**Probability of Statistics**
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**Lecture – 05**
**Random Experiment, Events**

Today I will introduce e the basic concepts of probability theory. So, probability it (Refer Time: 00:29) origins in the games of chance in early 16th and 17th century when the owners of gambling houses in Europe they became interested to explore that whether 1 can find out the probability of various events, which take place during the gambling game such as tossing of dies, rolling of a coins, roll at wheels etcetera.

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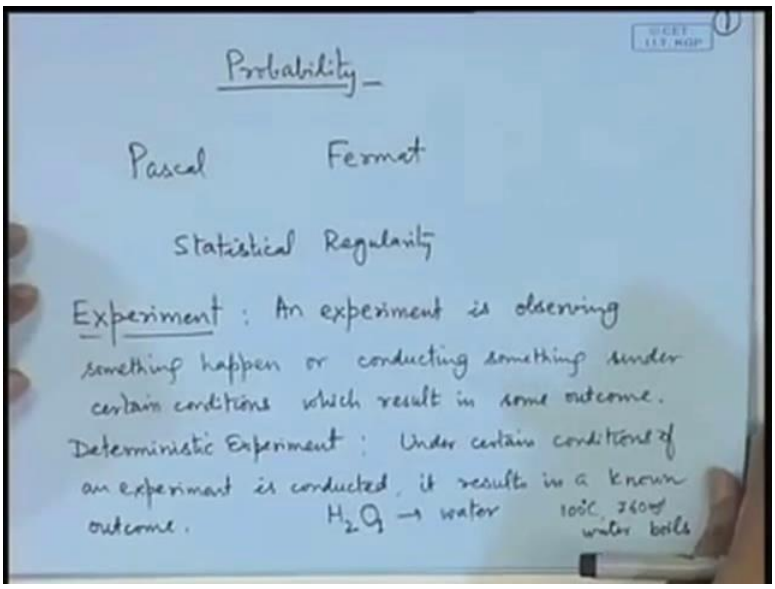


So, they contacted some of the prominent mathematicians of that time say Pascal and Fermat and through the correspondence between this mathematician the theory of probabilities are get to developed. One of the fundamental features of this probabilities that the phenomena which we are interested in r random in nature. So, for example, if you consider tossing of dies, then we do not know whether which face we are in number 1 2 3 etcetera will come, but in the long run if you toss a enough number of times then the proportion of the number of occurrences of say one of them say 6 will be 1 by 6, then there it mean is that the probability are the chance of appearance of 6 is 1 by 6.

Similarly, in the tossing of a coin, we do know that at each trail whether we will get a head or a tail, but if we toss a large number of times then we know that nearly 50 percent of the times there will be a head or 50 percent of the time there will be tail. This long term behavior is known as a statistical regularity and that is what encourages us to a study the subset probability. A similar kind of observation you can make in the experiments which are connected with the real life such as experiments in physics, experiments in genetics or virtually any phenomena in real life; considered for example, birth of a (Refer Time: 03:05). Now, suppose we considered human beings, then for each birth we do not know whether a the child will be a boy or a girl, but in the long run it is very well known what percentage of children will be boys and what percentage will be girls.

An insurance company while promoting a new policy would like to know how many of the improving survives up to the age of maturity. For example, if the policy matured at the age of 60, then it would like to know the percentage of people in the target group, who will be surviving beyond the age 60 and therefore, may get the benefits which are due to them. Now, for individual person is not possible to tell whether he will die at the age of 60 or not, but in the whole population on can tell the percentage of people dying before 60 or dying after the age of 60.

A similar kind of statistical regularity is observed and it is used for weather predictions, the prediction of the say growth of crop, the economic growth, the financial situation of a country etcetera. Here what happens that a most of these cases although the things may look that they are pre (Refer Time: 04:31) or pre design, but predetermined, but actually there will be several conditions which regulate the occurrence of the final phenomena, and therefore one can treat them as random phenomena.

Now, we will introduce some of the basic concepts, the first is experiment; the term which I have used repeatedly just in the discussion. So, an experiment is observing something happen or conducting something under certain conditions, which result in some outcome, let me explain this little wave definition. So, considered say rain fall. So, now, in rainfall is a consequence of several things, finally we observed that there is a rain fall. So, there is a cloud formation, there is some leno occurrence, there is a humidity, there are various factors which we need to there is a rain fall or there it is cloudy or it may not rain at all it may rain in some other reason.

Now, observing of this weather is a experiment. Similarly suppose we consider how much crop of a particular or how much yield of a particular crop say weight is there in a particular in a particular filed, now this is dependent upon the seeds the plot of the land where it is the irrigation procedure and other mechanical procedure which are used for forming. So, the entire process although we are not conducting, but this happening and it is a random experiment the outcome is recorded as the final yield of the crop. You must be very well familiar with lot of experiments, which are done in physical, chemical and biological sciences for example, we have various experiment in chemistry, were certain chemicals are mixed and they result in some compound being made.

So, broadly speaking we segregate the experiment into 2 types of a experiments: one is deterministic experiment; in the deterministic experiments under certain conditions if an experiment is conducted it results in a known outcome. So, many of the class room experiments in physics, chemistry, biology etcetera they are like this for example, if I have 2 molecules of hydrogen and the molecule of oxygen then we know that outcome is a water. Suppose we considered say we put water in a vessel and heated, then the temperature reaches 100 degree Celsius and that (Refer Time: 08:44) pressure say 700 mg then the outcome is that the water will boil.

So, these experiments are called deterministic experiments; however, we are not concerned with these experiments in the subset of probability, we are concerned with the experiments, which are called random experiments

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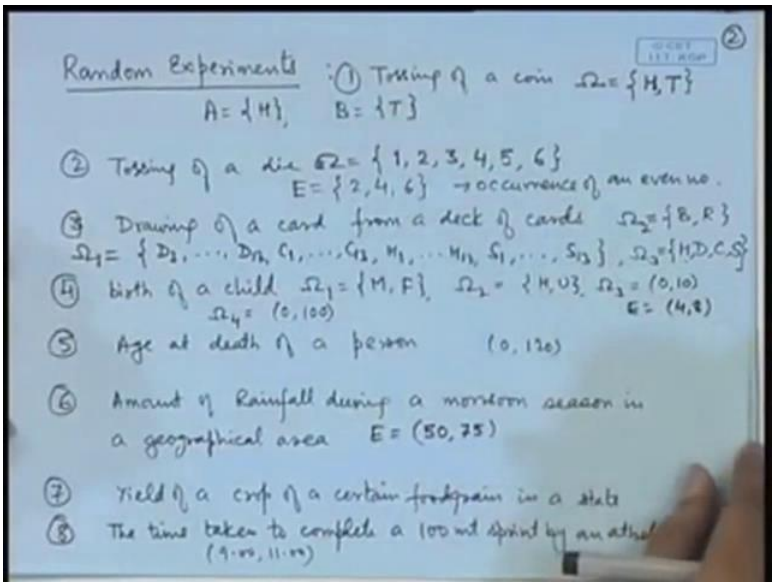


In the random experiments although we made fix the conditions under which the trails are conducted or the experiments is conducted, but the outcome is still uncertain. Considered for example, tossing of a coin.

So, although we may fix lot of conditions such as what kind of coin we are having how to hold it when you are tossing, but even then when we toss the coin and it calls the outcome is uncertain; it may be head or tail are in extreme situation, we may considered that is coins on which side also. Considers a tossing of a die; so again the condition are similar; it may fix the die in various ways, but when we toss it and if we are really tossing it then after falling in which face will be the upwards is not known.

Suppose we consider drawing of a card from a deck of cards; suppose we considers a birth of a child; suppose we considered age at death of a person; suppose we consider say amount of rainfall during a monsoon season in a geographical area; suppose we want to consider yield of a crop of a certain food grain in a state; suppose we consider the time taken to complete a 100 meter sprint by an athlete. All of this phenomena here the conditions of the experiment are fixed.

For example when we look at say the time taken to completed 100 meters sprint by an athlete, then the conditions are fixed for example, the ground is fixed, the starting time is fix, the athletic is in a perfect condition the person who will directories is prepared, the person who will record the time is prepared. However, how much actual time the spender

will take who compute the 100 meter race will always be uncertain; it may be 10 seconds it may be 10.1 second, it may be 9.7 seconds etcetera.

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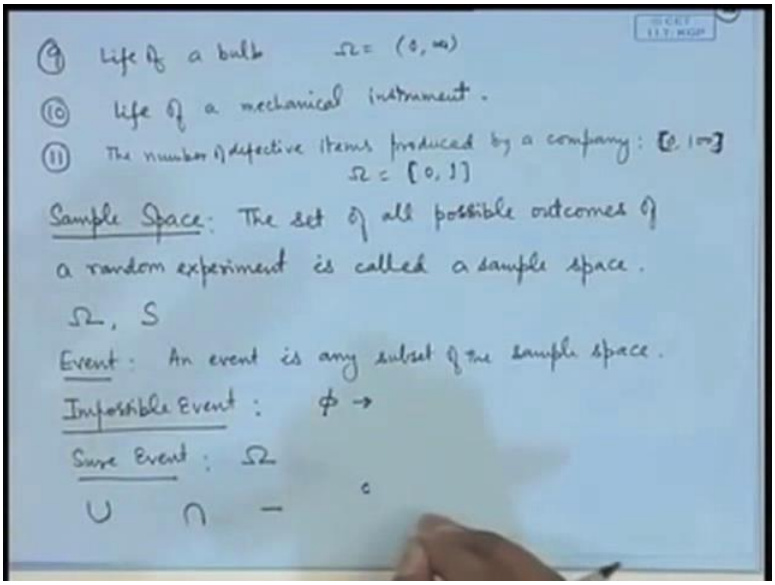


Suppose we are considering say a mechanical instrument such as a life of a bulb. So, when you purchase a bulb from the market and you light it then it may work for 1 hour, it may work for 10 hours, it may work for 100 hours.

So, although all these bulbs may be produced by the same company under the same conditions, even then the actual life of the bulb is not fixed it can vary. If you look at say working time of a say life of a mechanical instrument. So for example a certain turbine or a certain engine; although they are all produced by certain a standard process, but the actual life of that instrument will not be cannot be predicted in advance, we may consider say. So, in these cases all of these examples they relate to certain fix conditions for the conducting of the experiment. However, the final outcome is not known in the advance. So, all of these are known as random experiments and in the subject of probability we are concerned only with the discussion of the random experiments.

So, for example, if we why it is type of events are of interest for example, you considered birth of a child or age or death of a person. So, now, these phenomena are extremely useful in particle. For example, insurance companies when they propagate a life insurance policy, they are very much interested that what premium they have to fix. Now how to decide about the premium? The company charges a premium and in the case

of unlikely case of the person dyeing, before the age of maturity he has to be paid full benefits of the policy and plus some assured sum. Whereas, if the person completes the policy; that means, he does not circum before the age of maturity, then he face the premium till the maturity and then he gets certain benefit which are not that much as much as he would have got the person had and died before the age of maturity.

Therefore the company has to estimate how much premium it will be charging and how much will be actually the cost to the company in the event of the date of the person pre maturely. So, the age at death of a person in the target group is to be estimated, and therefore we will records and we keep the records of the ages of the persons in that particular target group, for which the companies trying to sell the insurance policy.

If you look at the amount of rainfall then extremely important phenomena because there are lot of policies of the government; the policy agricultural policy, the economic policy, which are based on actual rainfall which is going to be there in the country; the yield of a crop makes the government to be decide about how much ford grain they are going to purchase from the former, how much they are going to a store, what should be the price which should be given by the formers, what should be the price for the market.

So, all of these events the experiments although they do not look random beforehand; however, outcomes are not known and therefore, they are random and in the subject of probability and statistic we do study this type of phenomena. So, now, let me introduce the certain basic terminologies of the random experiments. The first of this is the concept of a sample space: the set of all possible outcomes of a random experiment is called a sample space, the usual notation we will use either capital omega or capital S etcetera could be note the sample space.

So, let us considered some examples suppose we are considering tossing of a coin; in the tossing of a coin we may be put the outcomes we may considered as head or tail, and if we denote by H the occurrence of head upwards or by T the occurrence of tail then the sample space can be describe as H T. If we considered tossing of a die, then the sample space we may describe as occurrence of the face upper most, so the sample space will be the set of the numbers 1, 2, 3, 4, 5, 6.

If we are considering drawing of a card from a deck of cards, now in a deck of cards where are 4 denominations: heart, club, spade and diamond and each card as a value 1 to

13. So, if we considered drawing of a cards then it will be consist of the set for example, diamond 1 to diamond 13, club 1 to club 13, heart 1 to heart 13 or a spade 1 to spade 13. So, the sample space consists of 52 points. If we look at we may also observe the sample expression and we like for example, you may only cover the color of that. So, in that case we may describe it differently we may call this one as omega 1 and if I recording only the colors then it may be say black or red or we may only record the denomination that is whether it is a heart or whether it is a diamond or it is a club or it is a spade.

So, the sample express will consist of only 4 points, this also shows that sample space is not a unique thing; in a given experiment what we are interested in will decide that what is a sample space. If you are looking at the birth of a child then it is a various statement or you can say where we have describing what is the random experiment. Now we made a call whether the child is a male or a female child, we made a called whether the child born has is healthy or not healthy. So, we may put it as healthy or unhealthy; we may look at his body weight at the birth, so the body weight may be some number is starting from say 0 to say may be 10 pounds, it may be the total life of the child. So, in that cases may be something like say 0 to 100.

So, it depends upon that what is our actual interest and we can write the sample space accordingly age at death of a person, so this may be you say 0 to say may be 100, 120 keep in to a counted there are some people who leave very long, there this unit of the time is years, amount of rainfall may be recorded in say centimeters, yield of a crop may be recorded in some metric tons, the time taken to complete a 100 meter sprint may be your time from say a 9 seconds to say 11 seconds about I am considering on international filed; life of a bulb it is a number say which is 0 to infinity although theoretically speaking it is not infinity, but it can be a large number life of mechanical instruments in the either you can do described; suppose we are looking at the number of defective items produced by a company. A particular kind of items you are looking at; suppose we are looking at certain boards then what will happen that? We will may define the defectives that if they do not conformed to certain describes standard of measurement.

So, now the number of effectives may be recorded in terms of percentage. So, the percentage can be say 0 to 100 percent or it could be say proportion in that case we may write the number as 0 to 1. So, the sample space will again be dependent upon the way we want to look at it. Next we define what is an event? An event is any subset of the

sample space; now this is a very broad definition and therefore, any subset of the sample space is qualifies to be an event, to be called an event as far as the probability theory is concerned.

So, let us look at the experiments now and the examples that we have already done. In the first case if you look at omega is equal to H T, we may consider a subset as consist of only H, we may consider as subset consisting of say only T. So, the set A denotes that head has occurred; the set B denotes that T has occurred so these are events; we may consider say the set E in the case of tossing of a die, we may write 2, 4, 6 this means occurrence of an event number; in the birth of a child suppose we are looking at say weight and birth in pounds and suppose we say E is equal to 4 to 8 then it means that the birth weight of the child is 4 to 8 pounds. If we look at the amount of rainfall an in centimeter during the particulars monsoon season and we may put say 50 to 75; that means, the actual rainfall is between 50 centimeter to 75 centimeters in that geographical area during that particular monsoon season. So, any subset of the sample space can be considered an event.

Now, we have various kind of events for example, impossible event since every subset is a subset of every event is a subset of the sample space therefore, the empty set phi that is a subset of omega therefore, this is will correspond to impossible event. Similarly we have sure event, since omega itself is a subset of omega therefore, this is denoting the sure event for example, we may consider tossing of a die and he say that 7 occurs. So, that will correspond to an impossible event because 7 is not a subset of this and that will correspond to 5 as far as this random experiment is concerned.

Suppose I am looking at the age at death of a person and we say 1000 years, then it is an impossible event. Suppose we say time taken to complete a 100 meter sprint by an athlete and we may put the time as say 5 seconds, then in the present circumstances or present age this is an impossible event. Similarly if I put that the time take into complete a 100 meter sprint by an athlete, why did he complete the race is less than 1 minute then this will be a sure event.

If we look at the life of a bulb and we says the positive number, then or a nonnegative number then it is a sure event. So, these are 2 you can say (Refer Time: 25:55) types of events which are possible. However, there are various set theoretic operations like

unions, intersections, differences, complementation, and therefore given any two events when we take their union's intersections differences complementation excreta there must correspond to certain events and we can describe them in the form of probabilistic explanation.

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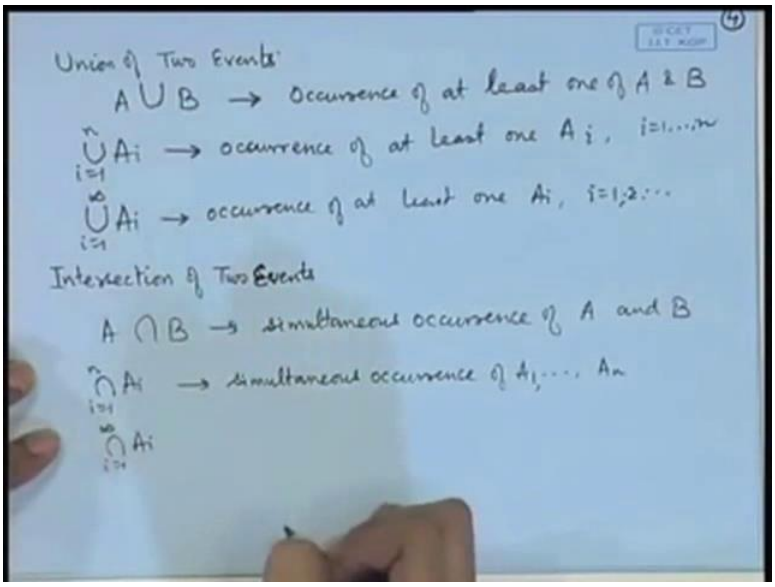


So, for example, union of two events; what does union of two events implies that? If I say A and B are true events, then A union B this will mean occurrence of at least one of A and B; that means, either A occurs or B occurs or both occur. So, in set theoretic representation A and B a union B means that set of aliments which are either in A or B or in both; in probability theory the event A union B will indicate that at least one of A or B as occurred. Similarly we may consider union of a n events (Refer Time: 27:16) to a n then this will be in occurrence of at least one Ai, I is equal to 1 to n; you may even considered an union of finite number of events union Ai, i is equal to 1 to infinity, this will mean occurrence of at least one Ai, i is equal to 1 to infinity; intersection of 2 sets denotes set of all those points which are common to the two sets.

Now in set theory that is the presentation; in probability theory A intersection B will mean the simultaneous occurrence of A and B; that means, both event A and B are being to have occurred, similarly we can considered intersection of n events Ai, i is equal 1 to n; that is simultaneous occurrence of A 1 2 A n; that means, all of the events A 1 A 2 A n

occurs and in a single way intersection of an any countable collection of events A 1, A 2 excreta.

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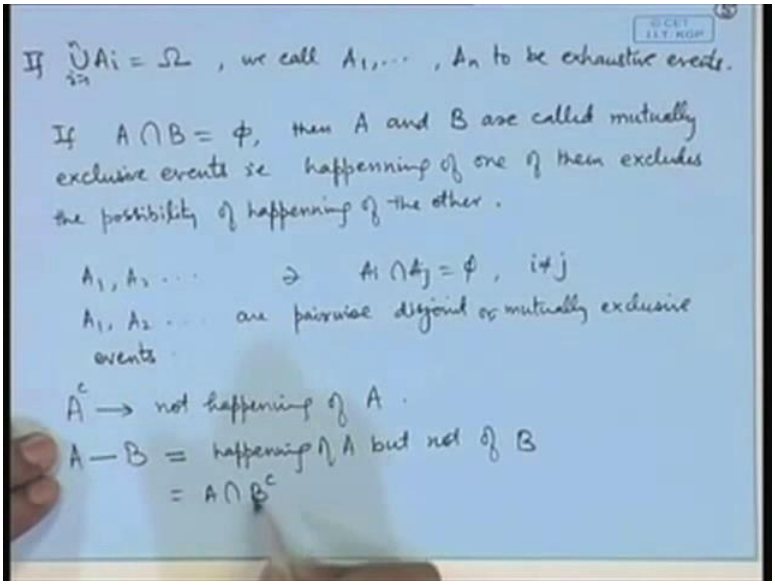


Here when we considered the unions or the intersections certain basic properties are clear. For example, we may have union of A i say i is equal to 1 to n is equal to omega; that means, all the points are of omega are contained in one other of A i's, such events are called exhaustive events. So, if even union of A i is equal to omega we call A 1 to A n to be exhaustive events, here in place of n we may have an infinite collection of events also.

Similarly, if A intersection B is equal to phi; now the set theory is means joint sets. In probability theory A intersection B is equal to phi denotes that the event A and B cannot occur together, they are called mutually exclusive events; A and B are called mutually exclusive events. That means, happening of one of them excludes the possibility of happening of the other.

We may also consider something like this that we have a collection A 1, A 2 etcetera events, such that A i intersection A j is equal to phi, then I is not equal to j; then we say that A 1, A 2 excreta are pair wise disjoint or mutually exclusive events; given an event A, the A complement will denote not happening of A.

In a similar way if I have event A and an event B then A minus B will denote happening of A but not of B. This is true because this is equal to A intersection B complement; that means, simultaneous occurrence of event A and B which means that A occurs and B does not occur. So, it is a simultaneous occurrence of A and B complement, and it is translated to occurrence of A, but no occurrence of B.