Ordinary Differential Equations (noc 24 ma 78)
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Week-02
Lecture-07
Ordinary Differential Equations

welcome students and in this week we are going to talk about what is OD and what are the different methods of you know studying OD okay first of all you see we are basically working with a function so we assume okay

Assume a function, let us say y is from AB and this AB is open subset of R, right?

AB subset of R to R. It can be R also, but basically y is from AB to R is an unknown function, is an unknown function, unknown function and essentially we are going to build a new kind of function using this unknown function and any relation, any relation

Of the form.

Of the form.

Capital F. Of.

So, essentially, first of all, it will depend on x, x is from a, b, right, it will also depend on y, y is, y is a function, the function, and it will depend on the derivative of a function, y prime, let us say, the derivative, and it will also depend on the second derivative, and it will go on, it will depend on y, the nth derivative, this equals to .

Let us say, you have a function like this, a relation, which depends on x, y, y prime, y double prime, and y power n, so this is called, is called

nth order nth order order means the highest you know power of the derivative which is exist in the equation that is called order so the nth order equation nth order equation okay where where what is f f is a function f is a function which is defined from a b clear cross

And this n is any n. n can be a natural number.

So, I will specify all that.

So, a b cross y. What is y?

y depends on c. For every x, y x is in r. So, this is in r cross y power the nth power of y. Sorry, the nth derivative of y. That is also in r. So, r to r.

is a smooth function smooth function what do i mean by smooth function i mean that it is um at least continuously differentiable function okay so at least continuously differentiable at least continuously continuously differentiable okay differentiable

So if basically what we are doing is we are basically looking at a relation like this and as such a relation we will call it as a nth order equation or to be very specific ordinary differential equation.

Ordinary differential equation.

differential equation.

I forgot to mention this ordinary differential equation, right.

And you see the thing is why this ordinary, sorry Mark, why this ordinary, why is it called ordinary?

Because ordinary, because the ordinary, this is because we have

Basically, we are looking for a function y, which is a one variable function and the domain is r, right, or a subset of r, okay.

So, basically, y, the domain of y, domain of y, of y is a subset of r, right, is a subset of r. Hence, hence, all derivatives, derivatives involved, derivatives involved, involved are

are ordinary derivatives, right?

Ordinary derivatives or simple derivatives, let's just say, ordinary derivatives, okay?

So let's say if you have a function which is from r to r, then you have derivatives, partial derivatives, all those things are there, and in this case, we call it a partial differential equation, right?

So, PDE is, what is PDE?

PDE is partial differential equation.

What is partial differential equation?

It is essentially a differential equation of the same sort of form, but here just the function, the unknown function which we will be talking about, that will be from R or Rn to R, partial differential equation.

So, deals with, deals with functions.

functions from f omega subset of Rn, n can be greater than equal to R.

Here n is greater than equal to .

So, basically the derivative involved will be, you see the derivative involved will be ux, uy and so on.

So, uxy such sort of derivatives will be involved.

So, it is called a PDD.

So, anyways for this course we are only going to concentrate on OD.

So, let us look at this.

You see here the function which we have defined, the definition is it correct?

It is not.

It is incomplete definition.

Why it is incomplete?

You see, this function, so let us say for an example, let us say for an example, first of all, A, the above definition, the above definition is incomplete.

Definition is incomplete.

incomplete.

Why it is incomplete and how to complete it?

Let us just see.

So, first of all, let us just assume and here also n is greater than equal .

So, first of all, we will just put n equals to here.

So, let us just see.

So, let us assume that

n equals to .

So, essentially we are looking at a relation

relation which will look like this no f of x y y prime equals to and f is c continuous to differentiable c right whatever the domain it is but it is c so essentially now you see the thing is and as an example you can write that let us say f of x y y prime is given by only y

Now, you see if you are putting y, now the equation which the relation which we are looking at is f is .

So, essentially if f of x y y prime is given by only y and then y is , then that is not an ODE.

So, it is not an ODE, because ODE has to contain something about the derivative.

So, the derivative should be involved in this.

So, see for n equals to , if this function

has to contain the derivative.

It should not be independent of y prime.

So, see, small note, f should not be independent, should not be independent, independent of y prime.

y prime, right.

See, if it is independent, then it is basically a relation between x and y and that is not an ordinary differential equation because there are no derivatives involved there, right.

So, it has to be, some derivative has to be involved.

So, essentially what we are doing is this.

You see, we have to, you know, modify the definition.

It is a smooth function and let me do it here, which is dependent

dependent.

So, let me put it this way, n greater than equals and del f del yn.

So, with respect to this variable, the last variable, this variable, if you think of it as z variable, the del f del z that should not be

So, basically it should be independent, sorry, it should be dependent on the derivative at least.

In this case, you see, if n equals to , it should be dependent on y prime.

So, basically there should be some, you know, so if it is a nth order derivative, it should not be independent of the nth order derivative.

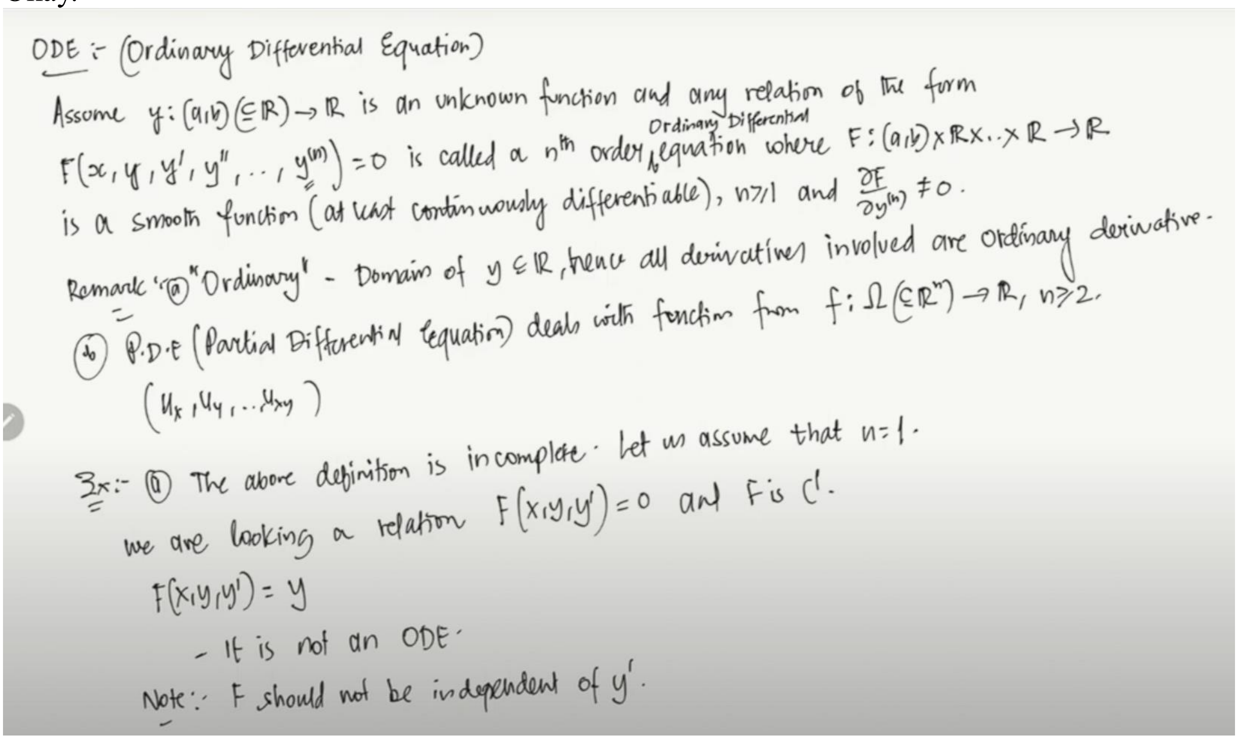
So, nth order ODE, then it should not be independent of nth order derivative.

Clear?

Okay.

So, I hope this is clear.

Okay.



ODE :- (Ordinary Differential Equation)

Assume $y:(a,b)(\subseteq \mathbb{R}) \to \mathbb{R}$ is an unknown function and any relation of the form $F(x, y, y', y'', \dots, y^{(n)}) = 0$ is called a n^{th} order Ordinary Differential equation where $F:(a,b)\times\mathbb{R}\times\dots\times\mathbb{R}\to\mathbb{R}$ is a smooth function (at least continuously differentiable), $n \geq 1$ and $\frac{\partial F}{\partial y^{(n)}} \neq 0$.

Remark :- (a) "Ordinary" - Domain of $y \subseteq \mathbb{R}$, hence all derivatives involved are Ordinary derivative.

(b) P.D.E (Partial Differential Equation) deals with function from $f:\Omega(\subseteq\mathbb{R}^n)\to\mathbb{R}$, $n \geq 2$.

$(u_x, u_y, \dots u_{xy})$

Ex:- (1) The above definition is incomplete. Let us assume that $n=1$.

we are looking a relation $F(x,y,y') = 0$ and F is C^1.

$F(x,y,y') = y$

- It is not an ODE.

Note :- F should not be independent of y'.

Let us look at some examples here.

So, what is this?

See, if I am putting, let us say, first order, f of x, y, y prime is y, y prime plus .

Okay.

Let us just say.

Then, the given ODE, the given ODE which we are interested, ODE,

is of the form of the form form y y prime plus equals to so basically you are looking at a od which looks like y y prime equals to minus okay so that's the first example let's look at a second order equation f of x y y prime y double prime and let's write down write this down as a

y double prime plus b y prime plus c y plus x square, okay.

Then what is the given O D?

Then the given O D corresponding to this f, right, O D, O D. See, this is independent, this is dependent on say two variables, right, because a I am assuming is non-zero, clear, okay.

So, in this case what happens is then the given O D is of the form

of the form a y double prime plus b y prime plus c y plus x square equals to .

Clear?

So, this is another form of equation and similarly you can also form any other equation that you can of course figure it out.

So, please try to construct a let us say fourth order equation.

So, this is a small exercise let us say exercise try to construct

Try to construct a fourth order equation, fourth order equation with variable coefficients, with variable coefficients.

coefficients okay so in this course most of the times whenever i am saying equation please understand it in terms of ode okay so it is ordinary differential ode ordinary differential equation okay because since we are always working this is the ode course right so it doesn't make sense to say od od all the time okay so this is the ordinary differential equation right

Anyway, so now there are two forms of equation.

So basically we need to study this equation.

So what is the main idea of how can we study this equation?

See there are two forms.

We classify this equation.

So classification.

So there are different different classification of this.

One kind of classification of ODE is this.

ODE, what is it?

So let us look at it.

I will say that there is something called a linear equation, linear equation, linear equation.

So, what is the definition?

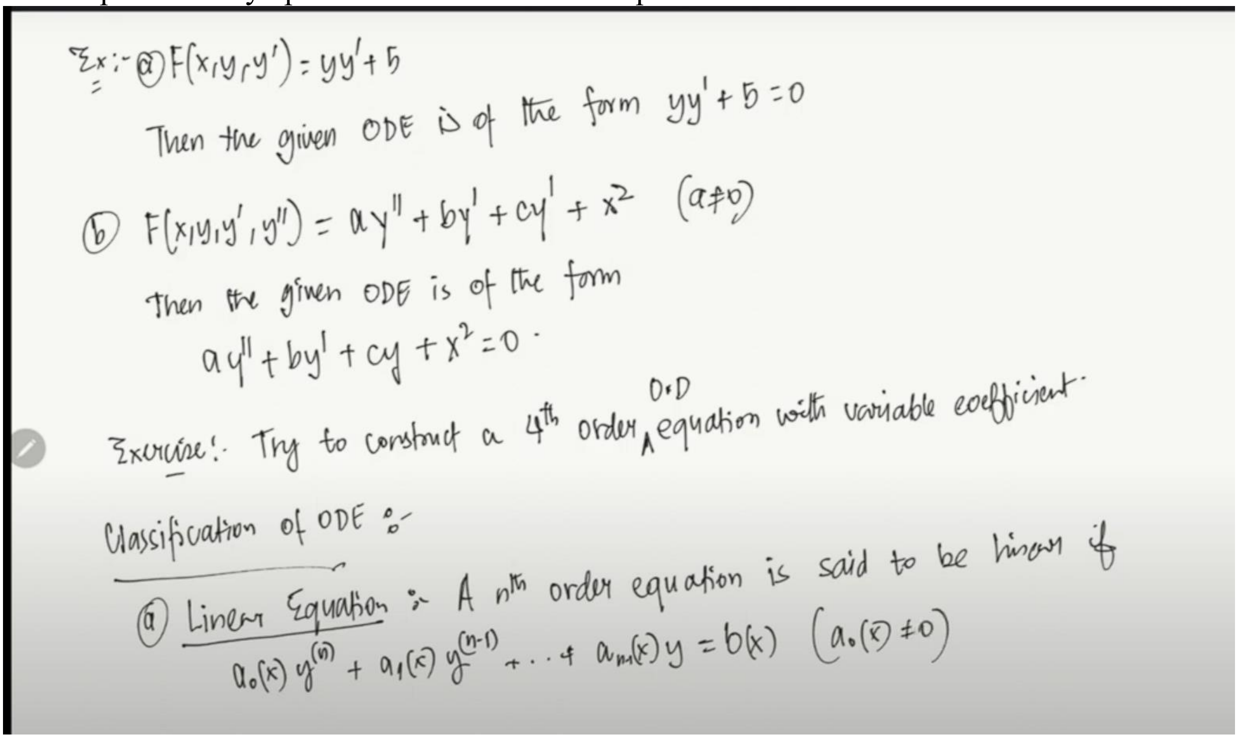
So, nth order equation, nth order equation again ODE this is equation is said to be linear, is said to be linear.

linear if the following it looks like this a naught of x y the nth derivative of x plus a of x y n minus and it goes on like this right and let us say a naught of x sorry plus

a n of x. So, it will look like this, y equals to .

So, if, equals to b of x, let us say, if b of x. So, if it looks like this, so basically you see whatever the coefficients of y n, right, the derivatives, the coefficients corresponding to the derivatives and y, that all those coefficients should only depends on x and should not depend on y.

okay or any derivative of y so basically it should be in this form okay and then we call it a linear equation so why basically is this you see what you can think of it in a more precise mathematical sense what you can do is this we think of it like this define an operator so define a linear operator linear operator okay operator so let's do it for n equals to huh



l from c to ab to c ab.

And how am I defining?

As l of y. So, you see, I am defining a map from c to ab to c ab.

So, twice contingency differentiable functions.

So, let us say y is a twice contingency differentiable function and what happens to y when l acts on it?

So, that I am defining it to be a of x

y double prime plus b of x y prime plus c of x y okay if i define it like this now you see l is a linear map is a linear map linear map you can of course check linear map right so please check this part you need to check this huh it is a linear map very easy to show that this is a linear map so if this is a linear map then

ly equals to some function depends only on x, okay, is a linear equation, linear second order equation.

In this case, why second order?

Because n equals to , I am choosing, you can choose n equals to whatever, , , whatever.

So, linear second order equation.

So, that is what it is linear, linear in y, okay.

So, this is linear in y. Here, we are assuming that

n minus minus .

So, here I am assuming that a naught is of x is non-zero.

So, this is a linear second order equation.

Now, any equation, any equation which is not in this form, which is not in this form, not in this form,

form will be called non-linear, will be called, called non-linear.

Is this clear?

So, here also please check this b does not depends on y, b explicitly will depend on x okay so any equation which is not of this form which form of the form this this form okay let's just call this form as a linear form of the of the form let me let me put it this way of the forms star will be called a non-linear equation

equation okay i hope this is clear non-linear equation non-linear equation okay right so that is clear that is clear now you see what we are going to do is essentially look at some examples that will clarify things examples

Okay, A, the first linear equation.

You see, let us say, let us write a first order equation, linear.

Okay, how does it look like?

You see, it will always look like this, y prime plus some ax times y equals to bx.

Okay, so this is the form of, form of any first order equation, first order linear equation, order linear ODE.

It cannot be of any other form.

That you can understand by the definition itself.

So, let us write down an equation.

Let us look at this.

You see, let us say y prime, yy prime plus xy equals to .

What sort of equation is it?

You see, y prime, the coefficient of y prime depends on y. So, this is a nonlinear equation for us.

non-linear equations.

And as you can understand, you see, intuitively also it is very, very clear that once you have a linear equation, it should be much easier to solve a linear equation than a non-linear equation.

Because it is a little complicated in structure.

Let us look at another example.

Let us look at this example.

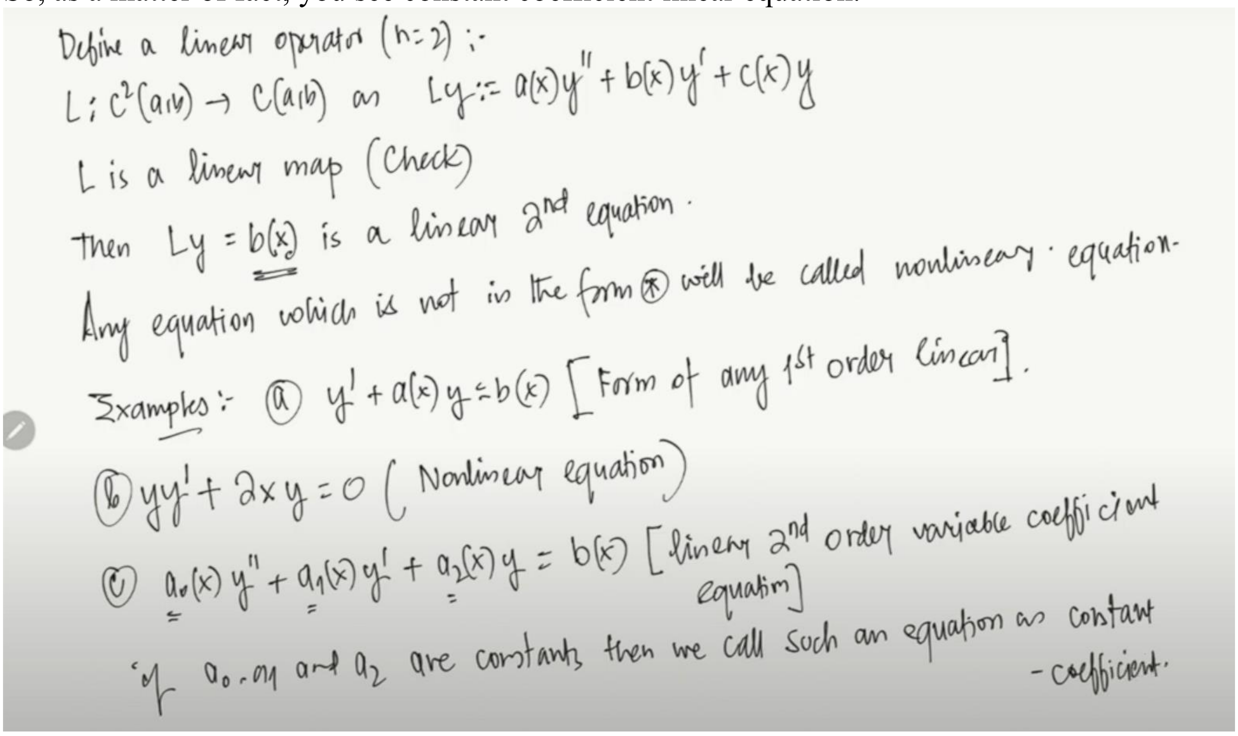
Let us say that a naught of x

y double prime plus a of x y prime plus a of x y equals to, first of all, let me write it as b of x. Now, if we do that, you see, then this equation is a linear second order equation, second order.

second order okay and we call it a variable coefficient okay because these coefficients you see depends on x okay variable coefficient equation linear second order variable coefficient coefficient equation ode and if you see if a a and a in this case okay are constant

constants, then we call such an equation as constant coefficient.

So, as a matter of fact, you see constant coefficient linear equation.



Define a linear operator $(n=2)$:-

$L : C^2(a,b) \to C(a,b)$ as $Ly := a(x)y'' + b(x)y' + c(x)y$

L is a linear map (Check)

then $Ly = b(x)$ is a linear 2nd equation.

Any equation which is not in the form (*) will be called nonlinear equation.

Examples :- (a) $y' + a(x)y = b(x)$ [Form of any 1st order linear].

(b) $yy' + 2xy = 0$ (Nonlinear equation)

(c) $a_0(x)y'' + a_1(x)y' + a_2(x)y = b(x)$ [linear 2nd order variable coefficient equation]

If a_0, a_1 and a_2 are constants then we call such an equation as constant-coefficient.

So, first of all, what is the degree of difficulty of solvability?

So, let us say we want to solve, the

Solve ODE.

Let us understand the difficulties.

Difficulty, solvability, I mean difficulty level.

Let us just put it this way.

The level of difficulty.

Level of difficulty.

So, that will actually clear up the fact that how to study this.

See, first of all, linear equations are easy.

Linear equations and non-linear equations, right?

Non-linear equations.

nonlinear equations.

So, this is easy, right?

This is complicated.

It can be easy, but generally speaking, generally they are complicated, difficult, okay?

It may not be solvable also, nonlinear equations.

Now, so linear equations is easy, but it is not that easy in all the cases, okay?

Now, you see linear equations, let us say that

Now, if one, let me put it this way, if one is dealing with constant coefficient linear equations, constant coefficient linear equations, then it is easy to solve, easy to solve.

Then this is also solved, right?

Okay, that is the given, I mean, we can always solve it without a problem, okay?

So, always solvable.

Let us just put it this way.

Always solvable.

And we will see how to prove this thing.

So, anyway, I hope most of you already know how to do this, but we will still go through this.

So, it is always solvable.

Now, this is , this is .

If you are dealing with a variable coefficient linear equation, so if is dealing with a variable coefficient, with a variable coefficient,

coefficient linear equation, then things are complicated.

Things are little complicated, little complicated, complicated and in some cases, some cases are solvable.

are solvable.

So, let us say if it is a Cauchy Euler equation, you can solve it or

There is something called a power series method.

So for certain equations, you can actually use power series method to solve it.

But please keep this in mind.

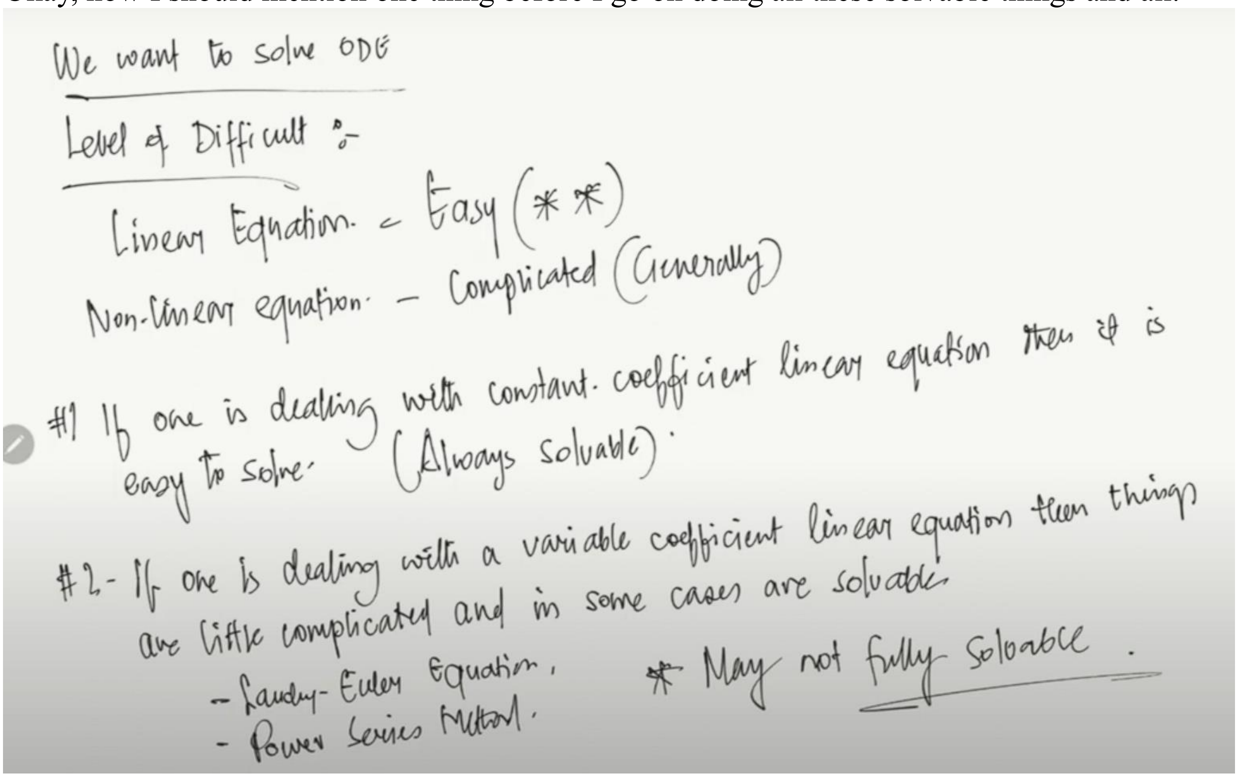
Generally speaking, variable coefficient linear equations are complicated.

It may not be solvable.

So let me put it this way.

May not be fully solvable.

Okay, now I should mention one thing before I go on doing all these solvable things and all.



The question is, whenever I am saying it is solvable, what do I mean by this?

Okay, that is one question.

Okay, so first of all, you see, given this equation, okay, so solution of an ODE, what does it mean?

Solution of an ODE, let us understand this.

So you see, I will take n equals to , okay?

So what is an OD?

Any OD will look like this, right?

If you remember, f of x, y, y prime, y double prime, y power n, this sort of thing, right?

Now the thing is, any solution of an OD, what I am going to do is, for this, I mean, for the purposes of understanding, I will take n equals to , okay?

So you see, first of all, given OD,

So, you can take n equals to also, n equals to also, does not really matter given an ODE.

How does it looks like?

f of x, y, y prime, y double prime equals to .

A solution y, what does it mean?

So, basically you are given this ODE, I want to find out what does it mean for a solution of this ODE to exist.

So, let us put it like star.

Y from AB to R, it is given, right?

AB to R is a solution, is a solution.

Sorry, this is unknown.

So, that unknown function, that actually once you, let us say by some method you solve, you found out something, that function, okay.

So, this function, any function, we will call it a solution of star, of star, if y is at least c of a, b. So, basically, first of all, y has to be at least twice continuously differentiable in a, b.

And y has to satisfy this equation has to satisfy star star for all x in a b. What does that mean?

It means it means it means you see f of x y of x

y double prime of x, sorry, y prime of x, y prime of x, y double prime of x, that should be for all x in a, b. If this is the case, then we say it is a solution.

So, let us look at an example.

So, let us say f of x is given by, sorry, let us say an ODE is given by this.

y prime minus y equals to .

This is an ODE, example of an ODE, of course, right?

And what is your corresponding f here?

f of x, y, y prime will be like y prime minus y.

So, if your F look like this then you have the ODE which looks like this for since F is this.

So, given a F which looks like this you have an ODE corresponding to that.

Now, you see for this ODE one can easily check one may easily check easily check that the

yx, okay, equals to, sorry, okay, is a solution, is a solution.

Okay, why it is a solution?

First of all, you see yx is .

So, in this case, what is open a b?

Open a b is in R here.

Essentially, basically, it is whole R, right?

Now, you see yx equals to .

Does it satisfy the equation?

Is it a white continuous differentiable function on R?

Of course, it is.

And does it satisfy the equation?

You see, if y is ,

identically equals to , y prime is also identically equals to .

So, basically y prime minus y will satisfy this equation for all x in R, right.

So, the second condition is also satisfies and this is an equation and solution.

Also, you can check that let us say y x equals to e power x also satisfy the equation, also satisfies the equation.

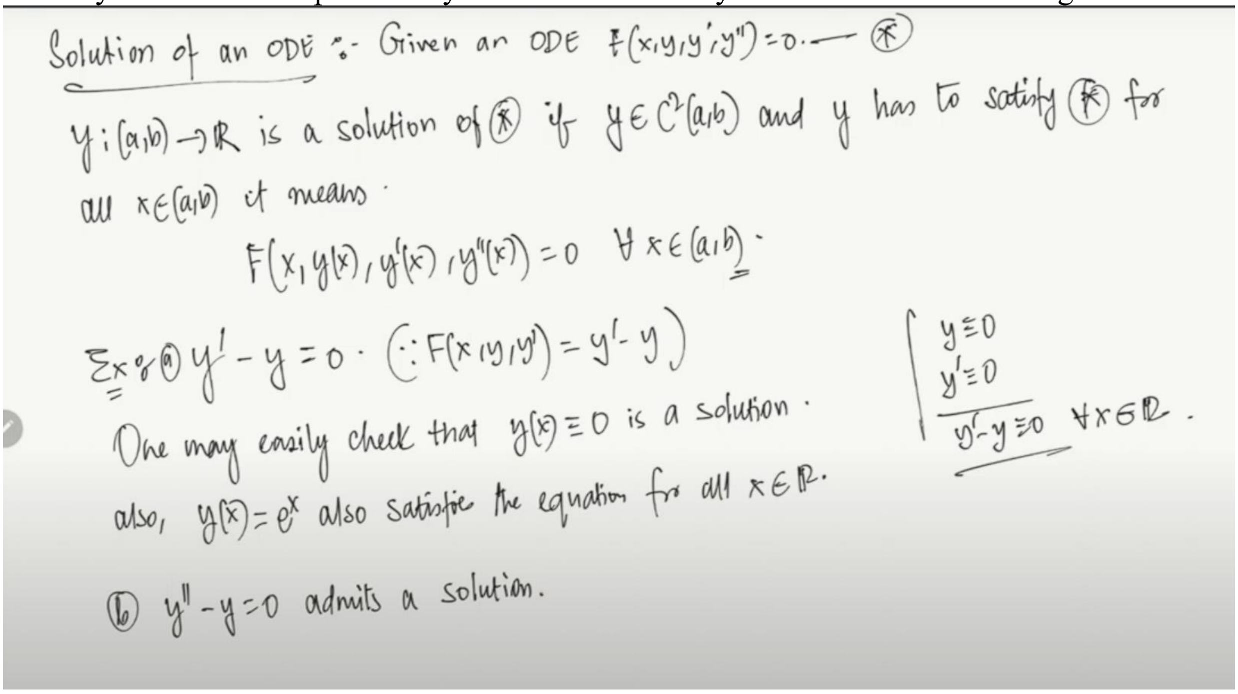
Yeah, for all x, for all x in R, right?

Because y prime is exponential x, exponential same thing is happening, minus exponential is going to give you , then this holds for all x. Again, exponential is twice the continuity principle, so you understand that is the solution in this case, okay?

Right, and now one thing I need you to check this, okay?

So, please check that y double prime minus y equals to admits a solution.

admits a solution okay you do realize that exponential x will be a solution can you find any other solution okay and the thing is solution in this sense okay so can administer solution first of all of course you do realize exponentially the solution so this you have to check check right now



As far as these equations are concerned, what I am going to do, so this is our main idea.

So, we are going to again classify it a little bit.

You see, if ly equals to bx, okay, and l is linear, l is linear, is linear.

linear operator, operator, remember linear operator from where to where, cab, so for now, I am only going to concentrate on second order, the same thing holds for third order, fourth order, nth order, does not matter, okay, so I am only going to do it for n equals to here, so let us say l equals to equal to bx, this is a, and l is a linear operator, is given, is given, okay, for n equals to , so basically what I am trying to say is this, that is, that is,

L of y looks like this.

a of x, y double prime plus a of x, y prime plus a of x, y. And here, of course, I am assuming a is not for all x in whatever a, b. a, b can be r also.

Now, you see, if b of x is , then

the equation equation is called is called a homogeneous equation homogeneous in this case it is called homogeneous linear second order equation so big name ah so homogeneous homogeneous

So, I will use a notation, this sort of notation in this course, homogeneous linear second order equation, let us write it as nth order equation, okay.

So, that is always given, order you can always figure it out, it is not a big deal.

So, it is homogeneous linear OBE, here, okay.

Now, if bx is not equal to , then that is a not equal to , then we are dealing, we

In homogeneous equation.

Homogeneous equation.

yes okay now so that is more or less the idea so essentially what we have to do is first of all what is the solvability thing first of all we have to solve for the linear equations linear homogeneous equation that is the most easiest form and then we will talk about linear non-homogeneous equation and then from there we will go to non-linear equations right so that is basically the equation part okay so now so this is the equation part now let us look at the system

system of od what is the system of od so equations is fine right now let's look at a system of od what is the system so basically we have so i will again i will do it for n equals to here we can of course write it for n equals to whatever right so you can of course generalize this thing this is just for understanding purposes right okay so what is the system of od so any

relation of the form f x y y y prime y prime equals to and f of x y y y prime y prime equals to .

here I will assume that del f del either one of the i's.

So, basically i equals to or i equals to that is non-zero.

So, basically it should be dependent on at least one derivative and again same here f of del yi derivative should not be zero for i equals to or i equals to is said to be

to be a system of ODE.

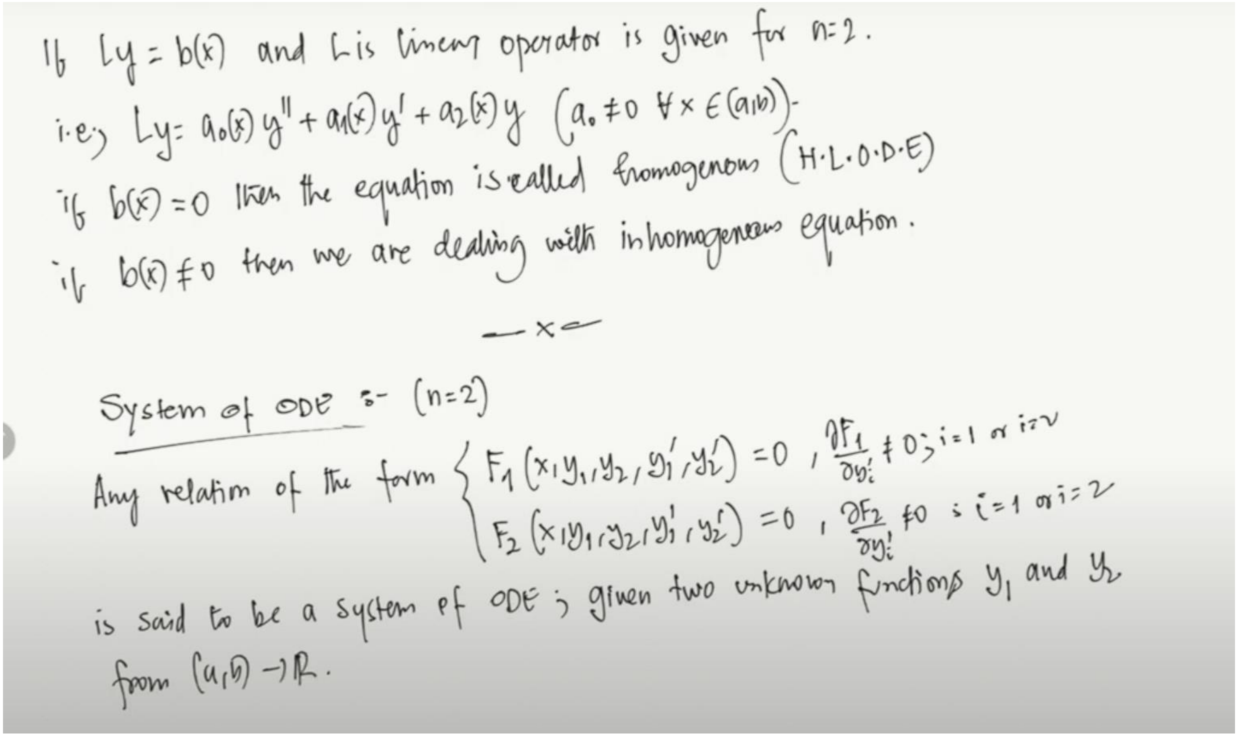
What do I mean by this?

I mean that basically you are looking at, initially we are just looking at one equation, right?

So, basically one y and then we have, you know, the solution, the equation written in terms of y and the derivative of y. Here we have two equations y and y, okay?

So, it is a system of ODE given two unknowns,

unknown functions so basically given or two unknown functions what i mean by this is basically you have two unknown functions y and y y and y from a b to r so anyways we are dealing with o d e's only it is just ordinary differential equation but here it is not even one equation now it's two equations okay so let's look at one example so basically the first easy example



You see, let us write down y prime equals to a y plus a y and y prime equals to a

y plus a y .

So, this, you can see that this is nothing, this is a function of f of subconstants, of course, and y and y .

And again f and f here is nothing but again function of y and y and so basically you can if you define f to be a y plus a y then you have this equation and y prime will look like this.

So this is so the above this this is an example of a cross linear system.

linear system, right, cross linear system.

So, essentially what we have is, you see here this sort of system, why we say it is a cross system?

So, basically let us say if one defines, if one defines y of x, capital Y of x to be

y and y of x, y of x, then we have already seen what is y, sorry, what is capital Y of x?

Capital Y prime of x is nothing but y prime of x and y prime of x, right?

So, then that will imply that basically you are looking at this equation, you see,

y prime of x is nothing but a, a, a, a of x.

y x okay so it is a cross matrix and hence we call it is a so and this can be written as this you see y prime x is a y of x right where a is a cross matrix cross matrix and then so that is why therefore it is called a cross linear system why linear because you see again here all the coefficients right it may be dependent on x but here so basically here let me put it this way

Here, here, aij depends only on x, depends only on x, only on x and not on y and y, okay.

So, you see, this is an example of a linear system, linear system, okay, linear system.

Okay, let us look at a nonlinear system then.

This is A, right?

So, look at a nonlinear system.

Again, I will do it for a, so okay, and first of all, what is the order of this system?

You do realize that only one derivative is involved.

So, it is a first order linear system, okay?

So, first order, first order cross linear system, okay?

So, we will define it like this.

This is just our definition.

Now, you see, the thing is, let us write down another linear system, which is a nonlinear system, not necessarily linear.

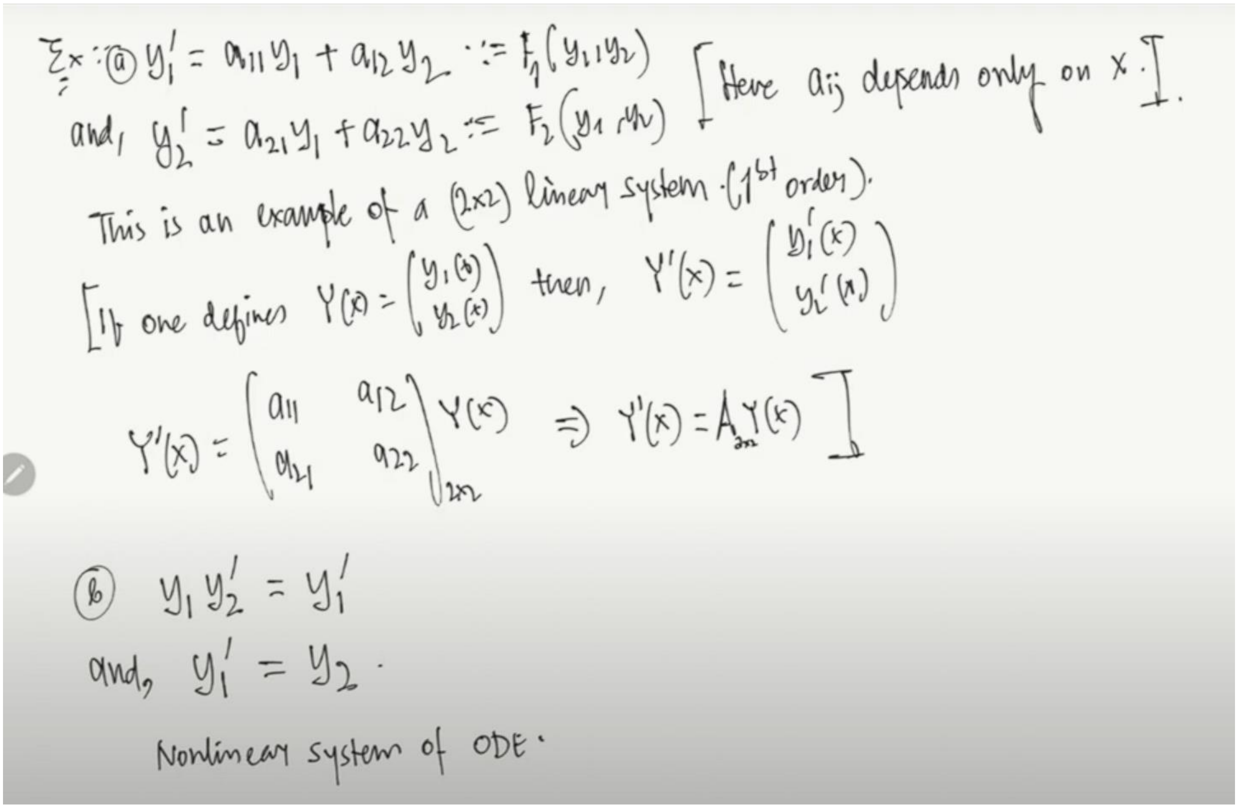
So, you see, let us write down y, y prime equals to y prime and y prime equals to y.

Okay, so you see this equation as you can see that this is the, if you look at the first relation, that is a, since y and y prime is involved here, so this equation, we call this as a nonlinear equation.

This is a nonlinear system, sorry, nonlinear system of ODE, system of ODE.

This is a nonlinear system of homology.

This is how we define it here because you see again as I told you the derivatives whatever it is even if it is a system or if it is just an equation the derivatives has to be independent of the unknown functions or its derivatives.



So here you see y prime contains y.

So that is why if let us say here there was no y

So, this y prime is y prime and y prime is y, then you have a linear system.

So, that is the example.

Now, let us look at, yeah, now let us look at this thing.

You see, the thing is the systems, they are actually a much, you know, generalized version of a equation.

So, basically, what I mean by this is, in this course, we are only going to, so in this course,

course, we are mainly interested in systems.

Interested in systems.

Okay.

And you see what I meant by this is, of course, we are going to look at equations from time to time.

But generally, we need to only study systems and then the equation part is taken care of.

How?

So, let us say example.

As an example, you see

Let us look at this example.

a of x, y double prime plus a naught of x, let us write down, a of x, y prime plus a of x, y equals to b of x, right?

Okay, so this is an equation, is a second order equation, is a second order linear ODE.

linear ODE.

Now, you see what I will do is I can actually write down this write this equation down as a system.

So, the equivalent system equivalent system.

system.

How do I write it?

You see, I will write z of x, you define z of x to be, define z of x to be y prime of x. If you do that, then our equation will actually give us that a naught of x, and of course, here I am assuming a naught of x is not .

Let us just assume that.

So, n of, this is for all x, for all x. So, you see, this equation will turn out to be z prime of x,

equals to b of x minus a of x z of x minus a of x times y of x okay so if you look at it properly what you can do is define another definition define okay r of x

What is the r of x?

It is z of x and y of x. So, therefore, r prime of x will be nothing but z prime of x y prime of x. The derivative is with component y. So, that will be given y and what is z prime of x?

z prime of x is nothing but b of x by a naught of x

right.

Let me write it this way, zx yx plus b of x a naught of x, okay, and .

Let me put it this way and z is nothing but minus a of x by a naught of x a naught of x okay and y is minus a of x by a naught of x okay and what is y prime y prime is nothing but z of x so basically it is okay so you see z prime of x is nothing but minus a of x by a naught of x times z

minus a of x by a of x times y plus b of x by a of x, right?

And what is y prime of x?

It is nothing but zx, okay?

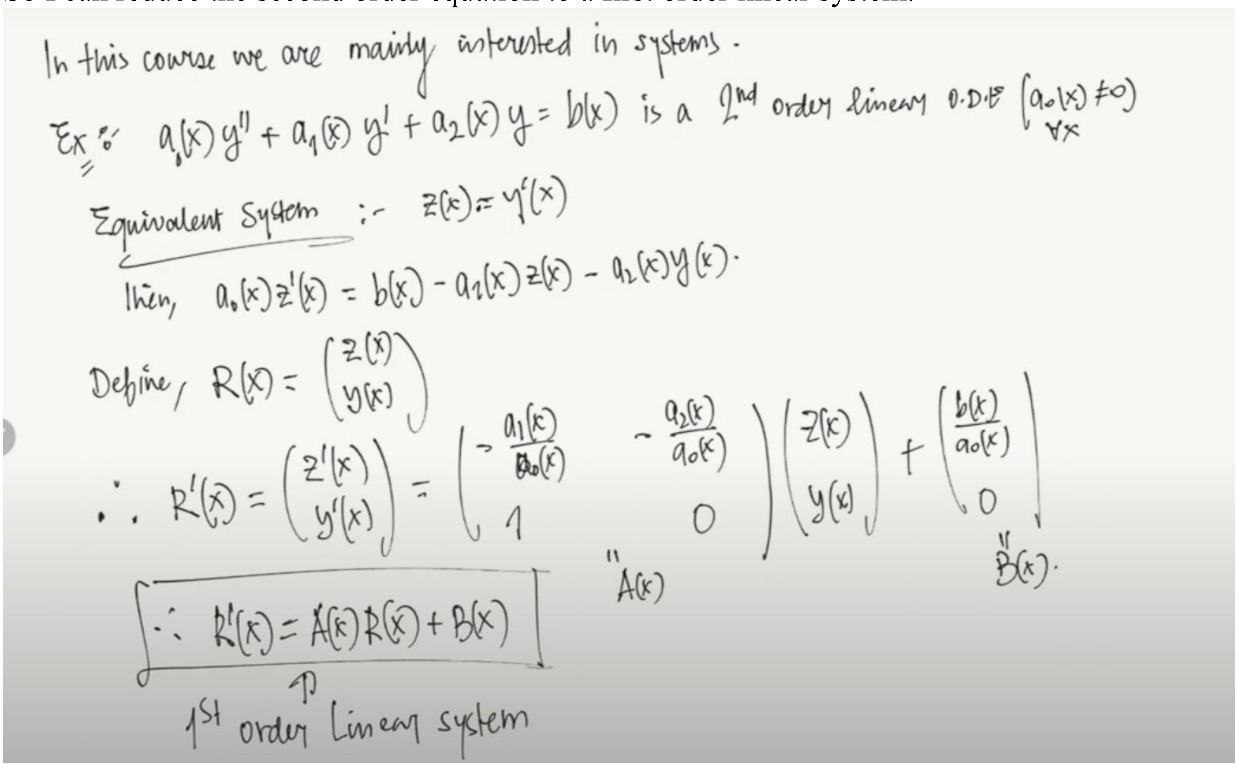
So, you see, if we, so basically, therefore, this is nothing but r prime of x equals to, let us just call this matrix as, I do not know, maybe let us just call it capital A of x, capital A of x, r of x,

plus b of x. Let us just call this thing as capital B of x. So, you see, I can actually reduce second order linear OD as a equivalent system.

So, in this case, this is a first order, please remember this, this is a first order linear system, linear system.

First order linear system.

So I can reduce the second order equation to a first order linear system.



So basically let us say if I give you a first order linear system, can I write it as a second order linear equation?

Now let us look at the converse.

So given a linear system.

Let us say cross system.

Can one write it as a second order linear equation?

The answer is not possible.

And you have to please find an example.

Find an example.

Okay, so what I meant is this, if you are given a second order linear ODE, you can only think of this exact same thing as a first order linear system.

Okay, so basically it is a linear system, you can think of it like this and you see, so basically what I am trying to say is this, if I know how to handle first order linear systems, I know how to handle a second order ODE because second order ODE you can always reduce it.

Okay, what about let us say third order linear ODE, if you are given a third order, so let me ask you another question.

Okay, question.

Given an nth order, nth order, n can be greater than equal, let us say .

Given an nth order, linear ODE, linear ODE, okay, can one convert it, convert it to a first order system, first order system.

system okay the answer is of course you can so please check this part again i i you can okay it is possible and please check again please check this how do you check it exactly the same sort of thing what i did for uh n equals to okay same thing works here but you have to do it for any n greater than equal so please try this part so i'm not doing that

Okay.

So, essentially what we understood is essentially this is the linear system which we need to understand.

Now, what about second order, third order linear system?

Of course, those things are there.

Those are very complicated things.

Linear systems can be anything, right?

I mean it does not really matter.

It has to be look like that.

But anyways, what we are going to do, sorry.

Yeah, linear systems, I mean, sorry, what I am saying, what I am trying to say is this.

First order linear systems looks like this, right.

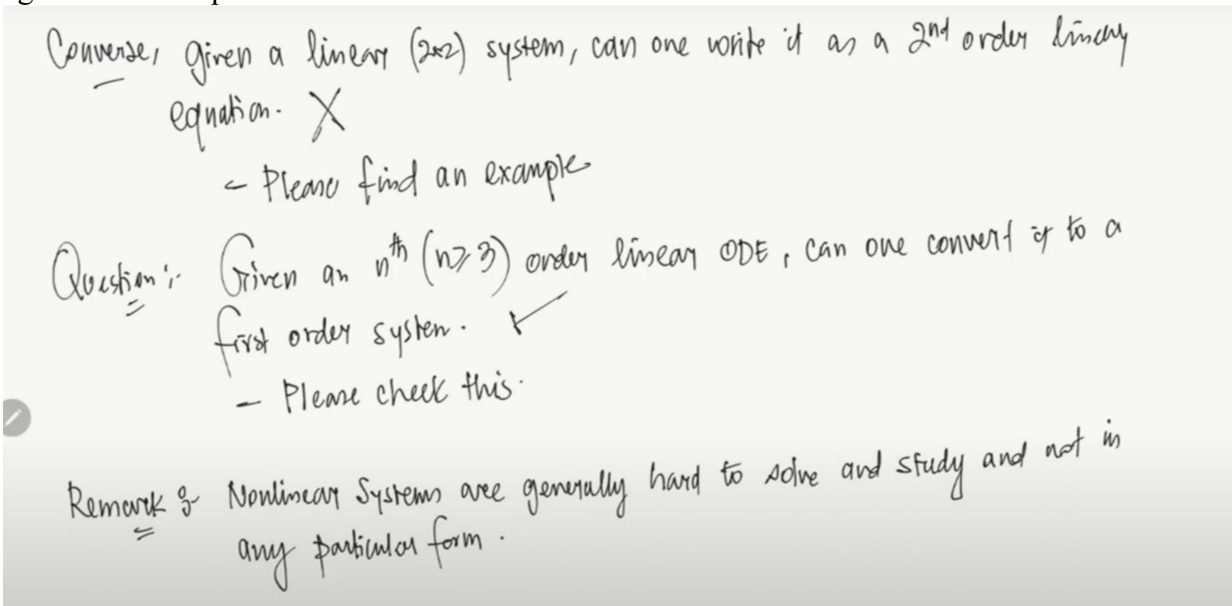
Now, you can of course complicate the system to a non-linear system, okay.

And then what happens is, so let me put a remark here, remark, remark.

Non-linear systems are generally hard to solve and study.

And not in any particular form.

particular form here okay so in this case what happens is gets very complicated and you have to do it one by one so basically ah you have to look at one one system individually you cannot put a you know general theory behind it okay except the existence and uniqueness for a particular non-linearity okay so essentially what we are going to do is in this case you see this sort of system again and let us put some nomenclature down so



If you have a system like this, linear system, linear system, let us say write it like this, y prime is f of, so let me put it this way, a of x y plus a of x y

a n of x yn.

So, there are n number of functions yi is from ab to r unknown.

I am dealing with this sort of functions.

We are dealing with this sort of functions and this is the equation y prime a of x y

plus a   of x y plus a n of x yn and it goes on like this and similarly yn prime is a n of x y plus a n of x y and then a nn of x yn.

So, if your system looks like this, then we can of course write it as equivalent system.

One may write it as y prime equals to a times y, right?

y prime equals to a times y plus b, clear?

And what is a?

So, what?

So, let us put a number here.

So, let us say b of x, b of x, bn of x. So, this is a, see b is called a homogeneous one.

If it is non-zero, then we have an inhomogeneous equation.

So, where a is the matrix aij, it is a n cross n matrix, right?

And what is y?

y of x is nothing but

y of x, yn of x, this matrix, sorry, this vector value function and what is b of x?

It is nothing but b of x, bn of x. So, this n cross and this is n cross .

Okay.

So, we can actually write this sort of system as a system which looks like in a compact form.

This is the form y prime equals to a y plus b. Of course, a depends on x, b depends on x. Okay.

So, that is there, but it is independent of capital Y. This is called a linear system.

So, we will call it a linear system, n cross n system.

So, we will call such a system as a

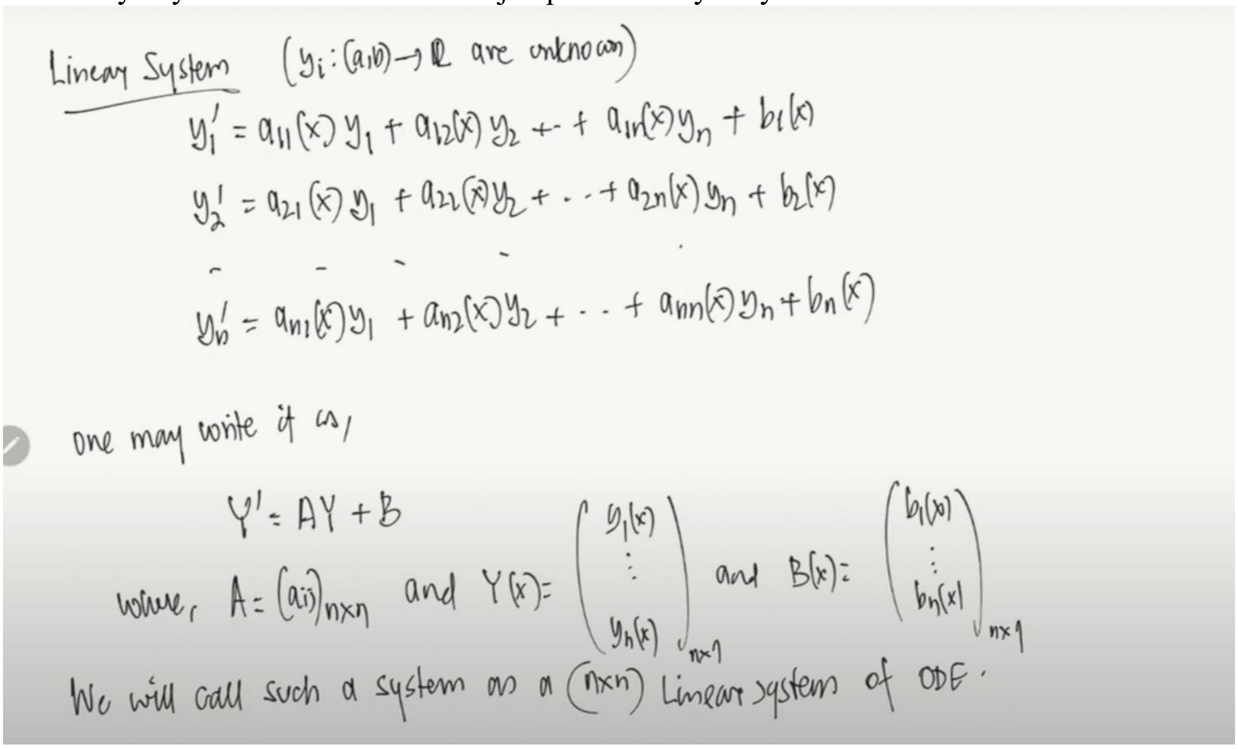
n cross n linear system, linear system of ODE.

Now, please remember this thing.

This is a linear system general form n cross n system.

Now, you can of course talk about nonlinear systems.

Let us say any of these coefficients a i j depends on x y or y derivative and all.



Then we have this nonlinear systems and all.

So, those are complicated things and this is the most easy one.

We will start our study by analyzing the linear systems first.

So, with this we are going to end this video.

Thank you.