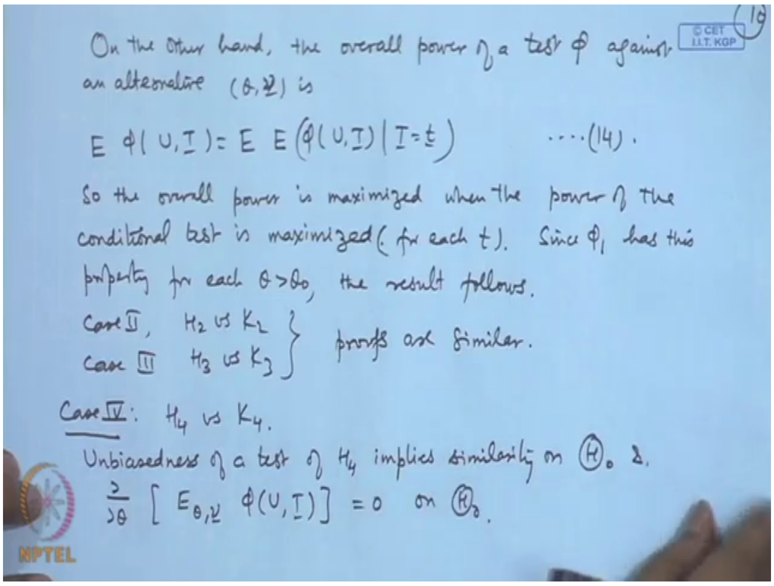
Statistical Inference
Prof. Somesh Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 44
UMP Unbiased Tests – IV

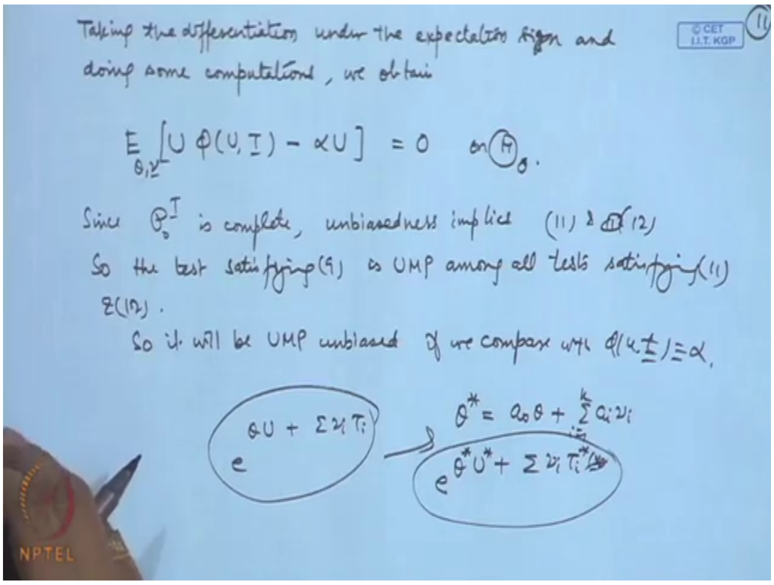
(Refer Slide Time: 00:17)



Against an alternative theta, nu is expectation phi U T that is equal to expectation of phi U T given T is equal to t expectation of this. Let me give the number here 13 and this will be then 14. So, the overall power is maximized when the power of the conditional test is maximized for each t. Now, phi 1 was already having this property; since phi 1 has this property for each theta greater than theta naught the result follows. I am not stating the case II and case III that is H 2 versus K 2 and H 3 versus K 3 so the proofs are similar.

Let me take case IV, that is H 4 versus K 4 here unbiasedness of a test of H 4 implies similarity on theta naught and del by del theta expectation theta, nu phi U, T that will be equal to 0 on theta naught. Now, we take this derivative inside the expectation sign.

(Refer Slide Time: 03:09)



Taking the differentiation under the expectation which will be permissible here because phi is a test function, so, it is a bounded between 0 and 1 here. So, and then what we do we carry out little bit of calculation doing some computation we obtain expectation U phi U, T minus alpha U is equal to 0. And, now this is U and this is coming because we are considering differentiation you are having the density function e to the power the theta into U. So, when you differentiate e to the power theta U with respect to theta you will get e to the power theta U into U and that is why this U has appeared here; this is on theta naught.

Now, since the family under theta naught this is complete we have already seen this thing unbiasedness implies the conditions 11 and 12 the conditions 11 and 12 which I stated for phi 4. So, these two conditions will follow because I can write expectation of expectation here. So, the test satisfying 9 is UMP among all tests satisfying 11 and 12. So, it will be UMP unbiased if we compare with phi U, T is equal to alpha.

A part which I have not covered here is the measurability of these functions. We should actually also show that phi 1, phi 2, phi 3 and phi 4, these are all jointly measurable functions. These are all functions of U and T. So, the joint measurability of this is also required. However, this proof I am skipping here and the readers can actually go through the detailed proof in the book of Lehmann and Romano.

We will consider further applications of this and when we are writing a distribution in the exponential family. So, for example, we are considering e to the power theta U plus sigma nu i T i. But one way consider different form of the parameters like we may consider reparameterization; we may consider say for example, theta star is equal to say a linear combination of theta and nu i.

So, what we can do, we can do little bit of readjustment of the coefficients the form of the distribution will still remain the same this will only be a exponential family in a slightly different form. We may actually write it as e to the power say theta star U star plus sigma nu i T i star. So, all these things will get little bit modified. However, it remains in the K plus 1 parameter exponential family.

What we have demonstrated here that the result for UMP and UMP unbiased tests which were stated for one parameter exponential family can be extended to the case of multiparameter exponential family; that means, we are still testing for one of the parameters. The we are having other parameters as a nuisance parameters the overall distribution is in the multiparameter exponential family. So, there is one exception here what is happening? The UMP test which was there in the one parameter exponential family now it is UMP unbiased and the test which were UMP unbiased they also remain UMP unbiased. So, in all the conditions we are actually getting UMP unbiased tests.

Now, in particular this helps us to resolve various problems like if we are dealing with the parameters of normal distributions, if we are dealing with the parameters when we are having say for example, if I am considering one Poisson's distribution or two Poisson's distributions, if we are considering binomial distribution, two binomial distributions, if we are considering say beta distributions and many of these cases.

So, these are all covered under this that as long as we are dealing with the distributions or whatever joint distribution of the observations is given; as long as that is remaining in the multiparameter exponential family it will be following; that means, for testing the problems of the nature H 1, H 2, H 3 and H 4 as I have defined here for each of these cases we will have UMP unbiased test the form will be given as there.

In the next lecture, I will be giving full working out of these tests that is UMP unbiased test for some of these problems that I will be carrying out in the next lecture.