Functional Analysis

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Lecture No. # 25

Hilbert Adjoint Operator

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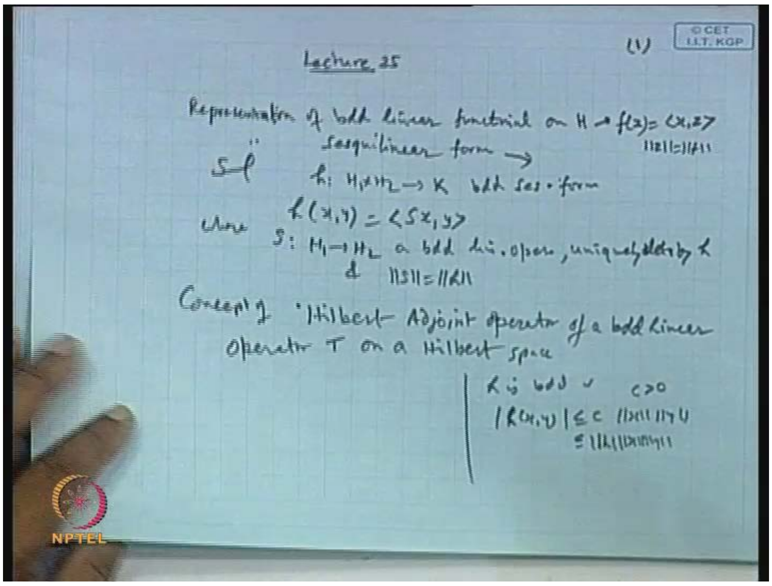

(( )) lecture, we have considered the representation of bounded linear functional on the Hilbert space H as well as representation of sesquilinear form, and we have seen that a bounded linear functional, every bounded linear functional f on a Hilbert space H can be expressed in terms of the inner product x z where z is uniquely retirement by f and norm of z and norm of f is the same. Similarly, a sesquilinear functional we have defined an h from H 1 and H 2 are the two Hilbert spaces, and h is a mapping from the Hilbert space H 1 cross H 2 to K, a bounded sesquilinear form, then h can be expressed as the inner product of s x y, where s is an operator from H 1 to H 2, a bounded linear operator and is uniquely determined by h, uniquely determined by h and have the same norm as the norm of h, is it not?

This we have seen these two represent. Now, these two representation will help you in introducing the concept of bounded Hilbert Adjoint operator, concept of Hilbert Adjoint operators. With the help of these two results, we will introduce the concept of the Hilbert Adjoint operator or Hilbert Adjoint operator of a bounded linear operator of a bounded linear operator T on the Hilbert space on a Hilbert space h. Now, this concept bounded Hilbert Adjoint operator. In fact, comes when we studied the matrices of an linear and differential equations. When we go through the linear differential equation or integral differential equation or the matrices and converted in the form of the operators, then the concept of these Hilbert Adjoint operator comes. In fact, we will see that in case of the finite just like we say the symmetric matrix, and hermitian matrix and all these thing these will be the part of means they can be related with this Hilbert Adjoint operators.

So, today we will discuss in detail, what is the Hilbert Adjoint operator of a bounded linear operator on a Hilbert space h. So, that is the condition.

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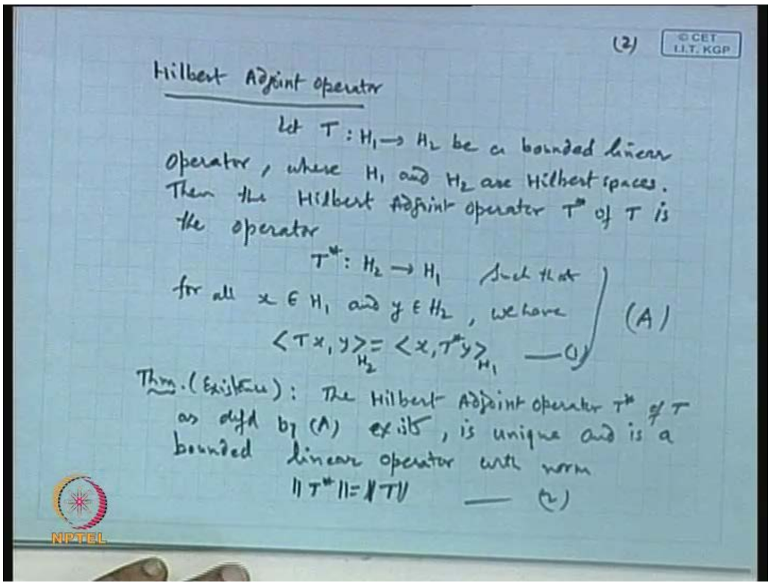

So, let see the Hilbert Adjoint operator. Let us see first we introduce define an Hilbert Adjoint operator h. Let us suppose T be an operator, let T from H 1 to H 2 be a bounded linear operator where H 1 and H 2 are Hilbert spaces, where H 1 and H 2 are Hilbert spaces (( )). Then the Hilbert Adjoint operator T star.

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Adjoint operator T star of T is the operator is the operator from H 2 to H 1 from H 2 to H 1, such that such that for all x belongs to H 1 and y belongs to H 2, we have inner product of T x y equal to inner product of x T star y. It means what is the Hilbert Adjoint operator, it is basically the conjugate operator type, if T is an operator from H 1 to H 2 then an operator from H 2 to H 1 will be called a Hilbert Adjoint operator provided these two inner products are identical. Here T x y, x is a map point in H 1, T is an operator which maps the element of H 1 to H 2, y is it. So, basically this is the inner product of H 2; x is an element of h, and T star is a metric from H 2 to H 1. So, this is the inner product of H 1.

So, this is a well defined inner product both; now, when these two values are identical, then we say an operator T star from H 2 to H 1 is an inner is a Hilbert Adjoint operator T star, clear. So, this is what we. Now, the question arise if t be a bounded linear operator whether this Hilbert Adjoint operator T star will exist, and if it exist will it be unique, and existence in uniqueness is granted whether both will have the same norm or the norm will be different. So, in case if both are equal and having the norm same, then obviously, it will be a useful thing. So, we have a result which gives the guaranty for the existence of this Hilbert Adjoint operator. So, what are the Hilbert Adjoint operator T star, the Hilbert Adjoint operator T star of T is defined is defined above. Let it be this is defined, let it be y A as defined by A; is defined above it defined by A exist is unique, and is a bounded linear operator.

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Bounded linear operator with norm with norm, norm of T star is the same as norm of T. Let it be this is say 1, this one is 2. So, what this theorem says says that if T is giving to a bounded linear operator, then there is a guaranty that Hilbert Adjoint operator will always exist. And not only exist it will be unique in nature, and the norm of T star and T will be identical. So, let see the proof, is it clear. So, let see the proof.

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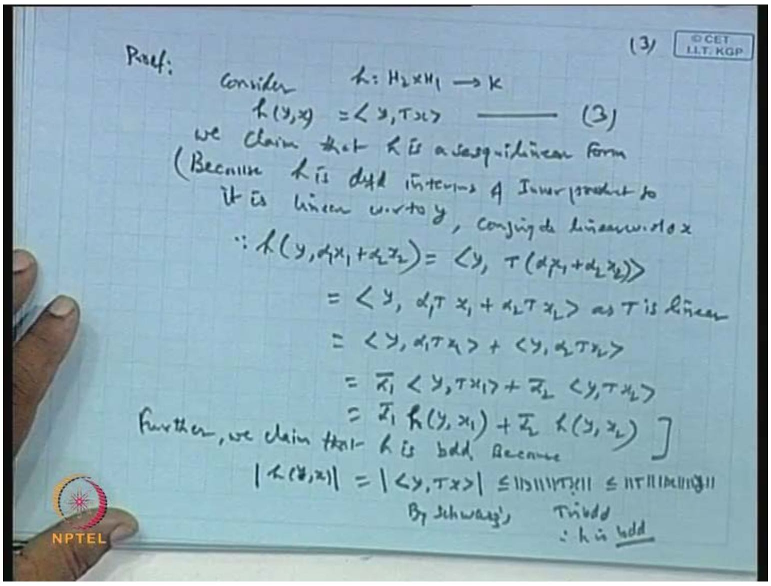

Let us start this is, suppose consider the inner product of y T x as h of y x; I am considering a function h define on the cartesian product of what y is an element of H 2, x is an element of H 1.

H 1.

So, h is a function defined on H 2 cross H 1 to K, because this is scalar. Now, this function is defined as (y,Tx). Now, let us define this functional h from H 2 cross H 1 to K by this method. We claim that h is a sesquilinear form, it means h must be linear with respect to the first coordinate, and conjugate linear with respect to the second coordinate. H is linear with respect to first coordinate; obviously, true.

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Why, because

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Because h is defined in terms of the inner product, h is defined in terms of inner product. So, it is linear with respect to y, with respect to the first coordinate y. Conjugate liner with respect to second coordinate y, it is conjugate linear with respect to x, because if I take h y alpha 1 x 1, alpha 2 x 2, then this can be written as y T of alpha 1 x 1 plus alpha

2, because this alpha 2 x 2. We cannot say directly, because of T is given, but since it is inner product, inner product is conjugate linear with respect to the second coordinate and T is giving to be linear, T is given that is why it will be conjugate linear with respect to x also.

This can be seen as it is y, T is linear. So, we can write alpha T x 1, alpha 1 T x 1, alpha 2 T x 2, as T is linear, is it. Now, this can be written as y alpha 1 T x 1, plus y alpha 2 T x 2 and this can be written alpha 1 conjugate, because it is inner product. So, alpha 1 can be taken outside. So, it becomes conjugate (y,Tx1) plus alpha 2 conjugate (y,Tx2), and this can be written as alpha 1 bar what is this h of y x is defined as the y T x. So, this will come accordingly h of y.

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h of y, what.

h of y.

x 1.

Right x 1.

Is it not?

Similarly, alpha 2 bar h of y x 2. So, this show the h is conjugate linear with respect to the second coordinate, is it correct? So, h is a sesquilinear form. So, if I introduce this, in this fashion, let it be say 3 then h becomes a sesquilinear form. Further we claim that h is bounded - h is bounded, it means bounded means a sesquilinear form is said to be bounded if mod of this is our, I said. A sesquilinear form h is bounded, if mod of h x y is less than equal to c times, norm x norm y, where c is some real number and the minimum value of c comes out to be norm h, is it not? So, using these thing we can say to find out the boundedness, let us take the mod of h x y and then prove that this is less than equal to constant times norm x into norm y. So, what is the mod of, because mod of h y x, is it correct? This is by definition mod of y inner product y T x.

But by schwarz inequality, there is relation between inner product in norm; it is less than equal to norm y into norm T x, T is bounded it T is bounded, it is given. So, norm of T x is less than equal to norm of T into norm of x into norm of y. So, c is nothing but norm T. So, h is bounded agreed.

Sir, we are taking the equation three, in our last last time prove nay.

Yes, because we have already proved the any function sesquilinear form will be given in form of the inner product, where inner product will be (hx,y) and x is an operator like this. So, I have consider a functional, functional h of y x in the form of inner product. So, far we do not know the h is a sesquilinear form, but if it is linear with respect to first coordinate and conjugate linear with respect to second coordinate, then it becomes sesquilinear. The linearity followed, because it is a inner product and T x that inner product is semi linear a conjugated with respect to second coordinate, but it is semi linear with respect to T x, but our interest in semi linear in with respect to x.

So, T because of the linearity, it will give this break up into two parts and immediately one can write this, that is why it is a semi linear. So, basically it is coming from the form of that using that form is it correct or not. So, we get that this is a bounded sesquilinear form.

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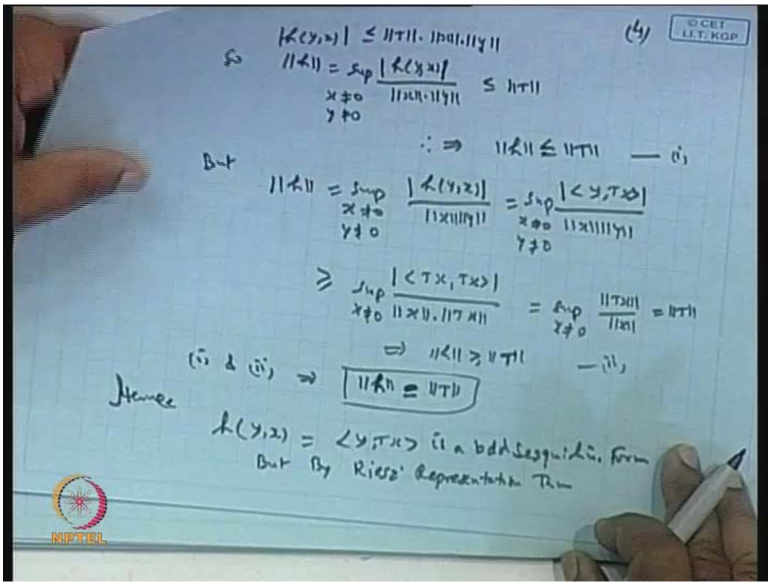

So, h is we what we get is norm of h, if I take this as h of y x mod of this is less than equal to norm of T, norm of x, norm of y, is it or not? This one now this will be equal to norm of h, this is equal to what? h of y x over norm of x into norm of y supremum is taken over what, x which is not equal to 0, y is not equal to 0, but when you divide it is less than equal to norm T. So, what we conclude is norm of h is less than equal to norm T, is it. So, this implies the norm of h is less than or equal to norm of T, but norm of h is equal to supremum mod of h y x over norm of x, norm of y, x is not equal to y, y is not equal to 0.

Now, if I this is true for all y which is different from 0, and all x which is different from.

0.

0. So, we can write it this y T this is equal to what is the y x, according to this third the h x y is nothing, but the mod of inner product y T x. So, this is equal to inner product of y T x divided by norm x, norm y, supremum is taken when x is not equal to 0, y is not equal to 0. Now, let us replace by y a particular value as T x. So, this will be entire thing will be greater than equal to mod T x, T x over norm of x into norm of T x and supremum is when x is not equal to 0, particular value. So, this is nothing, but the norm of T x square. So, this will give the norm of T x divided by norm x, supremum x is not equal to 0 and that is nothing, but what norm of T.

So, this implies norm of h is greater than equal to norm of T, combining this one and two. So, one and two gives norm of h equal to norm of T; that is what. So, what we conclude is that we have introduce an functional, hence this functional h which is defined as inner product of y T x is a bounded sesquilinear form not only sesquilinear, it is a bounded sesquilinear form. But by Riesz representation theorem, but by Riesz representation theorem if you remember what is this bounded sesquilinear form, this is of the form h of x y.

This is (sx,y).

Yes, this is yeah, this was the bounded sesquilinear form in this way; it means every bounded sesquilinear form will be represented in the form of inner product, where s is an operator H 1 and H 2; let us compare these two, what is the comparison here h x y and here y T x. So, basically you are writing that this y equal to T x is ok, this portion y we are writing in terms of some operator, is it not? So, if I write here s is an operator T star, then what happens to this; this will be equal to T star (y,x), because y x, is it not? y x is. So, h of y x is a y sesquilinear.

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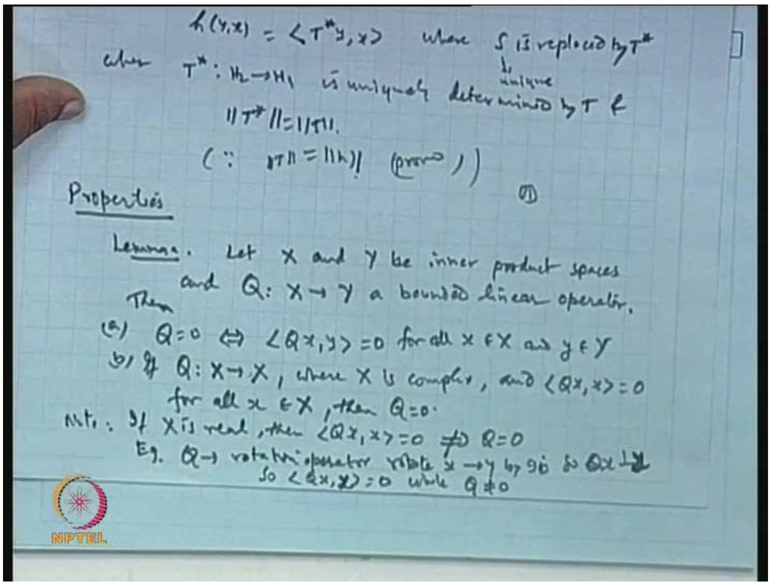

So, by Riesz representation theorem this h of y x, h y x can be expressed as inner product of T star (y,x), where S is replaced by T star, is it not? Just like here.

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No, you just see here you just see here, (x,y) then when you take h of this what is the change? Change is coming; it is nothing, but a inner product where x is replaced by S of x that is all. Here we are taking this this one h of y x, I do not know the right hand side I am not. If I compare the right hand side along with this, what should I write x will remain as it is, y will remain as it is plus an operator will come. So, that is why the s will replace by T star, but this image the domain of T star will be what H 2.

H 2.

Is it not? And range will be H 1. So, you can say that where T star is a mapping from H 2 to H 1, is it correct or not? Is a unique. Now, just like this S is replaced by this S is uniquely determined this is unique. So, this is also is uniquely determined by T, is it not?

By T and have the same norm as the norm of T, because norm of S equal to norm of T. So, it is clear, but norm of T is equal to norm of h, this is proved clear. So, what we concludes is that this.

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Now, first you say that norm of T star is equal to the norm of h.

And that is we have two to norm of h.

Yes yeah that is correct. So, we are getting this one, because of this clear this is proved norm of T star equal to norm of h and then we got it. So, this shows that norm of t star equal to norm of T, because norm of T star norm of T is norm of h and norm of h and this one. So, we are getting this one, is it.

Yes sir.

That’s fine.

So, this proves that combining these we get theorem. So, conjugate and the T star is an operator. So, that is proves that is here, is it not? Because what we require to prove is that was that Hilbert Adjoint operator T star of T exist. So, existence of T star is granted, because S is replaced by T star and existence of x is already proved in the representation theorem then it is unique, because S is unique. So, T star will be unique and it is a bounded linear, because S is a bounded linear operator, T star will also be a bounded linear operator and norm of T star in norm of T will be the same. So, this everything is showed and this result complete the proof, is it clear. So, we are using the sesquilinear form what we representation here to identify clear.

Now, there are certain properties of this Adjoint operator, Hilbert Adjoint operator. The in ordered to prove first lemma, we will prove the lemma first and then go for the property. What is the lemma is, let X and Y be inner product spaces.

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And Q is mapping from X to Y, a bounded linear operator operator.

Then Q is 0, if and only if; inner product of (Qx,y) is 0 for all x belongs to X, and y belongs to capital Y. And b part is that if Q is operator from x to X, earlier Q was X to Y, if Q is x to X where X is a complex X is complex, inner product space and inner product of Q x x is 0, means both the coordinates are same (Qx,x) 0 for all x belonging to capital X, then Q must be 0. So, first is of course, very simple because if we look the proof for the first

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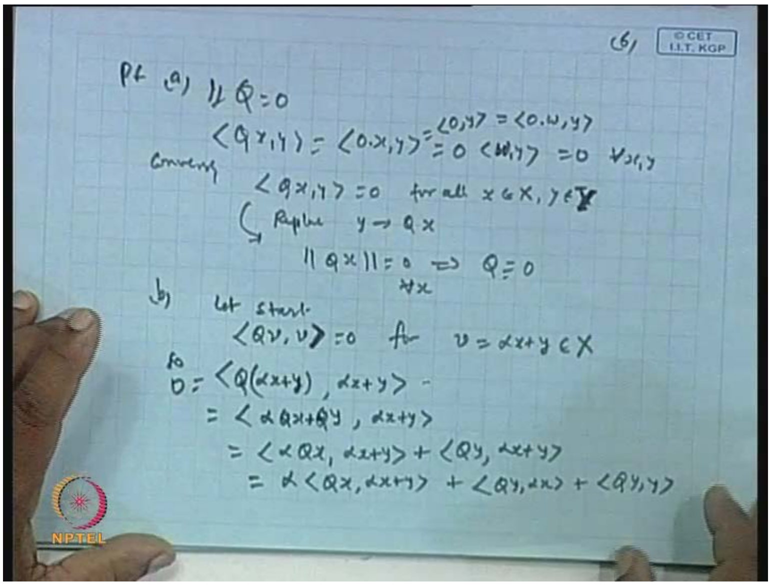

Then what is given Q is 0. So, if Q is 0 then basically the inner product of Q x y becomes 0 into (x,y), but 0 is a scalar. So, we can take x y 0 is scalar quantity. it can...

Sir, also Q is 0 means Q is 0 operator nay.

0 operator, so...

Also 0 operator always there are.

I did not, let us see can you write this thing as (0,y), 0 operator means it will send to 0.

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Now, it will be written 0 into (w,y); now, 0 into w; now. you can take, now this is 0.

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One of the coordinate is 0 means entire thing is 0.

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That's what. So, this is 0 for all x and y, that this is true for all x and y; now, conversely suppose this is 0 x y, conversely if (Qx,y) is 0 for all x belonging to capital X y belongs to capital Y. Then replace y by Q x. So, what we get from here is norm of Q x is 0 which improbable x. So, this implies Q must be identically 0 operator. So, this one part is very easy, what is the second part says if Q is an operator from x to x itself, and x is giving to a complex inner product space.

Then Q x x 0 will implies Q must be 0, that is the inner product of Q x x is 0 only when Q has to be 0, if x be a complex , if x is not complex then this condition may fail. If capital X is a real space then Q x x 0 need not implies Q equal to 0. So, note in case if x is real, then inner product of Q x x equal to 0, need not imply Q equal to 0; why for example.

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Yeah, if I take Q is a rotation operator operator, it rotate x to y by 90 degree rotates x to y by 90 degree. So, they are orthogonal. So, Q x is perpendicular to y therefore, inner product of Q x x will 0. So, inner product of Q x y will be Q x x.

Every the same (( ))

Q x, in a Q x y will be 0 x is it not. So, x here x is it correct or not? Say Q x x 0, while Q is not equal to 0, because rotation vector cannot be rotation operator cannot be 0 operator; it rotate an angle through pi by 2. So, x is a vector and Q x becomes orthogonal to x. So, (Qx,x) this inner product become 0, where Q is not 0. So, what it says is that if Q x x is 0 and if x is a x belongs to a complex plane, then Q must be 0, but if x is not complex then Q need not be 0 may or may not be 0. So, let see the proof for this. So, proof for the second part. So, what we do is let us assume Q v v 0 for v, let assume let us start Q v v this inner product is 0; for v equal to alpha x plus y, this an element belonging to capital X, because x belongs to capital X, y belongs to capital X and x is a vector space. So, the linear combination belongs to this.

Now, this will be Q v. So, what we get it inner product Q alpha x plus (y,alpha x) plus y is 0; and this will be equal to inner product, Q alpha times of Q x plus (y,alpha x) plus y, because Q is giving to be a linear operator, is it not? Q is a linear operator.

Sir, Q y alpha times Q x plus Q time of.

Q y, Q times y y Q is a linear.

That the Q y (( ))

Q y yes this is Q y exactly. So, this is Q y. So, this will be equal to inner product of alpha (Qx,alpha x) plus y plus Q y alpha x plus y. Now, this will be equal to alpha Q x and this alpha can be Q x alpha x plus y I am writing, then (Qy,alpha x) plus (Qy,y). Now, again this will be written as this alpha can be taken outside. So, alpha alpha conjugate (Qx,x) and then plus alpha (Qx,y) and plus alpha conjugate (Qy,x) plus Q y y is it. But Q x x is given to be 0. So, this part is 0 and this part is 0. So, what is left is now. So, we get from here is.

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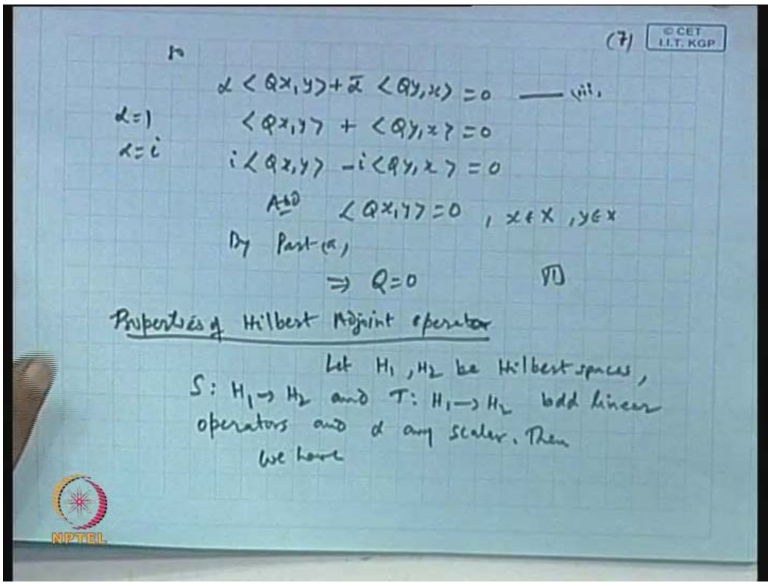

So, we get alpha inner product of (Qx,y) plus alpha conjugate inner product of (Qy,x) is 0, let it be third. Now, if I take because alpha is a complex quantity. So, take alpha to be 1. So, what we get is inner product (Qx,y) plus inner product (Qy,x) is 0 and when alpha is equal to i, what you get inner product of (Qx,y) minus (Qy,x) is 0, because I gets cancel alpha one means minus i. So, therefore, if I add then what we get is Q x y is 0 agreed. Where x belongs to capital X, y belongs to capital X. Now, this is the previous result a part if you remember what is the a part? A part says if Q x y 0 for all x belongs to x and y belongs to y then Q must be 0. So, if there is a same that x is replaced y that is all.

So, from by previous by part a this implies Q must be 0 clear. So, this proves the thing and when Q is not when x is not complex then you have get a contradiction which I have shown earlier. So, that is this clear.

Sir, when it is a problematical alpha.

Alpha is equal to 1.

That is first stage.

Problematical alpha is equal to i.

Then.

Then this will be the i, and this will be minus i, but i cannot be 0 nay. So, that is why.

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Clear. So, this. So, this proves based; now, based on this lemma, we can establish many properties of the Hilbert Adjoint operator. So, let see various properties of the Hilbert Adjoint operator. So, let H 1 and H 2 be Hilbert space.

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Hilbert spaces and S is an operator form H 1 to H 2, and T - T is an operator from H 1 to H 2 and both are bounded linear operators operators, and alpha alpha any scalars, then we have the following property.

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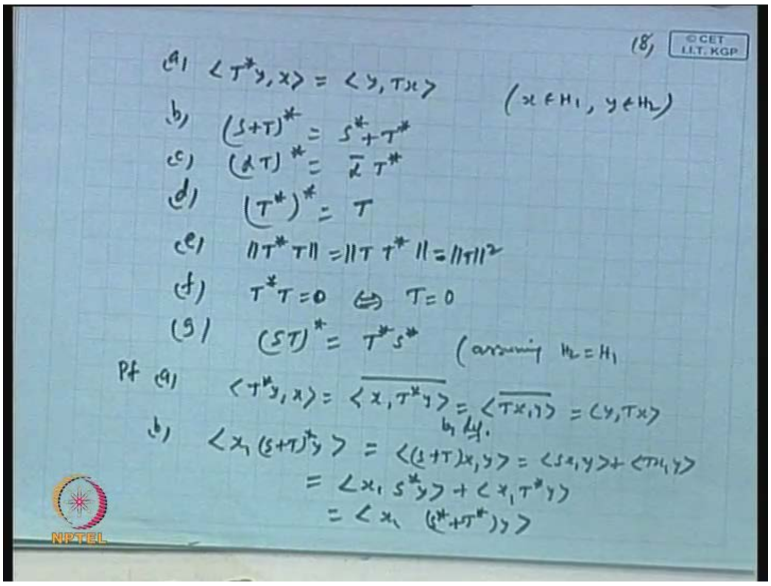

First inner product of T star (y,x) is the same as inner product y of T x, x belongs to H 1, y belongs to H 2. The do you remember this definition this is different from the definition definition says inner product (Tx,y) is x T star y, is it not? That was a definition of the Adjoint operator Hilbert Adjoint, here we are saying T star y x then T can be taken here and this. Second is S plus T star is S star plus T star both are bounded, then third alpha T star is the conjugate alpha T star, d T star star is T norm of T star T is the same as T T star which is same as norm of T square, F T star T equal to 0, if and only if t is equal to 0 and g S T star is equal to T star S star; assuming that H 2 equal to H 1. We are assuming H 2, because otherwise this product is not well defined. So, these are the properties following.

So, proof of this let us see the first property T star (y,x), we want this thing. So, we can write this as x T star y conjugate when we make the conjugate the order reverses, is it not? Now, we apply the definition of Hilbert Adjoint operator - as for the Hilbert Adjoint operator T x y equal to x T star y. So, this will give (Tx,y) by definition, but conjugate again give you y T x. So, this proves that clear. Second part x plus T star; so, consider x S plus T star y. Now, by definition of the conjugate, because if S is a Hilbert Adjoint operator, T is Hilbert Adjoint operator, both are then S plus T will also be a Hilbert Adjoint. Why because what is required is Hilbert Adjoint operator, it should be a bounded linear operator from S to a, if s is a bounded linear operator from one space to other T is also the bounded from H 1e to H 2, then sum of these two will also remain bounded linear operators and then you find the mapping from S star to H 2 to H 1, we will get the same.

So, this is by definition S plus T x y which is the same as S x y plus T x y, because linear property with respect to the first coordinate, and then you write it again x S star y plus x T star y and add them as. So, we get S star plus T star y. Now, this is equal to this can you say this product equal to this.

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X comma.

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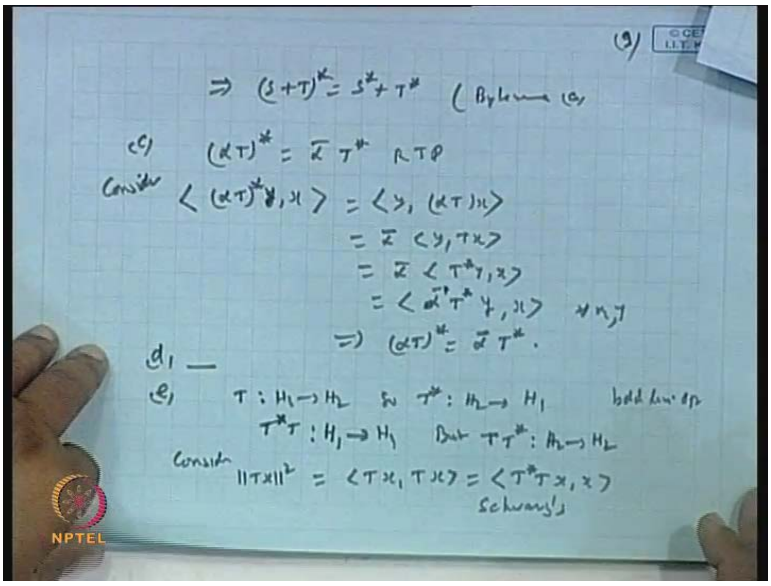

So, can you say from here S plus T star equal to S star plus T star.

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Yes, because by lemma first part lemma is saying what was the lemma? Yes lemma is saying this Q x y is 0 implies Q is 0. So, this is to for all x and y, is it not? So, if you take this one say take it here. So, (x,x) plus T star minus S star T star y equal to 0, and this is true for all x and y. So, this has to be 0 this minus this has to be 0 and we get this. So, this clear not. So, this by lemma. Then third part let us see the third c c is alpha T star, we wanted to show alpha conjugate T star this is require to prove, is it not? So, what we do is we start with consider consider alpha (Tstar,x) (y,x) is it not?

Now, this will be written as (y,alphaTx), but alpha is in the second bracket. So, conjugate sign will come, and this will be equal to alpha bar T star (y,x). So, this will be equal to alpha bar alpha bar sorry, T star (y,x) and then again this is true for all x and y. So, we get from here is alpha T star y is alpha bar T star similar way. Similarly, fourth d star d can be computed in a similar way then no problem, is it correct or not? Now, fifth one e what is e is T star t equal to T T star equal to norm. So, first we have to define accordingly, so, that the T star T and T what is the T? T is a mapping from H 1 to H 2, is it not? This was the H 1 to H 2. So, T star will be an operator from H 2 to H 1. So, when we write T star T, it means first it will take the elements from H 1 then it will go on T H 1 will be the element of H 2. So, basically it is from what H 1 to H 1, is it not?

(( )).

H 1 to H 2 and what is T T star H 2 to H 2.

So, it means the domain of this and domain of all are not the same, but what it says is that there norm will be remain the same. So, let us see the norm of T both are equal to the norm of T square. So, consider norm of T x square. Now, this can be written as inner product of T x T x, is it correct? And this will be equal to start with these. So, T star (Tx,x), is it not? T star (Tx,x). Now, T and T star both are bounded these are giving to be a bounded operators. So, they are bounded operators, T star will also be bounded linear operator. So, this is a bounded linear operator. So, by the schwarz's inequality modulus of this inner product is less than equal to norm of this into norm of this.

Sir, one minute sir T is bounded T star is bounded (( )).

But what is the proof that multiple is not these two would be a bounded.

Bounded operator that is norm of T 1 into T 2 is less than equal to norm T 1 and norm T 2. So, that will give the boundedness.

From there.

So that will be give that schwarz's. So, by applying the schwarz we get from here is now, this is less than equal to norm of T star T into norm of x into norm of x square, is it not?

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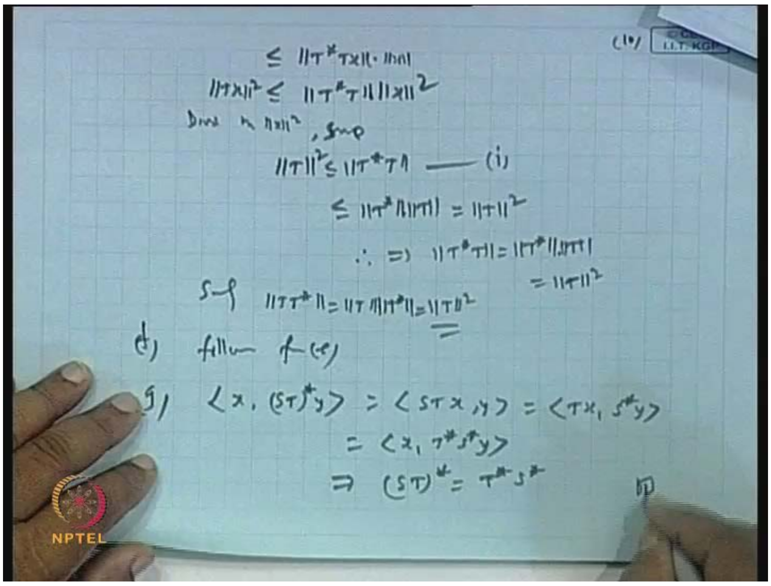

By this is norm of this into norm of this again this is less than equal to norm of this because basically this will be like this norm of T star T into norm of x further it will be less than equal to this and we get this part norm of T star T divide by norm x square. So, what we get it? This is equal to… So, divide by norm x square and take the supremum. So, we get norm of T is less than equal to… Is it not? Clear? Is it correct or not? Now this will be further less than equal to norm of T star into norm of T. Is it not? Normal, norm of T, T square this is norm of T square sorry norm of T square. So, this will be equal to norm of T star, T which is less than equal to norm T, but what is the norm of T.

Norm of T star.

T star is nothing but the norm of T.

No no.

It means these two are identical, when they are two equal. So, this implies norm of T star T must be equal to norm of T star norm of T. Nay and basically both are be equal to norm of T square. Clear? So, this proofs that result… Is it ok or not? Similarly, you can start with T T star, similarly start with norm T T star in a similar way you can show Is not it?

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If norm of t is divided by norm of a (( )) of them.

That's the reality of the norm of T x square.

Divided by norm x square. So, basically it is norm of T square by take the norm T x by T x and whole square then take the supremum inside. So, you get it nothing clear. So, this proved, now what else is that; now sixth. Then T T star equal to 0 this f follows from here from e y T T star is 0, T T star is 0 automatically T will be 0 and vice versa. So, nothing is there and g S T star this we have to show. So, consider x S T star y. Now, this will be equal to S T (x,y). Now, this is equal to what T x S star y and which is equal to x T star S star y and this implies S T star equal to T star S. So, order reverses and. So, this completes your this part agreed. So, this…

Now, there are as I told you earlier that this Hilbert Adjoint operator, we can connected this with our metrics operator, and in fact, is a particular case our self Adjoint operator symmetric operator, they can be converted or can be consider in terms of this (( )) and relation between these two. We the self Adjoint operator in a particular cases can be classify in T.

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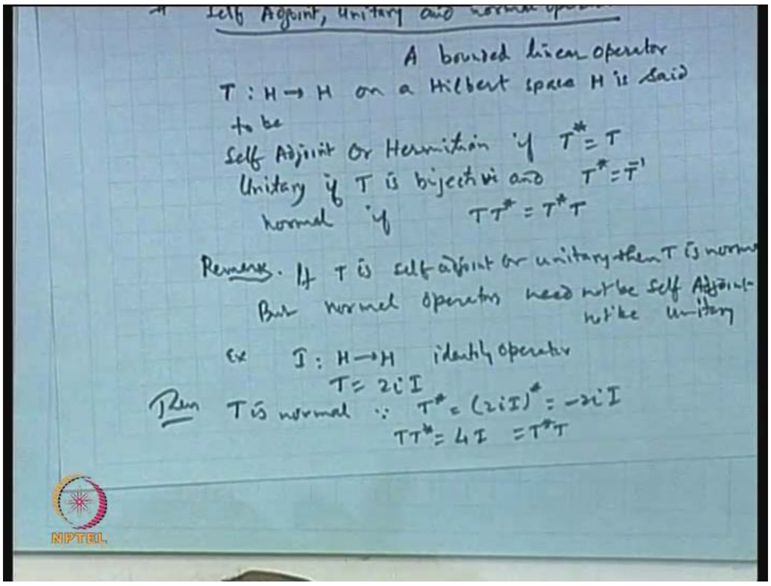

What we have now, self Adjoint self Adjoint unitary and normal operator, that definition will depend on this classes of bounded linear operators importance on this. So, we defined as A bounded linear operator T, A bounded linear operator T from H to H on a Hilbert space, on a Hilbert space H is said to be said to be self Adjoint or Hermitian, if T star equal to T. Then Unitary if T is bijective means one one on two bijective and T star equal to T inverse. So, when T is a 1 1 on two inverse exist; and normal if T T star equal to T star T. So, a bounded linear operator T on a Hilbert space is said to be self Adjoint, if the Hilbert Adjoint operator of T coincide with the operator T itself.

When T is a 1 1 on 2 and the Hilbert Adjoint operator is coincide with the T inverse of the operator T, then we say it is a unitary and when T T star is the T star T is a normal. So, the Hilbert Adjoint T is defined. Now, this is remark. If T is self Adjoint or unitary then T is normal. Every self Adjoint operator is a normal operator, every unitary operator is a normal operator, why what is the definition of the normal. T T star we wanted to T star T. So, if I write here t star if it is self Adjoint then t star become T. So, what is the T T star, T square and what is this T square same. So, self Adjoint operator will be normal operator, if it is unitary then T star is nothing, but T inverse. So, T T star becomes i and T star T also becomes i. So, basically every self Adjoint unitary operator is a it means self Adjoint implies the normal, unitary implies the normal T naught, but normal operator need not be self Adjoint or need not be the unitary, clear; converse need not be true an operator may be normal, but it need not be a self Adjoint operator or unitary.

For example if we consider an operator I from H to H an identity operator, an identity operator and T I consider to be 2 i I, consider T here, then what we claim is T is normal; why normal, because what is the T star? T star is 2 i I star, and this will be equal to what.

(( )).

Conjugate of this minus 2 i I clear. So, T T star becomes what 2 i 2 i 4 i square that is 4 into i and that is the T star T also. So, T is normal, but T is not self Adjoint

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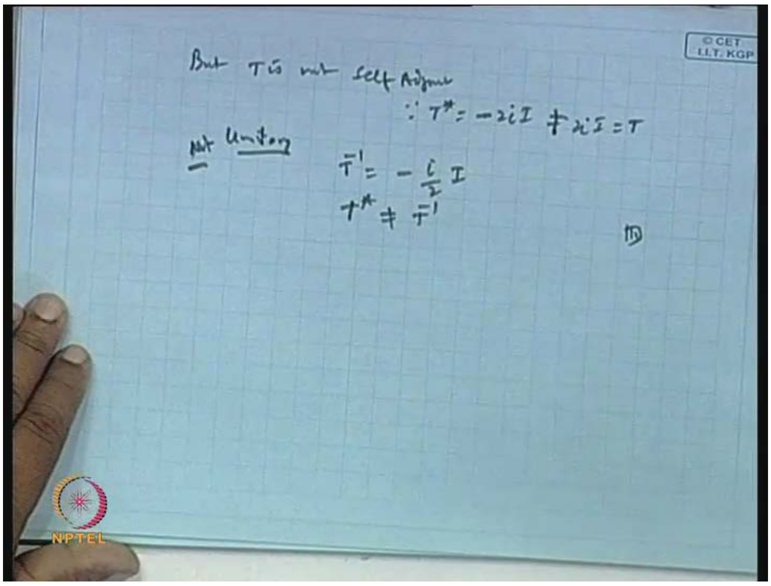

Why why self Adjoint, because T star is equal to T, because T star which is coming to be minus 2 i I is different from 2 i I that is T. So, it is not self Adjoint; it is not unitary why, because what is the T inverse T inverse will be minus i by 2 I, because T into T inverse should be I. So, this cancel to 2 and i square. So, T star which is not equal to T inverse. So, it is not unitary; So this, thank you.