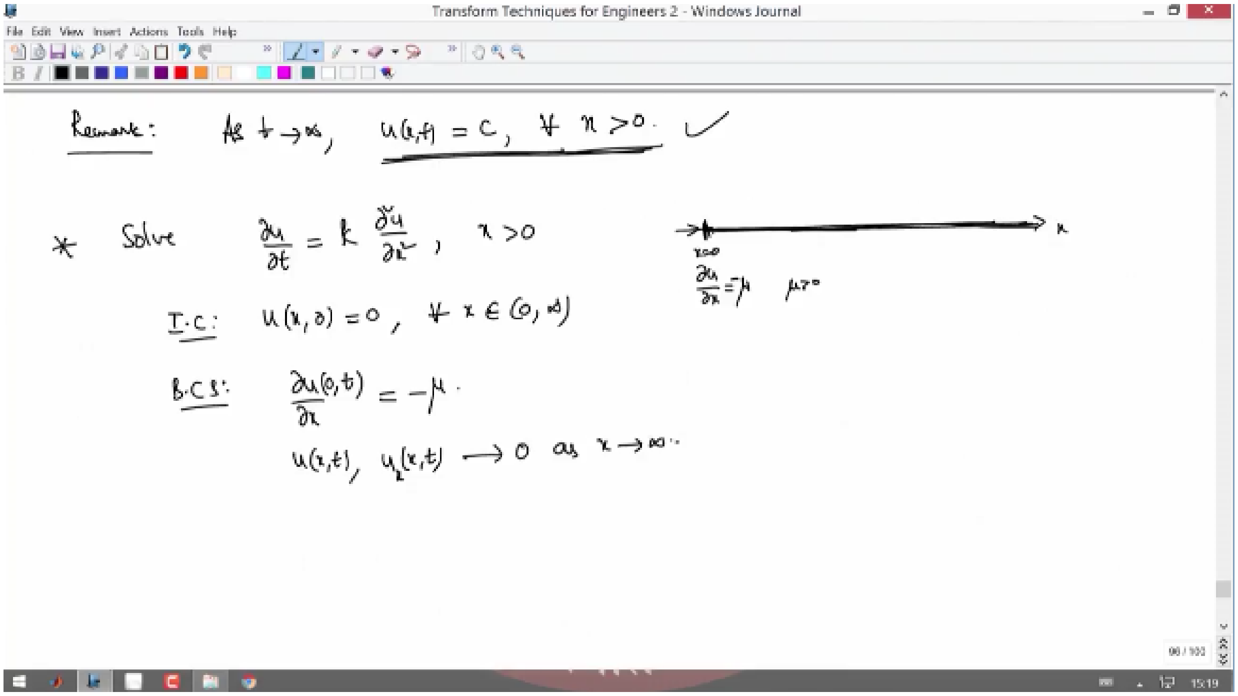


solution, solution of your differential equation, so this is your solution of a given problem, so this is what happens for all times when you keep one end of the rod at temperature C, initial temperature is 0 throughout the rod for all times this is what happens, okay, so as T goes to infinity this is becoming 0 sine xi/xi so that's going to be pi/2, so what you end up is finally as T goes to infinity, you can easily see what you can expect U(x,t) converges to C, that is you can easily see as T goes to infinity this quantity is 0 and you end up sine xi X/xi D xi, X is positive so that makes it pi/2, pi/2 goes and what you end up is going to be, that is what is expected so when you have a rod which is initially at 0 temperature and it keeping a temperature at C here eventually the heat will propagate whatever you pumping in, so where you're keeping the temperature constant so that diffuses, heat diffuses makes the temperature throughout the rod as C, even at infinity okay, so that's what happened so, so this is the solution, so this is your remark which is on par with the reality.

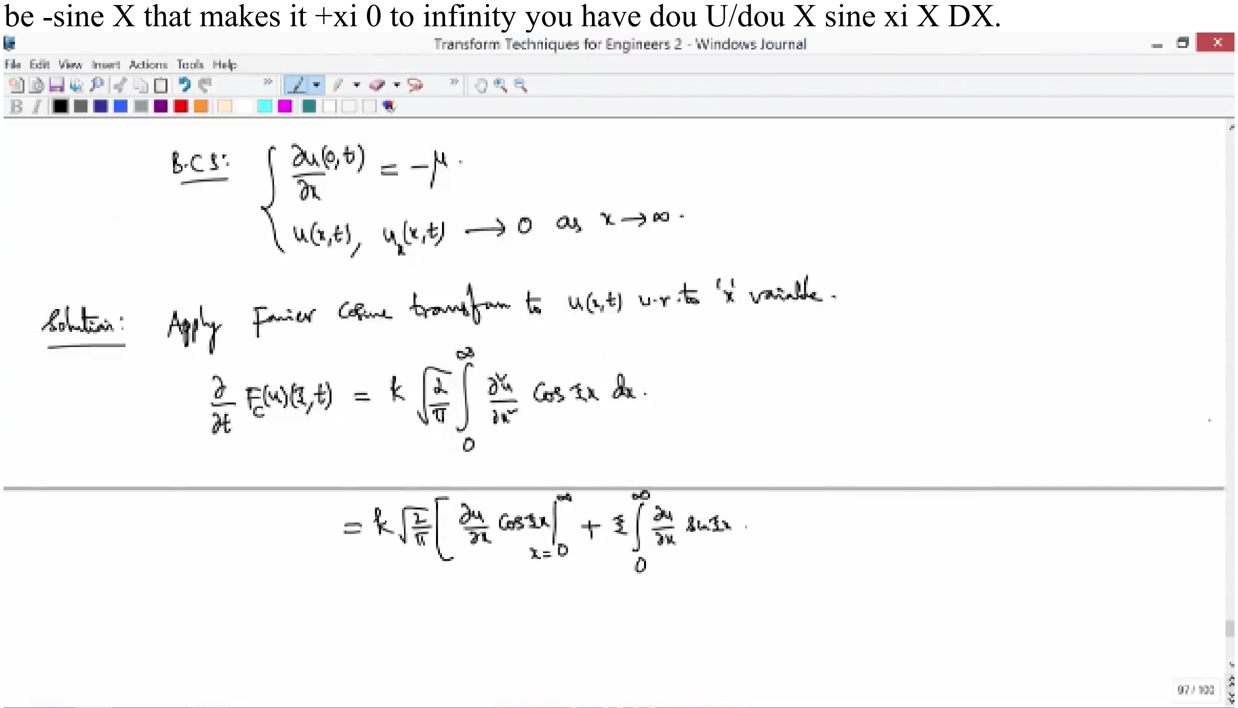


So temperature finally is C for all times, for all T positive, okay, now for all, for every X positive this is what is true, throughout the rod it becomes, it’s rod with temperature, constant temperature C eventually as T goes to infinity, as time passes, this is how we solve, if your domain is a semi-infinite domain that is between 0 to infinity.

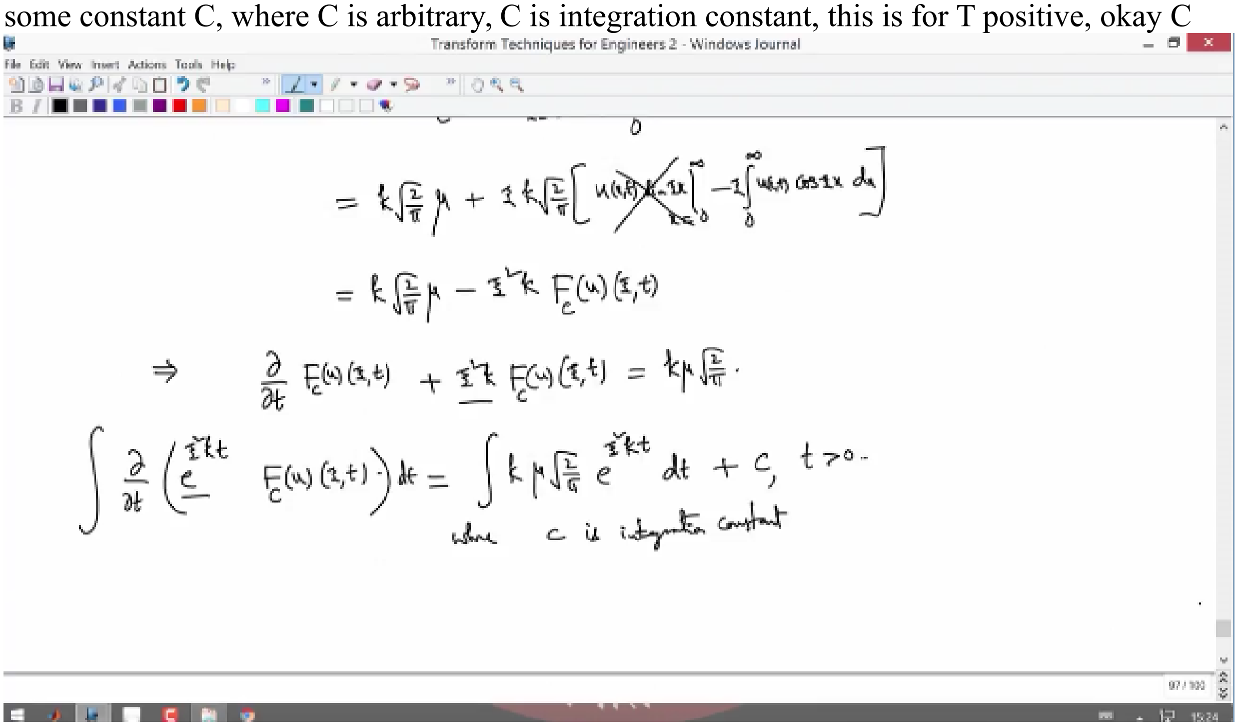
So let's see another example where this Neumann condition is provided for this heat equation, we will just see that and then we'll move on to other type of problems. Let's consider this problem, solve dou U/dou T = kappa dou square U/dou X square this is heat equation, one-dimension heat equation for an infinite rod, semi-infinite rod, so again we consider the same situation as X, X goes to infinity this is your semi-infinite rod and you have at X = 0, you have provided initial condition let me see, what is the initial condition? Initially again we keep the rod at 0 temperature, so U(x,0) 0 for every X, X belongs to 0 to infinity boundary conditions, so what I do is I try to, so if you use this flux, flux is dou U/dou X here if I give this as mu that is actually nor outward heat is going outwards with mu positive, if I give –mu with mu positive is actually you’re pumping in some heat, heat is your pumping in through this surface, some heat is propagating, so it's passing from outside into this rod, constant so flux is with this, with this rate it's going, it's coming inside, so let's see what happens so if I write this mathematically boundary condition is dou U/dou X(0), X is 0 and T which is equal to -mu let us say, okay, this is your boundary condition, and if you want to apply a Fourier sine transform or cosine transform because the domain is 0 to infinity U(x,t) and UX(x,t) this is where you are applying the Fourier sine or cosine transform, this has to go to 0 as X goes to infinity, so these are your boundary conditions.



So you can solve similarly such a boundary value problem for the heat equation, so let me, so we use because this Neumann boundary condition is given, so we use Fourier, we apply Fourier cosine transform to U(x,t) with respect to X variable, to see this what you end up is dou/dou T of like earlier for FC, Fourier cosine transform of U which is function of, X becomes xi and this is T, this is equal to kappa times, now if I apply both sides, apply write the definition of the Fourier cosine transform for dou square U/dou X square this is from 0 to infinity cos xi X DX, do the integration by parts like earlier so you get root 2/pi and this if you do integration by parts dou U/dou X cos xi X for which you applying limits -cos xi X if you differentiate that's going to be -sine X that makes it +xi 0 to infinity you have dou U/dou X sine xi X DX.

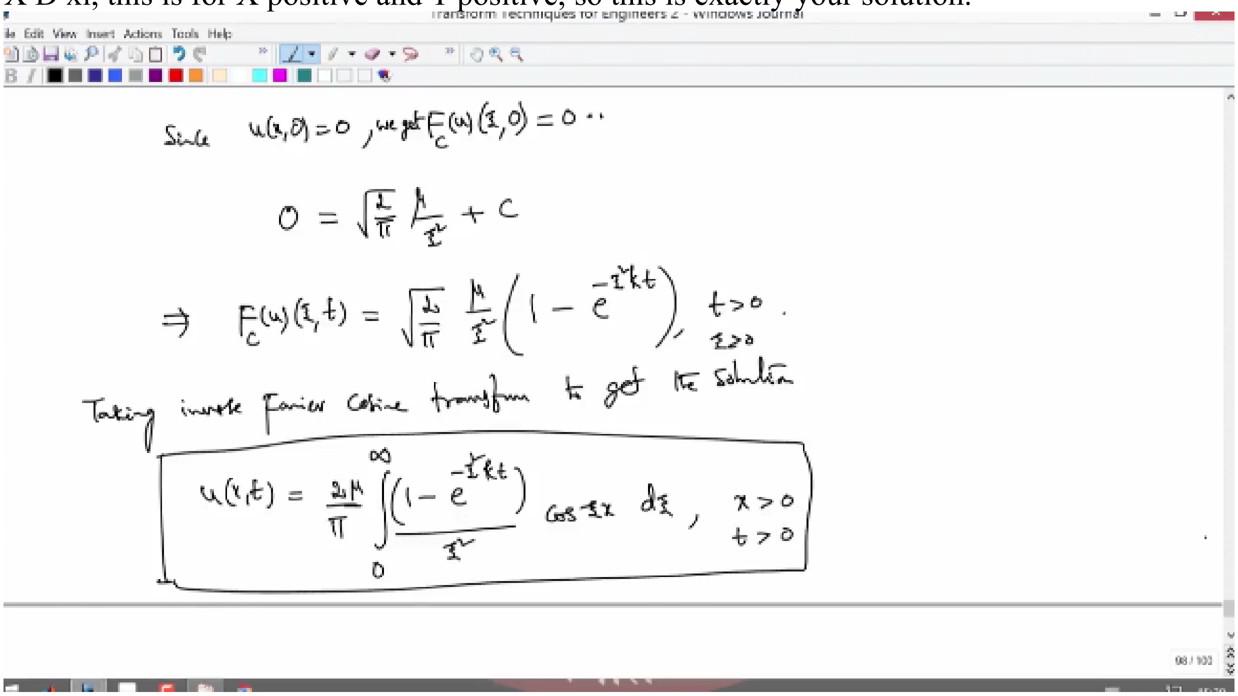


So since at infinity this is going to be infinity, at X = 0 cos xi X is 0, cos 0 is 1 and dou U/dou X at X = 0 that is given as your boundary condition, so you end up kappa root 2/pi that is – mu, -mu but minus minus plus here, okay so that's mu, and then you have + xi kappa root 2/pi, again this integral you can again do the integration by parts, so you see that U and U(x,t) sine xi X, you apply 0 to infinity limits and you have minus and here integral 0 to infinity one xi comes out because of the differentiation of sine xi X that makes it cos xi X DX and this becomes U(x,t), so this is equal to kappa root 2/pi mu + this here if you use the boundary conditions again U(x,t) as X goes to infinity that is 0, and sine xi X at X = 0 that is 0, so this is what happens, so this is a temperature assume that temperature at X = 0 is finite, so this is going to be, this nullified you end up getting –xi square kappa root 2/pi, so – xi square kappa root 2/pi this integral together that is Fourier cosine transform of U, which is function of xi with respect to X variable which is also function of T, originally U(x,t) so X becomes xi, other variable as this.
So your partial differential equation becomes ordinary differential equation like this, so dou/dou T of linear equation here like earlier F(u)(xi,t) + xi square T, xi square kappa F, Fourier cosine transform of U of function of xi,t which is kappa mu root 2/pi, it's just a constant, so what is it solution? Solution is F(u)(xi,t) I multiply E power integral this is your T xi square, if you do the integration here xi square kappa T, multiplication with respect to dou/dou T, I multiplied so this is what it, if I multiply this factor throughout, so you have a kappa mu root 2/pi E power xi square kappa T, so that is what happens, so if you multiply this factor both sides left hand side is this, so we can do the integration with respect to T, do the integration with respect to T + some constant C, where C is arbitrary, C is integration constant, this is for T positive, okay C

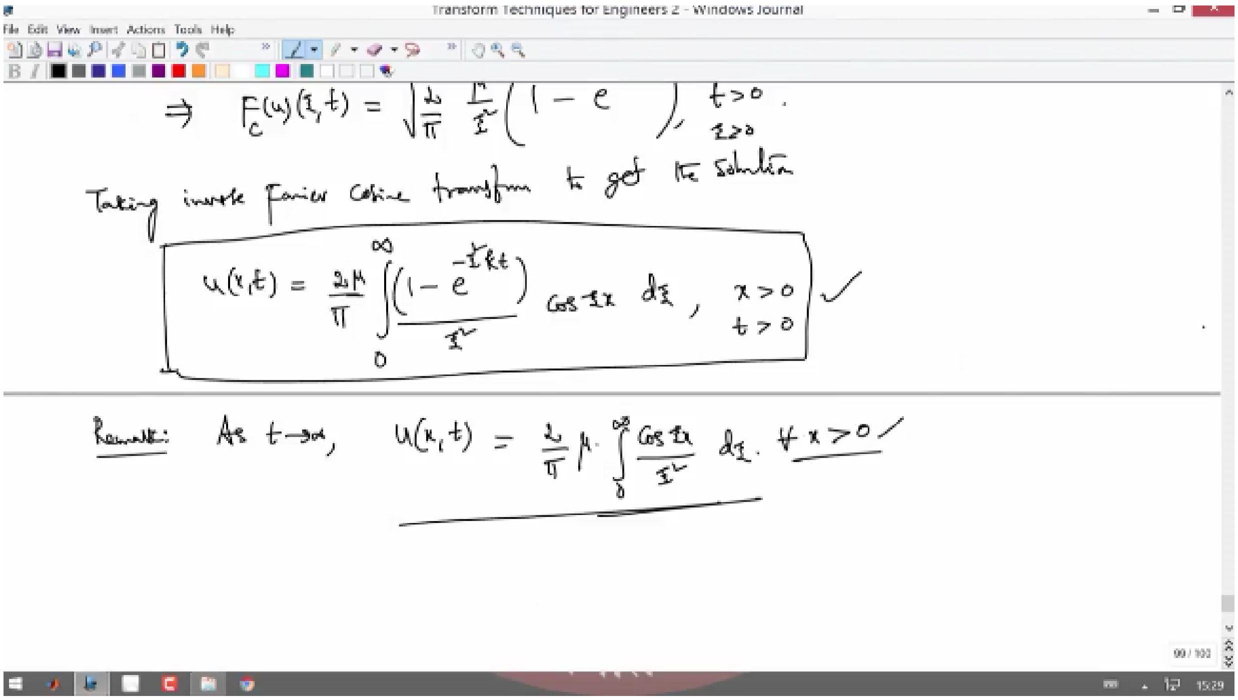


positive and xi is also positive, so this integration and this derivative goes you end up getting FC(u)(xi,t) which is equal to kappa mu root 2/pi is a constant and this if you do the integration you have kappa xi square E power xi square kappa T, that is a integration +C times, so you have this quantity here E power xi square kappa T this is what it becomes, so this makes it finally if I write U(xi,t) this is root 2/pi kappa mu and this gets cancelled, so you have to write here E

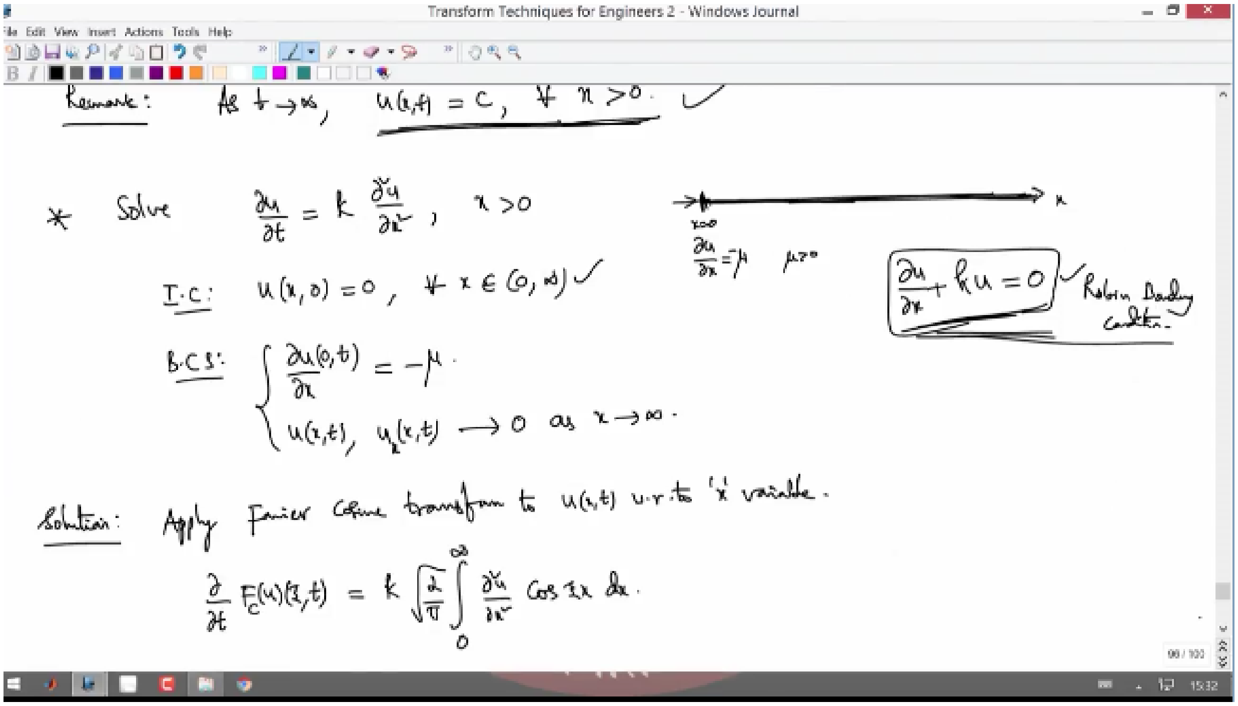
power -xi square kappa T, so this divided by KK goes mu, KK goes you end up getting mu/xi square.
And here + C times E power – xi square kappa T, so again you've used the, we have not so far used initial condition that is this, if you apply the initial condition which is 0 we kept the rod at 0 temperature, since U(x,0) initial temperature 0, you have FC(u) which is function of xi, at T = 0 this is also 0, okay, forget this, so if you apply this two here so left hand side is 0, right hand side 2/pi mu/xi square and this becomes C, so C is minus of that so that implies FC(u) xi T this is C is minus of that so that is common so 2/pi mu/xi square comes out which is minus, and C is minus of this, -E power –xi square kappa T, for T positive xi positive, so you are left with simply taking the inverse transform, taking inverse Fourier cosine transform to get the required solution, you add the solution that is U(x,t) that is if you do the definition of the inverse transform another root 2 pi, root 2 pi, that is 2/pi, mu is constant comes out, and what you end up is 0 to infinity for this function 1 - E power – xi square kappa T divided by xi square cos xi X D xi, this is for X positive and T positive, so this is exactly your solution.



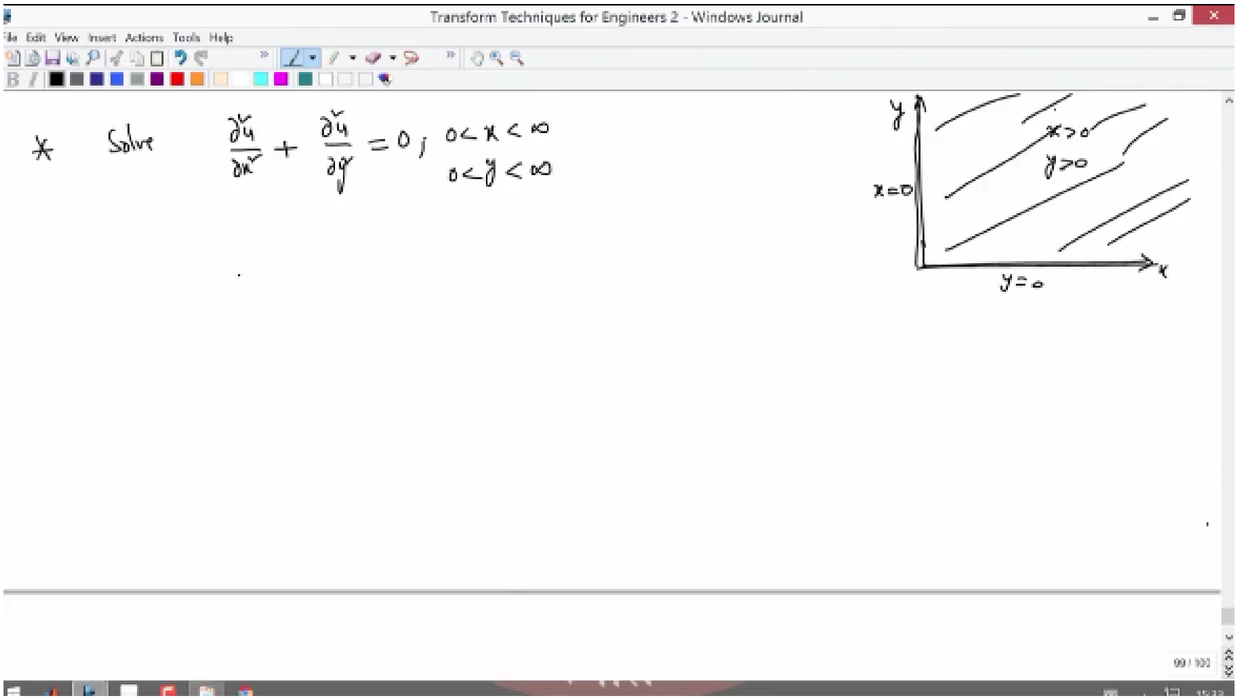
So you can calculate this integral as T goes to infinity, so again like earlier you can also have this remark as T goes to infinity what happens to this U(x,t) as you pump in this thing so you end up getting 2/pi times mu integral 0 to infinity cos xi X/xi square D xi, this integral so you have to evaluate and if you see you keep pumping you end up getting some, if you constantly do it as T goes to infinity so you end up getting a temperature it's infinity I think this quantity we really don't know exactly, so if you evaluate we know, so what you expect is this T goes to infinity or temperature this is true for every X positive, so this is what happens.



At X = 0 cos xi X is 1, 1/xi square is -1/xi and then that is 0 to infinity, so that is so it's infinity right so that is expected, I think this is going to be infinity, so this integral is not convergent integral, so as you expect is going to be infinity cos xi X/xi X square if you fix X I think it is going to be infinity, so this quantity so that is what is expected because you have a rod you're keep on pumping some heat inside what happens to the rod eventually? Eventually you'll see that is going to be U has to go to infinity, as clearly you can see at X = 0 1/xi square so whose integration is 1/xi, if you substitute to the limits this at xi = 0 that's going to be infinity.
And similarly at any X positive and maybe, so that we don't know so if it is converging integral we can just do this as and how do we evaluate for all X positive, X = 0 that is infinity here and then X positive you can rewrite, this is a even function inside -infinity infinity cos xi X/xi square D xi, this is half of that, so your integral is half of this, this you can do the contour integration, yeah so clearly so this is you have 1/xi square, xi square, 1/xi square and, at 0 you have this is not convergent integral, it's not the usual sense, okay, so that is what is expected, so you can easily see as you pump in the heat is eventually the temperature of the rod becoming more and more, it will become bigger and bigger, okay, temperature it just increases eventually to infinity throughout the rod, okay, that's what you can see from this example if you are given initial condition as some other things or the boundary condition as something else you cannot use anything, for example dou U/dou x + some H of, some H constant times U which is 0, for example if this is a mixed condition or Robin condition they say, Robin boundary condition we

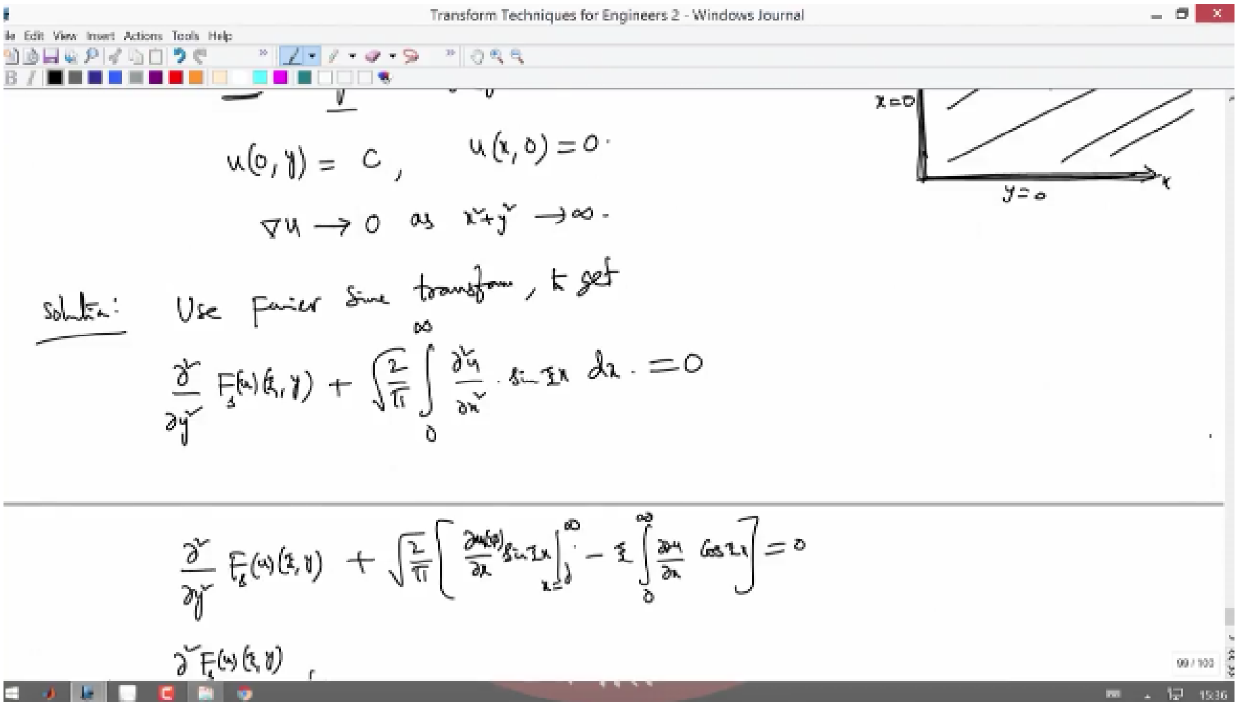


really don't know how to solve this, okay, we may have to use, we cannot solve by these methods, we may have to use some other methods, okay, that is a different issue, so we will not solve this by this method, if it's Robin condition, so this is how we solve these heat equation of a semi-infinite rod, so let me also solve some Laplace equation, so let me solve one more problem using this Fourier cosine transforms, let me consider a Laplace equation in semi-infinite domain let us say if this is your X, this is your Y, so this is a quarter plane problem if you domain is this one X positive Y positive, X positive and Y positive so we'll solve this, solve dou square U/dou X square so this is what is your, assume that this is your plate, same in quarter plane plate and which is in a steady state, so temperature of this at steady state dou square U/dou X square and dou square U/dou Y square = 0 this is X positive that means X is between 0 to infinity y is also between 0 to infinity, what is your boundary? This is your boundary, so that is Y = 0, this is X = 0.



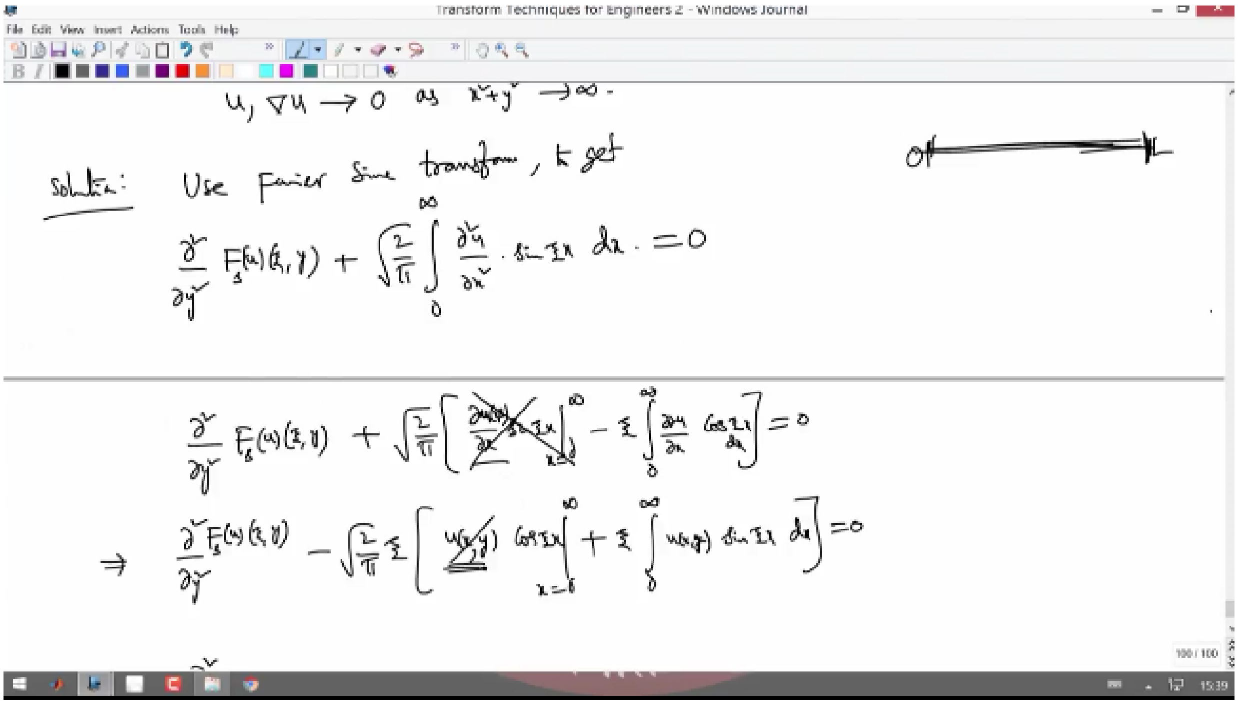
So I just need to give the boundary conditions, so what are the boundary conditions you provide, U at X = 0 that is 0, Y throughout Y, for all Y let me give some constant C and then U(x,0) Y = 0 along here I keep this temperature 0, okay, I maintain constant temperature here, I maintain 0 temperature here, is already in the steady state, okay, in order to apply the Fourier transforms I at infinity gradient of U that goes to 0, as X square + Y square goes to infinity, any direction if you go to infinity gradient, change in the temperature, temperature goes to 0, okay, so such is the problem the solution can apply same technique by Fourier sine transform we apply, because we have the boundary conditions are Dirichlet, so we apply Fourier sine transform to X, X if you apply Fourier boundary condition is this, okay, if I apply Fourier transform to the X variable boundary condition is at X = 0 so which is Dirichlet so that applies we have to use Fourier sine transform, use Fourier sine transform to get, what you get is FS(u) which is function of xi, Y, okay.

No, so this is yeah xi, Y for which dou square/dou Y square this is for this plus if you apply for this root 2/pi integral 0 to infinity and dou square U/dou X square times sine xi X DX, which is equal to 0, so this is what it becomes, so first one dou square Y/dou Y square of FS(u) which is function of xi, y + root 2/pi, so do the integration by parts again so like earlier dou X sine xi X from 0 to infinity - into xi times cos xi X dou U/dou X 0 to infinity which is equal to 0, so if you do this implies dou square FS(u) function of xi, y/dou Y square, this one plus what happens to these boundary conditions, dou U/dou X, dou U/dou X at X, this is the U(x,y), this is U(x,y) we're doing at X = 0 to infinity, so what happens to dou U/dou X? Because of this, no, what is

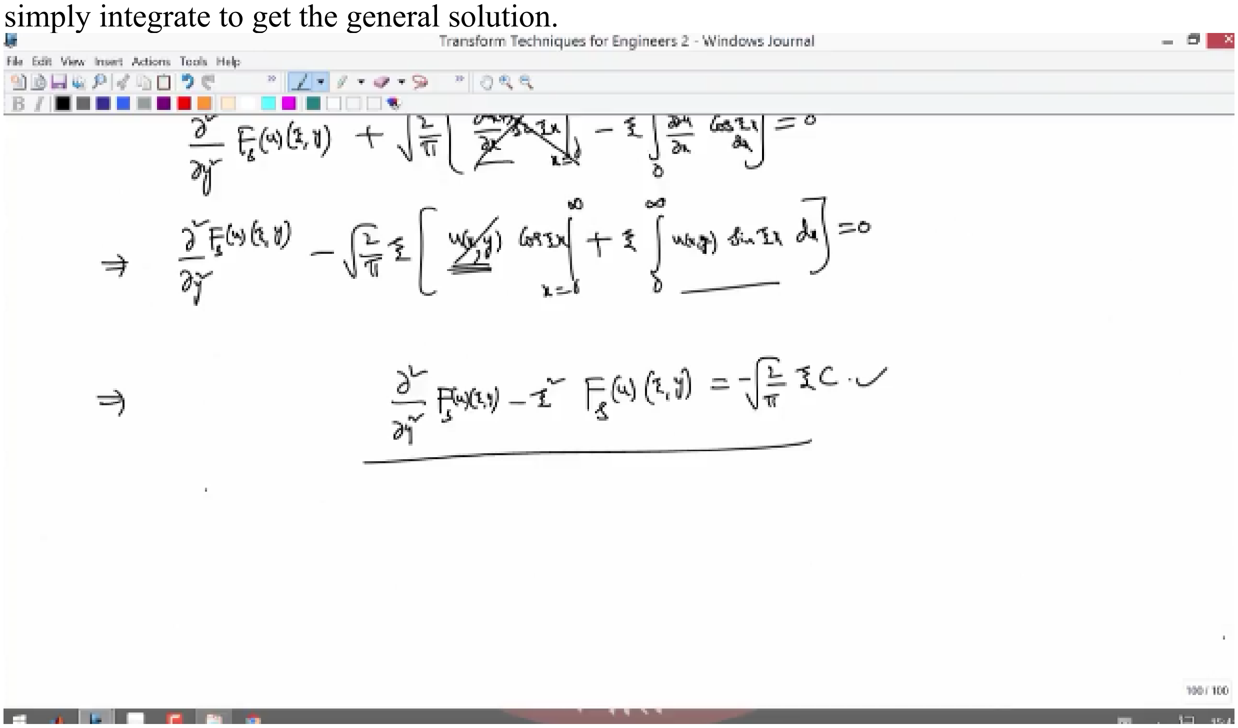


this one? So at X = 0 this because of sine xi X that is 0, what happens to dou U/dou X? At X goes to infinity if I use this one because this gradient, the temperature is gradient which is not, which is going to be 0, so at infinity if you maintain a C temperature you can expect the temperature as at as T goes to infinity, or in the steady state what you expect is the temperature here will be, will be there throughout the plate same, a same temperature that is why we don't say U goes to 0, as X square + Y square goes to infinity, in that case C is inconsistent, C has to be, that means you're keeping 0 temperature here, temperature C here and you expect this to be in the steady state that means throughout its eventually it has to, there's a temperature gradient so C and this side so it's a linearly it distributed to up to infinity, right, that is in the steady state. So anyway, so we just take this way so gradient is, gradient of U goes to 0 as X square that means dou U/dou X, dou U/dou Y that makes it this 0 so this contribution won't be there, so you get root 2/pi xi times, again if you do the integration by parts you end up getting U(x,t) cos xi X - another xi comes here, xi that's because of derivative of sine cosine that makes it –sine xi X that makes it this plus, and you have U(x,t) U(x,y), for X, Y sine xi X DX, okay, equal to 0, so again so here you have to apply the boundary condition at X = 0 to infinity, so this implies dou square/dou Y square of FS(u) xi Y minus again here so this goes to, at infinity again this, so you can say that at infinity this is also goes to 0, the temperature goes to 0 so if you apply you see that this is also 0 you need this, U(x,t) at infinity has to be 0, so eventually if this is C throughout at infinity is C linearly distributes finally so that it is in a steady state, that is what is expected.

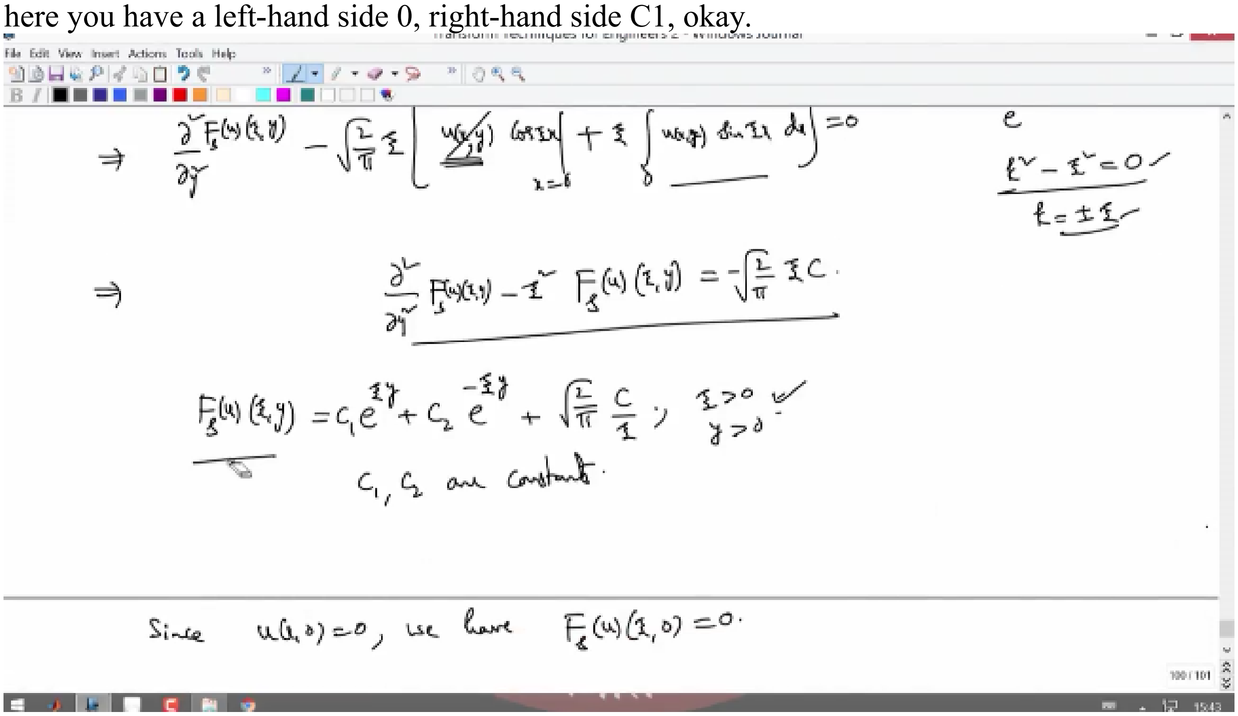
So if you have a semi-infinite rod which is temperature at 0 and L linearly between 0 to L so you have a linear function between 0 to L maintained, so in the same way that is the steady state condition, so that it doesn't depend on, that doesn't depend on T, so the same way the 2-dimensional plate you have the similar situation, so this goes to 0 and you have a cos X = 0T



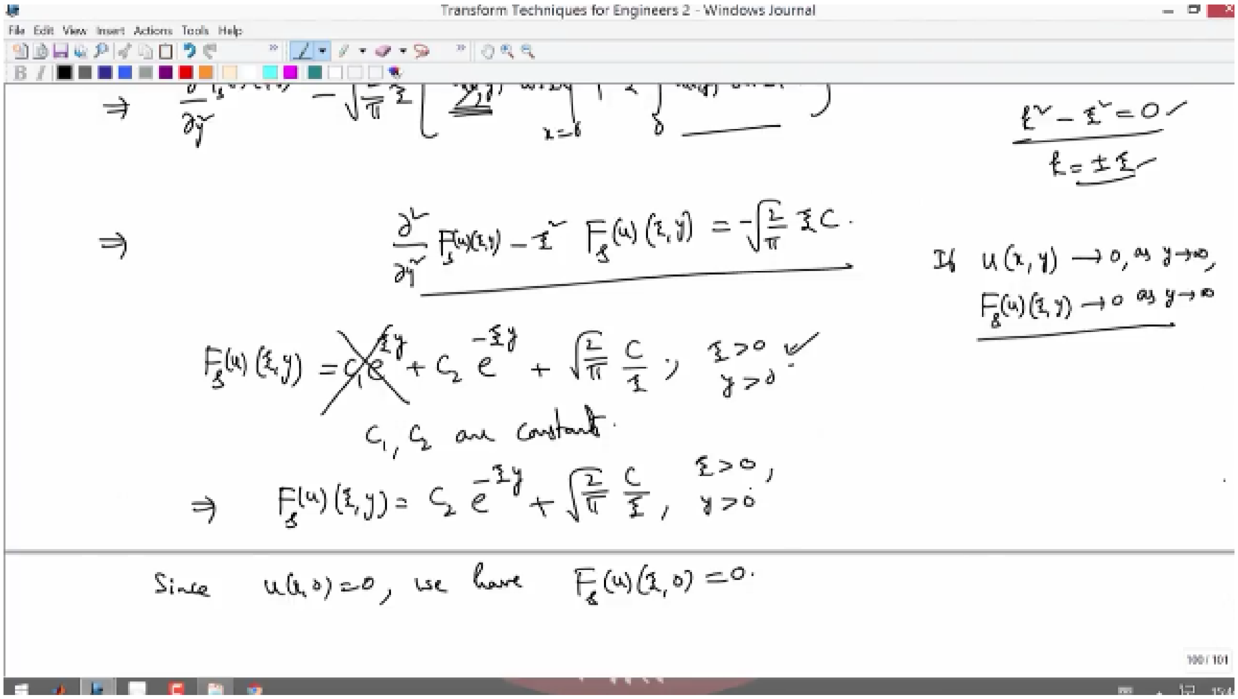
that is, this is not T this is Y, U(x,y) so X,U at 0,Y that is also C so you have a C, minus minus plus root 2/pi xi C okay. And then this becomes - root 2/pi, root 2/pi this integral will become Fourier sine transform, so you have xi square Fourier sine transform of U of xi,y = 0, so this if you write it outside minus this is equal to root 2/pi, - root 2/pi xi C okay, so you can write dou square/dou Y square of FS(u) xi(y) - this is your ordinary differential equation, so you can simply integrate to get the general solution.



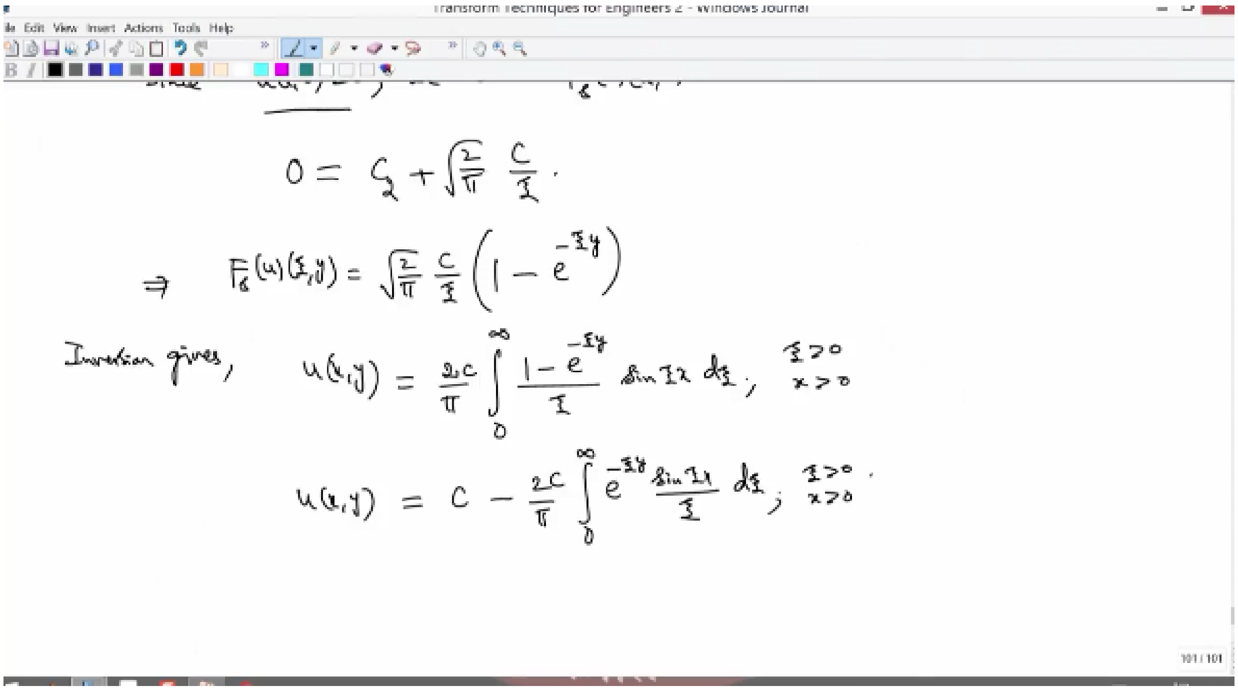
So what is the general solution? FS(u) (xi,y) is equal to, see if you see this is a second-order equation, you have if you write E power Y K type of solution if you look for, what you get up is K square -xi square = 0, so homogeneous solution if you look for this one you get K = + or – xi, so you have E power + or – (xi,y) into C1 + C2 times E power - xi,y, these are the two linearly independent solution for this homogeneous equation, and plus particular solution and you simply you can see that particular solution will be, minus minus goes so you can see that this is going to be 2/pi C/xi is your particular solution of this equation, you can easily calculate by methods in the differential equation, so by looking at this you can easily see that this is the constant that is 0 here, if you put it here in the place of this FS(u) xi, y is this place you end up that both are same, so is actually satisfying this particular solution, so you have a general solution is C1, C2 are arbitrary constants, and this is your solutions xi positive, Y is positive, so still we have not used again like earlier we have not used this initial condition, what is the initial condition, the other boundary condition we have not used that is U at X = 0, right, you at Y = 0, so if you use this you can see that Fourier sine transform of U, since you can write this since U at X0 = 0 we have Fourier sine transform of U(xi,0) is also 0, so this if you substitute here you have a left-hand side 0, right-hand side C1, okay.



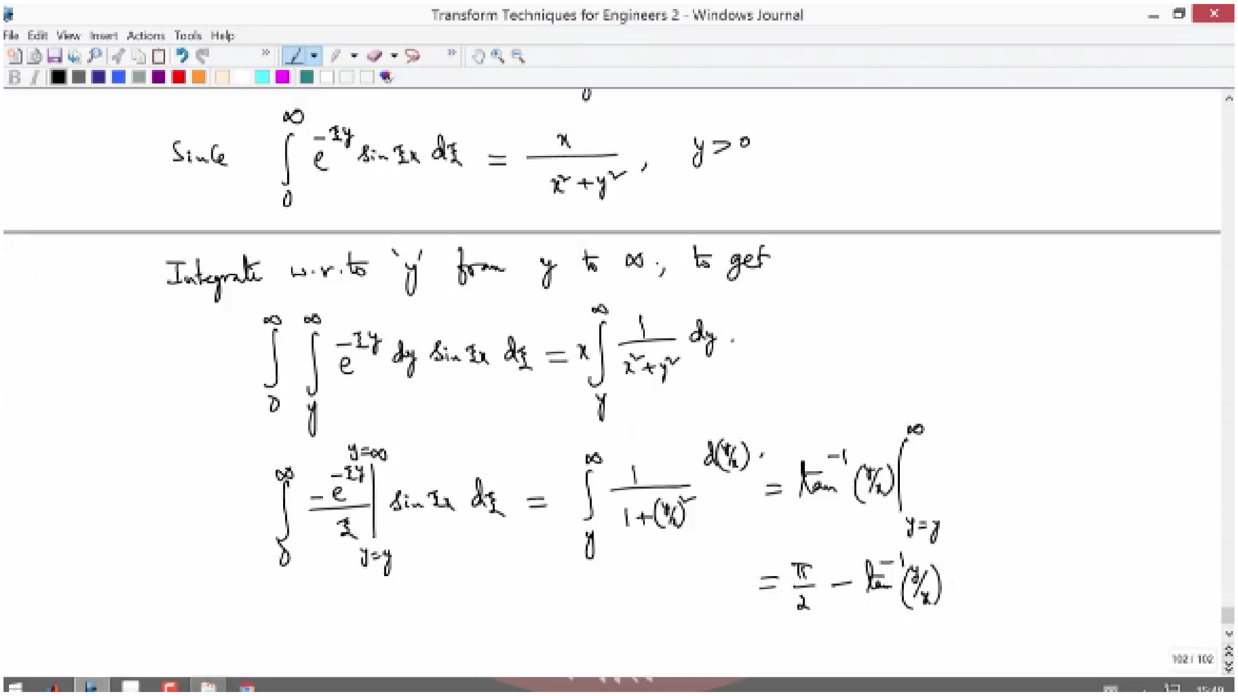
Another condition is as Y goes to infinity, as Y to infinity this Fourier transform also should go to 0, okay, U(x,y) goes to infinity, goes to 0 as X goes to, as Y goes to infinity, so if this is the case if you look at this if you take the Fourier transform for this FS(u) of xi,y this also should go to 0 as Y goes to infinity, okay, so because of this I can see that this C1 has to be 0, okay, so that means FS(u)(xi,y) is actually equal to C2 times E power –xi, y + root 2/pi C/xi, this is what is my solution.



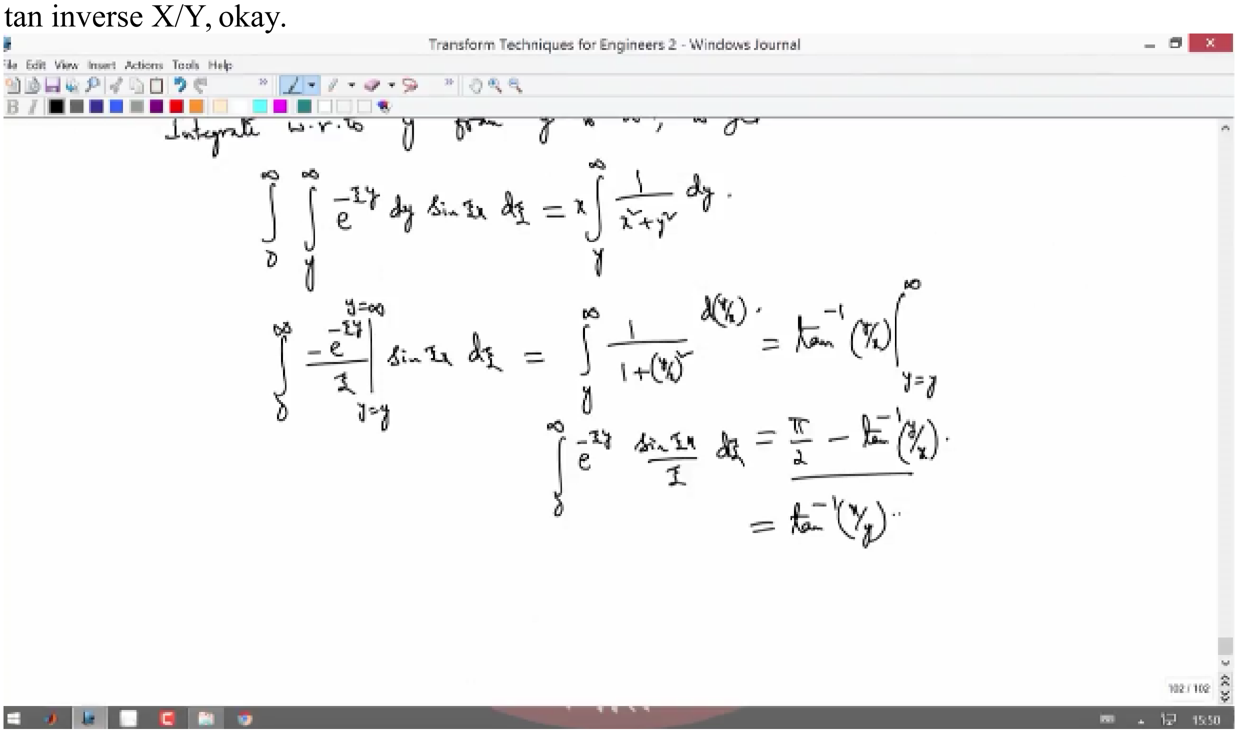
So now if you since the other boundary condition you have this, if you apply here if you apply the boundary condition here the left hand side that is 0, right hand side is C2, Y = 0 this is 1 + root 2/pi C/xi so that makes it C is minus of that, C2 is minus of that, that makes it FS(u)(xi,y) is root 2/pi C/xi is 1 - E power –xi, y okay, so this is what it is the case, so you take the inverse transform, inversion gives finally U(x,y) = 2/pi and C comes out integral 0 to infinity, 1 - E power –xi,y divided by xi sine transform we have used, sine xi X D xi, this is what is true so we have because you can split this into 2 integral, sine xi X/X positive and xi positive, so because X positive you have sine xi X/xi D xi that is going to be pi/2, so you have C, first one is C, the other integral is 2C/pi integral 0 to infinity, E power –xi, y sine xi X/xi D xi, okay, this is what exactly U(x,y) as xi positive and X positive.



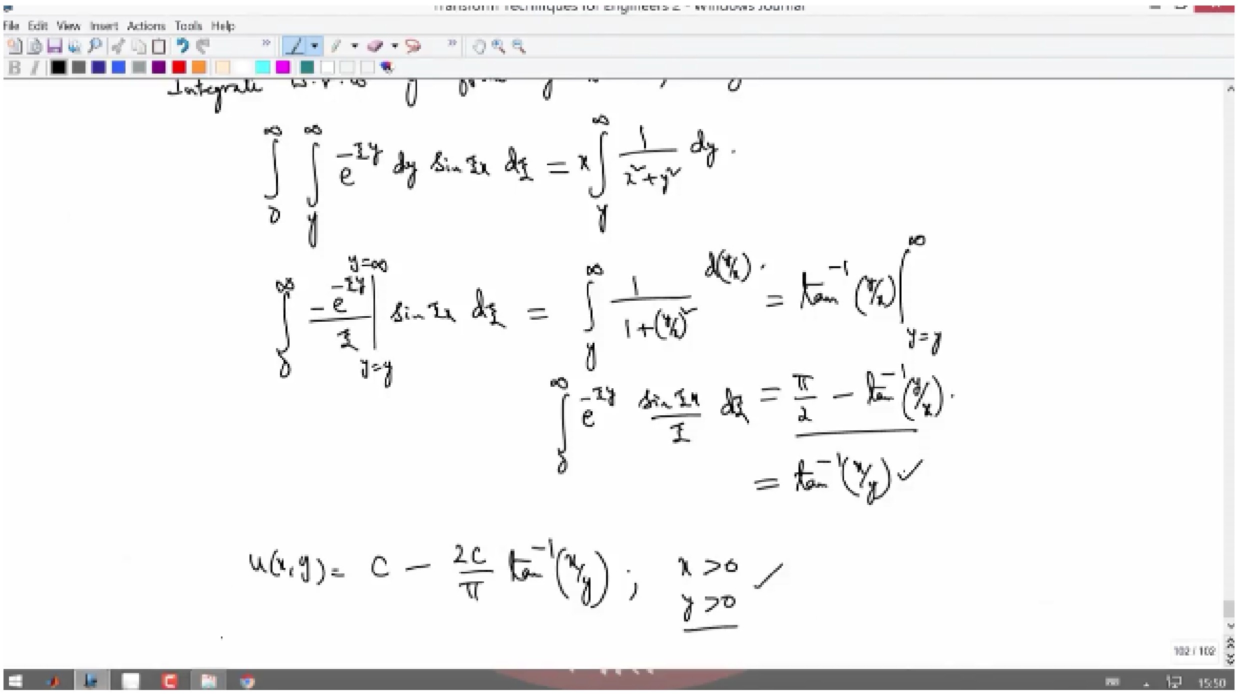
So what happens to this, how do I evaluate this integral? So you can do some simple technique, so let me start with this integral, so let me if I start this simply if I start, since I know since 0 to infinity E power –xi, y times sine xi X D xi, what is this one? This is actually equal to X divided by xi square + Y square, X divided by X square + Y square that's what you should have, okay, since this is the case, since this integral we can simply by integration by parts you can find this because of this what I do for this I'll try to integrate with respect to Y, I integrate with respect to Y, so integrate this because Y is also positive, integrate with respect to Y from Y to infinity, let me apply to get what you get is integral 0 to infinity, integral Y to infinity, E power –xi, Y DY sine xi X D xi = X comes out, Y to infinity, 1/X square + Y square DY, so what is this one? This is actually equal to integral 0 to infinity, if you do this divided by -E power –xi,y/xi sine X okay, and if you apply this to Y = Y to infinity, Y equal to infinity, D xi equal to X, X if you cancel here so Y to infinity X you take it out or you have 1/x and 1+Y/X whole square, so DY/X, so that we can 1/X we can put it inside, that is actually equal to tan inverse Y/X. Now we put Y = Y to infinity, so if you do that tan inverse infinity is pi/2 tan inverse infinity - tan inverse 1 that is tan inverse Y = Y, so you have Y/X, okay.



And the left-hand side it becomes 0 to infinity, and this is infinity it is 0, and minus minus plus this is going to be E power –xi, Y sine xi X/xi D xi, because this is also you can rewrite this as tan inverse X/Y, okay.



So this is from trigonometric inequalities you can get it, so finally you can easily see that this solution is U(x,y) = C-, if you do this one, this integral is 2C/pi times tan inverse X/Y, so this is what it happens for X positive, Y positive, so this is your solution, this is the required solution



for your problem, okay, so this is how you can apply your Fourier cosine or Fourier sine transform for any of these problems either steady state, heat equation, or one-dimensional heat equation, this kind of problems in a semi-infinite domains or when you see some one way, one of the domains X domain or Y domain when it is a semi-infinite domain that is 0 to infinity you can apply Fourier sine transform or cosine transform and solve the boundary value problems. So with this we end the applications of Fourier transforms, and so you can also, I'll give you an assignment some of the problems similar to this what I have solved. In the next video, we will introduce a Laplace transform from the Fourier integral theorem, start with the Fourier integral theorem so that is valid for an absolutely integrable function, and from which we define what is, we pick up from which we pick up what is called the functions that are exponential order and then we derive, we just modify this Fourier integral theorem and what we get up, end up is we're getting some kind of form that defines Fourier Laplace transform and it's inverse transform for and an exponential order functions, okay, this is what we will see in the next video. Thank you very much

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