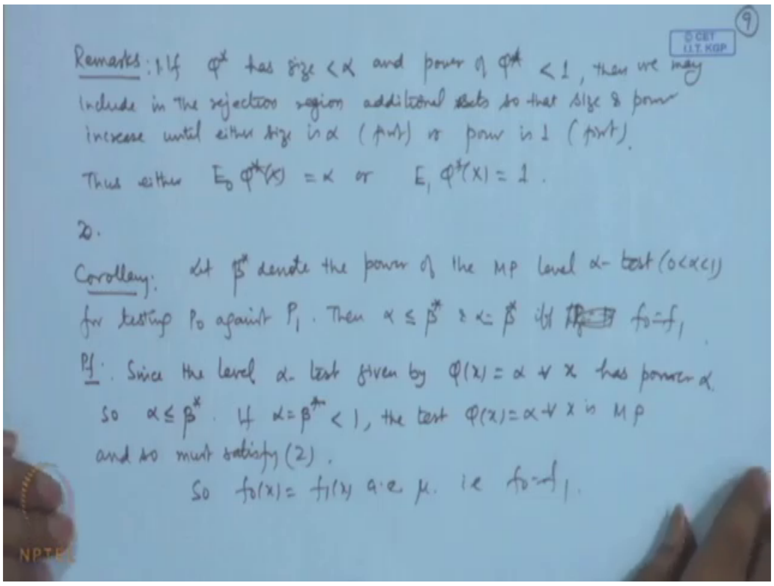
Statistical Inference
Prof. Somesh Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 34
Neyman Pearson Fundamental Lemma – II

(Refer Slide Time: 00:30)



Now, let us look at further remarks on this. If phi star has size less than alpha and power of phi star is say less than 1, then we may include in the rejection region. See power and the size both are related to the probability of the rejection region; so we may include additional points additional points or you can say additional space or additional set so that size and power increase until either size is alpha first or power is 1 first.

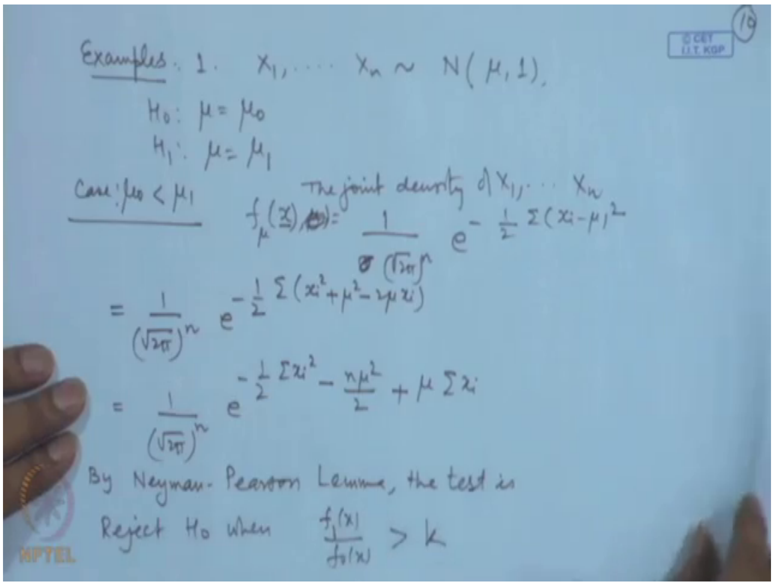
So, thus we will have either expectation of phi star equal to alpha or expectation of phi star X is equal to 1. So, another thing that we have noticed here that except the point f 1 x is equal to k f naught at other points the uniqueness of the in the definition of phi is there. And on the set where f 1 is equal to k times f naught, here there is a chance of shifting because of the value that we are having there; that is the difference that we are having there G C minus this point here because the C star may not be chosen uniquely and therefore, we can define arbitrarily. However, it means that it the size is still alpha.

So, therefore this does not make any difference here; let me give examples here and of course, we have the following corollary. Let beta denote the power of the beta star denote the power of the most powerful level alpha test for testing P naught against P 1 then alpha is less than or equal to beta and alpha is equal to beta if and only if P naught is equal to that is pi naught is equal to pi or we can say f naught is equal to f 1.

Since the level alpha test given by phi x is equal to alpha for all x this will have power alpha. So, alpha should be less than or equal to beta because this is one of the tests with power alpha and beta is the most powerful test power. If alpha is equal to beta star that is less than 1 then the test phi x is equal to alpha for all x is most powerful and so must satisfy 2 because of the necessity converse part of the Neyman Pearson lemma.

So, f naught will be equal to f 1 almost everywhere mu that is the two densities are same. Now, let me give applications of this Neyman Pearson lemma in deriving the tests for simple versus simple hypothesis case.

(Refer Slide Time: 05:31)



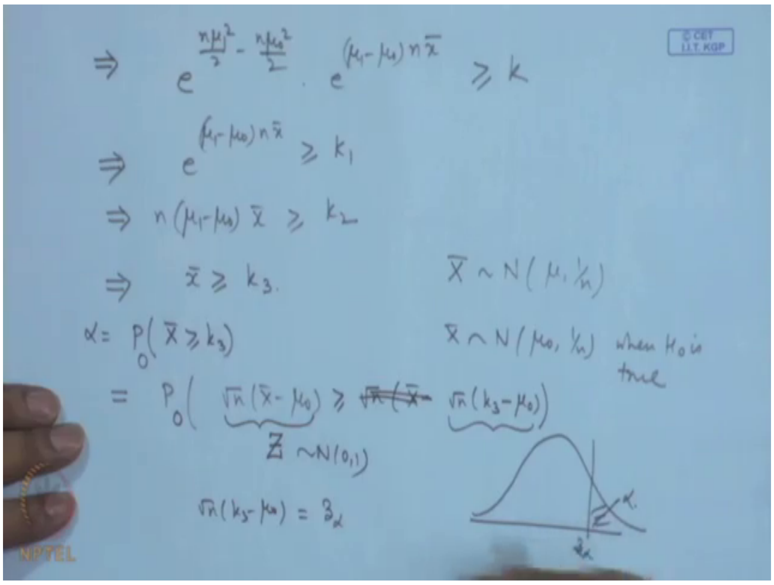
Let us start with say X 1, X 2, X n is a random sample from a normal mu 1 distribution. We are testing the hypothesis say mu is equal to mu naught against say mu is equal to mu 1. Let us take the case say mu naught is less than mu 1.

So, let us write down the joint distribution of x 1, x 2, x n the joint density of x 1, x 2, x n. So, that is at mu that we are calculating. So, that is equal to 1 by sigma is 1 root 2 pi to the power n e to the power minus 1 by 2 sigma x i minus mu square.

Now, this term we can simplify 1 by root 2 pi to the power n; e to the power minus 1 by 2 sigma x i square plus mu square minus 2 mu x i that is equal to 1 by root 2 pi to the power n e to the power minus 1 by 2 sigma x i square minus n mu square by 2 plus mu sigma x i.

So, if I write down by Neyman Pearson lemma the test is reject H naught when f 1 x by f naught x is greater than k. We may put greater than or equal to or greater it will not make any difference in this case because the distribution of x is continuous here, we are dealing with the normal distribution. So, the middle part of the phi function which is we given the Neyman Pearson lemma is not required in this case.

(Refer Slide Time: 08:07)



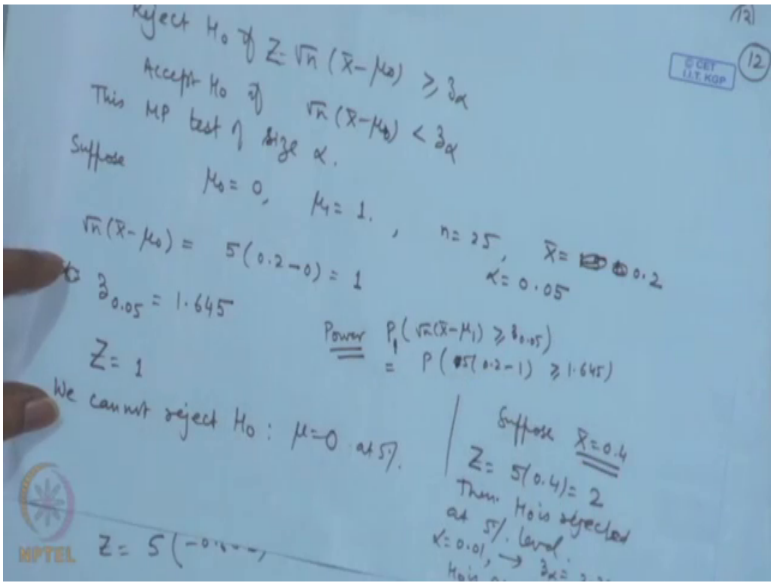
So, this condition is equivalent to now here we are having f 1 here mu 1 is coming and in f naught we have mu naught. So, when we write the ratio this terms gets cancelled out we are getting e to the power n mu 1 square by 2 minus n mu naught square by 2; e to the power mu 1 minus mu naught sigma x i we can write as n x bar greater than or equal to k.

Now, this is all constant mu 1 and mu naught or fixed constants. So, this is equivalent to e to the power mu 1 minus mu naught n x bar greater than or equal to some k 1. This is equivalent to saying if I take log on both the sides I get mu 1 minus mu naught n x bar greater than or equal to some k 2. Now, this is equivalent to now, I have here mu 1 minus mu naught positive. So, this is equivalent to saying x bar greater than or equal to k 3.

Now, the distribution of x bar is normal mu 1 by n. So, when we consider alpha that is the probability of rejecting H naught then here x bar follows normal mu naught 1 by n when H naught is true. So, this can be written as probability root n X bar minus mu naught greater than or equal to root n X bar minus sorry root n k 3 minus mu naught when mu is equal to mu naught. When mu is equal to mu naught this is following a standard normal distribution.

So, this value root n k 3 minus mu naught is nothing, but z alpha value where z alpha denotes the upper 100 alpha percent point on the standard normal distribution.

(Refer Slide Time: 10:44)

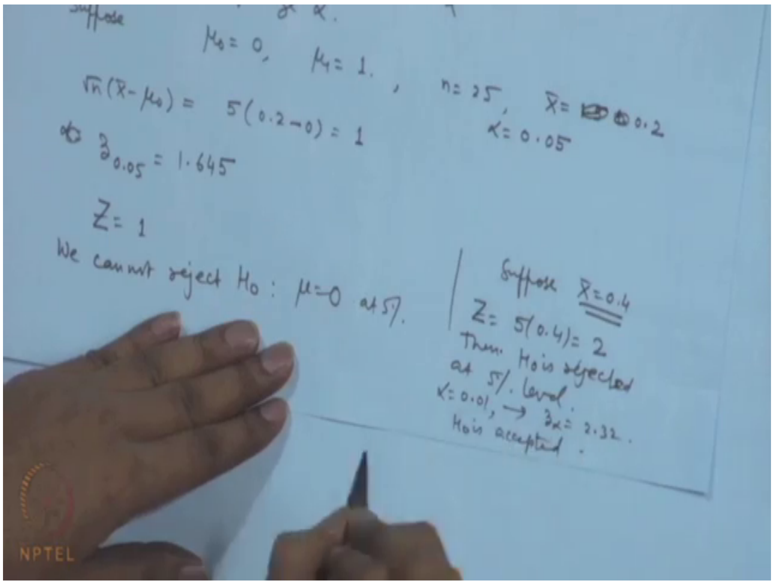


So, the test is reducing to so, the test is, reject H naught if root n X bar minus mu naught is greater than or equal to z alpha and accept H naught or you can say reject H 1 or do not do not reject H naught if root n x bar minus mu naught is less than z alpha. Inclusion of equality in this case or this case does not make any difference because of the continuity the probability of the equality will be actually 0.

So, this is the most powerful test of size alpha. Let us take an practical example here. Suppose, I take say mu naught is equal to 0 mu 1 is equal to say 1 and say n is equal to say 25. And in a given problem suppose my X bar is equal to 1.5 or say one point X bar is say equal to 0.2. In that case let us calculate this quantity root n X bar minus mu naught that is equal to 5.2 minus 0 that is equal to 1.

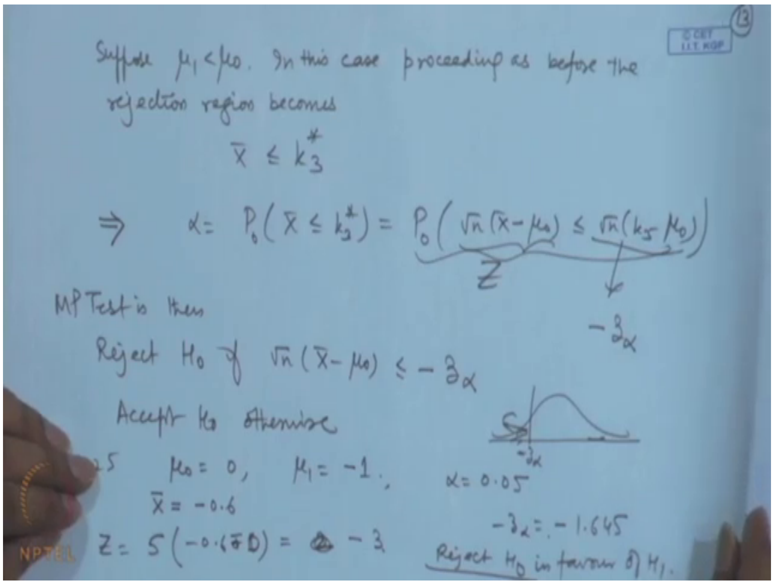
And, let us consider say alpha is equal to say 0.05 if I take alpha is equal to 0.05 then z of 0.05 is equal to 1.645. So, we are getting here this let me call this value as Z. We are getting Z as 1 and Z alpha value is 1.645. So, we cannot reject H naught that is mu is equal to 0, if x bar is equal to 0.2. On the other hand, suppose here I would have got say X bar is equal to 0.4 in that case Z value would be equal to 5 into 0.4 that is equal to 2. In that case this value will be higher, so, then H naught is rejected at 5 percent, but if I change the level of significance; it may still be possible to accept this.

(Refer Slide Time: 13:46)



For example, if I take say alpha is equal to say 0.01. If I take this then I will get z alpha is equal to 2.32 and here I will get H naught is accepted. Also notice here that I have considered mu 1 greater than mu naught.

(Refer Slide Time: 14:21)



Suppose, I consider mu 1 less than mu naught, suppose mu 1 is less than mu naught in that case from here if you consider the condition; it will change to x bar less than or equal to k 3 because if mu one minus mu naught is negative then the region will get reversed. In this case proceeding as before the rejection region becomes X bar less than or equal to k 3 let me call it k 3 star.

So, in that case if I consider alpha and this is then reduce to root n X bar minus mu naught less than or equal to root n k 3 minus mu naught. Now, if we are putting this is equal to alpha then this is Z here and this will become minus z alpha because this point here minus z alpha where the probability lower 100 alpha percent point here.

So, the test is then this is the most powerful test. Reject H naught if root n X bar minus mu naught is less than or equal to minus z alpha accept H naught or do not reject H naught otherwise. For example, here if I take say mu naught is equal to say 0 mu 1 is equal to say minus 1 and let us take say alpha is equal to once again 0.05.

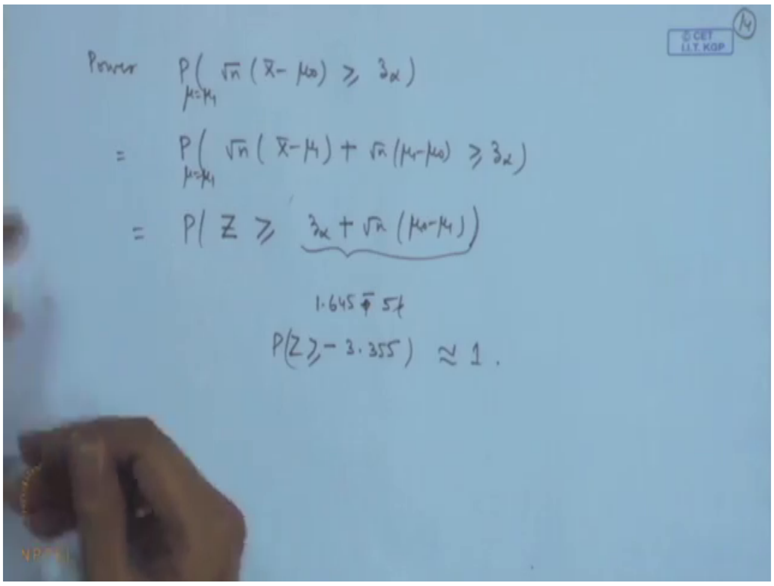
Suppose, the observed value of X bar turns out to be say point minus 0.6, then what will happen here this value? Let us call it Z and n is equal to 25. So, this is equal to 5, X bar is minus 0.6 plus 1 that is equal to 2 here.

So, once again you note sorry mu naught is 0. So, that is minus 3 and minus z alpha is equal to minus 1.645. So, here you are observing that this value is smaller than this. So, we will reject H naught; ao we will reject H naught in favor of H 1.

Now, the problem that I have discussed can be easily seen to have wider ramifications and of course, in both the cases we can calculate the power of the test also. What will be the power of the test? For example, in this case let us see power; what will be the power here? It is the probability of rejecting root n X bar minus mu 1 greater than or equal to z 0.05 because this point has already been decided here.

So, this is 1.645 that is probability of 25 sorry 5, X bar is 0.2 minus 1 greater than or equal to 1.645. Sorry, this is not the correct calculation let me do it again here.

(Refer Slide Time: 18:39)



Probability of root n X bar minus mu naught greater than or equal to z alpha under mu is equal to mu 1. So, that is equal to when mu is equal to mu 1 this does not have the standard normal distribution; rather we need shifting here X bar minus mu 1 plus root n mu 1 minus mu naught greater than or equal to z alpha for mu is equal to mu 1. That is equal to probability Z greater than or equal to z alpha plus root n mu naught minus mu 1.

So, this can be again evaluated for example, in this particular case we have taken z alpha is equal to 1.645 plus 5 times mu naught minus mu 1 is minus 1. So, this is minus 5 so that is equal to minus 3.355 that is probability Z greater than or equal to this that is nearly 1. So, the power of this test in this particular case is almost 1. So, it is good because in the normal distribution case this probability is almost 1.

In the next class, I will consider further applications of the Neyman Pearson lemma to derive the most powerful test in the simple versus simple hypothesis case. And then we will further extend these results to cover the case when the hypothesis may become composite and we will be discussing then certain results or certain conditions on the density functions which will give the results for those distributions so that we will be covering in the following lecture.