Introduction to Abstract and Linear Algebra
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Lecture – 01
Set Theory

Good everybody; so my name is Sourav Mukhopadhyay I am an associate professor head department of mathematics Indian Institute of Technology Kharagpur. So I welcome you all to this course Introduction to Abstract and Linear Algebra. So this is a very fundamental course on abstract and linear algebra; so first we start with the Set Theory. In this course so what is the set and then we talk about set operations, union, intersection, then will move to the set relations, mappings, so then we talk about different type of mappings then like bijective mapping, on to mapping, on to on mapping; so all this things we will cover under the set theory.

Then we move to the algebraic structure like group we define the group and before that we have to defined a binary operator, binary composition which is basically a mapping and then we move to the algebraic structure like group, ring, field and then finite fields, some of the application of the field finite field and then we move to the vector spaces that that is the part of the linear algebra.

So, vector space is then we talk about we define what you mean by vector spaces, then we talk about linear transformation about the vector spaces, then we introduce the metrics then how this metrics is related with vector spaces. So in the vector spaces we will talk about basis, we talk about dynamos of a vector space so all this basic details will cover under the linear algebra for; then we finally, move to the some of the advance topic on abstract and linear algebra, so there will discuss some of the application of both abstract and linear algebra ok.

So, again I welcome you all to the course introduction to abstract and linear algebra so this is 8 week course and each week at the end of each week there will be 5 models each model is around by 20 if 5 to 30 minutes. So at the end of each week there will be a online assignment and assignment will be MCQ, we have to submit the assignment and then after the 8 weeks the course is over there will be exam and again exam will be a online. So it will be again MCQ base so you have to give the online exam so and then you will get the; so exam is not mandatory for this course attending this course, but those who want to get the certificate they have to register for the exam and they have to sit for the exam and get the after passing the exam they I will get the certificate ok.

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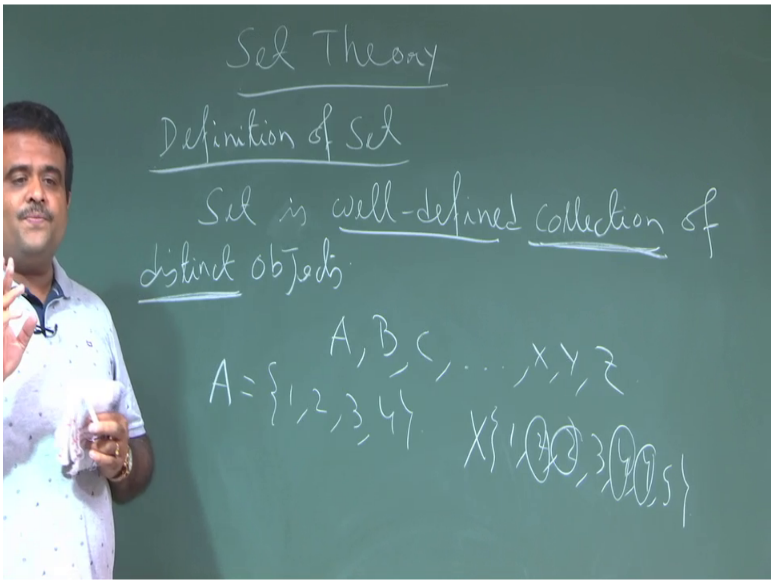

Let us start with the set theory so we start with the definition of the set; so what is a set? Set is basically a well defined collection of distinct object; set is a set is well defined collection of distinct objects ok. So this is the definition of the set; so there are few terms p q r are involve here like one term is it is basically a collection it is collection of objects is objects are also called elements. So these are the and there is another term well defined, so what do you mean by well defined will explain and there is a term distinct.

So, set should not contain equal elements, set is a distinct collection of object for example, if you take a set like consider represent by a symbolically we denote this set by capital what are like A B C D. So these are the capital letter alphabet are used to represent a I mean set like; if you define a set A which is the first 4 natural number like 1 2 3 4 this is a set; this is the collection we take a back and I put this number into that back that is the collection any collection, this is a collection so this collection is a set.

Now, we have to take this other properties like distinct; this is a distinct collection there is no reputation, but if you take this set say 1 2 2 3 4 4 5; if you take this collection this is not a set because this is not a distinct collection this 4 are repeating 2 times 2 are repeating 2 times; so this is not a distinct collection. So set has to be distinct collection there will be no reputation of the elements in the set so this is one property of this collection.

And another property is the well defined property well defined property means once we decline a set; that means, this is set what is the property of this set that it is the first 4 natural number natural number means 1 2 3 4 5 6 like this so it is first 4 natural number that is 1 2 3 4; now well defined means if you take any other object and these are the elements these are the elements these are called objects this is these are the objects or one object these are object or it is also called element, this is basically member of the set.

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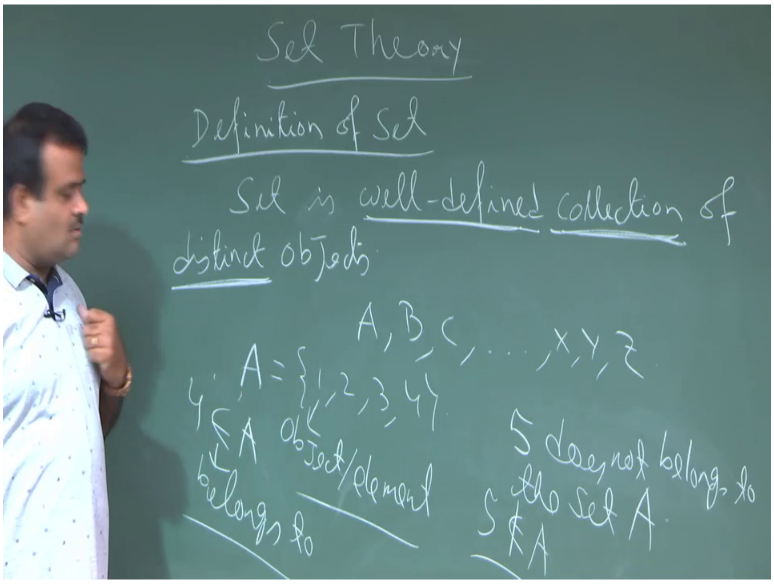

Now, if you take any other object say if you take 5 in natural number 5 then if we ask whether 5 is belongs to this set A or not the answer will be yes or no straight cut; it is not that may be may not be not like that is 5 belongs to A no; 5 does not belongs to the set A; this is the property I mean this is the this is the well defined property. If you take any other object from universe like me Sourav is Sourav belongs to the set no straight cut answer, but it is the natural number 4 belongs to the set yes; so 4 belongs to this set this symbol is used for belongs to this is the notation this is the notation we used for belongs to.

So, 4 belongs to this set, so this is the property and then 5, 5 does not belongs to this is the does not belongs to symbol 5 does not belongs to A, even me Sourav does not belongs to this set so this is called well defined property. If you take any object from the universe will define the universe.

Then we can say we can straight away able to say whether that object belongs to this set or not ok. So this collection this well defined collection is called a set now if you defined a set of all people belongs to this room my set is say S is a set this is set of all people all people in this room in this room ok.

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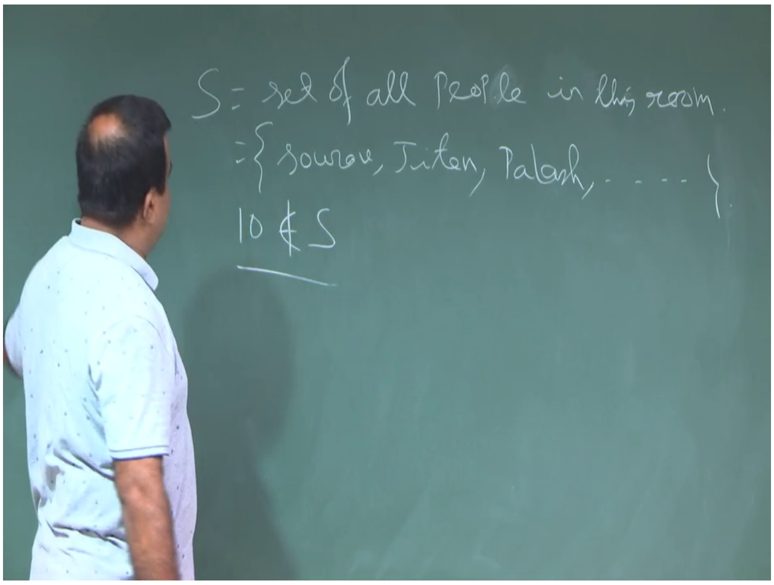

Now, if this is my set S set so the S may be I am Sourav then you are Jiten you are Palash and so on. So this is my set now what is my set my set is the set of all human being in this room. Now this is distinct I take this collection is a distinct collection I am not repeating Sourav twice or Jiten twice and this is well defined well defined means if I say if I take a person say Demol who is not in this room then we can say that Demol is does not belongs to this set, but if you take Jiten then you then we can say you Jiten belongs to this set or not; if you say natural number of 10 is this natural number 10 belongs to this set no natural number 10 does not belongs to this set.

So, this is the, this is called well defined property of a set ok. So this is the definition of a set now we talk about how we represent a set how to represent a set how we can represent a set.

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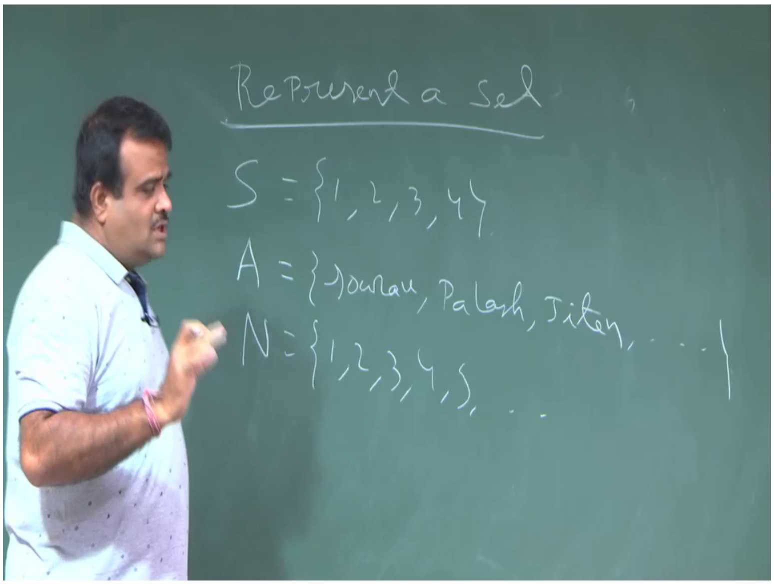

Say for example, if you have a set like this say every set has a property; say our set is say first 4 natural number that is a property so 1 2 3 4 we can explicit this is a explicit way to right a set. We write we mention all the elements of that set in this collection so that is 1 way to represent a set explicitly we write, but if we say that my set is all the human beings in this room.

So that is all the people here see there are say how many where around 30 40 people so you can just right the name of the people like this and we can explicitly write all the names of 40 people and this collection is a set. So this is one way to represent the set explicitly, but this way we have a issue with the size of this collection like if you have 100 people in this room then this set has to we have to write 100 names in this collection.

So, this is a this is if you have more if you have infinite number of like if you have a set of natural number set of natural number which is denoted by N this is say 1 2 3 4 5 so it is not possible to explicitly write all this elements so what we do we just write a short cut I mean we just write in a different way like this set.

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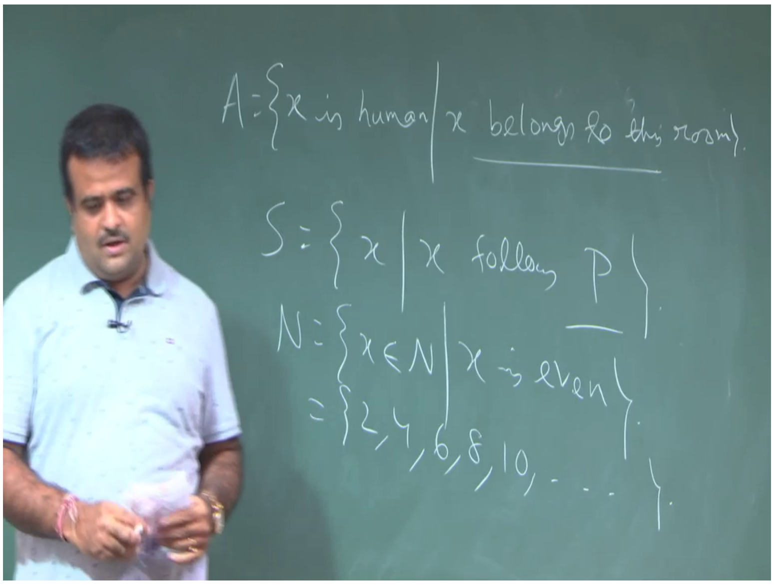

This is the x, x is a human being x is a human and then x satisfy a property what is property x is in this room x is in x belongs to belongs to this room ok. So this is the way we represent this set; so this is the property x not satisfy so; that means, any set we can write in this way x, x follows some properties x follows p; the property this is a property so for this set this is a property what is the property is the this person is belongs to this room that is a property ok.

Now if you take the set of integers; so set of integers so this is the N, which is the set of integers. Now if you take the set of even integer I not only integer natural number so this is x, x belongs to N and x is even. So this represents the set of all natural even natural number even natural number like 2 4 6 8 10 dot dot dot like this is the set of even natural number. So every set has a property we denote that set and followed by the property ok.

So, now we just talk about we just defined some set which are very standard set notation like set of has you said N is represent that set of natural number.

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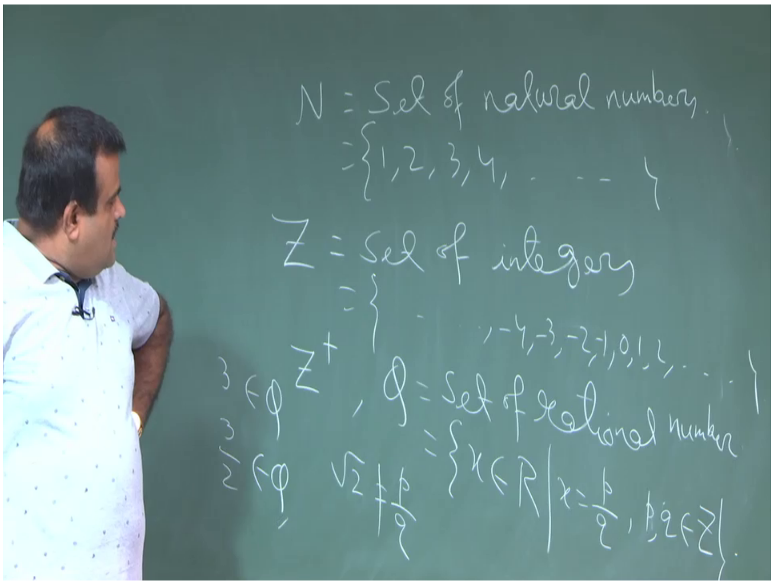

Natural numbers so N is basically starting from 1 2 3 4 dot dot dot then we represent Z Z is the set of all the integers these are standard notation we will used throughout our course; set of integers like its start its positive integer or negative integer also so we have starting from minus infinity then we have dot dot dot say minus 4 minus 3 minus 2 minus 1 0 1 2 3 4 like this ok.

So, this is the set of integers and then we will have the set of positive integer and then we have rational number Q; Q is the set of rational number. So what is the a rational number rational number is a number which can be represent has p by q form had p q are both integer. So it is basically a real number real number set is R such that x will be written in the form of p by q where p q are coming from integer set Z ok. So this is the property that p q there must exist p q such that so every every integer or every rational every natural number is a rational number, but not all like this is a ration 3 is a rational number 3 by 2 is a rational number.

So, what is the not rational number which is called irrational number we we know that root 2 is an irrational number; because for root 2 there is no such p q will exist root 2 cannot be written has p by q form. So this is with this, this is the notation standard notation and there is a set of real number which is R and the set of positive real number we denote by R plus.

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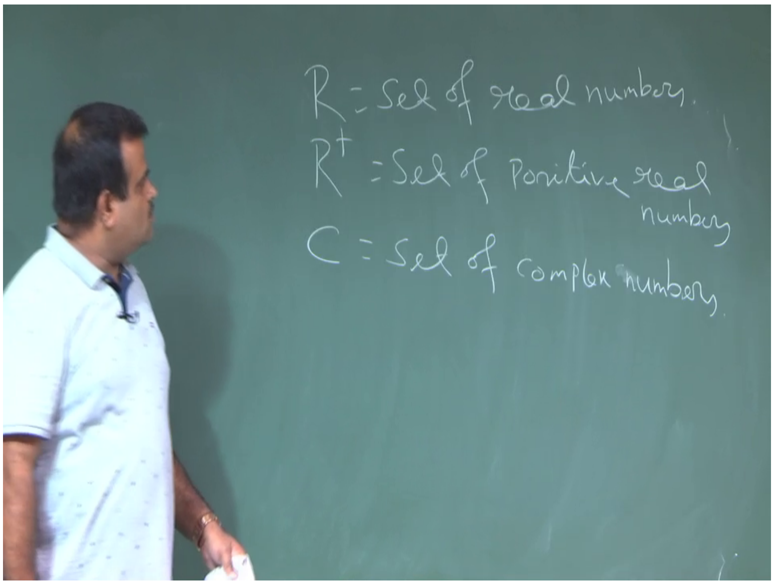

So these are the so R is the basically set of a real numbers and R plus is the set of positive real number positive real numbers and we can define another a set of complex number. So there will many others when we got we it will come will just defined at that time this is the set of complex number.

So, whenever the notion will talk we will discuss that time ok. Now we defined a set which are which does not contain any element is way convention to define such a set; so that is called empty set.

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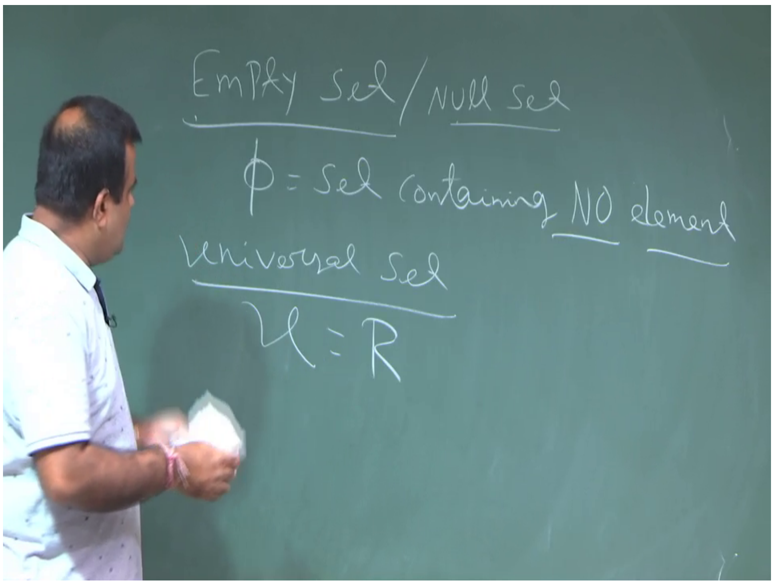

Empty set; which is basically a set having no element, so this is denoted by phi set containing no elements ok; so this is called empty set this is null, null or empty set null set sometime is called null set also ok.

Then we define another set which is universe set, universal set which is denoted by U. This is basically set of all universe like it is very relative things like if we say we can define our universe; suppose we are dealing with the numbers we are dealing with the numbers then we can set our universe to be R; we can set that then we can talk about we are in number system, real number system then our universe is R then we any set, any element, then any real number or any integer any natural number belongs to this set so this is the universe.

Now, if you in a complex plane then our universe is a complex number or if you are talking about the leaving elements; so our universe could be all the leaving elements in this world that is the universe so we can; so universe is very relative thing, so universe we can define depending on our applications ok. Universe is basically set of all sets like we will define the sub set so every set every set is the sub set of universe ok. Now, let us define the sub set which will be more clear ok.

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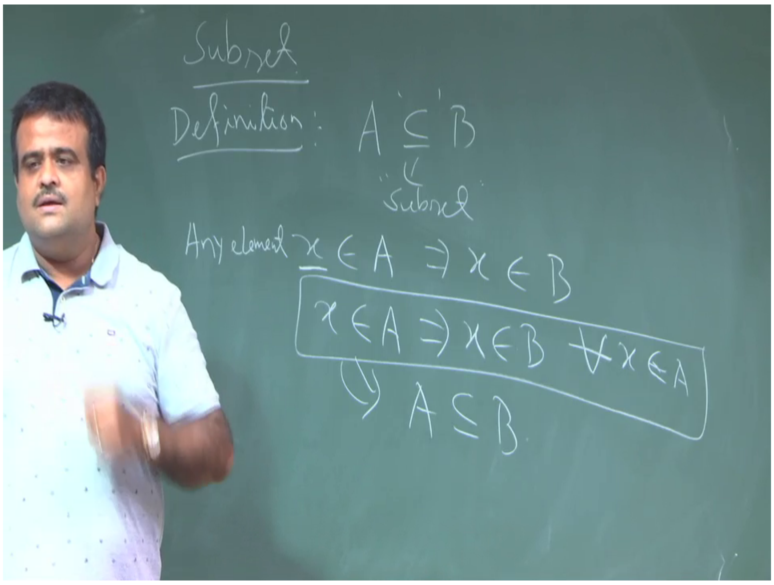

So, sub set; so what is the definition of subset? Suppose we have 2 set AB so we say A is a subset of B this is the notation to use subset notation. So we say A is a subset of B if and only if you take any element x from A this imply x will be in B so; that means, every element of A is an element of B; if that property satisfy then I called A is a subset of B we take any element so this is for this is any element any element any element x belongs to A imply this or in other word we can say an element x belongs to A if it is imply x belongs to B and this is if it is for all this is this is the symbol you use for all for all.

If this property satisfy then we called then we called A is a subset of B. So we take an element arbitrary element in A if you can so this is an element of B then that is sufficient to say is a sub set of B, any element. So every element has to every element of A must be a member of element of B then A is a subset of B. Now this symbol is includes so if we use only this like if we this is a proper sub set symbol.

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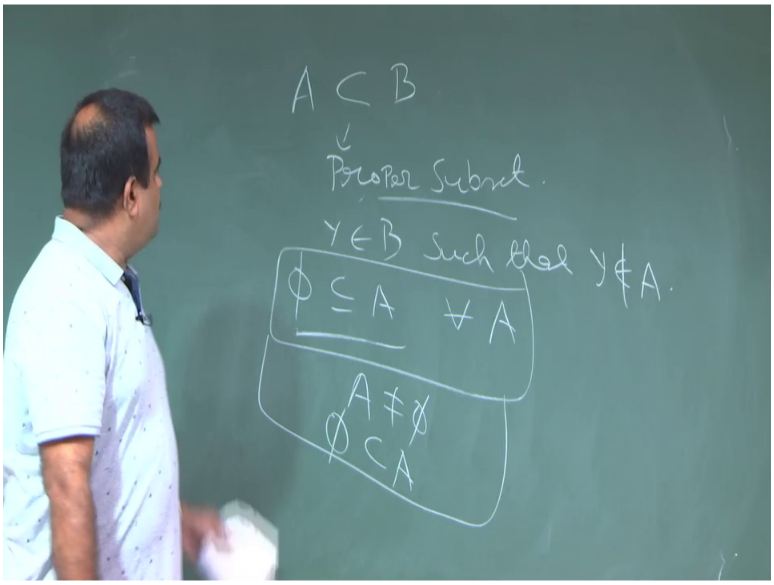

So, A is a proper subset of B; proper subset means so there is a element in B which is not in A so; that means, B is bigger than A in that sense so again bigger and smaller is relative to the cardinality of the set will come to that.

So, every proper this a proper subset proper subset means A is a subset of B, but if there is A element if there is A element there has to be at least one element y belongs to B such that y does not belongs to A ok. So then we call proper subset of B. Now we have some properties of this now every this empty set is a subset of every set; this is for all A so every set A this is, but this is the proper subset; if A is not empty then this is a proper subset ok.

And then if we have a A, you say which is subset of B then B is called super set of A like if A is a subset of B then this means A is a subset of B and in other word this is also called B is a super set of B is a super set of A.

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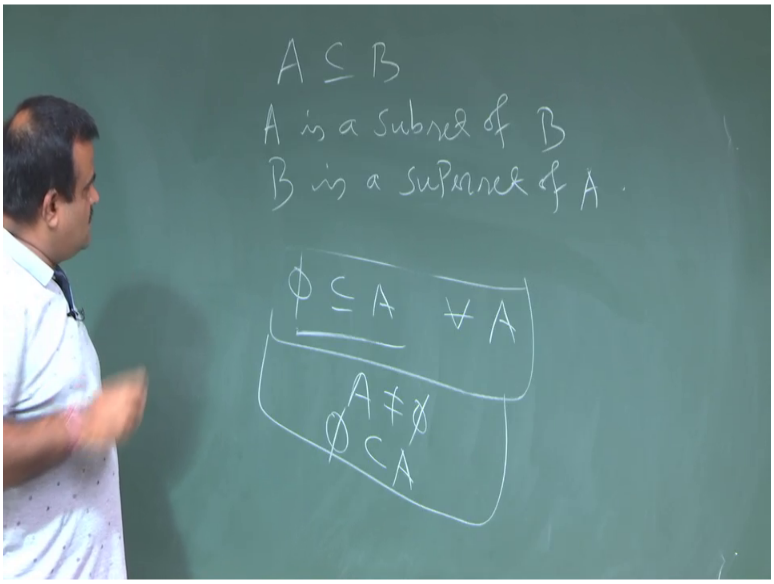

So these are the definition of subset as super set and so if we define the universe set; so every set is a subset of a universe set.

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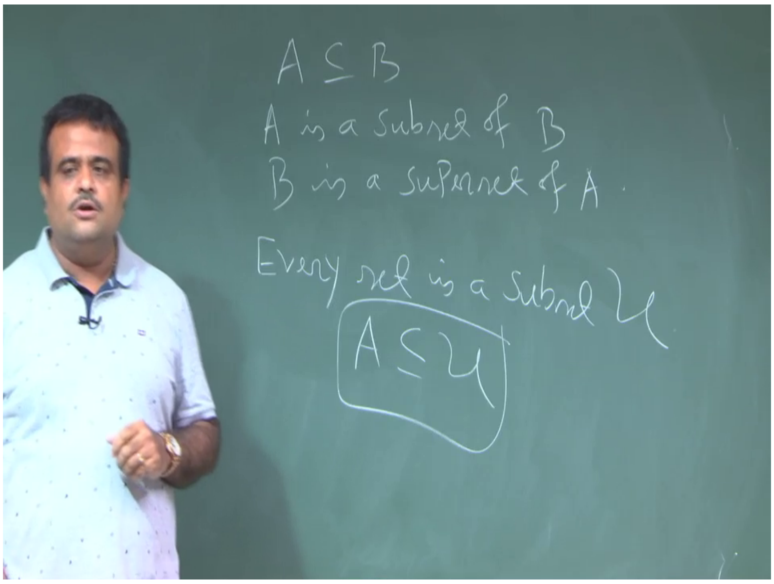

Universal set every set is a subset of the universal set. So if you take a set A these will be the subset of universal set. So this is true for every set.

So this is just a introduction and the definition of the set and in the next class we will talk about we will start the set operation like union, intersection and then will defined those operation and then will discuss some properties on this operations and will prove those properties in this way; so that will discuss in the next class.

Thank you.