Advanced Complex Analysis-Part1: Zeros of Analytic Functions, Analytic Continuation, Monodromy, Hyperbolix Geometry and the Riemann Mapping Theorem

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Lecture-12

Proof of the Implicit Function Theorem: The Integral Formula for & Analytically of the Explicit Function

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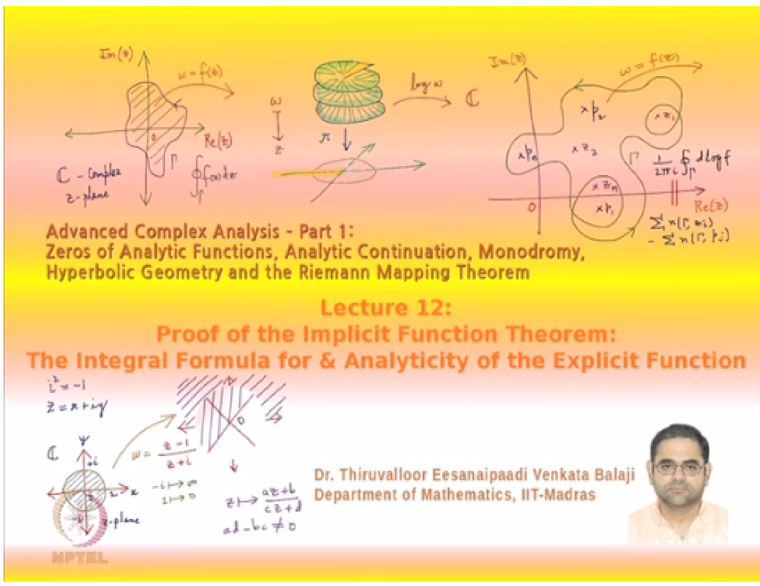

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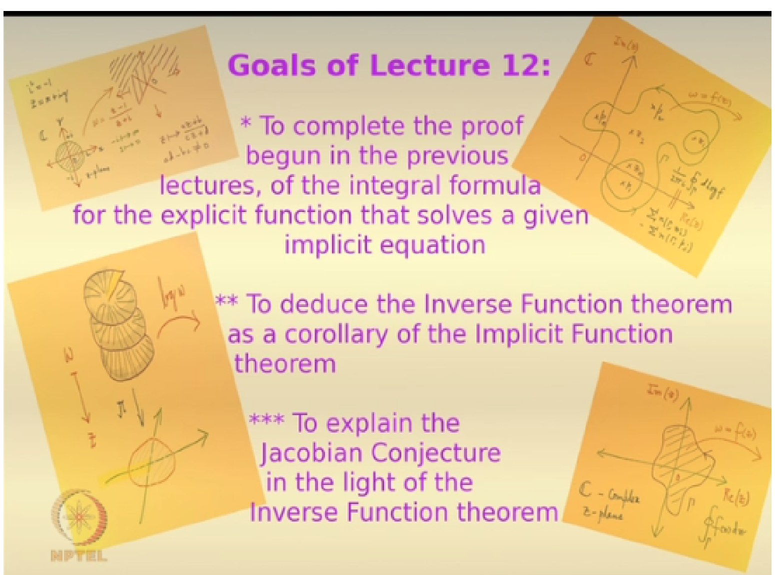

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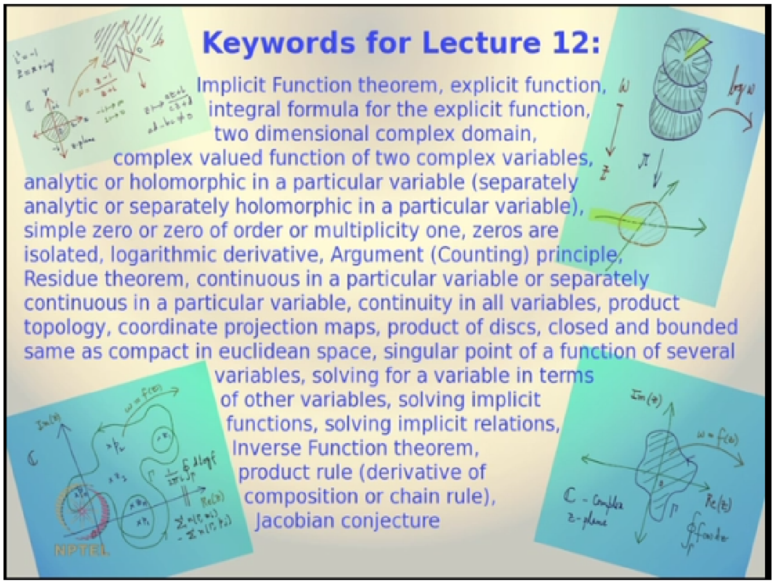

Ok so, this is the continuation of previous lecture, so let me recall what are the functions are.

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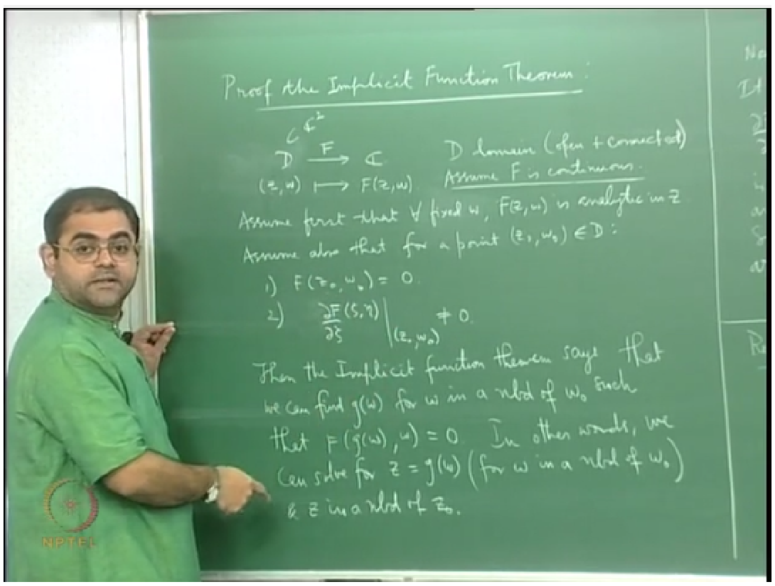

We have a domain in C2 the power of c2 itself, it is an open connect the substrate of C2 and we have a function F a complex valid function of 2 cos variance, refresh variable B denoted by z, second variable denoted by w and we assume that the function is continuous we assume that there is the point in domain where the function vanishes and the first derivative with respect to the first variable does not vanish and of course we assume that the function for each fixed value of the second variable in the domain .

The function is an analytic function of the first variable okay, see if this domain really confuses you for some reason you can simply assume that this domain is all of the complex the product of you know 2 copies of the complex type okay you can assume D=C2 if you think that it makes life more easier to understand okay.

So, here is the implicit function theorem which says that if the vanishes at the point at the first variable the partial derivative with respect to the first variable does not vanish okay which means by that I mean you considerate as a function the first variable finishing the second variable and then you substitute that point you take derivative with respect to the first variable and then substitute that point, that is the partial derivative with respect to the first variable at that point.

If that is not sure if the implicit function theorem says that the you can solve for the first variable enters the second variable. So, in this case you can solve for Z in terms of w, so you can write the explicit function g, Z=g, g of w which will D an explicit equation for f of z, w=0. In other words in an neighborhood of w0 what you will get is a function g of w says that if you write z=g of w and substitute it in this equation you will get f of g of w, w =0.

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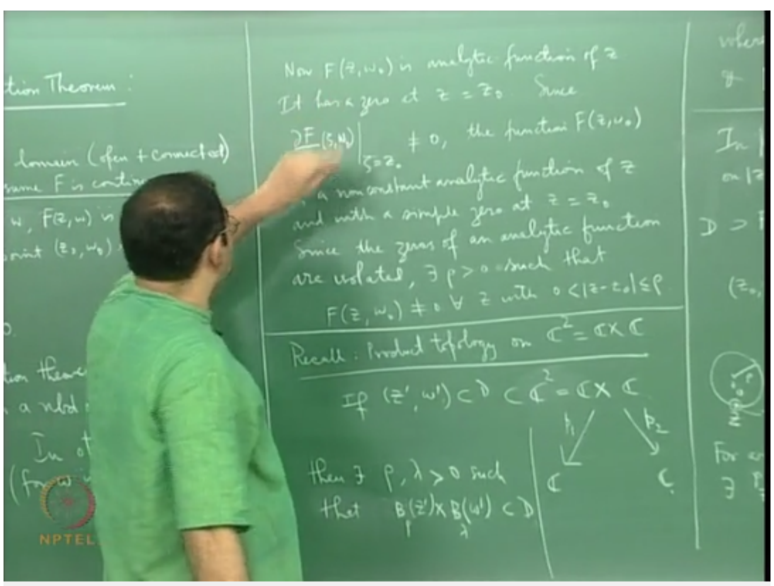

So, you are solve f of z, w =0 for z as g of w that is what the implicit function theorem says and now. So, what we do is as you may recall the the first type of this tells in that if with respect to the second variable free of portion w0, this is an analytic function of z and z = z0 this is 0 and because the first derivative does not vanish it is a simple 0 okay. And therefore by I

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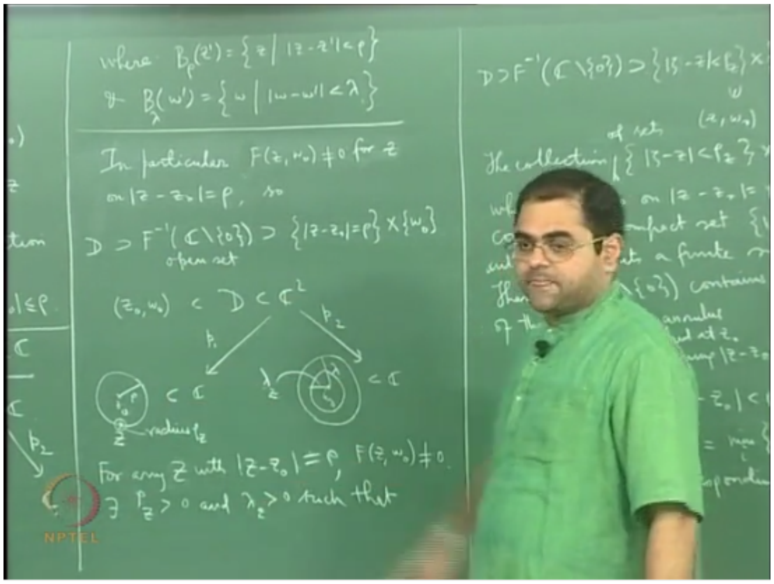

We proceeded after that to show that there is a there is an open annulus surrounding finite a circular finite radius centered at z0 an open disk surrounding w0 where f does not vanish okay.

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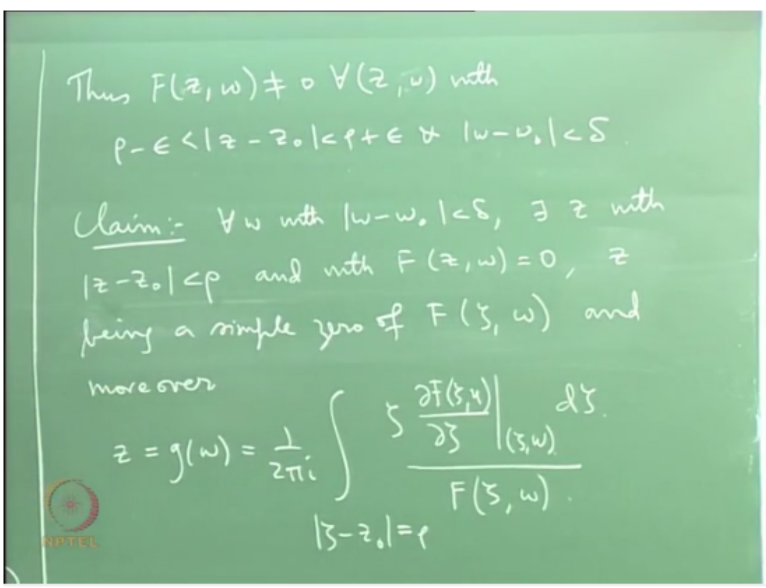

And for all for all the values of w in this disk surrounding w0, we said that we could solve for z as a function of w okay with z line inside a disc centered at z not radius rho okay. And of course this rho is chosen in such a way that it is the disc which isolates the 0 of the 0 z not of ffz, w w0 okay. So, this is the in this disk the function ffz, w0 which is a function of z has only one 0.

And that 0 is at the center that is how this rho is chosen and the rho divide the delta is chosen is already involved but that is what I explained in the previous lecture and now what I want to say is that for every w in this disk open disk centered a w0 radius delta if I define g of w to be this function okay. Then my claim is that and if I call that gfw z then F of z, that z, w is 0 okay.

In other words g of w solves the equation F of z, w=0 with z=g of w okay. So, the explicit the implicit equation F of z, w =0 is solved for the first variable z in terms of a second variable w if for all w in this disk alright that is what it says. Now I will have to prove I will have to prove this fact. So, the first thing I will tell you is that I want to tell you of something about this integral okay mind you for all points on this boundaries have 2.

F of the function doe not vanish that is because of this fact, that the set of points where the function does not vanish contain this contains an annular open annular region containing this circle cross this centered open disk centered at w0 okay. So, I am dividing by this term and this term does not vanish mind you here the first variable lies on the on this disk center of radius rho.

And the second variable is constraint by this equation it says that it lies to distance of delta from w0 okay. So, I am dividing by something that is not 0 alright. And mind you if you look at what is happening here the variable of integration is a called we tell as zeta because I do not want to use z right I am calling this variable of integration is zeta.

And zeta is the first variable the second variable is insert to w alright and if you take f of zeta, w you know that is it is already an analytic function of zeta. So, I take it is derivative with respect to zeta and then I substitute for second variable to be equal to w okay you can also like this tow F by tow zeta of tow by you can also write this is d by d zeta of F of zeta, w okay is the same thing.

Because you freezing the second variable to w and you are taking derivative with respect to the first way right, now the point I want to make z since and you know of course a derivative of an analytic function is also an analytic function okay. So, since F of z since F of zeta, w is an analytic function for fixed w it is derivative with respect to the first variable is also an analytic function for fixed values of the for the fixed value of w.

Therefore this is an analytic function of zeta this is also an analytic function of zeta this is says simply the which is the identity function and what we have the denominator is also an analytic function of zeta for this fixed w. so, this integrant is an analytic function of zeta and it it is and that denominator does not vanish, so this integral is very defined oaky.

And this integral is well defined and you are going to get a function of w because you are going to integrate with respect to see for a path integral to be well defined all you need is that the integrant is a continuous function of the path you do not need anything else okay. So, for this function for this integral to be well defined I just need the integrant to be a continuous function but in fact it is a quotient of analytic functions with that denominator analytic function not vanishing.

So, certain use of continuous function, so this integral exist and after you integrated out the variable zeta goes away and what is left out is only a function of w I am calling that as g of w, so this function is well defined alright. Now the claim is that this function solves f of z, w =0 okay that is settling. Now so and I have to also tell you that value of g of w if I call it as z then I have to tell you that the value z is taken by g at w only once okay, that is what I want to say.

That is I am just saying that z is you know it is a simple 0 of f of z, zeta, w for w in this neighborhood right alright. So, let us, so let me prove this , so what I am going to do is let me go to this side of the board and again the method of proof is essentially the same as we did for the inverse function theorem if you look at the proof of the inverse function of theorem you can see that and more or less following the same ideas.

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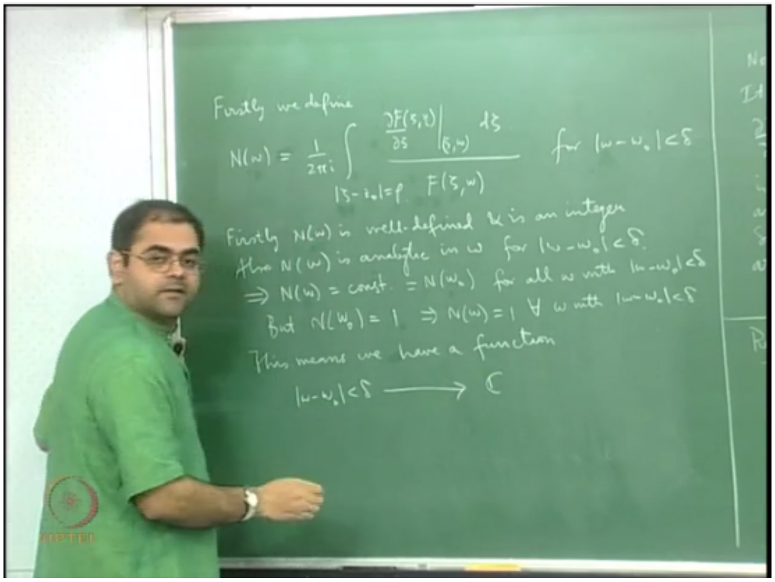

So, what I will do I will do the following this firstly we define 1/2pie i integral of modzeta-z0 z= rho and then I will write the same expression but I forget I won't write this zeta. So, will simply write do f zeta, theta by 2 zeta at zeta, w, p zeta by F of zeta, w okay. Suppose we define this for mod w-w0 mind this integral also make sense.

This integral make sense because again you know the integrant is the numerator is an analytic function it is just you take F and you freeze the second variable to be w then it is an analytic function of first variable you take it is derivative with respect to the first variable and you know the derivative for analytic for again an analytic function. Therefore this an analytic function and it is just the derivative of the function the denominator.

So, it is logarithmic derivative actually right and so in other words what this is just you are just taking the integral of the logarithmic derivative of F but then if you take the integral of a logarithmic derivative of F and divide by 2 pie i what you get is by the argument principle you are going to get the number of zeros-number of poles accounted with multiplicity. The first thing I need to show it is the following.

First of all M of w is well defined and is an integer okay it is well defined because the denominator function does not vanish on the boundary on this path on this circle where the variable of integration lines the denominator function does not vanish. Because that is how we chose this that is how you have chosen this rho and delta okay.

So, first of all this integral is well defined and since is the logarithmic derivative of the denominator it is going to be an integer it is a logarithmic derivative of the denominator divided by 2 pie square it is going to be an integer which is going to be number of zeros-number of pores counted with multiplicity. So, it is an integer valued function and it is an exercise that we have done before in proofs of several level theorems, it is easy check that M of w is actually an analytic function of w.

So, w in this disk okay, so let me write that also N of w is analytic in w for modw-w0 less than that okay. We have proved a similar statement earlier okay, so this is so you see you have an analytic function which is integer value and it is defined on this disk which is connected. Therefore it has to be constant okay, so this is our so idea which we have used earlier.

So, this implies N of w is = constant it is a constant integer and that constant will be equal to N of w0 for all w with mod w-w0 less than delta this is what you will get okay. And you see but what is N of w0 see n of w0 if you calculate N of w0 you are then looking at the number of zeros-number of poles of the function F of z, w0 in inside this disk but inside this it is an analytic and it has only one 0 the 0 is at the center at z0 and it is simple 0 okay.

So, N of w0 will be 1 okay, so what this will tell you is that N of w is 1 okay. So, **so** the model of the story is that you know for every w with in this disk I can find I find that the function F of zeta, w has only one single it has only one simple 0 at a point z which lies inside this disk that is what it is 6. Because what is N of w, N of w is N of w is 1 okay will tell you that you know it will have only one 0.

And that has to be a simple 0, if it is a 0 of a higher order then this number will go up alright. So, model of the story is that for every w in this disk I can find the z which is lying inside the disk bounded by this circle such that F of z, w is 0. So, what this tells you z I have a function from this disk to the z plane okay, so this means we have a function mod w-w0 less than delta from this set to C with w going to let me call this as g(w) and call this is g of w okay if you want let us call let us said okay .

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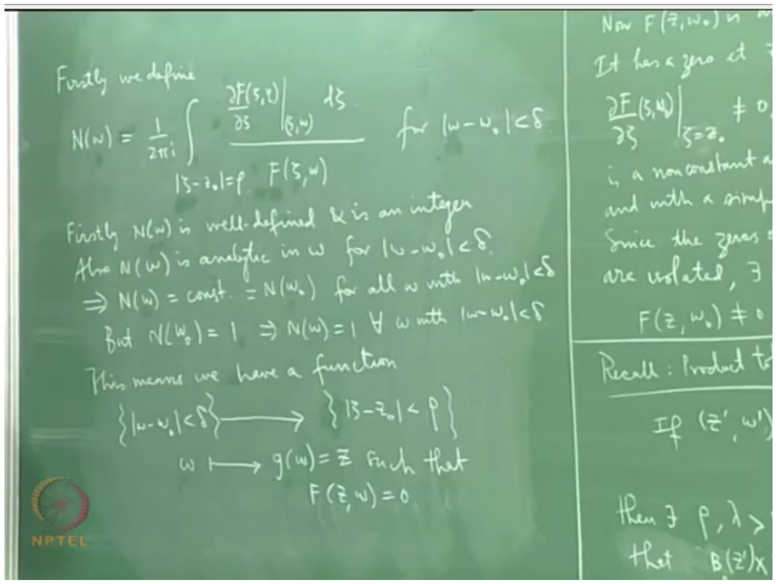

And in fact where it goes into is it is goes into this disk the set of all mod z-zeta0 z0 is less than rho it goes into this. So, basically I have a function like this, you know what this function is it sense to it is w the point z said that F of z, w0. Such that F of z, w=0 okay, this is the it sense for every it sense every w to the unique simple pole of the unique simple pole integrant which is essentially the unique simple 0 of F of F of zeta, w namely it is unique value z.

Such that F of zeta, w is a such that F of z, w is 0 okay, for every w I am getting a z right this is my function okay. And in fact there is there is also a another way of looking at it and which is as follows namely I have to tell you that I mean there is something here that I have to prove I will have to say that this see after I am getting a function here w going to some function of w which I am calling is that.

But I am calling that function is as g and I am written in that the formula for g is this okay. So, I will have to tell you that this that g is given by this formula that g is given by this formula it gives a value z satisfying F of z, w=0 okay. So, I will have to only show that this formula is correct alright, so how do I do that, that is again very simple application of residue theorem.

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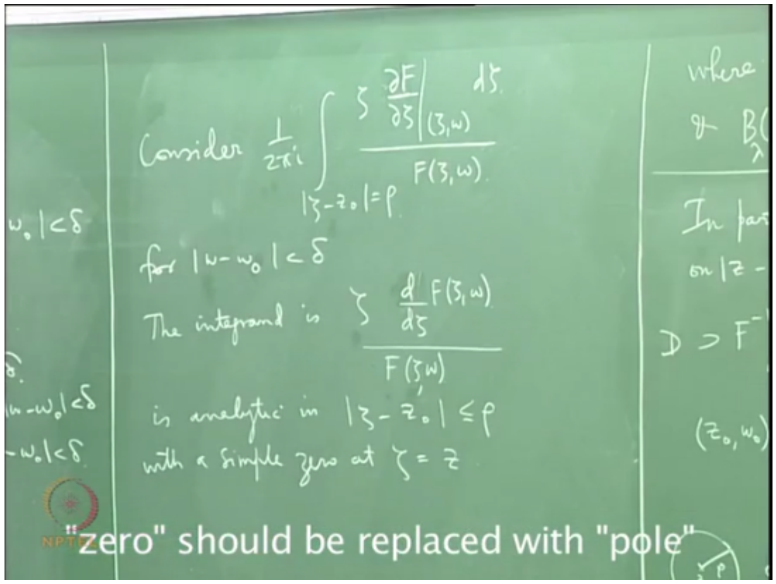

So, you see consider, now let us consider the that integral ½ pie i integral over mod zeta-z0=rho zeta tow zeta by tow sorry tow F by tow zeta at zeta, w d zeta by F of zeta, w for mod w-w0 less than delta consider this okay. Now they what is the integrant the integrant is zeta actually d by so you know if you want I can okay so let me write like this d by d zeta of F of zeta, w divided by F of zeta, w this is the integrant.

And consider the of course w is fixed okay this is defined for fixed w lying in this open disk. So, you consider this integrant as a function of zeta because the variable of integration is zeta okay if you look at this integrant you will see that it is an analytic function okay and it is analytic not only it is analytic in the on this disk and it is interior alright.

And you will see that it will have only one simple pole namely that will correspond to a simple zero of the denominator where that 0 is zeta=z which comes because of this okay. So, this if this is analytic in mod zeta-z0 less than or equal to rho with a simple 0 at zeta equal to z okay. There is a value z where F of z, w is 0 first that is that comes from here okay.

That is because of N of because this N of w is 1, so there is a z and there is only one z okay and the for that said F of z, w is 0 for the given w and the 0 is simple of order 1 okay, that is all completely you know buried in this statement that N of w is 1 right take that z, that z will be the only pole for this function and this is a simple pole. So, you know if you take ½ pie i integrate over a curve a function then by the residue theorem will tell you that you will get simply that residue for the function and that point.

And that at it is it will get actually generally we will get the sum of residues at various poles. So, in this case I will simply get this residue at the simple pole okay.

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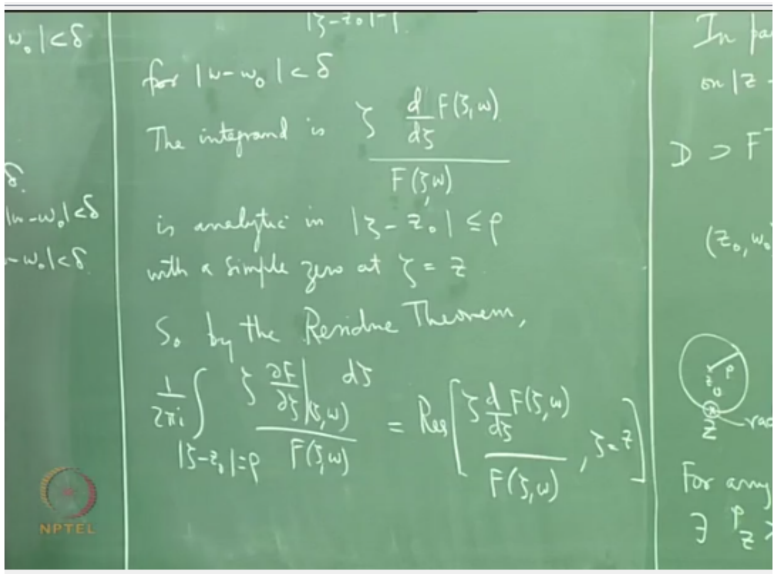

So, by the residue theorem what will happen is ½ pie i integral over mod zeta-z0=rho of zeta tow F by tow o zeta it zeta, w, d zeta by F of zeta, w is actually residue of this function zeta if you want d/d zeta F of zeta, w by f of zeta, w at zeta=z this is the residue and this residue at simple pole and you know there is a you know how to compute the residue at a simple pole, you will just have to multiplied by the variable-that pole. And then take the limit as the variable tends to that pole, that is how you find the residue at a simple pole.

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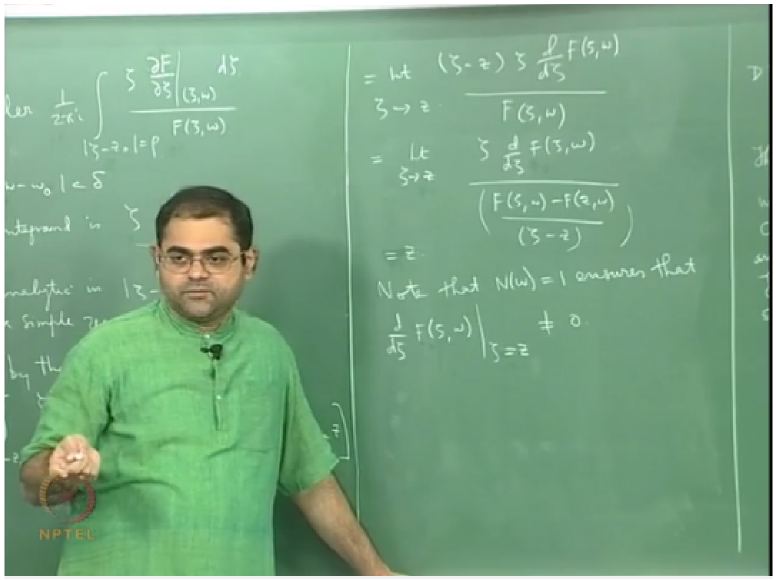

So, so that will be equal to limit zeta tends z zeta-z times the integrant which is get zeta d/d zeta F zeta, w by F zeta, w okay this is what I will get and well the and I claim that this is equal to z which will prove that this quantity = z and but z is g of w if you want and so it tells you that this is the formula of a g of w okay. So, it proves that formula that I have written here right.

So, why is this 2 of this is pretty easy because this is limit zeta tends to z of zeta d by d zeta of F of zeta, w by see the denominator I am writing this is F of zeta, w and F of z, w minding this is 0 F of z, w is 0 by zeta-z I am just pushing this zeta mind z to the denominator and you know now if I take the limit here I am going to get derivative I am going these I what I am going to get here is just this, these step going to cancel I am going to just get limit zeta.

So, z of zeta and that is going to be z okay that is proves the formula only thing that you will have to think about this how I can cancel this without ensuring that this derivative is non 0 okay. That this derivative is non 0 at z okay and that again follows from the fact that N of w is 1 okay, note that so let me write that down note that. So, let me write that down note that n of w equal to 1 ensures that d/d zeta F of zeta, w at zeta=z is not equal to 0.

This is non derivative is non 0 okay. Because it is a derivative at saying that the derivative is non 0 tells you that z is a 0 of order 1 of F of zeta, w which is what N of w=16 okay. If this derivative was 0 then N of w, the value of N of w would have sort up, it will not be 1 it will be some it will be an integer greater than 1 that is not the case okay. That is why I can cancel these 2 guides I will simply get limit zeta tells you said that zeta which is z okay.

So, the formula of our and what is this z, this z is the unique 0 of a F of zeta, w, so it is unique value lying in this disk in detail of this is of this disk where F of z, w is 0 okay. So, **so** that finishes the proof of this claim okay, now I come to the question more technical question has to when this function g that I will define okay which gives the first way it will be explicitly in terms of the variable, when is that function g also analytic okay.

So, the answer to that is it will these rho when the function capital F is also analytic in the second variable, so far what we have assumed is that the function capital F is continuous in both variables okay put together. It is continues a function of 2 variables and we have assumed that it is continuous in the first variable for every fixed value of the second variable okay. But what we have not assumed is that if you freeze the first variable.

Then consider as a function of second variable then it is analytic with respect to the second variable that we have not assumed. So, for we have assume only continuity with respect to both variables put together which will give you continuity with respect to each variable separately okay. So, what I am saying is that if capital F is also analytic in the second variable for every fixed value of the first variable. Then this g that you are defined it is actually an analytic function of w okay.

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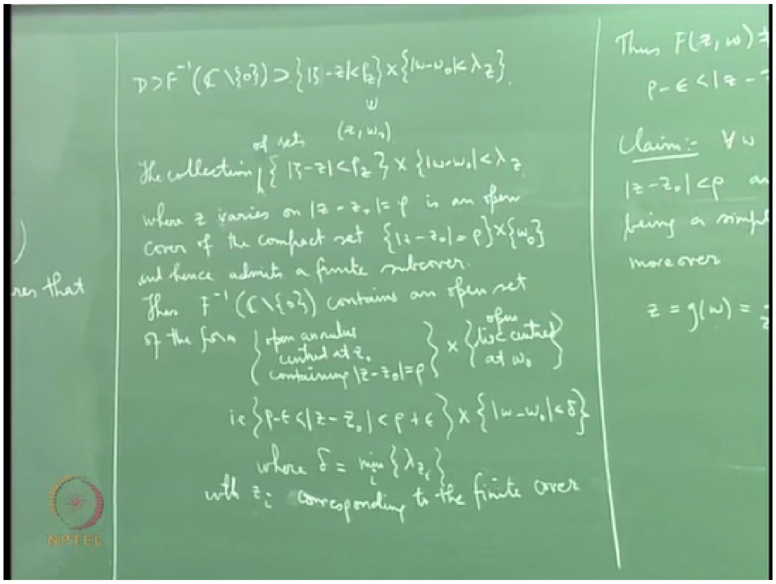

So, let me write that down so facts number 1 if F of z, w is analytic in w for every fixed z for every fixed z then g of w is analytic in w this is the fact okay.

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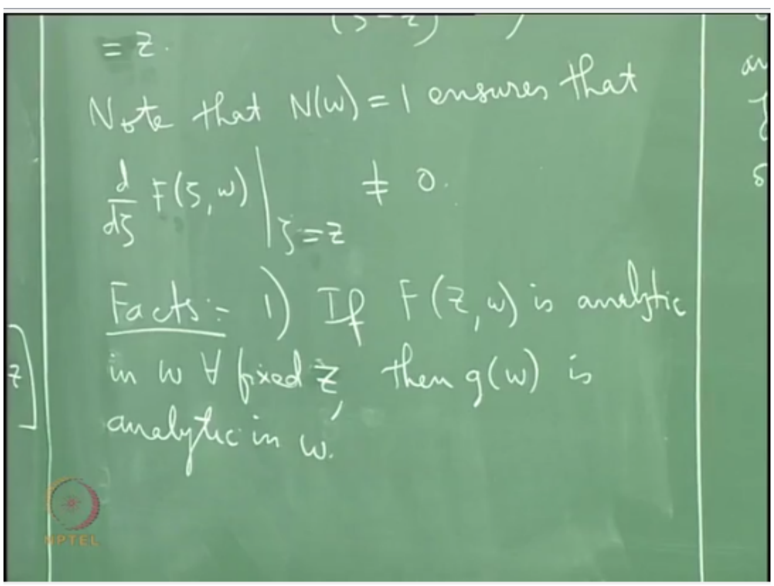

Proving this is again an exercise which involves techniques similar to once it we have used so far I will probably leave it as an exercise and in fact if a function is analytic then the natural c you have function which has which is given by a formula g is given by a formula and the moment you say it is analytic then you then it is natural to ask what is the formula for the derivative okay.

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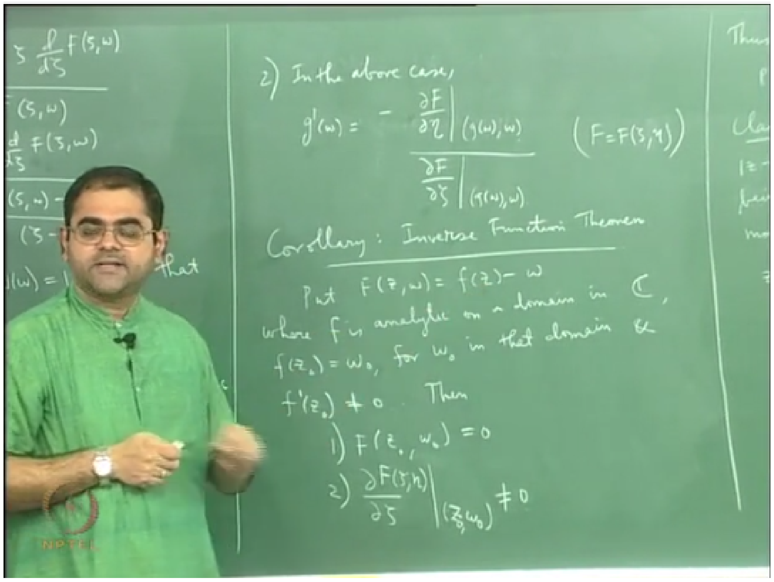

So, there is an answer to that and that is remark number 2 fact number 2 in the above case that is when F is a function analytic function in the second variable for every fixed value of the first variable and g is analytic the derivative d dash of w is actually given by – of tow F by tow second variable divided by evaluated at gw, w divided by tow F by tow first variable evaluated at gw, w where you know F is written the F of w.

This is partial derivative with a second variable and this is partial derivative with respect to the first variable and then you have evaluated this point z, w but now z is g of w okay. And this is everything, so this is the formula for g dash of w in terms of capital F and g of w okay and so I leave it as exercises for you to check that these formulas are these formulas hold but the point I want to say is is finishes and the statement of proof of the implicit function theorem okay.

But the what I want to next say is I want to say why this implies in inverse function theorem. So, you know as a corollary you have the inverse function theorem what you do is put you put F of z, w= F of z-w okay, where you know F is analytic on a domain in C okay. And F of z0=w0 for w0 in that domain and F dash of z0 is not equal to 0 okay.

So, the inverse function theorem says that you know you take an analytic function defined analytic function now of 1 complex variable assume that the function takes assume that the derivative of function is not 0 at a point then you know what it says in sufficiently small neighborhood of the point the function is 1 to 1 and you can write out an inverse function okay.

And so, you know if you put capital F of z, w to be small F of z-w then you can see that capital F of z0, w0 is 0 okay. Then number 1 you will see that capital F of z0, w0 is 0 because it will be F of z0-w0 which is 0 and second thing is if you take the partial derivative of this with respect to the first variable okay, then you will get F dash of z okay and if you substitute z0 you will get F dash of z0 and that which is not safe.

So, tow F by tow zeta of zeta F of zeta, theta at z0, w0 is not 0 which is you know which is the condition that we need to apply the implicit function theorem. So, now if you apply the implicit what is tell you is that I can I can write I can solve for z as g of w such that F of g of w capital F of g of w, w is 0 which means you are saying that small f of small g of w-small w is 0 which means you are saying small f of small g of w=small w, which means you are just saying g is inverse of f okay. So, let me write that down, so you get the inverse function theorem and .

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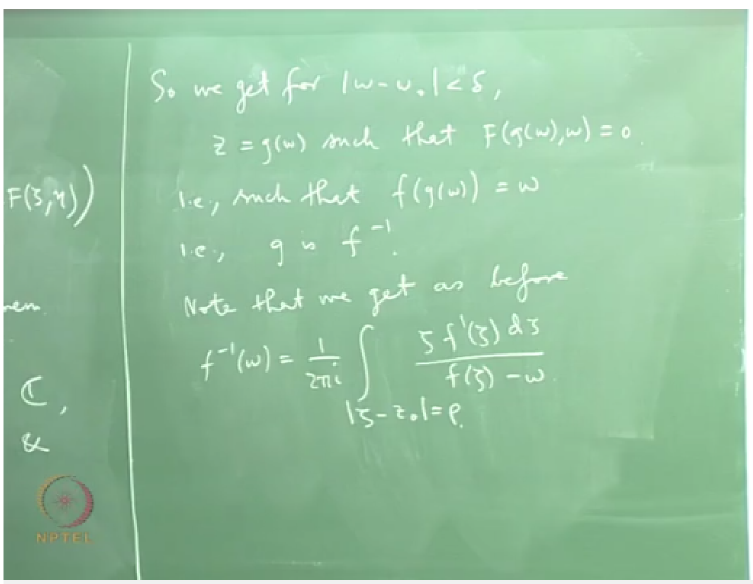

So, we get for mod w-w0 less than delta suitably small chosen suitably small you will get z=g of w such that F of g of w, w is 0 that is such that F of g of w=w that is it means that g is inverse g is f inverse. So, you get the inverse function, so g becomes f inverse okay. And in fact you can see that for fixed if I fix a value of z in F of z is a constant I will get constant-w and constant-w is certainly analytic function of w when I write F of z-w if I fix the w.

Then this is F of z- a constant and that is analytic because F of z is analytic, so it is analytic in the first variable if I fix the if I freeze the second variable. Similarly if I freeze the first variable z then F of z becomes a constant, so I get constant-w and constant-w is certainly analytic function of w the derivative of -1 alright. So, it is analytic in both variables right, so according to the implicit function theorem this g that you get will also be analytic, the g that you will get will also be analytic, it will be given by this formula.

And you will see that this is if you plug in capital F equal to small f-w in this formula you will get back the formula that we got in the inverse function theorem you will get back the same formula and you will also see that in get more if you plug if you use this formula note that we get also of course we get as before f inverse of w is ½ pie i integral over mod zeta-z0=rho, zeta f dash of zeta d zeta by f zeta-w okay.

This gives to this is the formula for the inverse function theorem it is get from here and and also and further you get this from this you get a very nice formula you get f inverse derivative with respect to omega with respect to w is you take look at this you take partial derivative with respect to the second variable. So, if I take partial derivative with respect to the second variable I will get -1 okay, so I will get -1 divided by.

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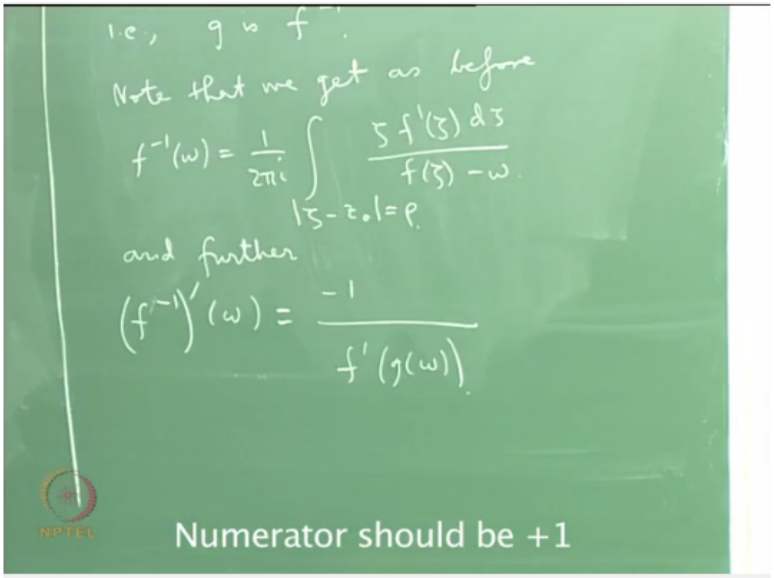

I take partial derivative with respect to the first variable if I take partial derivative with respect to the first variable I will get a f dash of z and then if I substitute for z g of w I will get f dash of g of w. so, what I get is I will get f dash of f w, so you get this formula okay. And this formula is not very surprising.

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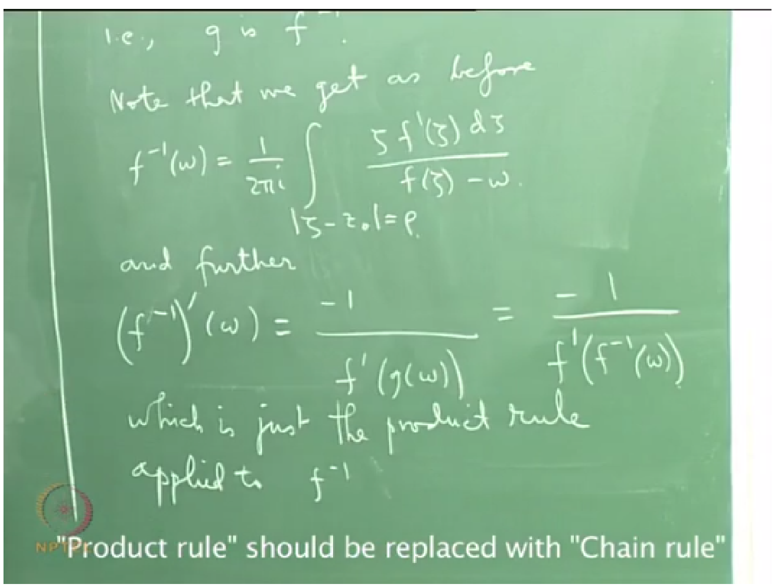

Because it actually which is actually which is just product the product rule applied to such con. So, here I have I have written g of w but I have to replace that g by f inverse. So, this is -1/f dash of f inverse of w and of course you know f inverse of w is z oh I am sorry okay yeah there is a – here I forgot.

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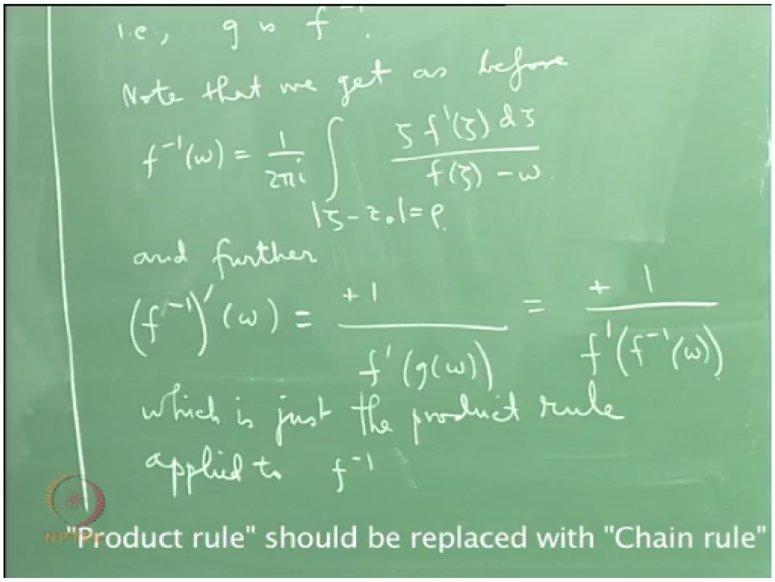

so I alright actually get a + yeah thanks for reminding that okay. There is already a – here but then if I take partial derivative with respect to the second variable I will get -1, so it will become + thanks for pointing that out. But yeah so essentially what you will get is you know it is f dash of z times g dash of f of z okay is equal to 1 okay. And that is just trying to it is just got by applying product rule to g circle f of z is identity.

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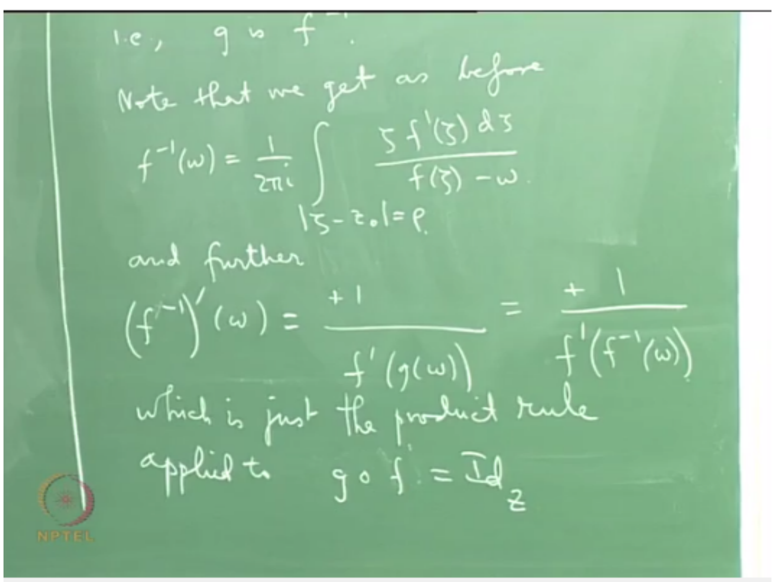

So, it is just g circle f = identity answer see g is f okay, so what does it say this says g of f of z is equal to that this is g of f of z = z differentiate it you will get g dash of f of z into f dash of z =1. So, it will tell you g dash of f of z it is 1 by f dash of z that is what you have got, so it is here to prove it applied write g is f okay, that is not for the subtraction.

So, okay, so now let me let me end this lecture by saying something interesting about an open problem which is an only open problem both in complex analysis and algebraic geometry.

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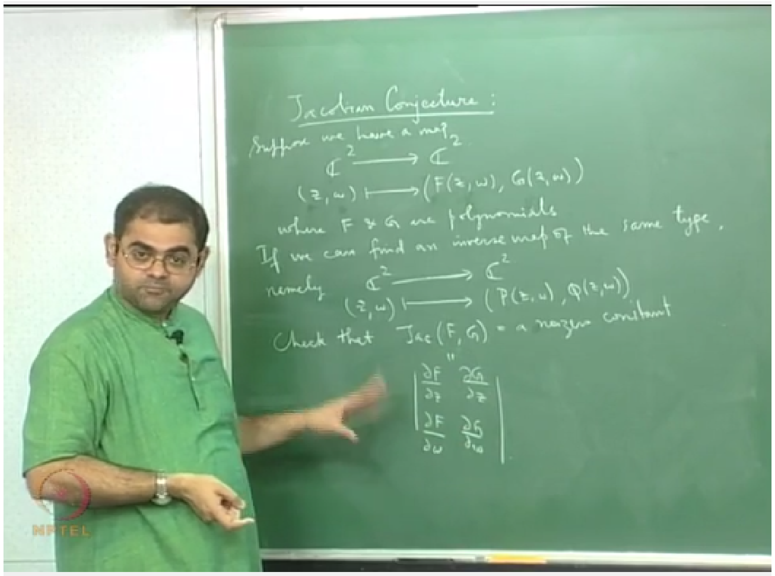

A very important jacobian conjuncture okay, so let me explain what this jacobian conjuncture is so what you do is you take c 2 to c2 okay map given by z, w going to F of z, w g of z, w okay. So, in general you know if you want a function that is going from c2 to c2 added take a point 2 complex coordinates and I have to produce 2 complex coordinates, so I have to give you 2 functions each function will vary will depend on both coordinates.

So, I call them as capital F and capital C okay, in general the best kind of function you will think of that you know both F and g are for example in analytic in both variables okay. And in particular what I want to do is I want to look at the case that F and g are actually f(j) are polynomial okay , suppose we have a map where capital F and g are polynomics.

So, F and g are polynomials in the variables z and w okay and if we can find an inverse map an inverse map of this same type namely c2 to c2 which is given by z, w going to say let me use something else and let me use some of that notation so I get I use p and q okay. So, assume that you have a map like this which is called a polynomial map because it is given by 2 polynomials.

And assume that of course you know if you give me a polynomial map I I do not know whether it is injective I do not know whether it is adjective but assume it is by adjective and assume that there is an inverse map this is also polynomial map okay assume that the inverse map this is also polynomial map which is given like this okay you check that jacobian of this pair F, G= a non 0 constant number non 0 constant complex number.

And that is hold also for the jacobian of p and q where of course Jacobian of F and G means you take determinant of you take partial derivative of F with respect to both variables tow F by tow z, tow F by tow w then you do it for the other function tow G by tow z, tow G by tow w, this is the Jacobian determinant mind you F and G are polynomial in 2 variables, if you take partial derivatives again you are going to get polynomial in 2 variables oaky.

And the fact is that this determinant therefore will be a polynomial in 2 variables but the fact is that if you have an inverse like this if this polynomial map has an inverse map is also polynomial map then I want you to check that this determinant which is a polynomial end and z and w is not actually at polynomial it is a constant and it is a non 0 constant okay, this is probably just follows by using essentially a product through the kind of argument okay.

So, you should be able to show that this is very this can be shown trivially okay, it is a basic knowledge of calculus. Now jacobian conjuncture is actually the converse okay, a jacobian conjuncture is a converse which says that you know conversely if you give me a pair polynomials F and G with the property that the Jacobian is a non 0 constant then the result is that is an inverse map which is again given by a pair of polynomials okay.

There is an inverse map which is again given by a pair of polynomials that is a Jacobian conjuncture and the fact is the this jacobian conjuncture I have said it for 2 variables but you can said it for any N number of variables of course you can check that if I try to write this for 1 variable it is trivial to check that it is true okay. So, the statement is really serious from the 2 variable case onwards.

So, this is a jacobian conjuncture for 2 variables and for you can write that for N variables okay. So, but the fact is that it is still unsolved even for 2 variables and it is statement only involving polynomials which are so easily understood okay. So, it is a very deep question and it answers to answer this question people who have gone into algebraic geometry they have gone into complex algebraic geometry.

They have gone into lot of complex analysis but the question is still open so it is a kind of question that if you solve is some young person who is taking this course of lecture solves he will surely he or she will surely get a prize equivalent to the noble prize in mathematics. So, you will suddenly get a feels medal if you solve this okay see such a deep problem but it is so easily stated.

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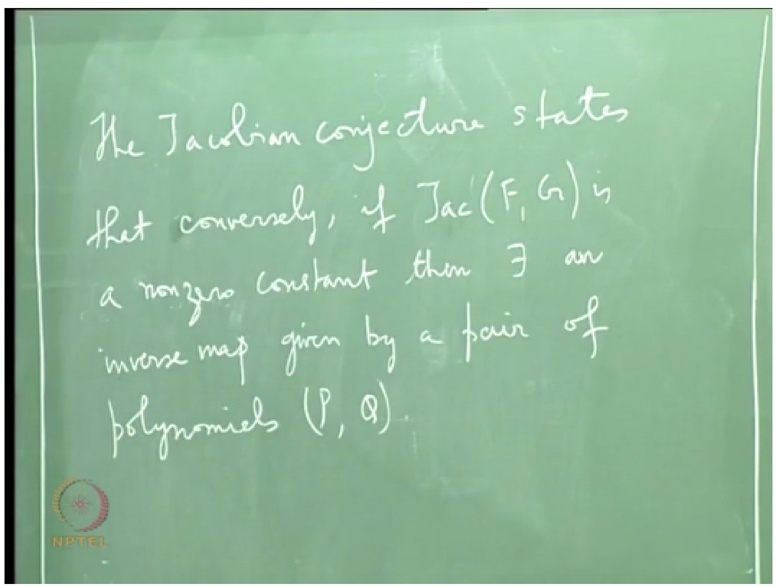

So, let me stated the Jacobean conjuncture states that conversely if Jacobean determinant of F, G is a non 0 constant complex number because non 0 constant complex number then where x is an inverse map given by a pair of polynomial P and Q okay. And I should say that this is not only for 2 variables it is also for N variables for N greater than 2 as for that is also the general case of jacobian conjuncture.

But the fact is even for 2 variables it is not solved okay and the reason why I am talking about this when I am talking about the implicit function theorem and the inverse function theorem is that you see what is the inverse function theorem that we have seen says it says that see whenever you know the derivative is not 0 then locally you can invert okay, whenever the derivative is non 0 at a point then you can find sufficiently small neighborhood of the point where the function is 1 to 1.

So, the you can write the inverse okay and this is the inverse function theorem for 1 variable but you can make the same statement for you can make the similar statement an inverse function theorem for several variables. So, you know for example if you want for 2 variables the inverse function theorem for 2 variables will be give me a function like this and assume that F and G are if you want analytic do not even assume they are polynomials.

Of course if they are polynomials they are analytic okay but assume F and G are more general analytic functions in both variables okay. Then calculate the jacobian okay, the inverse function theorem tells the 2 variable case that at every point where the jacobian does not vanish okay I can invert the function in a neighborhood okay. So, what I want to tell you is that from the view point of the inverse function theorem the jacobian non vanishing will tell you that locally this map can be inverted, locally I can invert this map.

And the inverse by the implicit if you use the inverse function theorem or the implicit function theorem locally the inverse functions that you get they will again the analytic functions okay. So, what analysis is that this jacobian non vanishing will allow you to invert the map locally and the inversion is achieved by analytic functions okay that is what analysis tell you but the conjuncture is very strong, the conjuncture says that you can find a global inverse.

And that global inverse is given by just 2 polynomials the 2 variable case and similarly if it is N variable case it tells you that that is a global inverse which is given by N polynomials okay. So, you know what you will have to prove is that somehow the local functions that you get which invert you to show that all the local functions you know the all can be chosen, so that they all glue together to give a global function.

And you have to show that global function is given by 2 polynomials the 2 variable case and it otherwise it that dustbin that is given by N polynomials in the N variable case. So, that is the gap between what we know in analysis and what the jacobian conjuncture demands and of course I must tell you that you know if you prove that there is a situation where you know the Jacobean is a non 0 constant.

But the map is not byjective then also you have given a counter example to the I mean if you are able to find you know if you can find a contradiction to this statement namely you find a counter example then also it give me a great result but no such counter example has been found okay. So, you are invited to try this or think about this.