Probability and Statistics

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Lecture - 71
Testing for Normal Variance

Now, in a similar way one can go about testing for the variance of a normal distribution. Now once again we have seen here that the variability of the normal distribution was tested in the Neyman Pearson Lemma setting by sigma X i square; that means, how much the value of sigma X i square is becoming, is it more or less kind of thing. So, when we consider the composite hypothesis for sigma square, we will be basing the decision on the distribution of sigma X i square as we know the distribution of sigma X i square by sigma square chi square. So, we will actually be getting a chi square test. So, let us consider 2 different cases - Testing for the Variance.

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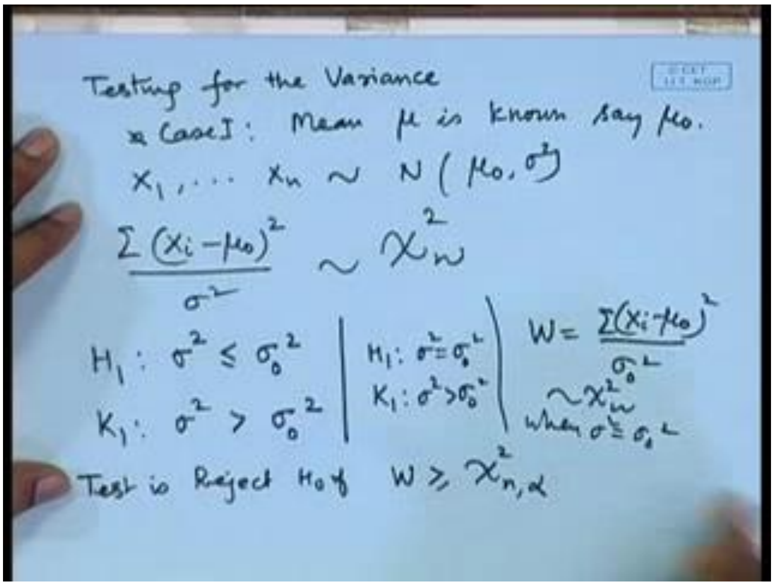

When we are doing the testing for the variance, let us write down the model here say mean is known say mu naught. So, we are having X 1, X 2, X n following normal mu naught sigma square. So, if you go back to the setting of normal 0 sigma square, we can actually consider X i minus mu naught square. So, summation of this that divided by sigma square that will follow chi square distribution on n degrees of freedom.

We may look at the hypothesis of the form say H 1 sigma square less than or equal to sigma naught square against say K 1 sigma square is greater than sigma naught square. So, when sigma square is equal to sigma naught square, we may consider sigma X i minus mu naught square by sigma naught square. So, this will follow chi square distribution on n degrees of freedom when sigma square is equal to sigma naught square. So, we may use the test as reject H naught if W is greater than or equal to chi square n alpha, the similar argument will be valid if I am considering sigma square equal to sigma naught square against sigma square is greater than sigma naught square.

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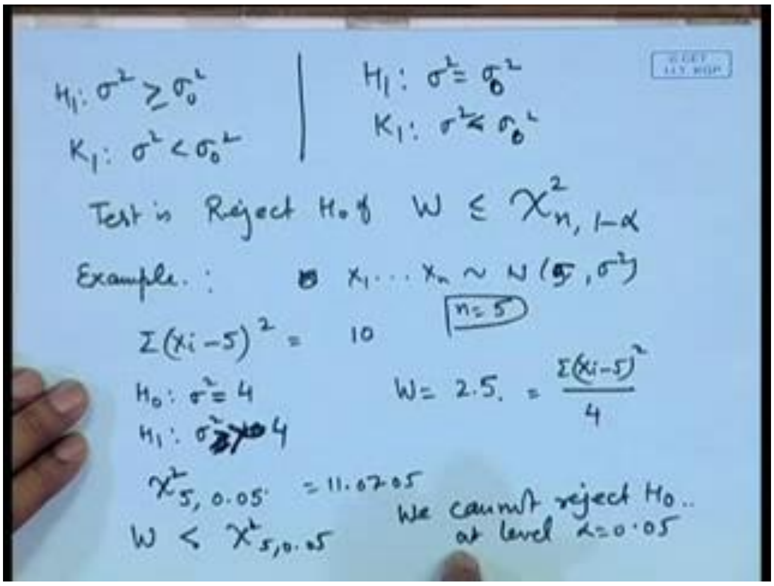

If we consider carefully then the reverse of this say sigma square is greater than sigma naught square against sigma square is less than sigma naught square then the test will be reject H naught if W is less than or equal to chi square n 1 minus alpha a same test will be valid if I use sigma square is equal to sigma naught square against K 1 sigma square is less than sigma naught square because what we are saying is sigma square is less or more and therefore, this is going to be useful.

Let me take one example here, suppose the value of suppose we are considering say X 1, X 2, X n, following normal 0 or by in about to say 5 sigma square and we are observing here sigma X i minus 5 square is equal to say 10, we are testing the hypothesis say H naught sigma square is equal to say 4 against H 1 sigma square is equal to maybe 10. So, if you look at the value here W that value will become equal to 2.5 that is sigma X i minus 5 square by sigma naught square that is 4 that is equal to 2.5. Now we have compare with the chi square value on suppose I am taking n is equal to 5 then I have to see this chi square 5 at alpha suppose I say alpha is equal to 0.05. So, from the tables of the chi square distribution one may look at chi square 5.05 as equal to 11.0705, naturally you are getting W to be less than see since we are testing here. So, we may put here greater than or equal to 4 say this is equal to 4 and here greater than 4 suppose I am putting.

The first is to reject for larger values. So, here W value that is 2.5 is less than chi square 5.05, you can see here we cannot reject H naught at level alpha is equal to 0.05, let us look at the interpretation of this, we are testing about the variability here that variability is equal to 4 or more than 4, now what is happening here is that sigma X i minus 5 whole square that is turning about to be 10. So, if you look at sigma X i minus 5 whole square by 2 that is sorry by n which is the maximum likelihood estimator that will be equal to 2. So, if you are using that as an estimator, naturally it is less than 4 there therefore, sigma square greater than 4 is not supported very much and whether it is significantly not supported that we have to see from the probability of the type one error and here it turns out that actually we cannot reject H naught.

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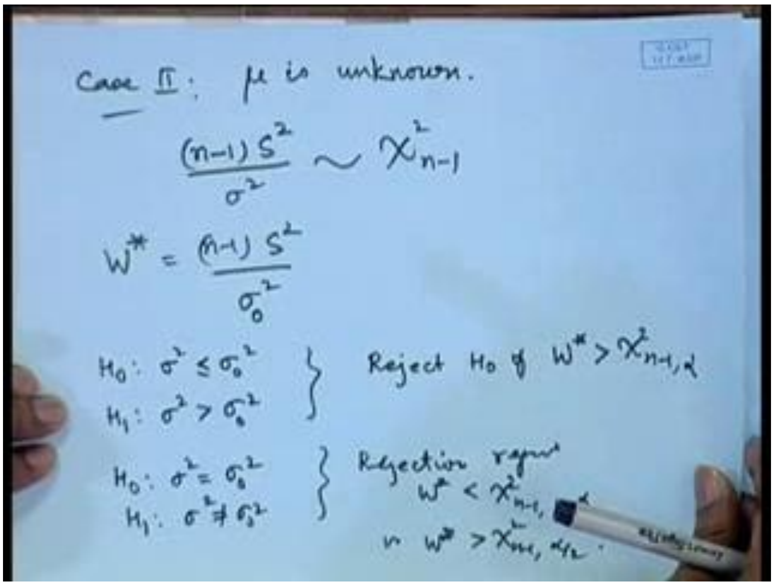

Now, let us consider the case of when mu is unknown, when mu is unknown we cannot making use of W, in place of that we have to make use of the distribution of S square.

So, n minus 1 S square by sigma square follows chi square on n minus 1 degrees of freedom. Let me call this statistic W star as n minus 1 S square by sigma square sigma naught square. So, if you are testing the hypothesis say at the form say H naught sigma square is say less than or equal to sigma naught square against say H 1 sigma square is greater than sigma naught square. So, what will be the test function reject H naught if W star is greater than chi square n minus 1 alpha.

Suppose we are looking at 2 sided hypothesis sigma square is equal to sigma naught square against say H 1 sigma square is not equal to sigma naught square then the test function will be the rejection region will be W star less than chi square n minus 1 1 minus alpha by 2 or W star greater than chi square n minus 1 alpha by 2, let us look at the examples here.

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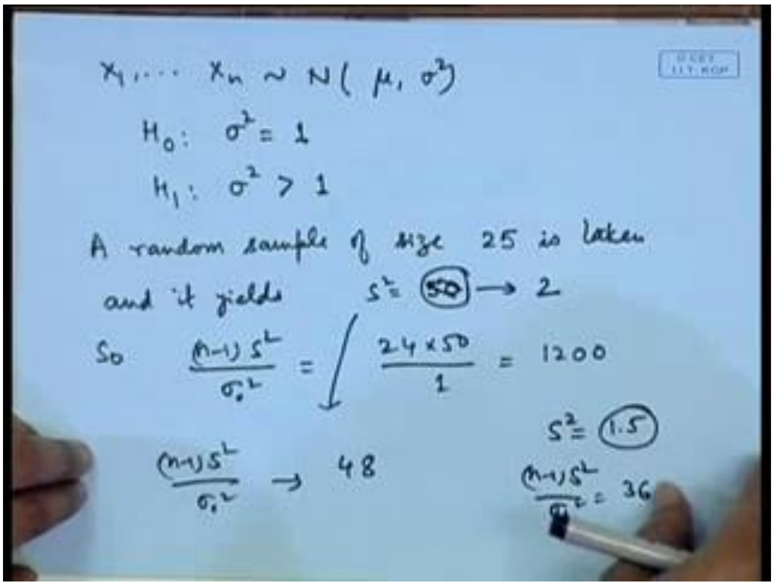

We want to test the hypothesis whether sigma square is equal to 1, again sigma square is greater than 1. So, a random sample of size 25 is taken and it yields S square is equal to say 50.

If we are considering say n minus 1 S square by sigma naught square then this value turns out to be 24 into 50 divided by 1 that is equal to 1200, you can easily see that this value is extremely large because S square is actually used as an estimate of sigma square and this value 50 is much bigger than 1 therefore, at any level of significant which is practical the value of chi square will always be a much smaller than 1200. On the other hand suppose I am considering here the sample which gives say S square is equal to in place of 50, this value is turning out to be say something like say 2 then there is a change here because in the value of n minus 1 S square by sigma naught square in that case will become simply equal to 48.

Now, if we are looking at the tables of the chi square distribution, one may easily observe the difference because chi square value and if I am looking at say 24 degrees of freedom and here at a certain level of significant after 0.05, the value will cross this 1. So in fact, with little luck suppose I am taking say S square is equal to 1.5, then what will happen? Here n minus 1 S square by sigma naught square would become equal to 36 and here you can see even at say one percent level of significance the value is 36.19 and therefore, we will not be able to reject H naught here.

Easily you can see that the acceptance or the rejection of the null hypothesis dependent upon the probability of the type one error as well as the point sigma naught square or you say theta is equal to theta naught in the null hypothesis that is used as a control here.

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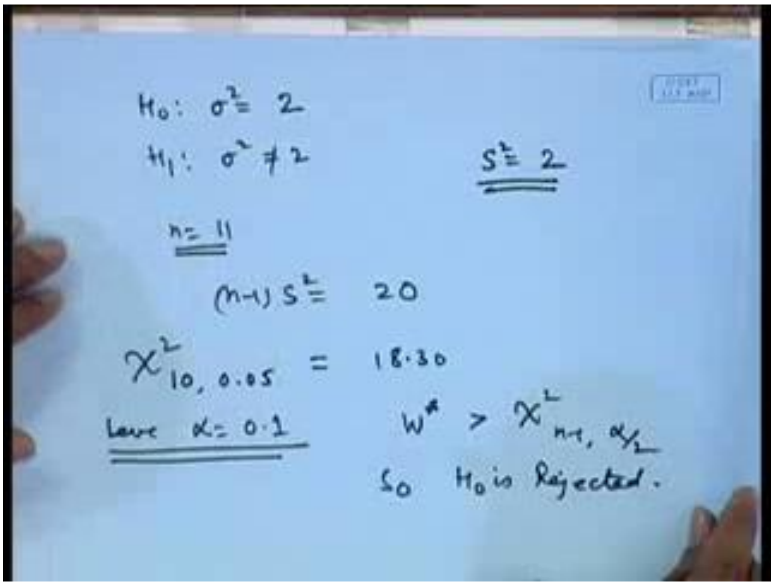

If we are considering say 2 sided here hypothesis in that case we may have something like say H naught sigma square is equal to say 2 against say H 1 sigma square is not equal to 2. Now in this case, what will happen? Suppose I have observed say S square is equal to 2 itself then if I have considered a sample of size n is equal to say 11 then you will have a n minus 1 S square is equal to 20. Now we look at chi square value on 10 degrees of freedom say at 0.05.

Now, this value chi square 10 at 0.05 is equal to 18.30. So, here if I am considering say level alpha is equal to 0.1 then you are seeing that your W star is greater than chi square n minus 1 alpha by 2. So, H naught is rejected, now we may slightly wonder here that S square was 2 sigma square is equal to 2, even then you are rejecting this, the reason is that for the sample of size 11, in if you are getting this value then there is a slight discrepancy because chi square on ten degrees of freedom is giving you a value which is slightly smaller than this the you are getting 18.30. So, this is the even turning out to be in the rejection region.

Even a smaller value than this would have been supported here now in the 1 sample problems we may consider tests for proportions.

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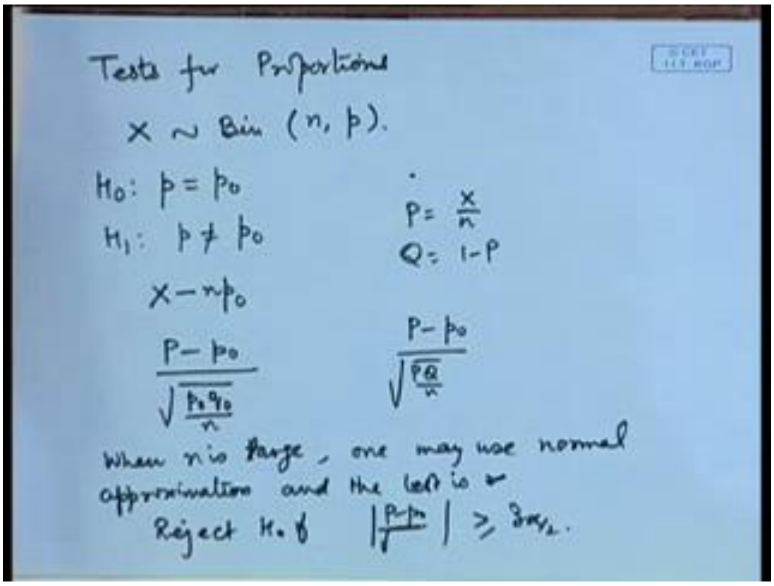

The setup is that we are observing a sample from Bernoullian trials. So, we may consider a sum and we have a setup like X following binomial n p distribution and we may be interested in testing about say H naught say p is equal to p naught against say H 1 p is not equal to p naught. Now here, we have seen we may consider the test statistic based on X because the distribution of X is binomial say if you consider X minus n p naught. So, we may divide it by this. So, we are dividing by P here. So, P is equal to X by n. So, if you consider the value here to be P minus p naught divided by root p naught q naught divided by n then you can use it as a test, on other hand one may use here the estimates of this also P minus p naught divided by root P Q by n where p is this, q is 1 minus p. So, one may use either of these things and when n is large one may use normal approximations and the test is reject H naught if z by alpha by 2.

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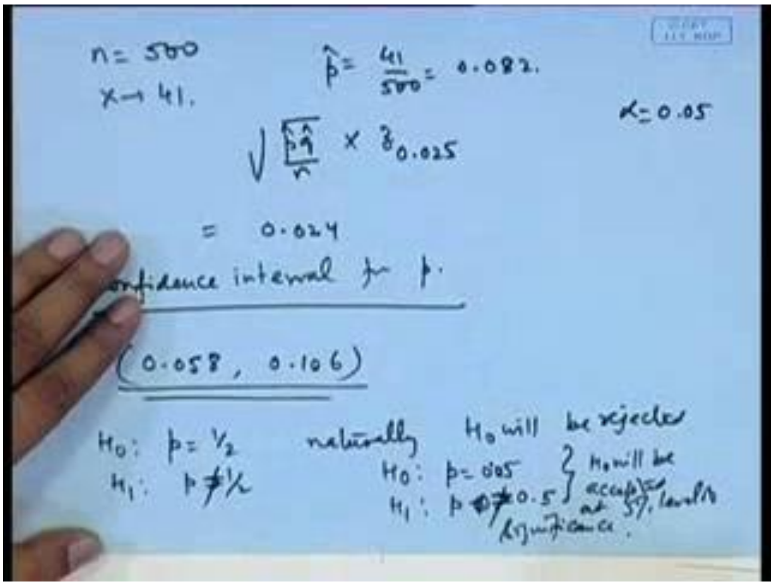

Let me consider an example here. Suppose a random sample of 500 customers was considered and here X is observed to be 41, let us look at say p hat and q hat here. So, p hat turns out to be 41 by 500 that is 0.082. So, if you consider p hat q hat by n into say z 0.025, suppose I take alpha is equal to 0.05 then this value turns out to be 0.024 say so if we are considering here the confidence interval say 95 percent confidence interval for p then that will be equal to 0.058 to 0.106. So, easily you can see suppose I am testing here the hypothesis H naught p is equal to say half against H 1 p is greater than half or p is not equal to half.

Then naturally H naught will be rejected, you can easily see here the value is actually lying between 0.05 to 0.1. So, if I am testing the hypothesis p is equal to half then naturally this will be rejected on the other hand if you are considering the hypothesis say p is equal 0.05 against H 1 p is not equal to say p is greater than 0.5, sorry, 0.05 or p is not equal to 0.05 then this will be accepted, this H naught will be accepted at 5 percent level of significance, let me take one more example here.

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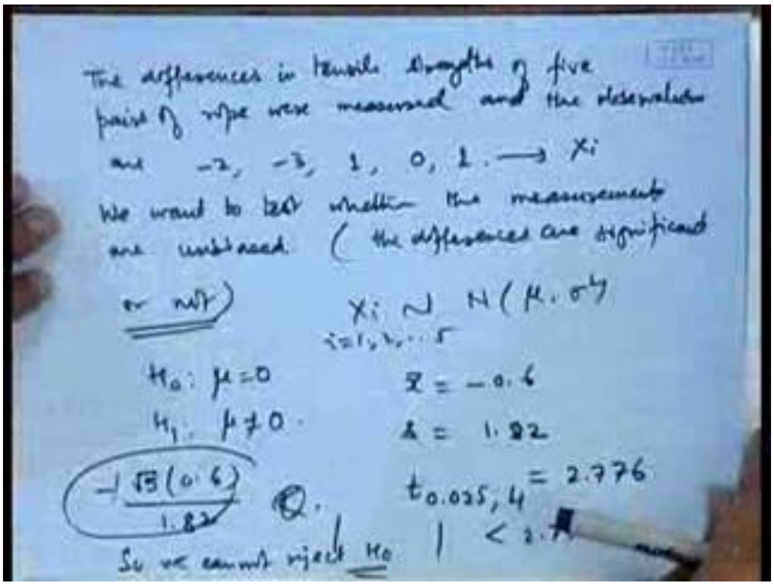

The differences in tensile strength of 5 pairs of rope were measured and the observations are minus 2, minus 3, 1, 0 and 1 we want to see whether the measurements are unbiased; that means, the differences are significant or not.

If you consider these values to be say X i's, we may consider the model of X i follows normal mu sigma square and the testing problem becomes for i is equal to 1 2 5 H naught whether mu is equal to 0 or h 1 mu is not equal to 0. Now you see we need to calculate the mean etcetera. So, X bar is equal to minus 0.6, here say standard deviation turns out to be 0.81, say 2. So, we need to look at root 5, since I am telling you mu is equal to 0 then the test is about minus 0.6 divided by 1.82, easily you can see suppose I am seeing the value of t at say 0.025; that means, I am making a test at level of significant alpha is equal to 0.05 then this value is 2.776, easily you can see that this value is less than this, sorry. If I am taking absolute value of this, absolute value of this is less than 2.776. So, we cannot reject H naught that is also supported by this because here you are having 2 values which are slightly negative and 3 value; 1 is equal to 0 and 2 values which are slightly positive.

Here you can see that if I calculated 95 percent confidence intervals for mu then that would have included this value here, now this idea of the extension of the Neyman Pearson Lemma to composite hypothesis can be applied to other distributions as well. So, suppose we are taking the example of say.

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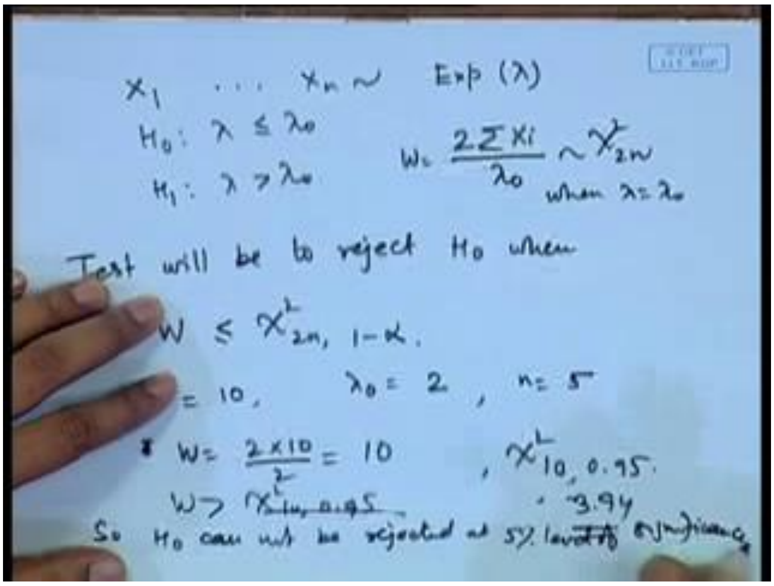

Exponential distribution that you considered that X 1, X 2, X n, following say exponential distribution with parameter lambda and we are testing say hypothesis lambda is less than or equal to lambda naught against say lambda is greater than lambda naught, we had already seen the interpretation of the lambda here which is the reciprocal of the mean. So, we had seen that for the larger value of the mean, larger value of the sample mean, we will be having a smaller value of the rate; that means, we will tend to accept H naught and for a smaller value of the mean we will be tending to reject H naught; that means, tending to go in favor of H 1 and the test function, we have devised in terms of a chi square distribution because we had made use of sigma X i. So, we consider twice sigma X i. So, if you consider twice sigma X i divided by lambda naught then let me call it W then this will follow chi square distribution on 2 n degrees of freedom when lambda is equal to lambda naught.

The test function will be to reject H naught when say W is so since W is favoring 1 by lambda higher value. So, smaller value of 1 by lambda will become corresponding to less than or equal to chi square 2 n 1 minus alpha. Let us state a example here where sigma X i turns out to be say 10, you are testing about say lambda naught is equal to say 2 and say n is equal to say 5 and say then this test statistics could become equal to sigma X i. So, W will become equal to twice into 10 by 2 that is equal to 10.

Now if you look at chi square value of 10 at say 0.95 then from the tables of the chi square distribution, this value turns out to be equal to the value of the chi square at this will turn out to be at 10 degrees of freedom, if we look at 0.95 values 3.94. So, easily you consider this W is bigger than this chi square ten point nine 5. So, H naught cannot be rejected at 5 percent level of significance here lambda naught is 2 and if I am considering an as well estimator for 1 by lambda that will be equal to X bar that is equal to 2. So, that value is not significantly different from this. So, we are testing whether this is less or not. So, then that is not supported by the data.

In the forth coming lecture, we will be considering when we are comparing 2 populations. So, we will be discussing tests for the differences of the means or the ratios of the variances for 2 normal populations as well as we will take up cases for comparing the portions of 2 binomial populations. We will also look at the chi square test for the goodness of fit, etcetera.