Modeling Transport Phenomena of Microparticles
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Lecture – 19
Modeling transport of particles inside capillaries

Hello! In the previous lectures we discussed about composite porous channels where we have a partially filled porous coating and then partially filled free flow and then coupling conditions. And also we discussed about recent interface condition that is called stress jump condition and its role. So in the previous lectures we discussed mostly rectangular geometries of the composite channels.

So while these have applications for example one of the important applications is flow inside glycocalyx okay endothelium and glycocalyx etc. So what is endothelium? These are the vessels where so there is a lumen inside and then porous layer. So typically they are varying cross-section but the best initial approximations. For example cylindrical completely regular circular shape cylindrical.

And then suppose somebody would like to further study planar cross-sections so then no one would go for a rectangular. So in that sense of the previous lecture we discussed about composite porous channels. So in this lecture we are going to discuss about more improved model like cylindrical composite porous channel okay.

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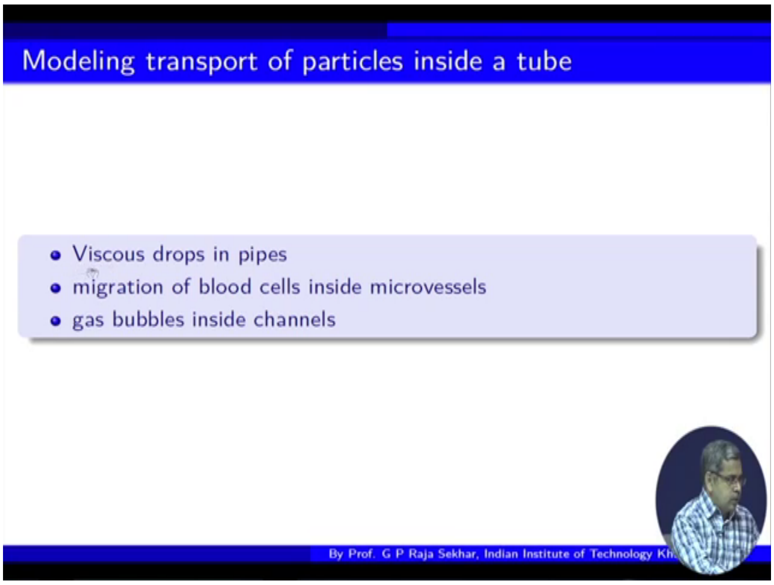

So you will see this has a lot of applications. Because viscous drops in pipes so they also has a similar configuration, then migration of blood cells inside micro vessels and gas bubble inside channels. So these are all examples found where one can model transport of micro particles passing through pipe okay. But it is not very obvious to model these you will see why at the end of this lecture.

So to start with we are not considering any micro particle so we just consider a composite pipe and then try to discuss okay.

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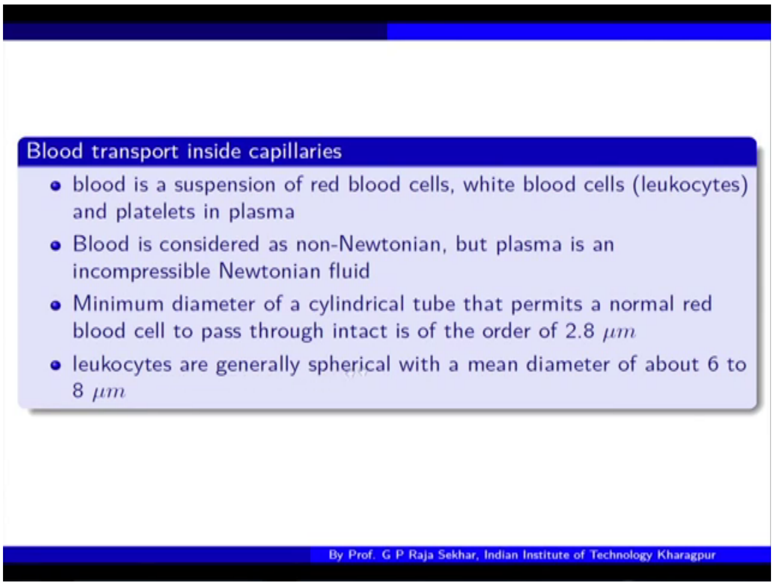

So for example one of the application is blood transport inside capillaries okay. Typically if somebody would like to model this they have the consider various parameters. For example one has to treat blood as a suspension of red blood cells, white blood cells which are leukocytes. Then if you see the common literature else that blood these considered as a non-Newtonian okay.

But plasma is incompressible Newtonian okay. And you will ask question so whether we have the model as a non-Newtonian or Newtonian. So typically in the capillaries it can be approximated as Newtonian okay. So before we model particle migration we would like to focus on capillary flow kind of okay. So when you say blood transport what are these leukocytes.

And what could be the order of their size so that they pass through within the lumen. If you see minimum diameter of a cylindrical tube that permits a normal red blood cell to pass through it intact is 2.8 micron then leukocytes are generally spherical with a mean diameter of 6 to 8 microns okay. So this is so keeping sphericity, minimum is this so they allow little larger range.

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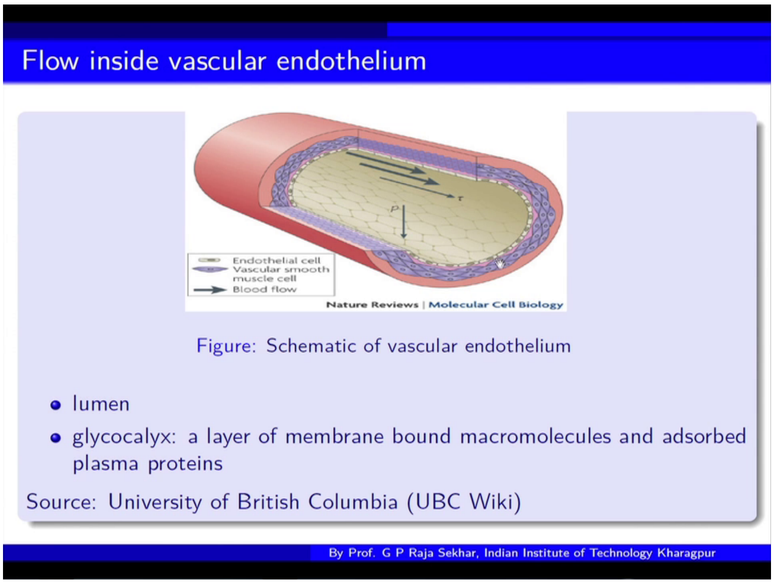

So this is a application I was referring flow inside vascular endothelium. So if you see so these are the porous coating and this is the lumen okay. So this is vasculature and then these are endothelial cells okay. So these are completely deformable porous okay. But throughout these lectures we are considering porous media as a rigid matrix okay. So what is here shown is lumen and the glycocalyx. What is glycocalyx?

Layer of membrane-bound macromolecules and absorbed plasma proteins which are this is the coating porous coating, cylindrical configuration.

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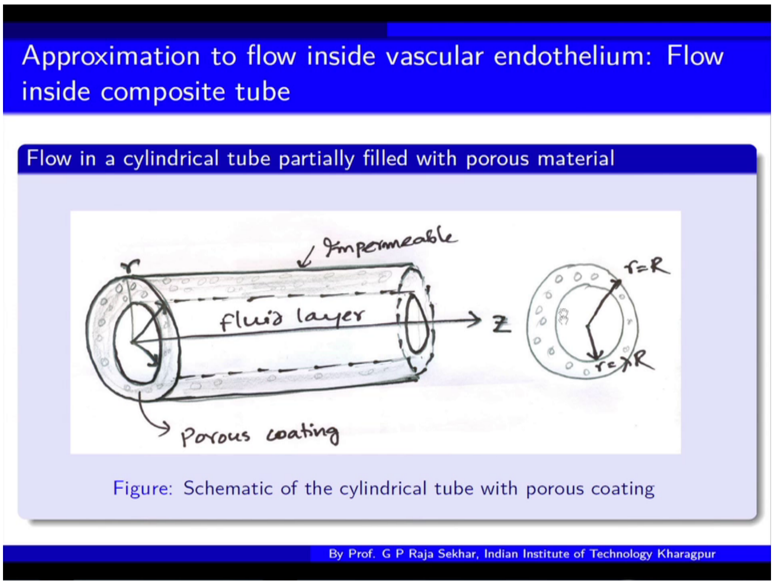

So we would like to approximate this as I indicated so approximation to flow inside vascular endothelium. So you have a cylindrical configuration. this is the porous coating and then this is the lumen okay. So if you see one cross-section view so the outer one is a of radius capital R and then inner one is of radius of Lamda R. So Lamda is constant so we can control the layer thickness with the parameter Lamda okay.

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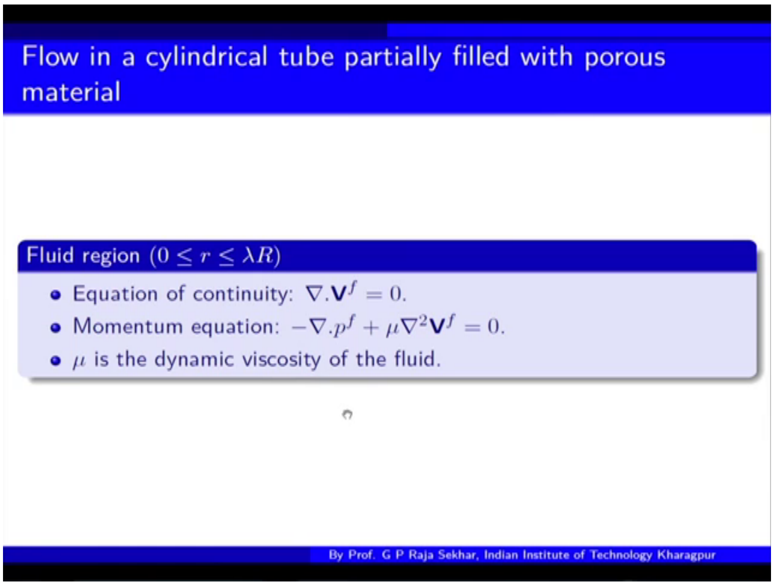

So within the fluid region we have stokes equations. So this is equation of continuity and momentum equation as usual. So Mu is the dynamic viscosity of the fluid. So this is the inner region.

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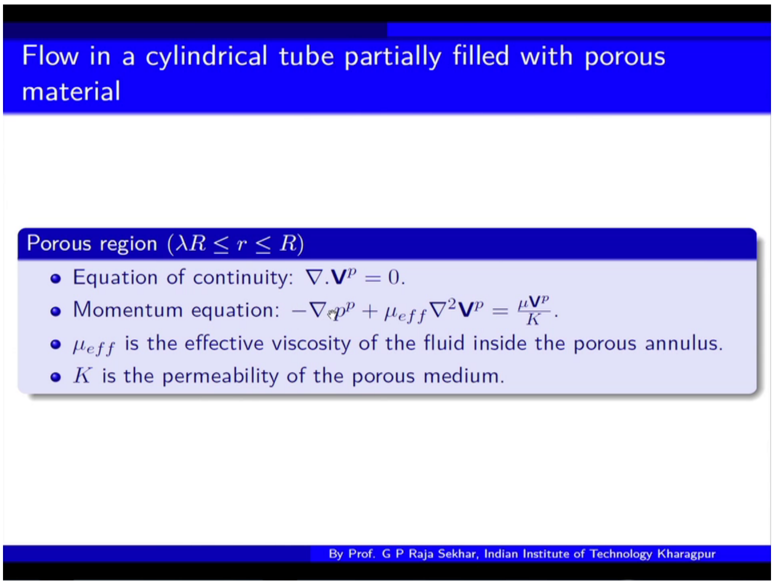

Then in the analyst region which is the porous region so we have again equation of continuity then we have Brinkman equations with so there should not be so with the pressure forces and then the corresponding effect of viscosity and then Darcy the damping term okay. So this is a Brinkman equation okay. So this dot should not be there, gradient.

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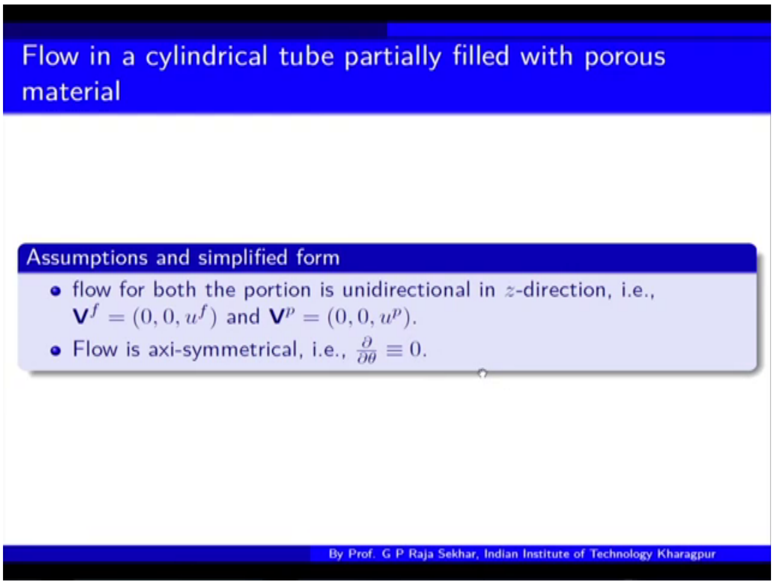

So then assumptions and a simplified form so we are assuming a unidirectional okay, so therefore, flow is a fluid flow and then correspondingly porous flow like this with the corresponding superscripts. Then we assume flow is axisymmetric okay.

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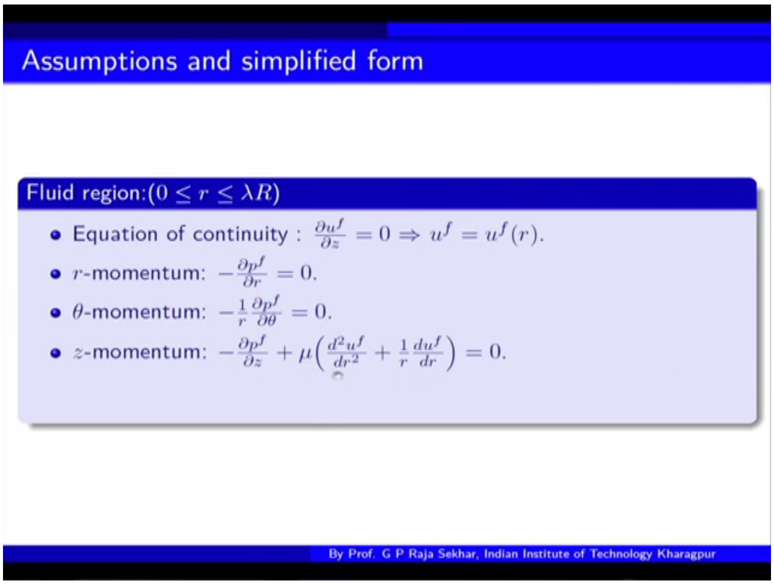

So correspondingly we have seen earlier so due to axisymmetry and unidirectional from conservation of mass we get the velocity is function of r alone okay. And the corresponding momentum equation. So these two indicates the pressure is function of z and we have z momentum equation okay. So this is already I have indicated. So similar thing one can have in the porous region.

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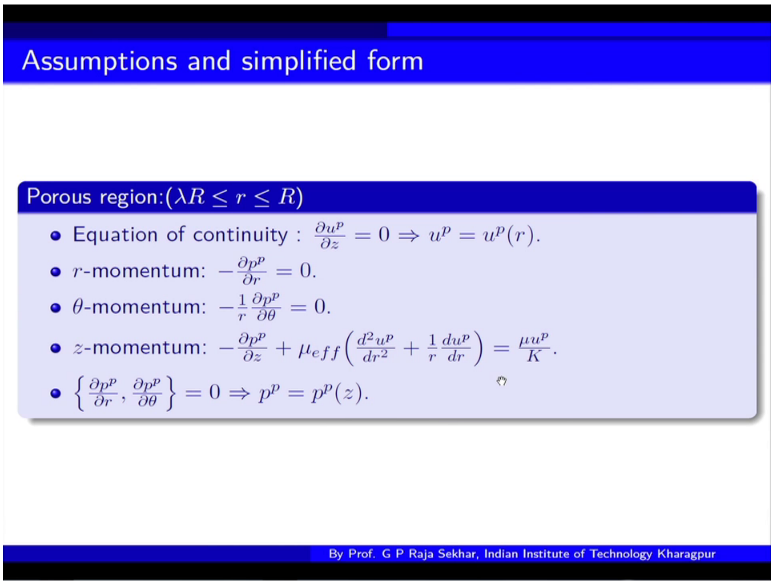

So equation of continuity indicates this then r and Theta z components. So here we have the corresponding permeability playing additional resistance and then we have effect of viscosity. As usual for the current study we assume that effective viscosity is equals to viscosity. So now before proceeding we have to fix interface conditions.

So here I would like to mention so even though in the previous lectures we have introduced the so-called stress jump condition, it is just to make you understand that recent developments etc. But for simple cases and then for keeping the algebra simple one can consider continuity of stresses so that the algebra is simple.

So those who are interested to see the impact of stress jump coefficient one can always take the corresponding stress jump condition and then analyze the results okay. So otherwise we are restricting to continue for tangential stress okay in this problem okay.

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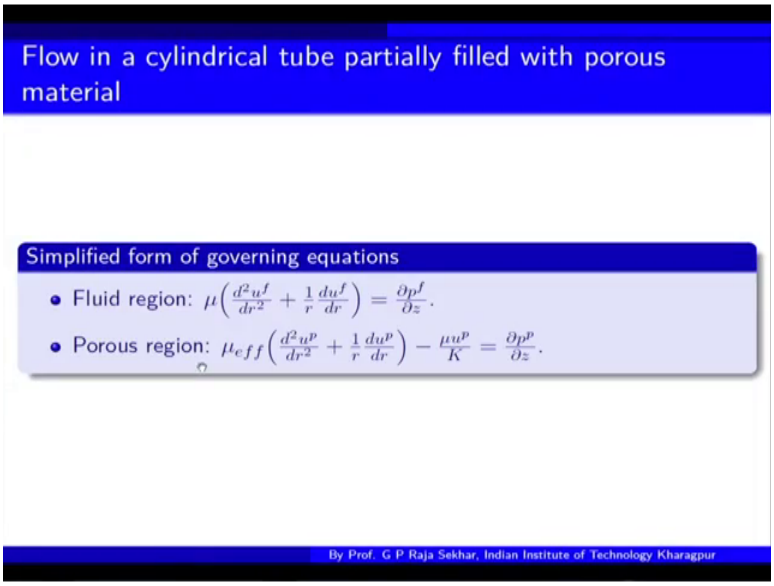

So simplified form of governing equations we have the corresponding z momentum equation in fluid region and the corresponding z momentum equation in the porous region okay.

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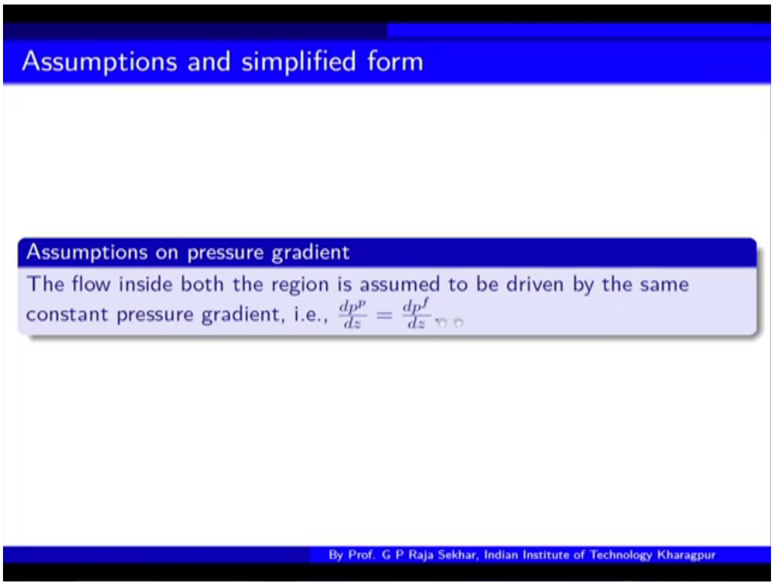

So the assumption is pressure is the same okay in both the region.

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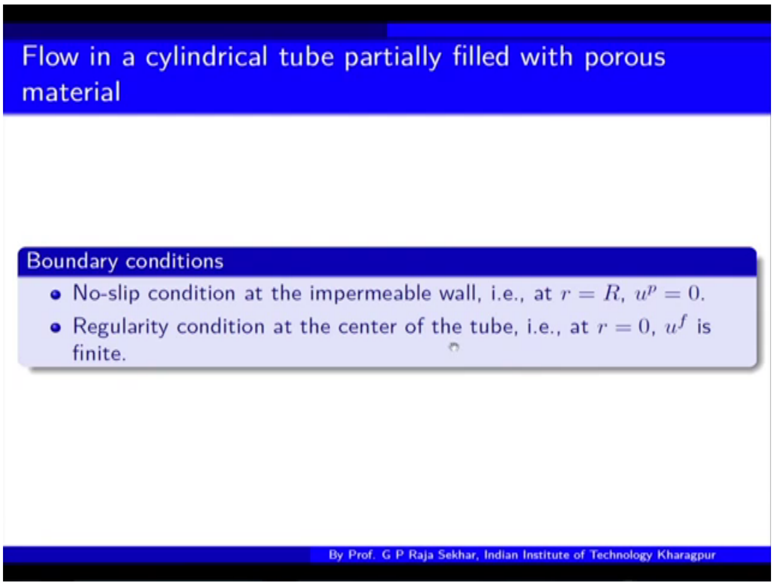

Now boundary conditions so the outer wall is impermeable therefore the porous velocity is zero no slip. Then the regularity condition at r = 0 therefore uf is a finite at r = 0 okay.

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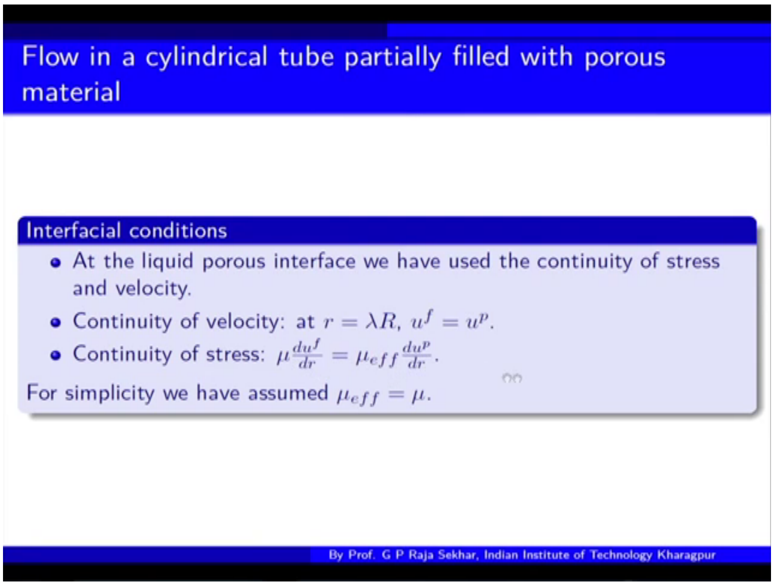

Now interface conditions, so as I indicated we use the continuity of stress and velocity. So therefore, at the interface your continuity of velocity and the continuity of stress okay. But we assume effective viscosity equals to viscosity.

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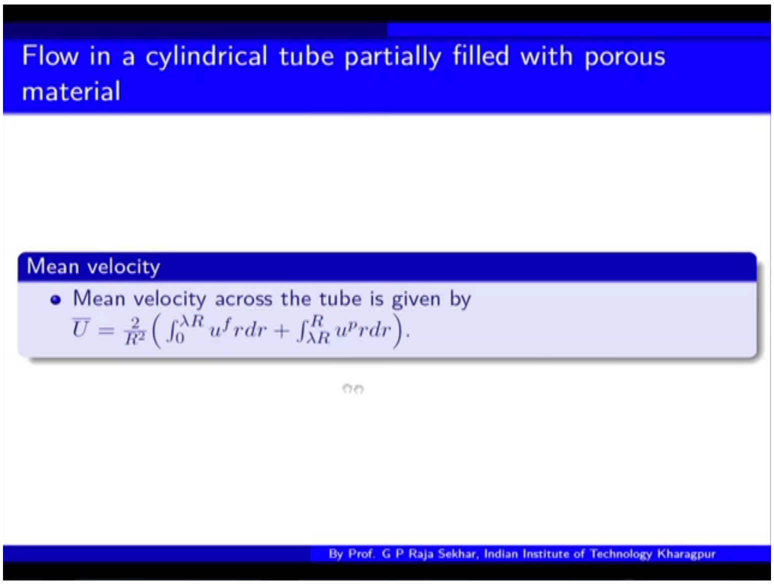

So the mean velocity is a defined by this because the fluid is ranging from 0 to Lamda R and then porous is Lamda R to R and corresponding area averaging we are doing okay.

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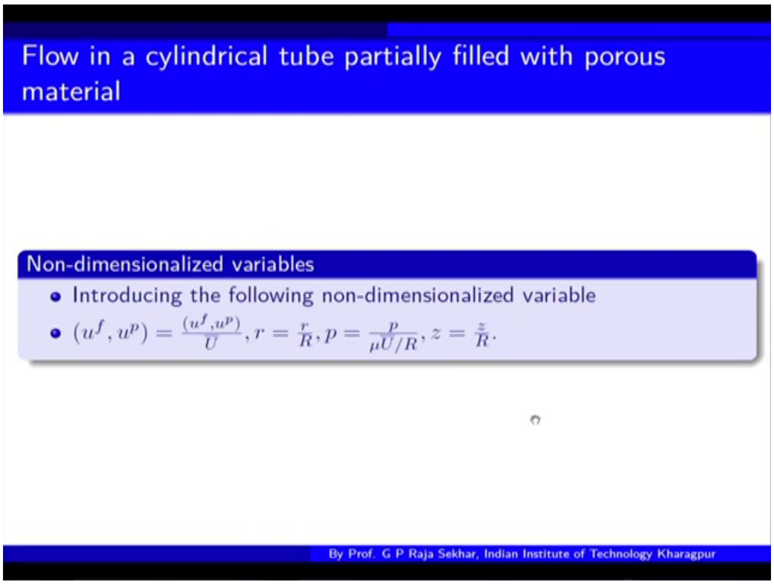

So then we introduce a non-dimensionalization. There is the radius of the tube is used for non-dimensionalizing the length scales for both R and z as well and once we non-dimensionalize so pressure is also as I indicated both r equals to a constant. So then non-dimensionalized momentum equation in the fluid region and in the porous region.

So we have seen in the previous lectures this is the damping term due to the permeability which is a alpha square is 0 one over Darcy number where Darcy number is this okay. So if permeability is a small Darcy number is small if permeability is large Darcy number is large okay. So non-dimensionalized boundary conditions and interface conditions.

So we have a seen for the clear flow solution so similar using similar method one can easily compute the solution. So this is an expression for the mean velocity so now if we find the general solution fluid reason we get this and the porous region we get this where these are the modified Bessel functions okay. So now we have regularity condition, therefore, in the lumen we have regularity condition at r equal to 0. So therefore, we expect that this will be 0.

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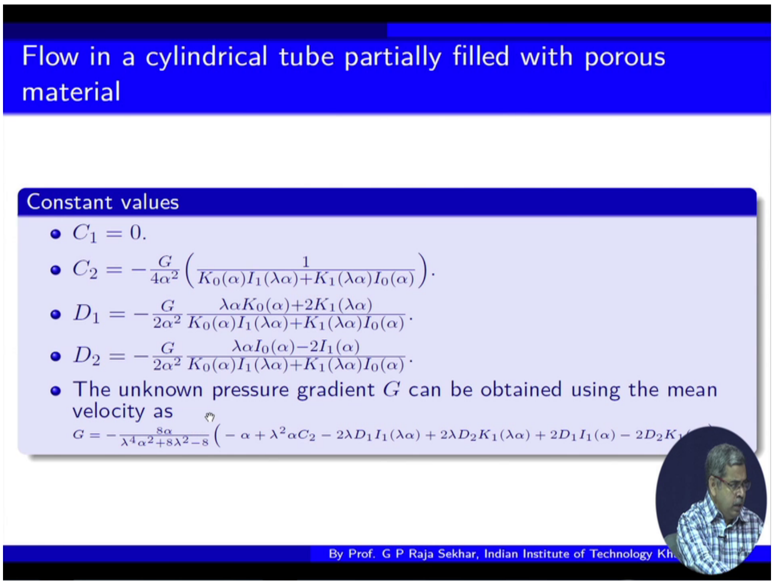

So correspondingly if you use the boundary conditions and interface conditions we get the corresponding solution and the volume flux balance can be used to get the pressure okay. So solution is obtained. So then we use Mapple or MATLAB to plot this okay.

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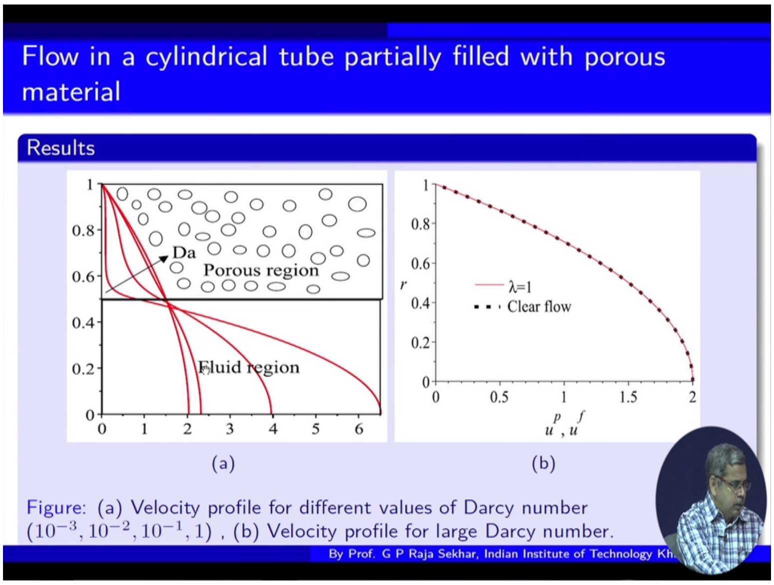

So you see we have shown with some nice editing so that you understand better. So we are showing only the symmetry. So for the porous portion you have we have fixed specific Lambda okay. So it is a luminous upto 0.5 and then from 0.5 to 1 you have porous okay. So as you can see in the figure (a) for a different Darcy number we are showing these curves and the direction is given. So it is increasing from 10 power -3 to 1.

So let us consider 10 power -3. That means Darcy number is very low. So since the permeability is very low you expect a very less velocities because a very small amount of fluid can percolate into this region. But you can see the total volume flux is balanced so therefore whatever is lost here; lost in a sense because it is very minimal; that has to be adjusted in the free flow region. So that is why the velocities are more here okay.

Now suppose you increase the Darcy number slightly so then the velocity in the porous region is increased and correspondingly in the fluid region it is a reduced. Further if you so that what we are getting. Then this is a, almost large Darcy okay. So almost this is a limit of fully clear flow region. So the total volume flux is a getting adjusted like this therefore, the velocity is getting adjusted like this okay. So this is indicating what kind of protein structure in the in the glycocalyx.

So corresponding to the protein structure we can estimate the permeability range and depending on the permeability range we can get the corresponding Darcy number and then we can study what kind of volume flux is getting accumulated within the within the porous layer. And if there are high volume flux then one can analyze what kind of tissue deformation happens. Of course this is approximate model with rigid porous matrix. But in case of a deformed porous matrix.

So these kind of estimates can be obtained okay. So this gives some correlation with the tissue lining and then the corresponding velocity profiles etc. So for example if somebody has to estimate some drugs and then the corresponding velocity to be controlled okay, so then we have to study the glycocalyx parameters like it is rigidity and then permeability etc. And then correspondingly one can get some estimate on the velocity of the drug. So these are some applications.

So now let us see the limiting case as always to validate one has to show some limiting cases. So this is for a large Darcy number Lambda 1 okay. That means a fully clear flow so it is agreeing with the clear float channel okay. So this is a Hagen Poiseuille.

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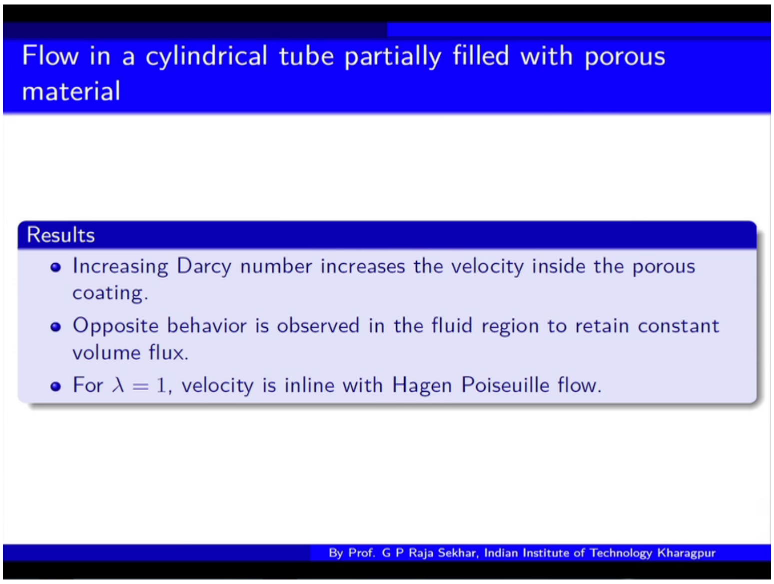

So that is the inference we are making increasing Darcy number increases the velocity inside the porous coating okay and opposite behaviour is observed in the fluid region to retain constant volume flux. What I meant, increasing the Darcy number increases the velocity in the porous region. Opposite behaviour is observed in the fluid region because for this it is more increase in Darcy number, velocity is reduced in the fluid region.

That is what we want. Then for Lambda 1 velocity is in line with the Hagen Poiseuille flow. This is what we have seen okay. So this is an approximation to investigate particle migration in a in a capillary okay. So in order to get some understanding about a similar problem we are going to discuss now some discussion about how one can model particle migration in a pipe okay.

So the corresponding algebra is a little involved so it cannot be just done in a lecture. So but the concept can be explained and what are the difficulties involved in modelling this okay.

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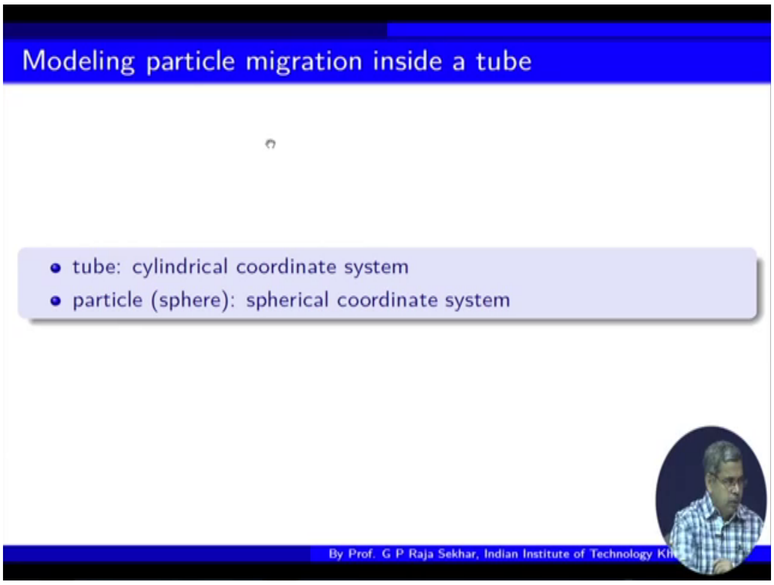

So we are interested in particle migration.

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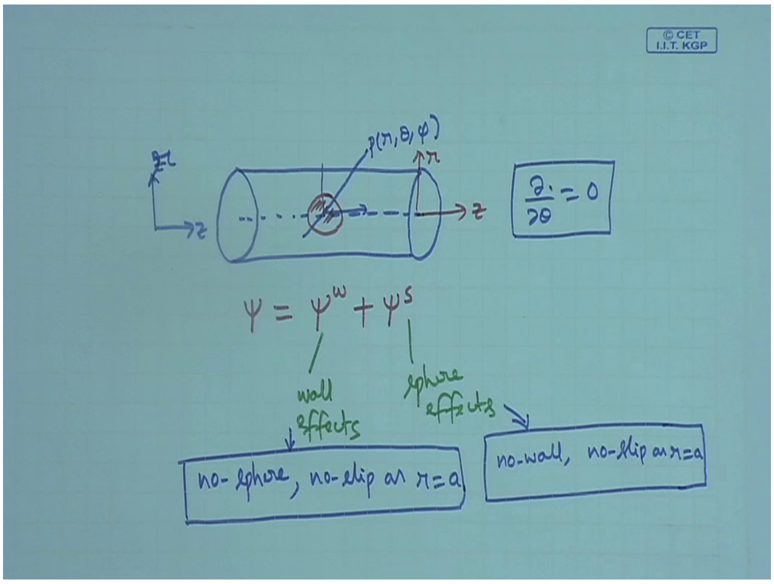

So consider a capillary okay, a tube, so we fix r and z. Now we are interested in modelling particle migration right. So you take a sphere so if you consider spear in order to satisfy boundary condition on the sphere we require spherical coordinate system to handle the sphere okay. So which means if you take any point with respect to spherical coordinate system.

This is the point where as with respect to this cylinder this is z axis and then this is r axis okay. So that means there is a interaction of the coordinate system okay. So it is very straightforward if somebody is concentrating on boundary conditions, suppose you are in a rectangular system so then we can reach plates okay.

Suppose you are in cylindrical coordinate system then we can reach the corresponding surface with r equal to constant and then axis with a z equal to constant plane okay. So if you want to represent say sphere surface r equal to constant using Cartesian, so it will be difficult okay. So that is what so depending on the problem we have to fix the corresponding coordinate systems okay.

So depending on the physical configuration. So here the difficulty is you have a spherical particle so where you have the handle the spherical coordinate system. But you have a cylinder that you have to use a cylindrical coordinate system. But then sphere is travelling inside a tube. So therefore you have an interaction of the coordinate system. So how this can be handled? So that is the task okay.

So as an indicator tube is cylindrical coordinate system and particle that is a sphere is the spiritual coordinate system. But considering Stokes flow which is linear the equations are linear. So one can use some superposition principle.

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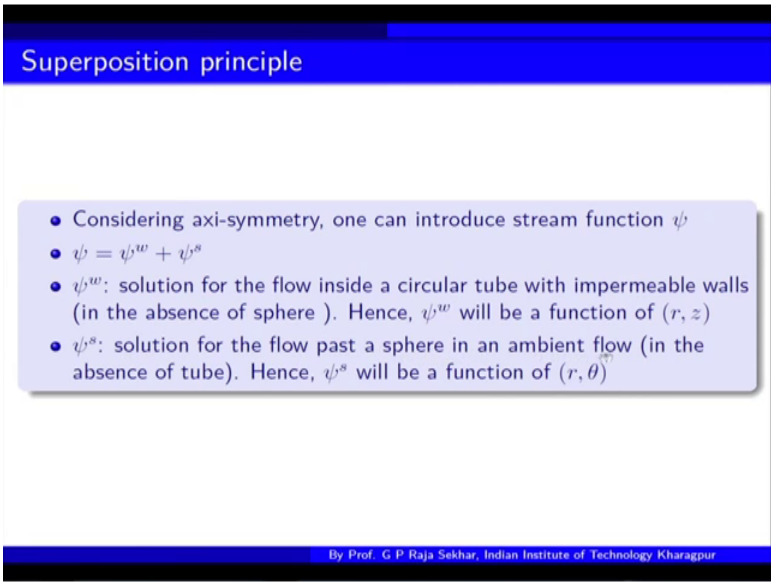

So what we do is we consider axisymmetry and introduce stream function okay. Then once we introduce stream function the total stream function is decomposed into two parts, where this takes wall effects into account and this takes sphere effect. That means we are trying to solve for Psi w using cylindrical coordinate systems in the absence of sphere okay.

So just like it assuming axisymmetry of course we are assuming axisymmetry so then we can introduce stream function. So once we introduce stream function we are considering Psi in the absence of sphere. Just a pipe flow satisfying no slip okay. So wall efforts means no sphere okay. Then no slip on r = a. So this is corresponding to Psi w. And sphere what we are considering is no wall no slip on r = a okay.

So these two are independent problems. So why we could do this and why this is justified? Because the problem is linear okay. So this linearity is allowing us to do this superposition. So we solve these two problems independently then combine Psi okay. So that is Psi w is solution for the flow inside circular tube with impermeable walls in absence of sphere. So then this can be a function of rz.

Then Psi s will be solution and then this will be function of r. So here when I say no sphere no slip on r equal to a. No wall no slip on r equal to a. So therefore, first compute the solution in sphere so this. So no sphere case will be Psi w will be Psi w of rz in cylindrical. And no wall means Psi s will be Psi s in r and Theta. Because axisymmetry for cylindrical is this and axis symmetry for spherical is this okay.

So we have computed without sphere okay. So just the solution but we have not yet forced it. Then without the walls just with sphere we have computed this solution.

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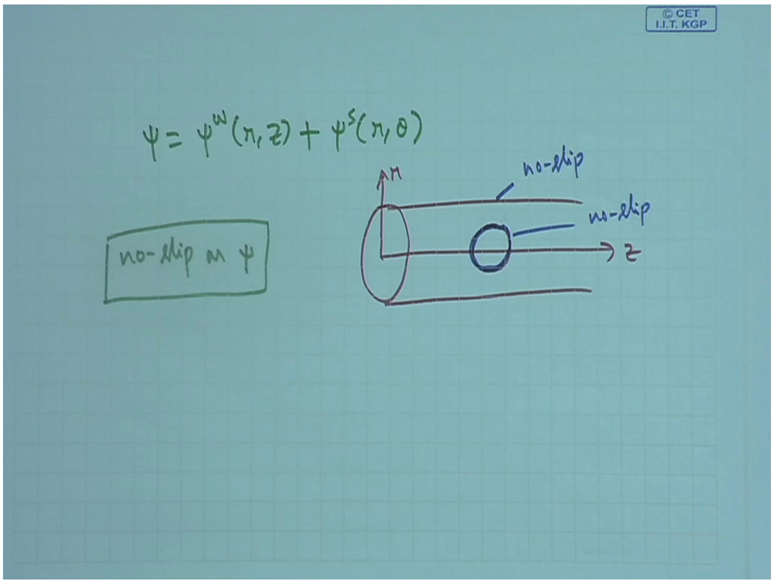

So once you compute then the resultant Psi will be of the following form rz + sphere r Theta okay. But then what is the configuration we have? So the configuration that we have is okay. So for the combined since individual we expect that but we have to solve the impact of the particle. So we do not enforce this on individual stream function okay. We do not enforce no slip we do not enforce no slip on the sphere on individual solutions.

We compute the general solution then superpose Psi, then we have to force no slip and no slip. So that means we are forcing no slip on Psi resultant Psi okay. So once you force so we can determine the corresponding arbiter coefficient involving this okay. So this is a very unique problem in that sense there are alternate methods for this. For example one can use say bi-polar coordinate system.

And then the bi-polar coordinate systems are separable in a linear case okay. So therefore, one can use but we are exploiting the linearity structure of the problem and then getting two individual components superpose and force the boundary conditions on the cylinder as well as of sphere on the combined stream function okay. So this approach can be done.

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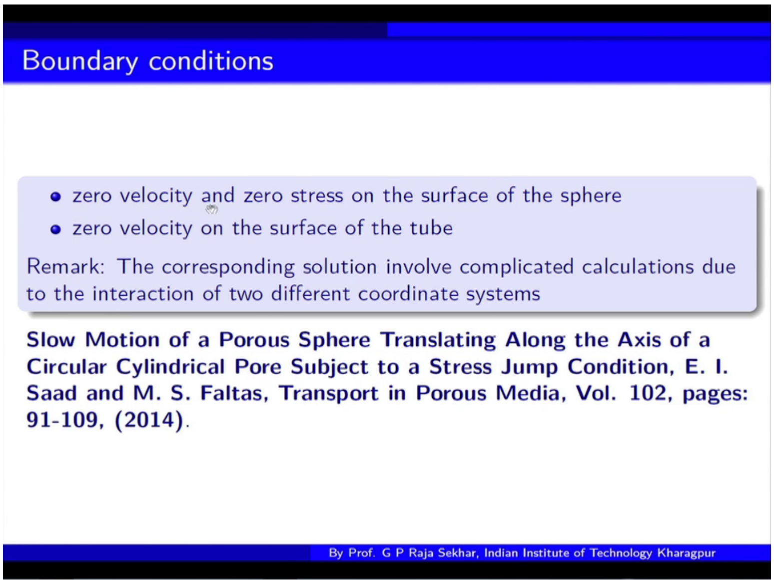

So that is what I said zero velocity and zero stress on the surface of the sphere, zero velocity on the surface of the tube and the complication is interaction of the coordinate systems okay. So this similar technique is used by these authors in a different context. So they were modelling porous sphere translating in a circular cylinder okay. So they have used this. So we thought we will just make you understand how particle migration can be modelled inside a tube.

As I indicated the algebra is very much involved because you have to compute individual stream functions and then forces and surface conditions. So one can do it but not because if you do it in a lecture will be very routine, lengthy algebra okay. So I am sure you get some idea about particle migration and then overall how porous tubes and porous composite porous channels can be handled okay.

So maybe in the next lecture we discussed some applications so that you get to know various situations where whatever we have learnt so far can be applied okay. Thank you!