Functional Analysis.

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Module No. # 01

Lecture No. # 27

Tutorial - III

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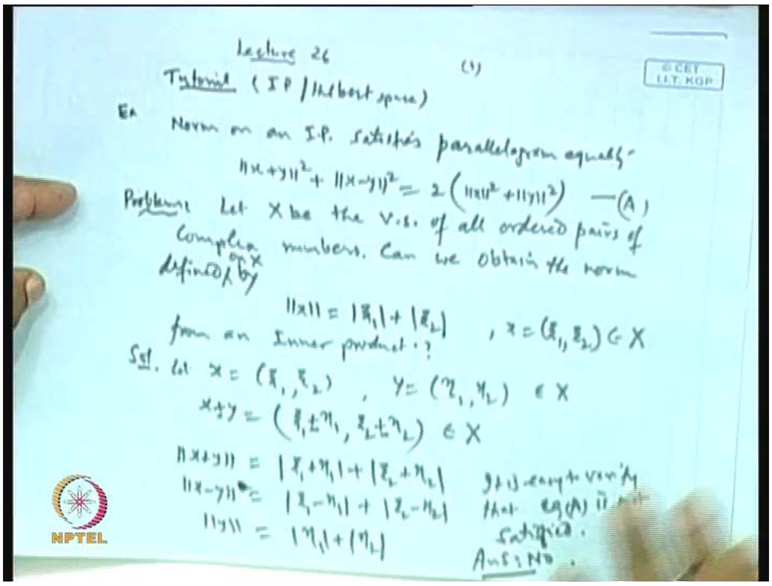

Problems based on, tutorial class, based on the inner product space, Hilbert space and the corresponding functionals defined on the Hilbert spaces. So, let us take to this problem, very basic problem. We know the norm on a inner product will always satisfy the equality, that is known as the parallelogram law. So, norm on the inner product, on an inner product, product space, satisfy the important parallelogram equality, satisfy parallelogram equality; that is, norm of x plus y whole square plus norm of x minus y whole square is two times norm of x square plus norm of y square, clear; and, that is what, norm. From here, if a norm does not satisfy this parallelogram equality, it means, it cannot be derived from the…Inner product Inner product.

For example, the c a b, which we have seen that, this c a b does not satisfy this parallelogram law, that, therefore, it is not a, cannot be derived from the inner. So, similar type problems here is, the problem is, let x be the vector space, vector space of all ordered pairs of complex numbers, of complex numbers. The question is, can we obtain the norm defined by, defined on x by y. The expression norm x is, mod x i 1 plus mod x i 2, where the x is x i 1, x i 2; this belongs to capital x, from an inner product. This is the question. The solution is, let us see, whether this satisfy the inner product or not; this, this norm satisfy the parallelogram law or not, equality. So, what is our… x is x i 1, x i 2 and let y be our eta 1 Eta 2.

Then, both are in X. So, x plus y will be, x i 1 eta 1, x i 2 eta 2 and x minus y will also be like this and they will also be the point of X. Now, the norm of x plus y, this is defined in this fashion. mod of first term plus mod of second term. Similarly, the norm of x minus y whole square is defined as mod of first term plus mod of second term.

Sir, it is not whole square?

Where, whole, it is not whole square, norm of this, ok. So, norm of y, from here, is defined as mod of eta 1, plus mod of eta 2. Now, if we substitute these values in the expression 1 a, then, we see immediately, this, this square will be this square plus this square plus two times this. So, as soon as you remove this square, you get less than equal to sign, ok. So, this cannot be equal to the two times of norm of x square plus norm… So, obviously, it is easy to verify that, equation a is not satisfied, is it clear or not. One can easily follow. So, answer is, no, clear. That is the one, is it clear or not? You just expand it and that… Even if you square, you are not able to get this equality sign and then, both things will…

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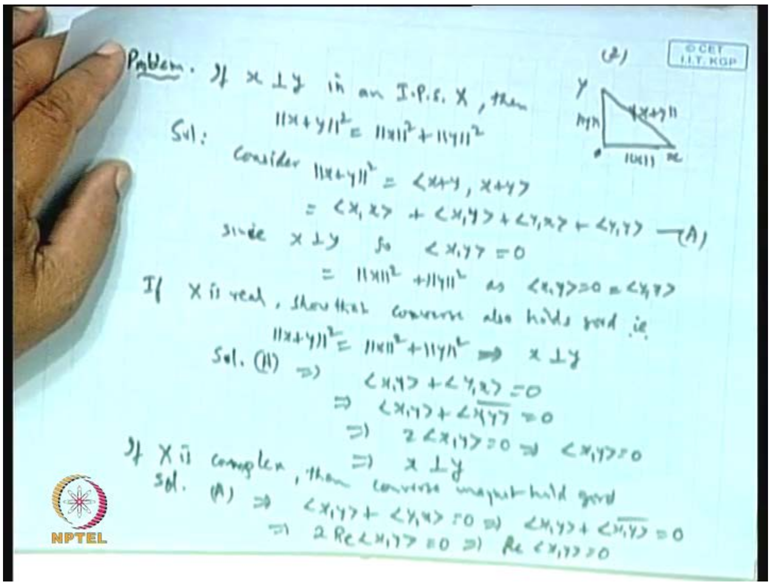

Now, another thing, we know the Pythagoras, if in case of the real, if the triangle is right angle triangle, then, the sides, this is the x, this is y; so, this side length is norm x; this side length is norm y; and, this side length is norm of x plus y. Pythagoras theorem says the square of this is the same as sum of the square of these two sides. In fact, this can be immediately proved, with the help of the concept of the inner product. So, the question is, if x is perpendicular to y in an inner product space X; this is a generalized formation, generalized thing over the Pythagoras, then, norm of x plus y whole square equal to norm of x square plus norm of y square. And, this can be extended to the n mutually orthogonal vectors, because these two vectors, (( )) by orthogonal, two vectors, and one can prove. In fact, the solution is very simple. Just you, if we go for the solution, what you do is, you start with the left hand side.

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Norm of x plus y whole square. So, this can be written as x plus y, x plus y and one can easily obtained as x comma x x comma y plus y comma x plus y comma y. Now, since x is perpendicular to y, so, inner product of x y as well as inner product of y x will be 0. Therefore, from here, you are getting, norm of x square plus norm of y square, as others part is 0. Is it ok, now? So, there is nothing to prove. Now, the interesting problem is, if x is real; that is real vector space, then, so that, conversely, the given relation implies, so that, converse also holds good; that is, norm of x is, x plus y whole square equal to norm x square plus norm y square is given. Then, it will imply, the x will be orthogonal to y; that is, norm of x plus y whole square equal to norm x square plus norm y square will implies, x is orthogonal to y, if x is real.

The reason is, because, if we use this one expansion, from here, norm of x plus y square is given from here. So, let us take this expression as A. So, if we look from A, what you get, will implies, the inner product of x y plus inner product of y x is 0, is it not; because, this is norm x square, this is norm y square; above norm of x plus y whole square is given to be this. So, this sum must be 0. But, this is the same as, x y plus the conjugate of x y is 0; but since the X space is real, so, inner product will be same as this. So, this will be the same as two times inner product x y 0; that is, inner product of x y is 0. So, this implies, x is orthogonal to y, ok. So, in case of the real, these two; but if x is complex, if x is complex, then, this may not hold; then, converse may not hold good, why?

Sir, this inner product of x comma y will become some complex number…

Because, because the reason is, from A, if I take, then, what you get it is, inner product of x y plus inner product of y x is 0. And, that implies, inner product of x y plus inner product of x y conjugate is 0; and, that gives the two times real part of the inner product x y is 0. It means, real part of the inner product x y is 0, does not mean the inner product x y is 0; because complex part may not be 0. So, this will not show the x is orthogonal to y, clear. So, complex part may not be 0. Therefore, so, therefore, the x may not be perpendicular to y; that is why, these are reason. Now, this…

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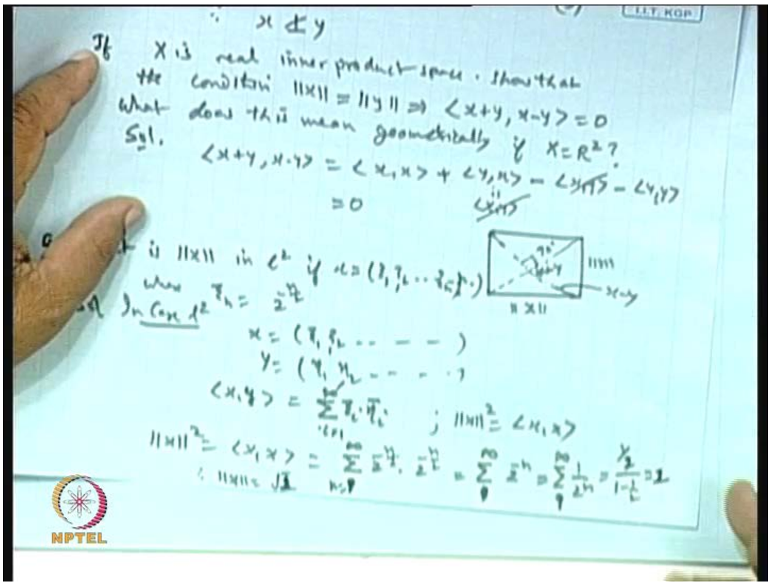

If an inner product space X is here, show that, norm x plus norm y implies the inner product is 0, what does it mean by this? This is, similarly, if, same problem, I am continuing the same problem. If X is an inner product, real inner product space, inner product space, then, show that, the condition, condition norm of x equal to norm of y equal, implies the inner product of x plus y comma x minus y is 0. And, what is the geometrical significance of this? And, what does this mean geometrically? If x is R, R 2, what does it mean by this? So, solution is simple, because, this is given. So, what is that…Start with this. Norm inner product of x y comma x minus y; this can be written as x x plus inner product of y x, then, minus inner product of x y and then, minus inner product of y y, is it not. And, this will be, since it is real, x is real, so, basically, these two will be identical. So, this is the same as inner product of x y. So, we are getting, these cancel here; this is equal to this. So, total value is equal to be 0, ok.

Now, what does it mean geometrically? If I take, this is our square, then, this is the length is norm x; this length is norm y. So, square, both are equal; it means, x plus y vector is this; x minus y vector is this, is it not. The inner product of these two vector is 0 and that is obvious; it means, the diagonal of the square are at right angle to each other, is it not. So, this shows, the diagonal are at 90 degree. So, this is the geometric ((meaning)); that is all. Then, let us come to…

Sir, is it true for higher dimension also?

In case of the (( )). If your inner product, this is ok, this, we have nine, as such, there are, these cases I do not know, whether it is a…Once you know the inner product, norm is obtained with the help of the inner product. Then, we can also find out the corresponding inner product, with the help of the norm. And, that identity is known as the polarization identity; that we have discussed. So, this is no point of (( )). Let us take this problem. Question is, what is the norm x, in, if x is this one? What is norm of x, in this, in l 2, if x is, x is x i 1, x i 2, x i n, x i n, where x i n is 2 to the power minus n by 2. You know the l 2 norm.

Yes, sir.

The l 2 norm, in case of l 2…

Sir, x is equal to x i 1, x i 2, upto x i n (( )).

No, no, up to infinite, infinite term. So, in l 2 norm, if x is x i 1, x i 2 and so on, y is eta 1, eta 2 and so on, then, inner product of x y is defined as sigma, sigma x i i eta i, i is 1 to infinity and conjugate of this, x i i eta y and when it is finite dimensional, then, we say simply, i equal to 1 to n, is it not; norm is ((left)) ok. Now, here, x i is given to be this. So, what will be the corresponding here, and norm of x square is the inner product of x x, is it not. So, here, x is given x i 1 is given to be this; so, conjugate of this will be the same as this, because it is a real. So, what you are getting is, norm of x square is the inner product of x x; that is sigma of x i i eta i. So, x i i is 2 to the power minus n by 2 into 2 to the power minus n by 2, n is 0 to infinity; because, conjugate of this is the same as this. So, this will be equal to sigma, 0 to infinity, 2 to the power minus n; and, that will be equal to what, sigma 1 upon 2 n, 0 to infinity, a geometric series, whose sum will be 1 over 1 minus half; it means, this is nothing, but the 2. So, this will be equal to 2. So, what is norm x, is under root 2. Therefore, norm of x will be root 2, clear? Just apply the definition, you get this; then, we get…

Sir, sir, in the definition we have, i is equal to 1 to infinity; then, you are telling…

It no problem; i equal to 0 to infinity or i equal to 1 to infinity, it makes no difference. If you take n is equal to 1 to infinity, then, what is the difference will come here; 1 to infinity, here it will come half and then, 1 minus half. So, total value will come out to be 1; that is all.

(( ))

1.

1.

That is all, 1. So, here it may be 1; it depends on, from where you are taking. So, if it is 1 to infinity, then, it will be half, you know, that is correct, clear. So, a over 1 minus r and 1 minus… So, 1. So, it is ok, clear. So, this, this one. So, this will be…

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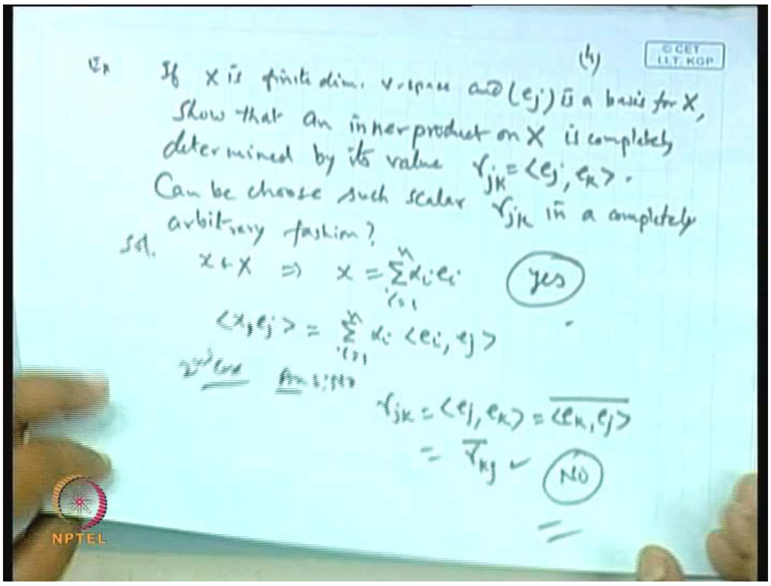

Now, this question, if x is a finite dimensional, finite dimensional vector space and e j is a basis, e j is a basis for X, for X, then, show that, show that, an inner product on X, inner product on X is completely determined, determined by its value, by its value, gamma j k, which is equal to inner product of e j e k. Can we choose such scalars, choose such scalars, can we choose such a scalars gamma j k, in a complete arbitrary fashion, completely arbitrary fashion, arbitrary fashion? X is a finite dimensional vector space and e j is a basis for X. Show that, the inner product on X is completely determined by its value, this one.

Now, X is finite dimensional vector space. So, any element of this, if x belongs to capital X, it can be expressed, x can be expressed in the form of alpha i e i, i is 1 to n, clear; now, where alpha 1, alpha 2, alpha n are suitably chosen, so that, you get x. Now, since this is inner product also, in an inner product, x is also inner product, so, we can take x e j; if I calculate this x e j, then, what you get is, this is equal to sigma i equal to 1 to n, alpha i inner product of e i e j, ok.

These only get, only the alpha, alpha i naught.

This is a alpha. So, what you get from here is that, this alpha i can be obtained.

From this inner product of this one.

From inner product. If these are the basis element and orthogonal complement, so, we can get the corresponding basis for this. So, we can find out the alpha i in terms of the basis elements. Now, this alpha i is nothing, but the gamma j I, you can find out. So, each element x, we can find out in terms of the gamma j i. Now, this, it means, this can be determined with the help of gamma j i. Now, this is not the answer. So, answer is yes, first case. The second case, the answer is no, why? Why it is no, because, what is our gamma j k? Gamma j k is e j e k. Now, this will be same as e k e j conjugate, inner product; but this is basically what, gamma k j conjugate. It means, we can choose that type of gamma only, whose gamma j k is the same as the conjugate of the gamma k j; we cannot too choose arbitrary the constants. This condition must be satisfied. Gamma j k must be equal to gamma bar k j; otherwise, we cannot. So, that is why, the answer is no. So, that is all. Clear?

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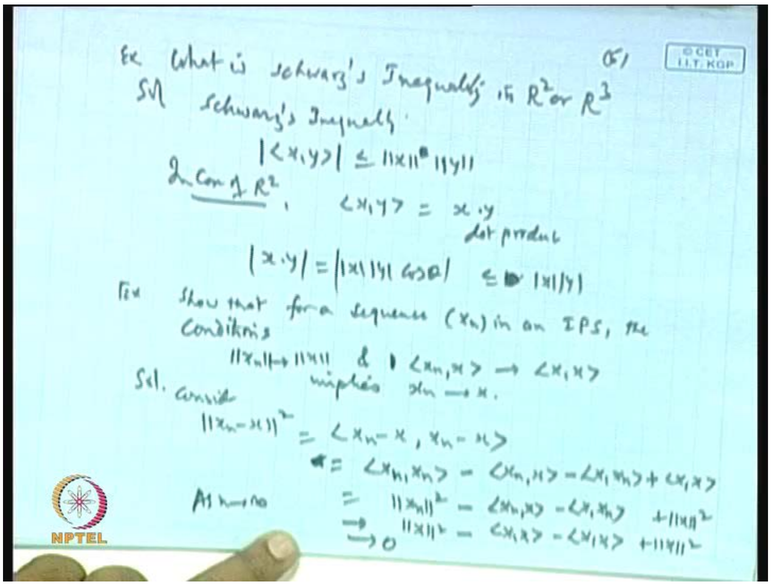

Come to, another question, what is the Schwarz inequality in R 2 or R 3? Can you tell me? What is the Schwarz inequality in R 2 or R 3?

Sir, we can start (( )).

Yes. So, Schwarz inequality is, what is the Schwarz inequality?

(( ))

Schwarz inequality says, modulus of the inner product x y is less than equal to norm of x square, norm of x into norm of y. This is the Schwarz inequality. Now, in case of R 2, in case of R 2, the inner product x y is nothing, but the dot product of this. This is the dot product. And, we know this dot product x dot y is mod x mod y cosine of theta, is it not. So, we can say, the modulus of the inner product x y is less than equal to modulus of this, which is less than equal to mod x into mod y, is it ok or not. So, because, mod cos theta will be less, bounded by 1. So, that is one. So, in case of this R 2, the inner product, Schwarz inequality is not, is nothing, but the dot product, mod of dot x y is less than equal to mod x plus mod y; that is what it is. Then, let us take this problem.

Sir, here in case of R 2, it is based, R 2 or R 3 is based, (( )) to find out some type of theta (( )). In higher dimension, we can define the angle from the Schwarz inequality.

No, we can define the, not, what we can do for the higher order, find out the dot product, divide by length of the vector x and length of the vector y; that, you will get the cos theta.

(( )) inner product of x comma y, (( )) higher dimension (( )) divided by norm x norm y…

Yes, length of x…

(( )) is that one.

Yes, no, it will be the cosine theta; it will remain less than equal to 1.

Less than, less than or equal to 1, means, you can define, it is some type of the function of either cos theta or sin theta. So, how then, you can find out (( ))…

Show that the sequence x n, this, show that for a sequence x n in an inner product space, in an inner product space, the conditions, the conditions norm x n goes to norm x and inner product x n x goes to the inner product x x implies, implies x n goes to x, ok. So, let us see. It is given that, norm of the length x n goes to norm of x and inner product x n x goes to x x; then, you have to prove x n converges to x in the…Of course, the norm defined by the inner product. So, what we do is, we consider this thing, norm of x n minus x, ok. Let us take the square. So, what you get is…Then, you write this thing, is equal to inner product of x n comma x n, then, minus inner product of x n x minus inner product of x n x n and then, plus inner product of x x, is it not; that is what we go for this, achcha.

Now, this is nothing, but the norm x n square. This is nothing, but the norm x square and this will remain as it is, let it be. Now, it is given that, x n x goes to x x. So, as n tends to infinity, this will go to norm x square, is it not; this will go to x x; this will go to x x and this will be same as norm x square. So, will it not go to 0?

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Sorry, is it not; because this is norm x n square; this is x n x and this is norm. Now, as n tends to infinity, this will go to the norm x square; then, x n x goes to x x, because of this given condition; this goes to… So, basically, this 2 x square minus two times of the inner product x is 2 x square. So, both will go to 0. It means, when n tends to infinity, this x n converges to x in the norm defined by the inner. So, this shows that, therefore, x n goes to x in the norm. Proved.

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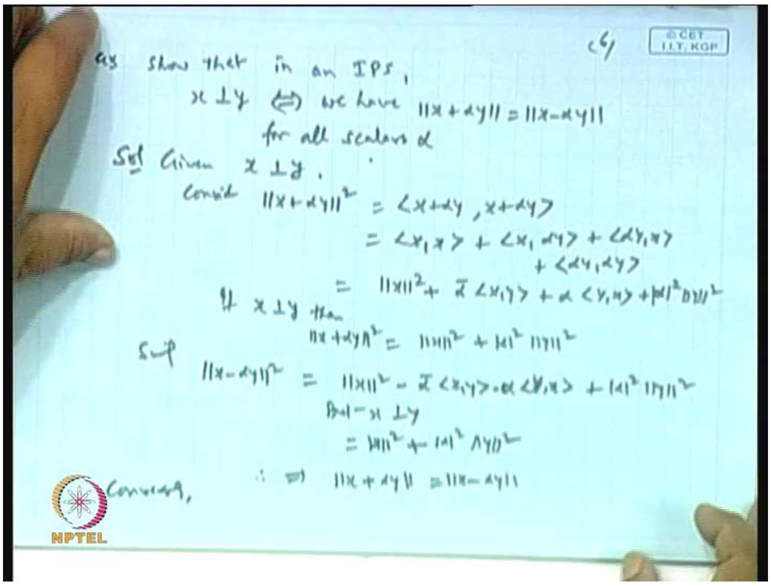

Show that, in an inner product space, show that, in an inner product space, x is orthogonal to y, if and only if, if and only if, we have, we have x plus, norm of x plus alpha y equal to norm of x minus alpha y, for all scalars, for all scalars alpha. This is, is it ok. Now, let us see the solution. Let us, given x is orthogonal to y; x is orthogonal to y. Now, we want to show this result. So, consider, and given, nice, given this. We wanted to show, this is equal to this. So, consider, norm of x plus alpha y whole square; this is equal to what, and this will be equal to x x plus x alpha y plus alpha y x plus alpha y alpha y. Now, this is equal to what, norm x square plus alpha conjugate x y plus alpha times y x plus alpha alpha conjugate, so, mod alpha square norm y square.

Now, if x is orthogonal to y, then, this inner product will be 0, this inner. So, we get, this equal to norm of x square plus mod alpha square norm y square, ok. Now, if, similarly, we can go for this, what is the difference will come here? If I open it from here, you will get first term norm x square; there is no problem. Only thing, difference come here is, here minus alpha y and then, you will get somewhere here also, minus alpha 1 minus alpha. So, that will be positive. So, there will not be difference; only the difference will come here with the minus sign; that is all. But x y is perpendicular; so, this will go to 0, this will go to 0. In fact, what you are getting is, minus alpha bar minus alpha plus mod alpha square; but x is orthogonal to y, given; therefore, this is equal to… So, both are equal; therefore, this implies…Clear? Now, let us see the converse part; if an only part is there.

Yes, (( )) it is given (( )).

If this is equal, it means, these two things are equal; then, we have to show.

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Yes. So, basically, when you equate these two…

So, these two cancels.

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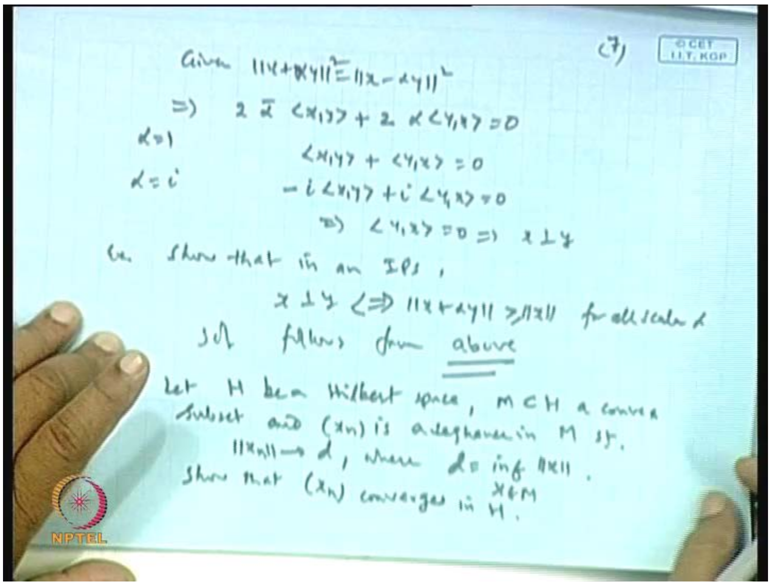

So, these two gets cancelled. Only thing, you are getting the alpha bar plus of this thing. So, conversely, conversely, what you are getting is, conversely, given, norm of x plus alpha y equal to norm of x minus alpha y, is it not. So, square of this, square of this and that will lead to, the two times alpha bar, inner product of x y plus two times of alpha inner product of y x is 0, ok. Now, if, 2 is gets cancelled. Now, this is given to be, x be any inner product space, for this, we can find, x be any inner... So, suppose, I take alpha to be 1. What you get? You are getting, inner product of x y plus inner product of y x is 0; if alpha is equal to y, then, what you get it, minus i inner product of x y plus i inner product of y x is 0; that is from here, if I add, i minus i gets cancelled.

So, basically, what you are getting, inner product of y x is 0; that is x is orthogonal to y; that is what. Clear? That is what. Or otherwise also, we can say, because this is the conjugate of this, if this is the conjugate of this. So, two times the real part of this will be 0. If alpha is real, then, basically, the inner product is coming, which is 0. So, inner product means, x is perpendicular to y. So, that is why you are getting, ok. Now, this will also leads one another problem, that is, show that, in an inner product space, inner product space, x is orthogonal to y, if and only if, norm of x plus alpha y is greater than equal to norm of x, for all scalars alpha.

Now, this immediately comes from here. What is the solution? Norm of x plus y alpha whole square is nothing, but this. Now, whether alpha is complex or real, mod alpha square will be a real quantity, positive quantity. So, this will always be greater than equal to norm x square. So, basically, it follows, solution follows from above, is it ok or not. Need not to bother about, ok. Now, in this, 0 operator, we have already discussed. So, need not to go for this. Now, this another problem. Let H be a Hilbert space, H be a Hilbert space; M is a subset of H, M is a subset of H, a convex subset, convex subset. Then, and, x n is a sequence in M, sequence in M, such that, norm of x n goes to d, where d is the infimum of norm x, when x belongs to M, x belongs to m.

Now, show that, show that, x n converges, sequence x n converges in H. Now, let me… What is this is? H be a Hilbert space; means, complete inner product space. M is a subset of H, a convex subset and x n is a Cauchy's, is a sequence in M, such that, norm of x n goes to d, where d is the infimum of norm x; then, show that, x n converges in H. So, how to prove this? We know, every Cauchy sequence, if, if a space is complete, the definition say, every Cauchy sequence must be convergent. So, if I prove that, x n is a Cauchy in H and since H is a Hilbert space, it, so, it must be a complete space. So, our aim is to show only the Cauchyness of this x n. So, let us start with this Cauchyness and we get from here is…

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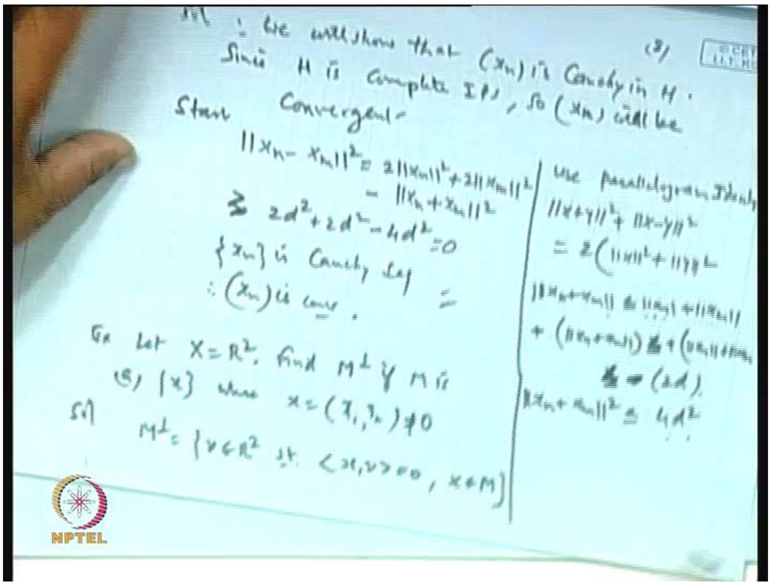

So, let us take... So, what we do is, we show, we will show that, x n is Cauchy in H and since H is complete inner product space, so, x n will be convergent, is it not. This is our ((trick)). So, for the Cauchyness, start with this. Cauchyness means, difference between any two arbitrary terms of the sequence must be, go to 0, when n m sufficiently lost. So, consider norm of x n minus x m whole square. Now, here, I use parallelogram law. What is the parallelogram identity? Norm of x plus y whole square plus norm of x minus y whole square is two times norm of x square plus norm of y square. So, use this identity. So, this will be equal to two times norm of x n square two times norm of x m square and then, minus times norm of x n plus x m whole square, is it ok or not.

Now, this will be, what is given is, now… This one is given, norm of x n goes to d, where d is the infimum of norm x. It means, d, if it is infimum norm x, then, d must be less than equal to norm x, for any x; because it is the infimum value. So, let us take this. The norm of x n will go to d, this will go to d, is it not; and, this will be, what is this norm of x n plus x, x m, where will it go? So, this is less than equal, this is, sorry, this will go to d square. Let us take, let, I must put the sign later on; 2 d square, then, plus 2 d square and then, this part x n plus x m, this thing, we can write like this - norm of x n plus x m is less then equal to norm of x n plus norm of x m, ok. So, minus times of this, minus times of this, will be greater than equal to minus times of this. So, minus of this is greater than equal to minus times, is it not?

(( ))

This, let us see this, what. This is taken. Now, this part, this is equal to, this is greater than equal to, now x n converges to d. So, it is minus 2 d. So, if we take the square both side, then, what happened is, this square, norm of this (( )), you do not put it, minus sign right now; you just take this plus less than equal to, less than equal to and then, square; otherwise, it will be problem. So, we get, x n plus x m, norm of this square is less than equal to 4 d square. Now, you put the minus sign. So, you are getting this from here is, this is minus 4 d square, but this is exactly, yes. So, it is the minus 4 d square, minus 4 d square. Why it is less than equal to is coming? Will it be less than equal to or not, or greater than?

Sir, if this is, this x n plus x n, this value is less than equal 4 d, then, the minus of this will be greater than equal to…

What is this is, what I did is, parallelogram law, I have used. So, parallelogram law says, in the x plus y whole square x minus y whole square is two times norm x square plus norm y square. So, I have put it that, norm of x n plus y, that is what I am mistaken. This is x n minus x m, will be equal to what, square will be two times this, plus two times this, minus this. So, here I have to put, yes, this is minus of this. And then, I used again, norm of this square, because this goes to d, this goes to d. So, you are taking this norm of this and norm of this and this converges to d. So, this converges to 4 d square. So, minus of this… Do not, I think do not go for this. We will, without even going for this… x n and x n. So, this is 2 d. 2 d means, four d square. So, minus of 4 d square. So, we get, this is greater than equal to. In fact, this one 0, this is 0, is it not? Whether this will be 0? Once it is 0 means, you are getting x n and x m, they are very close to each other, after certain stage.

So, we take, x n sequence is a Cauchy sequence. So, greater than equal to, because, here is minus sign, but why minus is not… If I put it equal to, greater than equal to, then, this will be automatically greater than 0. So, it is less then equal to this part, this part minus this is. So, it is a Cauchy sequence, ok. So, what we see here, that, we are getting this thing as two times of this minus 4 d square; two times of this two time, minus four d square and less than equal to. So, we get it, showed this one. Once it is Cauchy sequence, therefore, x n is convergent. That is all. So, let us see. Then, next is, let x belongs to R 2, x; norm of x n goes to d, d is the infimum of this. So, it is ok. Let x belongs to R 2, the x is equal to R 2. Find M, find M orthogonal, if M is, a, the singleton set x, singleton set x, where x is x i 1, x i 2.

Sir, I think, that sign should be greater than equal to. That sign must be greater than equal to.

This one...

This, here.

Because of this, minus sign, so, minus is greater. So, if, if you get the greater than equal to sign, then, this is 0.

Sir, basically, sir, norm of x n plus x n plus less than equal to 2 d square. If we prove 4 d square, we indicate that, we are subtracting something more. Then, the left hand side will be somewhat higher.

Higher. So, it is greater than equal to this.

Greater than equal to…

Greater than equal to 0. So, this will be, x n minus x m will go to 0, after certain stage. That will be greater. So, we get here. Achcha, now, x is (( )) perpendicular, if M is x, where the x is x 1, x 2, which is not equal to 0, 0. Let us see, what is the m perpendicular.

(( )) perpendicular to the set of M.

M perpendicular is the set of those vector v, belongs to R 2, such that, inner product of x v is 0, where x is a element of M, is it not. Now, M is this set, M is a set x, x i 1, x i 2, which is in R 2. So, what is the inner product here, R 2.

Inner product of…

R 2.

(( ))

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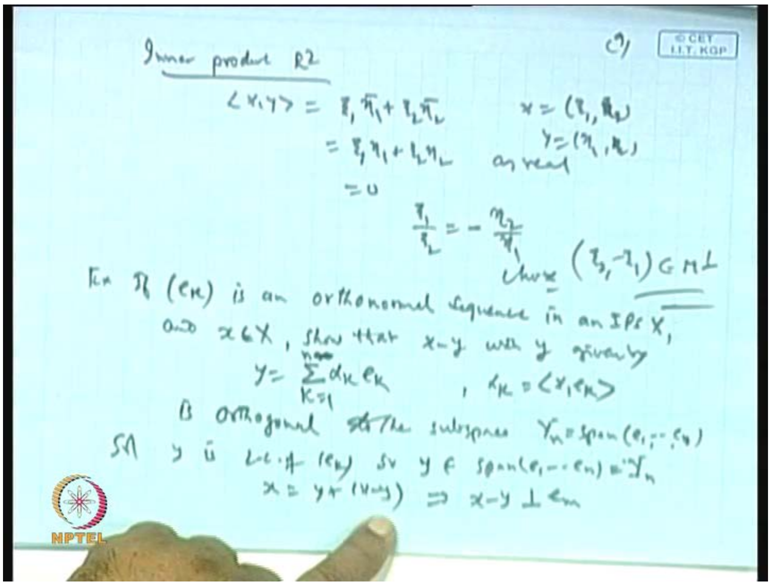

Inner product in R 2, R 2, is x y, is equal to what?

x 1…

x 1 x i 1. If x is x i 1, x i 2, y is eta 1, eta 2, then, inner product will be…

x i 1 eta 1 bar…

x i 1 eta 1 bar plus.

x i 2 eta 2 bar.

But because it is real, so, this inner product will be…

x i 1 (( )).

x i 1 eta 1 plus x i 2 eta 2, as real. Now, this we wanted to be 0. So, it means, what we want is, the x i 1 y x i 2 must be of the form minus eta 2 by eta. It means, even if I reverse the order, then, it will be helpful. Suppose, x i 2, I put it, x i eta 1, I put it to x i 2 and eta 2 is a minus x i 1,then, it may be 0. So, we can take like this, x i 2 minus x i 1, this is the point, if I choose, then, this will satisfy, is it not. This will satisfy. So, that will be the point in M perpendicular, clear. So, this one. So, that will be the point, ok.

Now, next, in case of this continuity, what is the… If, if e k, e k is an orthonormal sequence, normal sequence, is an orthonormal sequence, in an inner product space X, in an inner product space X and x belongs to capital X, then, show that, show that, x minus y, with y given by sigma alpha k e k, k is of course, 1 to infinity, where alpha k is the inner product x e k, x e k is orthogonal, is orthogonal to the subspace, to the subspace Y n, which is the span of e 1, e 2, e n. Let us see. What is, this is x 1, is this… So, how to get it?

Sir, exactly, what you have to find (( )) orthonormal sequence and (( )).

e k is a orthonormal sequence and x be any element of this, so that, x minus y, with y given by this, is orthogonal to the subspace Y n. So, first thing is, what is Y? The Y is, is the linear combination of e ks. So, Y belongs to the span of e 1, e 2, e n, that is, the elements of Y n, is it not.

Sir, (( )) infinite (( )).

Which one infinite, this?

(( ))

1 to n, sorry, this is 1 to n, 1 to n. It is a span of this. Now, any element x, if I take, x can be written as, y plus x minus y. This y is an element of Y n and this x minus y, according to that projection theorem, we can express any element x, as a direct sum of the y and y perpendicular.

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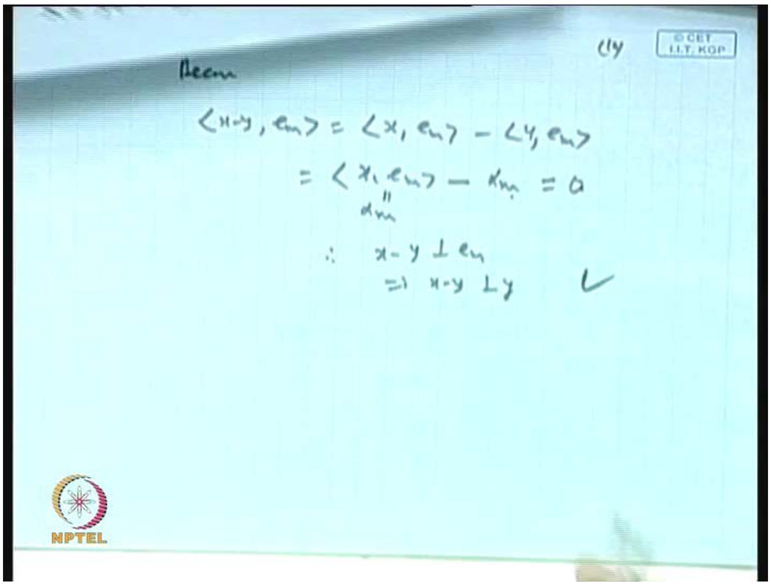

So, basically, this will imply that, x minus y must be orthogonal to this Y n; that is, it must be orthogonal to e m, is it not. And, that can be seen from here, that can be, because, because, what is the inner product of x minus y e m? This is the inner product of x comma e m minus y comma e m, ok. Now, x comma e m is nothing, but what, x minus y minus e m. If I take y, this is the linear combination; when you take the inner product with e m, it is e alpha m only. So, we are getting is, alpha m and this alpha m, x e m minus alpha m, when you take x e m minus alpha m, then, what is x? x is this element. So, inner product of this will be 0, ok.

So, we get, so that, x minus y (( )) by this orthogonal to this space. y it is coming to be x e m minus alpha m, is 0. What is alpha m? Alpha m is this x e m. So, basically, this is also alpha m, is it not? And, this is also alpha m. So, we are getting this. So, this shows, x minus y is orthogonal to e m. So, if it is orthogonal to e m means, it is orthogonal to y, because it is a linear combination. So, it is proved. Thank you. Thanks.