Essentials of Data Science with R Software-1

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Lecture 65

Basic Tests of Hypothesis and Decision Rules

Hello friends. Welcome to the course essentials of data science with R software-1 in which we are trying to understand the basic fundamentals of probability theory and statistical inference. So, from this lecture, we are going to now start the last chapter of this course which is about a test of hypothesis. So, the first question comes here that what is this test of hypothesis? So, let me try to take a very simple situation and you try to give your reaction to yourself because I cannot watch it. Suppose I make a statement, my age is 20 years will you agree? You will say no like this big no and if I say my age is 90 years, again you will say like this no but if I say my age is 35 years possibly you will say really no.

But if I say my age is 50 years you will say yes possibly yes but if I say my age is 51 years then what will be your reaction? In both the cases 50 and 51, your reaction is going to be the same. Now, in case if I ask you what are you trying to do inside your mind? Why you are saying this big no when I try to say my age is 20 years or 90 years? What are you trying to do I have make I am trying to make a statement that my age is 20 years, my ages 90 years or my ages 35 years or my age is 50 years.

What are you trying to do? You are trying to compare my age the which I am informing you in the form of a statement, you are trying to compare it with some hypothetical value and what is that hypothetical value? That you have gathered looking at my structure body structure phase structure right and you are sure that okay my age cannot be 20 years old or I cannot be 90 years old but for 50 and 51 yes both are acceptable. So, my first question to you is that what exactly are you doing in your mind so that you can take a decision that my statement is correct or wrong.

Now, second question, second question is that when you whenever you are trying to do say sampling or you are trying to draw a sample from a population, every time you draw a sample you will get a different value of the statistics. For example, if you are trying to compute the sample mean if you try to take two different sample means of the same size from the same population still you will get different values of the sample means.

Now, how can you say that this sample mean is correct and this sample mean is wrong? Well, in case if you try to take a sample of the students of say class 12 and you try to draw a sample of size say 10 student and you try to find out their sample mean and this comes out to be suppose 18 years and you try to draw another sample then the sample mean comes out to be 18.5 years.

So, the question is whether 18 is correct or whether or this 18.5 is correct? You will say okay both are nearly the same. Nearly the same means what you are trying to incorporate the variability into it, although from mathematics point of view 18 and 18.5 years they are entirely different to values but statistically you are trying to say that okay there is not much difference so I can accept this one.

On the other hand, in case if I have two samples one sample is telling you the age is say 12 years and there is telling you 20 years, then you will say no there is something wrong and you will not accept. So, what are you trying to do? You are trying to compare the distance between these two populations and you are trying to compare that if there is a homogeneous class of a similar age of students in the class, then two sample means cannot be so different and then once again you are trying to compare, you are trying to do something in your mind so that you can take a conclusion that whether 18 and 18.5 are the same or different or 12 years or 18 years or say 22 years they are the same or different.

So, what are you trying to do? If you try to say in case if you try to see what I am doing here, I am trying to make here a statement and you are trying to create another statement in your mind and you are trying to compare it and for that you are trying to follow a process but this whole process this entire process is so quick that you possibly never realize that what type of calculations are you trying to make inside your mind.

In very simple words I can say that if you try to draw two samples from the same population and suppose the ages are 18 and 18.5 years, so you think that okay this variation can be due to the random variation and possibly I can accept it up to certain limit right but in case if this random variation also becomes too large like as 12 years and 22 years, then you will say no there is something wrong and I cannot accept that 12 years and 22 years are the same values or they are trying to represent the same population. So, now in this chapter we are going to answer such questions.

So, in this lecture I am trying to take up some smaller definition concepts and topics which are related to test of hypothesis and one by one I will try to explain you and then I will try to club them together so that I can create a proper decision rule so that I can say whether this statement is correct or that statement is wrong in a scientific way in a statistical way. Well, this lecture may be little bit long, so if you want to take a break you can pause the video sometime whenever you want and can continue after that. But I want to because I have to club many things together so I wish to finish it in a single lecture. Hope you would not mind it. So, let us begin our lecture.

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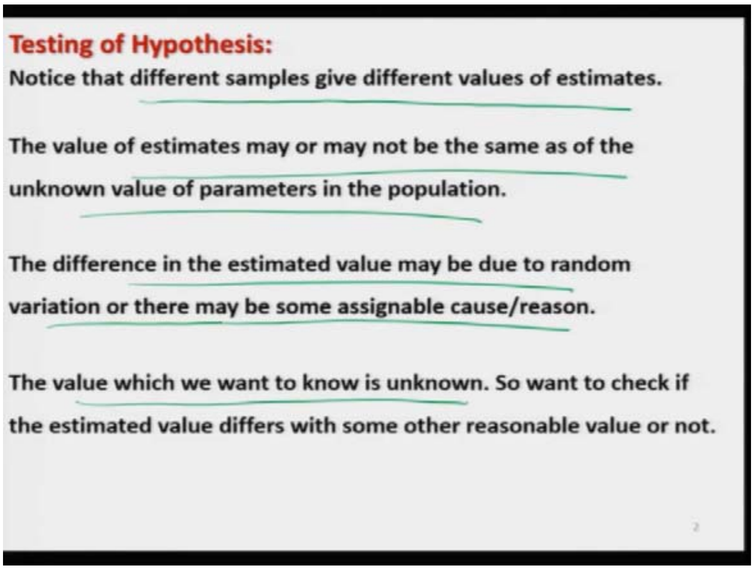

So, whenever you are trying to estimate the value of a parameter, you try to take a sample and you observe that the different samples give different values of the estimates and the value of the estimates may or may not be the same as of the unknown value of the parameter in the population. And the difference in the estimated values which are obtained from different samples may be due to some random variation, some reasons which are beyond your control or they may be due to some assignable cause or assignable reasons and moreover the value which we want to know that is the unknown parameter of the population. So, we want to check if the estimated value differs with some other reasonable value or not.

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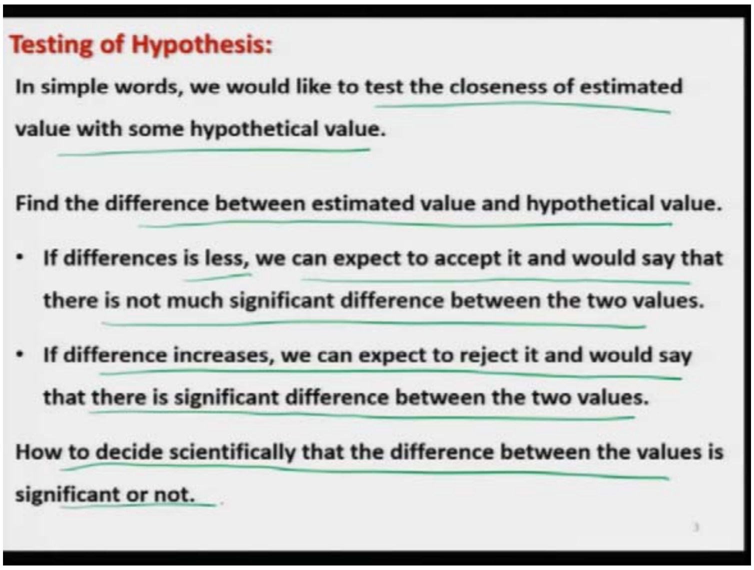

So, in simple words, what we want? We just want to take the closeness of estimated value with some hypothetical value and then what are we trying to do? We are trying to find the difference between the estimated value and the hypothetical value and if this difference is small, this is less then we can expect to accept it and would say that there is not much significant difference between the two values.

For example, when I made a statement that my age is 20 years or 90 years, then possibly inside your mind you expected that my age may be around 50 years also. So, you said no, the difference is quite large and you cannot accept my statement but you accepted your statement that the age is going to be close to 50 years but when I said my age is 50 years or 51 years then you again compare the difference between the two values and you said the difference between the two values is less, so you can accept my statement.

So, that is what I am trying to say here that if the difference increases we can expect to reject it and would say that there is significant difference between the two values and what will happen if I start saying that okay my age is 55 years, then my next statement can be my age is 60 years my age is 65 years. So, in some range you will accept my age, suppose, you try to say that okay in case if I say my age between 50 years and say 53 years possibly you will accept it but below that and above 53 years you will not accept my statement.

So, this is what I am trying to say that as we are getting away from the value 50 the chances of accepting my statement are becoming smaller right but the question is this, this is what we are trying to understand through some logic but my question is how to decide scientifically that the difference between the two values is really significant or not?

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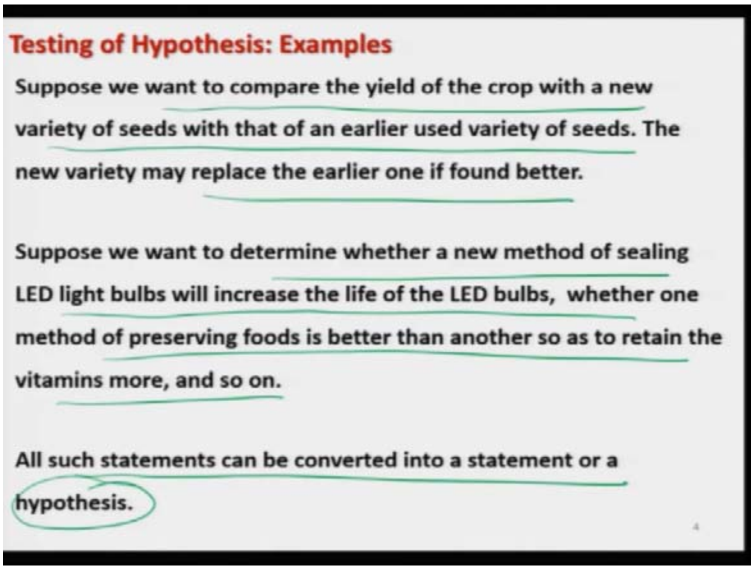

And suppose we try to take one more example and let us try to understand that suppose we want to compare the yield of a crop with some new variety of seeds with that of an earlier use variety of seeds. So, there is a new variety of seed and that claims that it can increase the yield of the crop. Now, in case the new seeds are better, then they are going to replace the earlier seeds otherwise not.

So, this is the type of statement and the decisions we want to make in real life and similarly if we want to determine whether a new method of sealing the led light bulbs will increase the life of the bulbs and similarly if there is another method which claim that it can preserve the food in a better way for a longer time then and it can retain the vitamins mineral etc. then we would like to compare the new method of preservation with the earlier one and this is the type of decision that we want to take on the basis of this acceptance or not acceptance of the statements.

So, all such statements can be converted into a statement or a hypothesis, you know what is the meaning of hypothesis? As soon as I say hypothesis it looks a very complicated word but if I say statement it looks a very simple word, so hypothesis is the same as a statement. So, that is why I am using the word here in the beginning as a statement, once you are convinced that the statement and hypothesis are the same thing possibly I will start using only the hypothesis.

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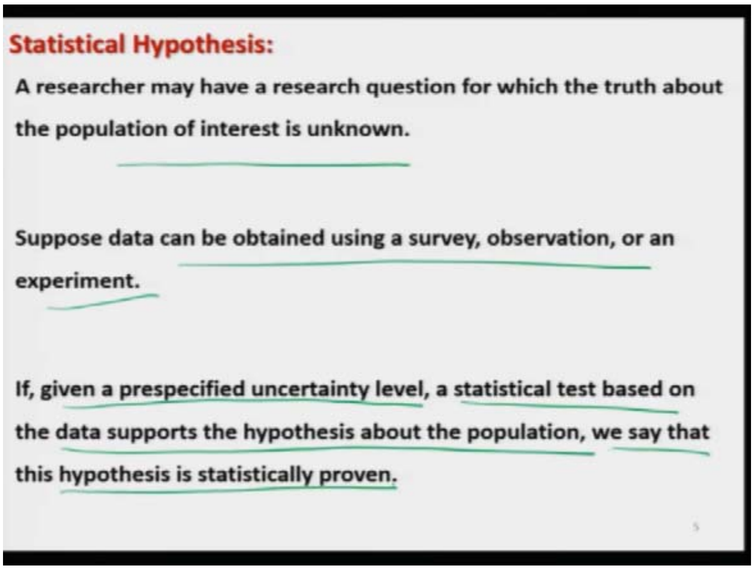

So, what is the statistical hypothesis? So, whenever we are trying to conduct an experiment then there is a question and that we want to answer. For example, if somebody is trying to develop a new type of fertilizer, the question may be whether the new type of fertilizer is better than the earlier fertilizer or not.

So, in order to get the answer some experiments can be conducted that in a given field some part of the field can be given the new type of fertilizer whereas the remaining part of the field can be given the earlier type of fertilizer and at the end, the crops from both the fertilizers can be compared. In case of the crops the difference in the yield of the crop is not much possibly I can say that okay the fertilizer is not effective and it is as good as the earlier fertilizer but if the crops increases significantly then I can say that the new fertilizer is better.

So, this is the type of decisions we want to take on the basis of the experiment and we want to know the truth about the population of interest which is unknown to us and we try to obtain the data using a survey experiment or getting some observation from anywhere and then we try to answer the question but definitely when you are trying to answer the question, there is going to be uncertainty because there is a random variation but definitely if this uncertainty is very high, you cannot accept the conclusion confidently. So, what we try to do, that we try to specify the uncertainty that okay this much I can expect.

For example, in case if I say my age is 50 years or 51 years, you can accept this amount of uncertainty but if I say my age is 50 years and 70 years possibly you may not accept this much uncertainty in my statements. So, if for a given pre-specified uncertainty level a statistical test based on the data support the hypothesis about the population, then we say that this hypothesis is statistically proven.

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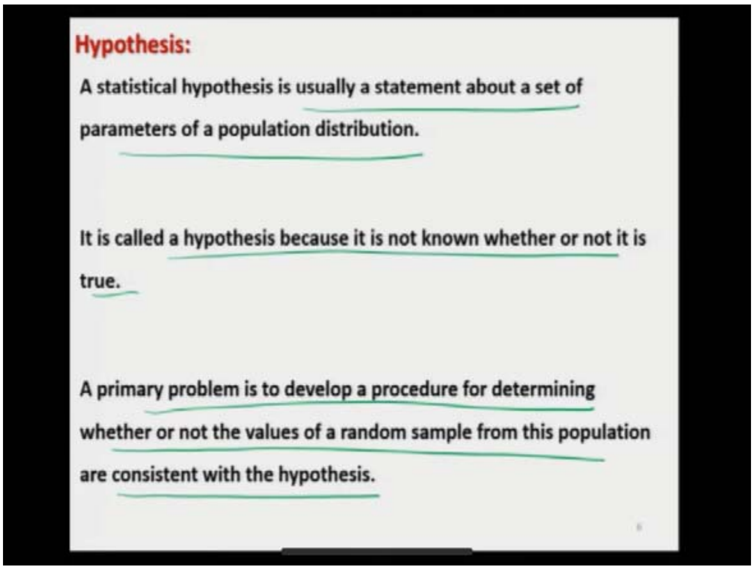

Now, the question is what is a hypothesis or what is a statistical hypothesis? So, this statistical hypothesis is usually a statement about a set of parameters of a population distribution because you see you have assumed that any experiment or the data that is coming from that experiment for is coming from a certain probability density function of probability mass function. So, and since we are working in a parametric setup to this score this p.d.f or pmf they are going to be characterized by the parameter of the distribution.

And this statement is called as hypothesis because it is not really known that whether it is correct or not. The statement is a statement but a hypothesis can have some uncertainty that you create a hypothesis that I will get 90 percent marks in the examination but this statement is going to be correct or not that will be known to us only after getting the marks in the examination. So, a primary problem is to develop now a procedure for determining whether or not the values of a random sample from these populations are consistent with the hypothesis or not? Well consistent is not the consistency of the estimator consistent in a dictionary meaning.

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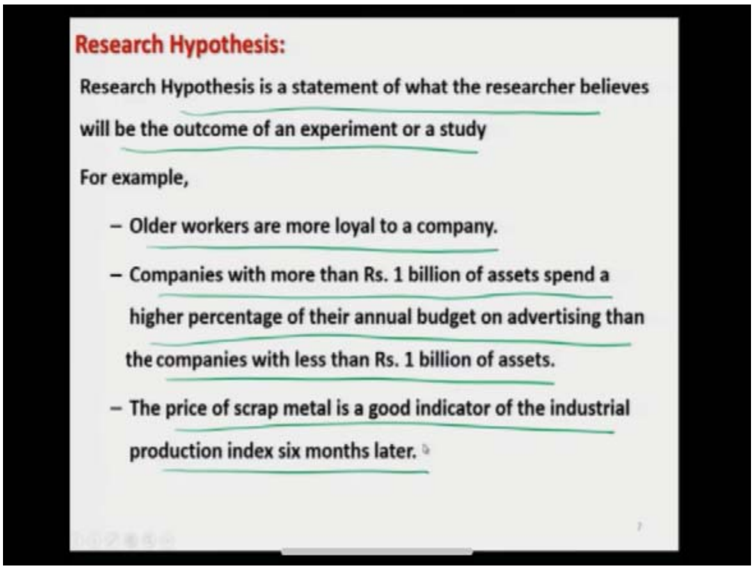

So, now for about this hypothesis whenever an experiment is being conducted, the involved experimenter tries to frame the statement in the form of a research hypothesis and this research hypothesis is a statement of what the researcher believes will be the outcome of an experiment or a study. For example, there can be a study in which people are trying to know whether older workers are more loyal to a company or not. Companies with more than Rs.1 billion of assets spend a higher percentage of their annual budget on advertising than the companies with the less than Rs.1 billion of assets. The price of scrap metal is a good indicator of the industrial protection six months later. So, all these things are trying to be converted into a statistical hypothesis and then they are going to be solved.

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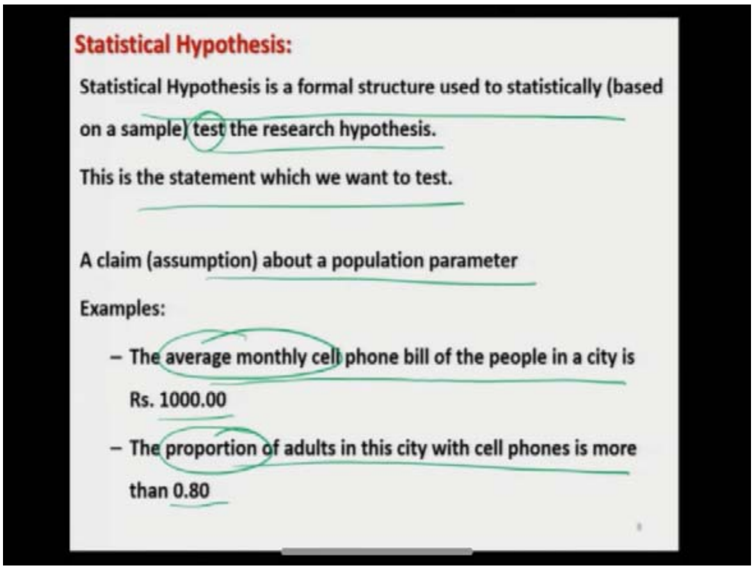

And statistical hypothesis is a formal structure which is used to statistically test the research hypothesis. Well, research hypothesis is a terminology which is basically used by many experimenters. So, that is why I have given you this idea and this test is based on the sample of data that you have obtained and this is actually the statement which you want to test and this is actually a claim or an assumption about the population parameter, for example I can make a statement here the average monthly telephone bill of the people in a city is Rs.1000 or the proportion of the adults in the city with the cell phone is more than 0.8. So, now you can see here you are trying to talk about the average monthly cell phone bill that is your value of mean and then you are trying to work here for the proportion. So, now you can know that what type of parameter can be involved over here.

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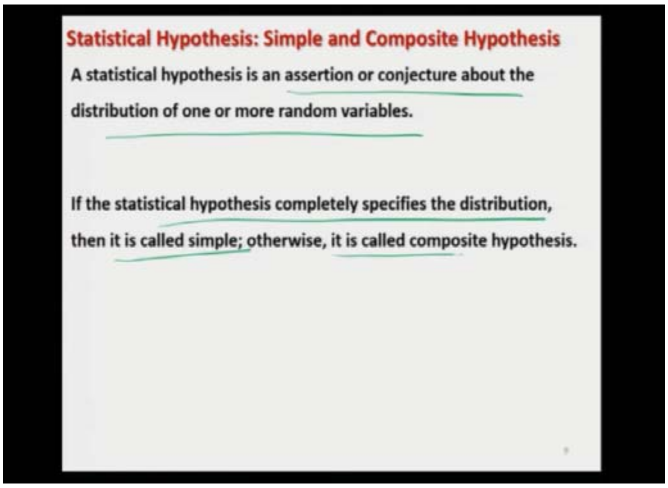

So, now when we are talking about the statistical hypothesis which is an assertion or conjecture about the distribution of one or more random variable, there are two types of situations and they are trying to be indicated by the simple hypothesis and composite hypothesis. In case the statistical hypothesis completely specifies the distribution, then it is called as simple hypothesis otherwise it is called as a composite hypothesis.

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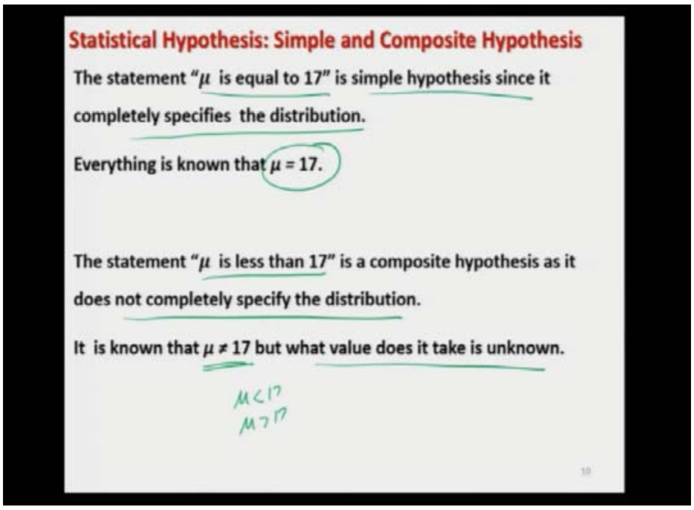

What does this mean for example if I am trying to make a statement that $\mu = 17$, right then this is a simple hypothesis because it completely specifies the distribution. This means what for example when I am trying to write down here μ equal to 17 then everything is known about the population. There is nothing is left but in case if I am trying to make a statement like μ is less than 17 then it is a composite hypothesis.

Why? Because it does not completely specify the distribution, for example when I am trying to write down here μ is not equal to 17, then I know that μ is not equal to 17 but whether μ is less than 17, μ is greater than 17 or what value does it take this is unknown to us. So, in that sense this is a composite hypothesis. So, this is a terminology that we used when we try to work with the test of hypothesis.

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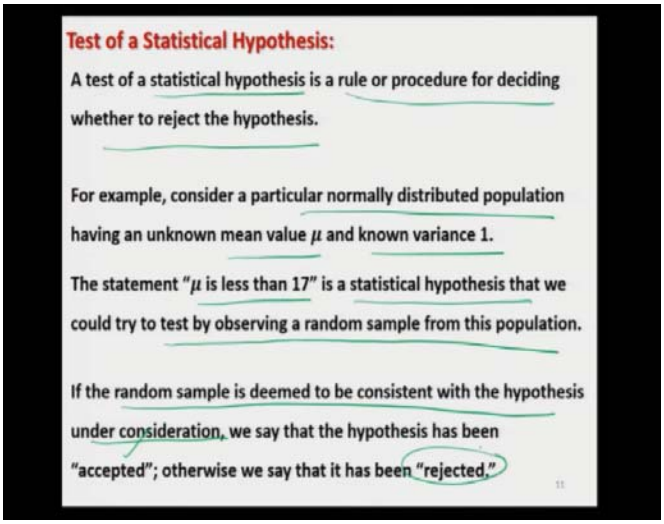

Now, the question comes here what is that test of a hypothesis and whenever now I am using the word hypothesis this is actually a statistical hypothesis. So, a test of a statistical hypothesis is a rule or a procedure for deciding whether to reject the hypothesis or whether to accept the hypothesis. For example, if you try to consider here a normal population whose mean is μ and variance is known to be as 1.

So, now in case if I am trying to make here a statement μ is less than 17, then this is a statistical hypothesis and we would like to test this hypothesis by observing a random sample from this normal population with mean μ and variance 1. And if the random sample is deemed to be consistent with the hypothesis under consideration then we say that the hypothesis has been accepted and if not then we say that the hypothesis has been rejected. So, that is our terminology.

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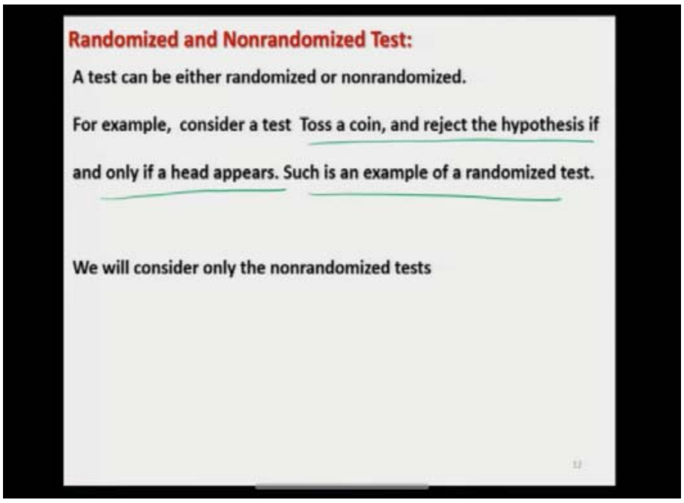

So, now whenever we are trying to create such type of test, then there are two types of tests which can be formed. One is a randomized test and another is a nonrandomized test. So, for example in case if we consider a test that toss a coin and reject the hypothesis if and only if a head appears. In that case we do not know when are we going to get the head in the first toss, in the second toss or in the third test so and so on. So, such is an example of a randomized test. But definitely in this lecture and in this course, we are not considering it, we are going to consider only the nonrandomized test. Remember this thing okay.

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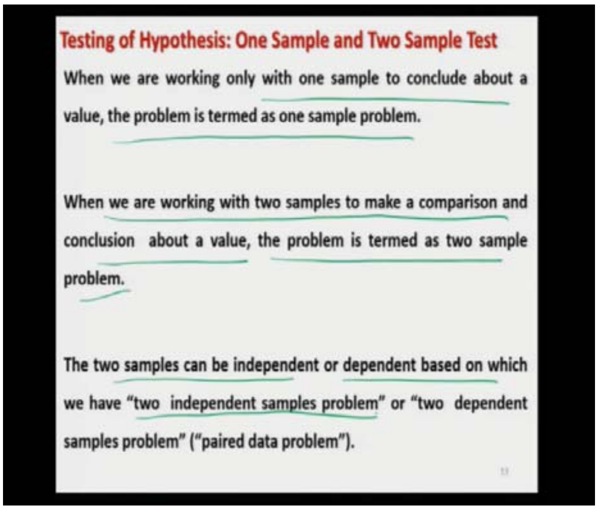

So, now the question comes here that whenever we are trying to test a hypothesis, we are going to obtain a sample of data and based on that, we have two types of test one sample test and two sample test. So, whenever we are trying to work only with one sample to conclude about a value then the problem is termed as one sample problem and when we are trying to work with two samples to make a comparison and conclusion about a value, the problem is termed as two sample problem and this two-sample problem can be divided into two parts when the two samples are independent or they are dependent. So, in case if they are independent, then we call the problem as two independent sample problem and if the samples are dependent, then we call it as a two dependent samples problem or this is also called as paired data problem.

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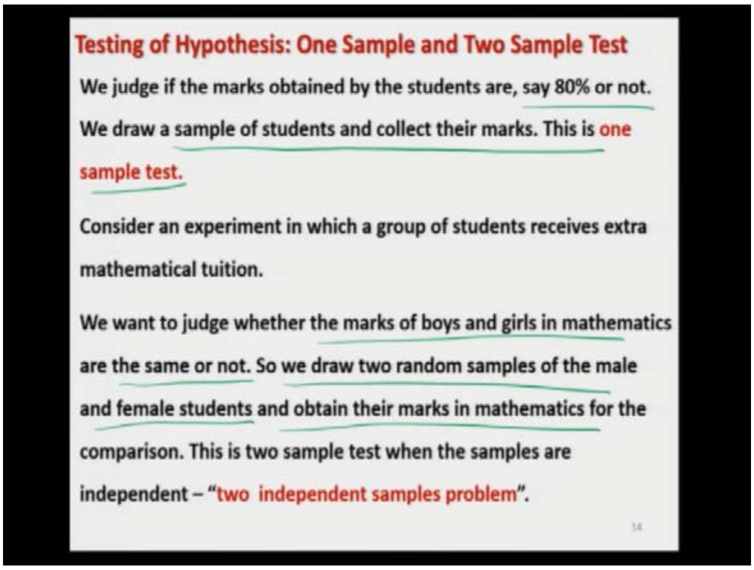

So, what are these things? So, for example, we judge if the marks obtained by the students in the class are say, 80 percent or not right. So, we are talking of the class. So, some student might have got 82percent somebody may have got 79 percent and so on. So, what we try to do? We try to draw here a sample of the marks of the students and then we try to do something, some test procedure to conclude about the statement.

So, in this case, we are going to draw only one sample of the students and then we are going to ask their marks. So, this is called as one sample test. And similarly consider an experiment in which a group of students received some extra mathematical tuition. Now, we want to know whether this tuition has been effective among the boys more or among the girls and we want to judge whether the marks of the boys and girls in mathematics are the same or not after getting that tuition. So, what we try to do?

We try to draw here two random samples, one for the male students and another for the female students and then try to obtain their marks in mathematics and we try to compare them. So, in this case we are trying to draw two different samples. The samples are coming from the same population in this case but the populations are different in the sense that one population corresponds to the boys and another population correspond to the girls. So, this type of situation is called as two independent samples problem.

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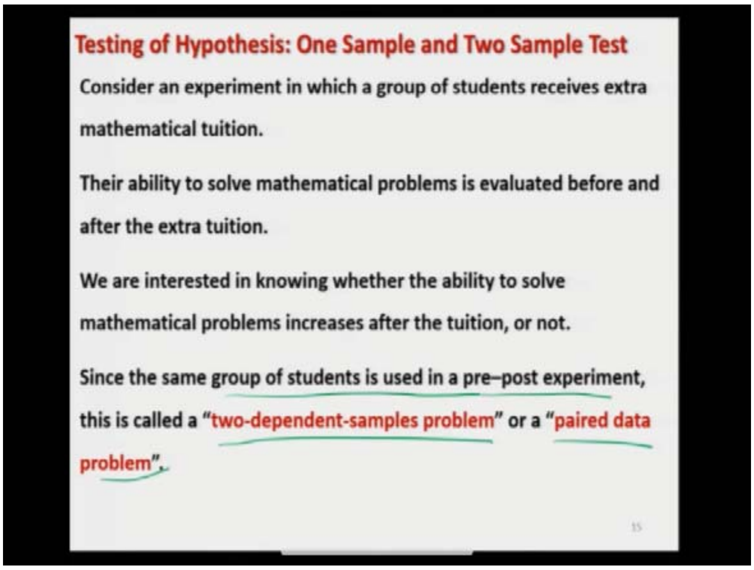

Now, there can be one more condition that suppose the group of student has been given some extra mathematical tuition and we want to know whether this tuition has been effective or not. So, what we are trying to do that we are trying to first take a test and then we are trying to find the marks of the students and then the students are given this tuition and after that they are once again given a similar test of the same level and then their marks are recorded and now we have the two sets of marks but they are on the same set of students.

One student before the tuition and after tuition, second student marks before the tuition after the tuition and so on. So, in this case we are trying to obtain here two samples but those two samples are on the same unit. So, in such a case we are trying to get the marks from the same group of students pre and post experiments so they are called as two dependent sample problem or this is called as paired data problem.

So, this type of data can also another example of paired data is whenever we are trying to conduct a clinical experiment and we want to test the efficacy of a medicine. So, the medicine is given to a group of patients and the body parameters of those patients are recorded before giving them medicine and after giving the medicines and based on that, it is concluded whether the medicine was effective or not. So, in this case also the medicine is given to the same set of people for example the patient number one, body parameters before giving the medicine and after giving the medicine. Patient number 2, the body parameters before giving the medicine and after giving the medicine and so on. So, this is again an example of a paired data problem.

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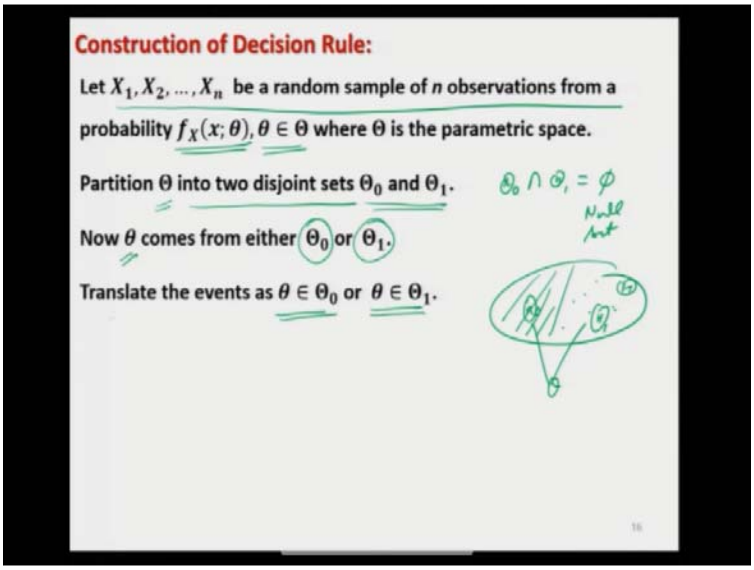

Now, the question comes here whenever we are talking of the decision rule, what are we really trying to do from the statistical point of view? So, from the statistics point of view, we are trying to say here that let $X_1, X_2, \ldots, X_n$ be a random sample of size small, an observation from some probability function say this $f_X(x;\theta), \theta \in \Theta$ because f(x) can be a discrete corresponds to a probability mass function or a probability density function. So, what are we going to do?

We want to partition this parametric space Θ into two disjoint sets because that this Θ_0 intersection Θ_1, this is equal to phi null set. Now, what we want to know? We want to know that whether this θ is coming from Θ_0 or Θ_1. So, essentially if I say here this is my Θ, so we are trying to partition it into two parts one part like this and another part here like this and we want to know that now random sample has been observed and based on that whether the θ will belong to this Θ_0 or will belong to Θ_1 like this. So, now whatever we want to know whether $\theta \in \Theta_0$ or $\theta \in \Theta_1$, this has to be translated as an event.

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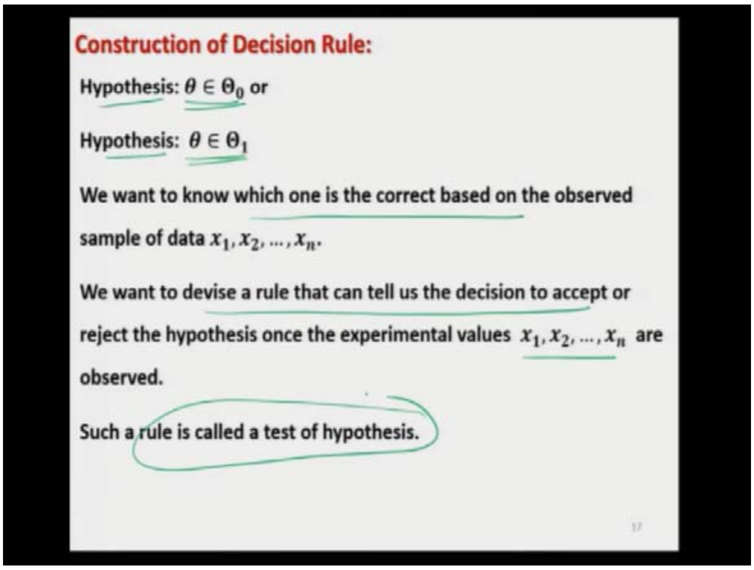

And then we try to write down these events in the form of hypothesis. So, I can write down here two hypothesis like as hypothesis is $\theta \in \Theta_0$ or hypothesis is that $\theta \in \Theta_1$ and we want to know on the basis of the given sample of data that which of the hypothesis is correct and for that we want to devise a rule that can inform us the decision to accept or reject the hypothesis once the experimental value like as $X_1, X_2, \dots, X_n$ are observed and such a rule is called as test of hypothesis. So, we simply want the decision rule to know whether this hypothesis is correct that : $\theta \in \Theta_0$ or this $\theta \in \Theta_1$.

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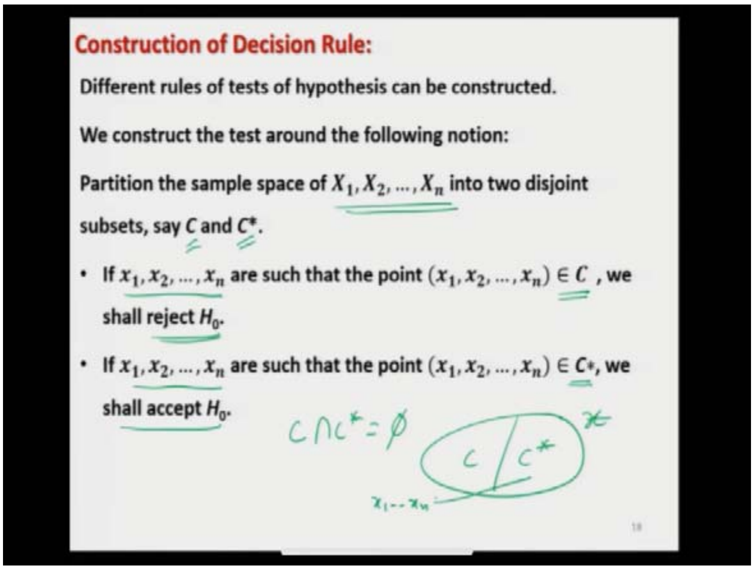

So, now for this what we try to do that there are different ways to obtain such rules of test of hypothesis and they can be constructed but we are going to construct such a rule around the following way right. We will try to partition the sample space of this $X_1, X_2, \ldots, X_n$ into two disjoint subsets say C and C* and we will simply say that if the observations $X_1, X_2, \ldots, X_n$ are such that that these observations are falling in the region C, we shall reject H_0 and in case of this observation $X_1, X_2, \ldots, X_n$ they are falling in the region C* we shall accept H_0. So, it is something like this we have a space of here x and then I am trying to divide into two parts, C and C* and both this is C and C* are their intersection is a null set, phi.

So, that there is no overlapping. Now, what we try to do we try to observe here a sample $X_1, X_2, \ldots, X_n$ and then we psi and based on certain rule we try to decide whether this set of observation belong to this region or this region and based on that I can say whether the hypothesis is going to be accepted or not.

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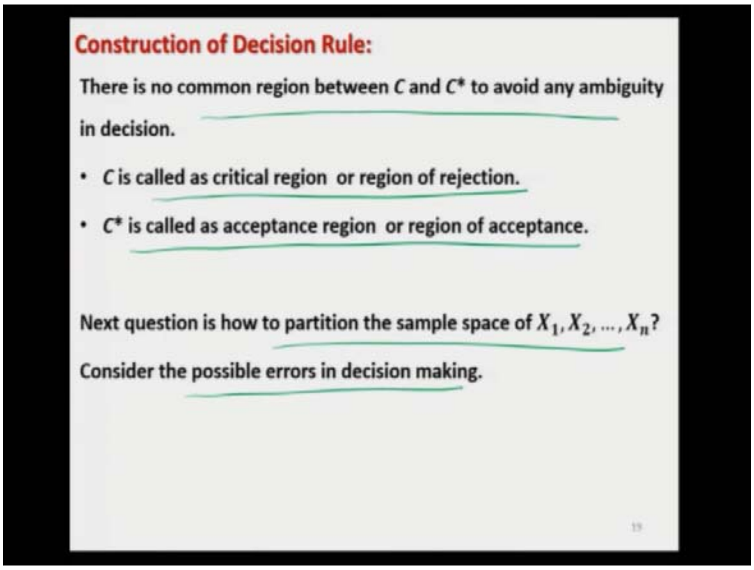

That is a very simple thing, this details are actually more and the job is very less right. So, there is no common region between C and C*, so that there is no ambiguity in decision and this C is called as critical region or the region of rejection and C* is called as acceptance region or the region of acceptance. So, now the next question is how to partition the sample space of $X_1, X_2, \ldots, X_n$?

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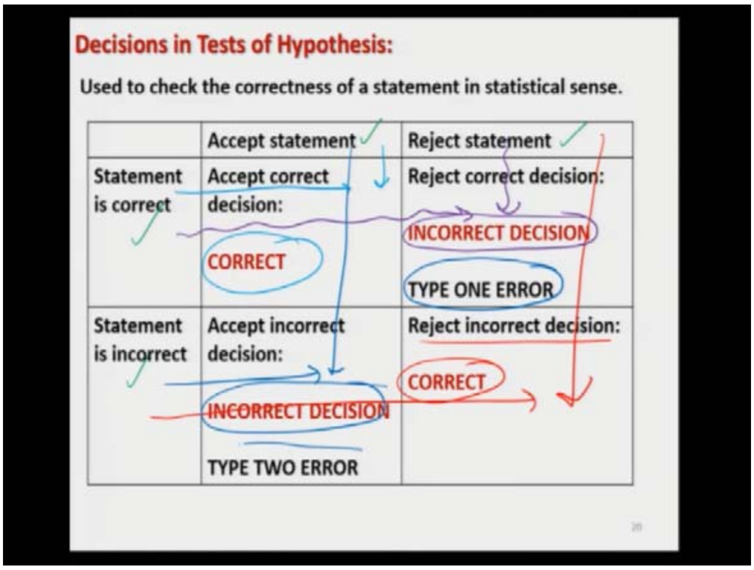

Now, in order to do this, we can do this by considering the following aspect that whenever we are trying to make a decision, there is a possibility of making an Error and we try to minimize that Error. For example in case if there is a statement or a hypothesis, there are two options whether we are going to accept the hypothesis or the statement or we are going to reject the statement. And on the other hand, there are two possibilities that your statement is correct or statement is incorrect. Now, what we try to do? We try to consider all the combination.

If the statement is correct and we accept it, there is no problem this is a good decision and this is a correct decision. In case if the statement is incorrect and we reject it, then this is also correct that is a good decision that you are trying to reject an incorrect decision. Now, in case if the statement is correct and we are trying to reject it, then this is incorrect decision and similarly in case if you try to consider that the statement is incorrect and you try to accept it, then this is also an incorrect decision.

So, out of these four decisions, two decisions are correct and two decisions are incorrect. So, now in order to give it a name that because there are two incorrect decisions, so we try to give them a name. This decision where we are trying to reject a correct statement this is called as Type One Error and when we are trying to accept an incorrect statement this is called as Type Two Error.

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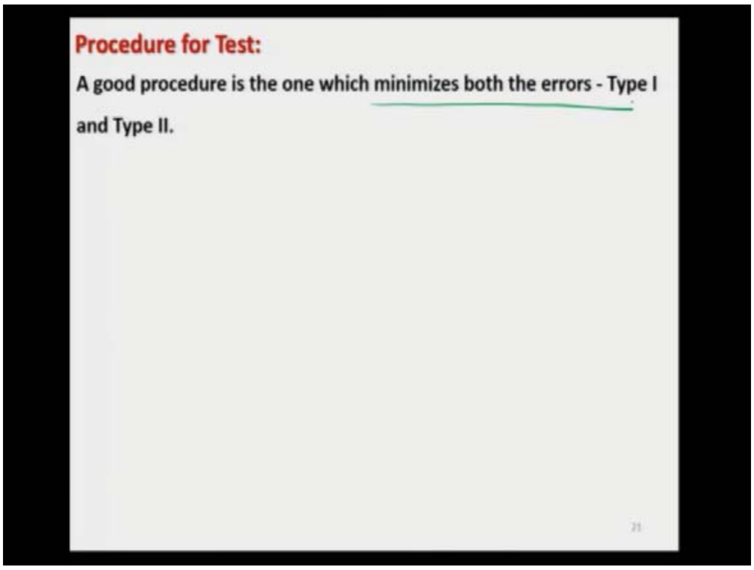

So, now a good procedure to know which of the hypotheses is correct can be a procedure which is minimizing both the errors that is Type One Error and Type Two Error.

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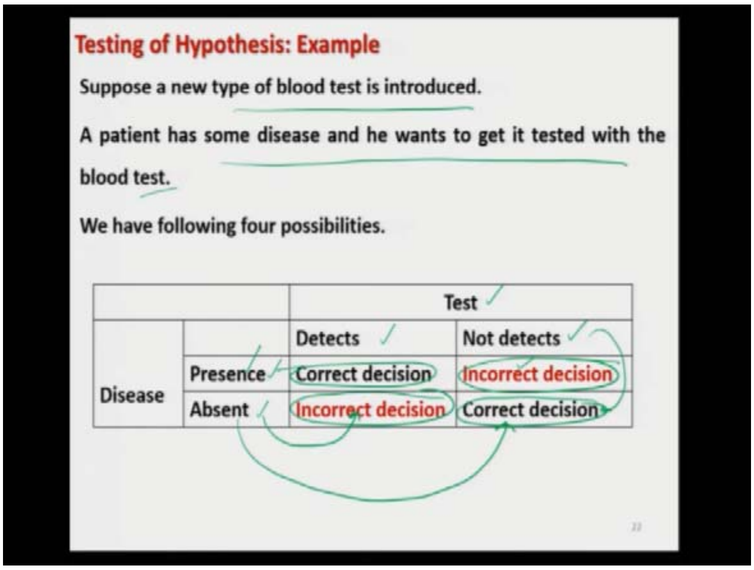

Now, to understand this thinly, let me try to take a very simple example to explain you. Suppose a new type of blood test is introduced and a patient has some disease and he wants to get it tested with the blood test. So, now there are following four possibilities that the test can detect the disease or not and the disease can also be present in the patient or absent in the patient.

So, now in case if the disease is present and the test detects it that means the test is good and this is the correct decision and similarly in case if the disease is absent and the test does not detect it this is also a correct decision but in case if the disease is present and the test does not detect it, that means this is an incorrect decision and similarly if the disease is absent and the test detects it this is also an incorrect decision.

So, there are two such incorrect decision but think and tell me which of the incorrect decisions out of these two is going to have severe impacts? So, surely there are two options that if a person has disease and the test says no, this will have a more serious impact on the patient because if the there is no disease and that test says yes, the patient has a disease then at the most patient will have to take some medication, that may have some side effects but comparison to the first Error the risk of life is extremely less.

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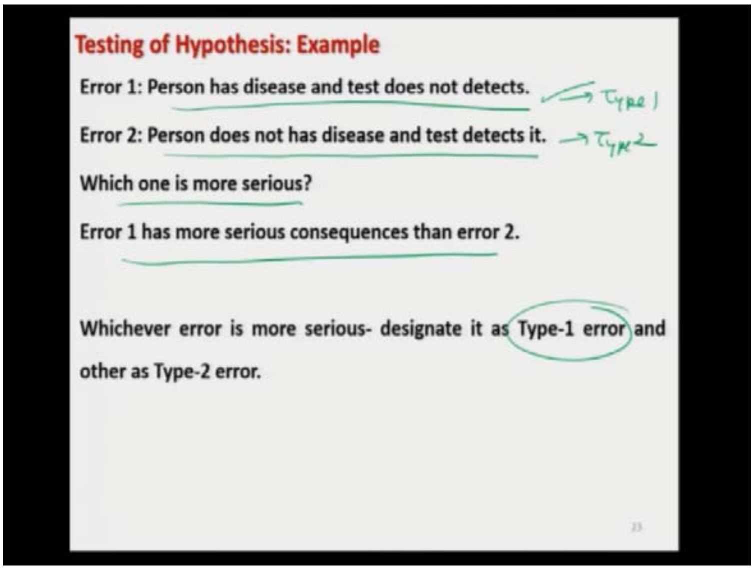

So, in this case, we can say that there are two types of Error same person has disease and the test does not detects it and the person does not has disease and the test detects it. So, now we try to find out between the two that which one is more serious and whichever is more serious this is called as Type One Error. So, in this case this Error one is more serious than the Error two in terms of the consequences. So, we are calling this Error one as Type One Error and this Error 2 as Type Two Error. So, this is your here Type One Error and this is your here Type Two Error. So, this is how we try to define and designate the Type One Error and Type Two Error.

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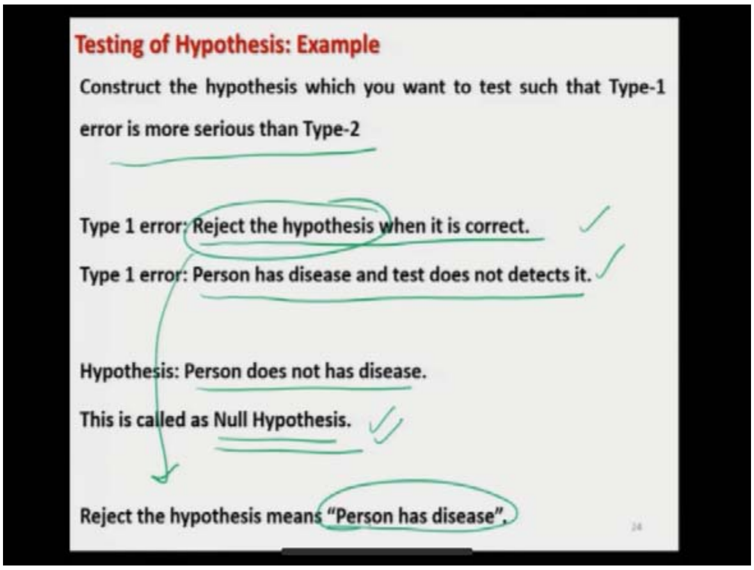

Now, we try to construct the hypothesis which we want to test such that Type One Error is more serious than the Type Two Error. Well, the question comes how does this constraint comes into picture? So, that will be clear to you later on that whenever we are trying to develop the test of hypothesis, then those tests have been developed on this basic assumption that the Type One Error is more serious than the Type Two Error. Why this is happening? That will be clear to you in the forthcoming slides. So, if you try to see here what is this Type One Error reject the hypothesis when it is correct and if you try to translate it in terms of your experiment, then you are trying to say the person has disease and the test does not detect it.

So, in case if you try to compare these two Type One errors from the definition and from what is really happening, you can say that the hypothesis is the person does not has disease and this statement that we want to test this is called as null hypothesis and this statement has been constructed in such a way such that the Type One Error is more serious than Type Two Error. Why? Because when you are trying to say here reject the hypothesis that means the person has disease. So, this is how we try to construct the null hypothesis and we call this hypothesis as a null hypothesis.

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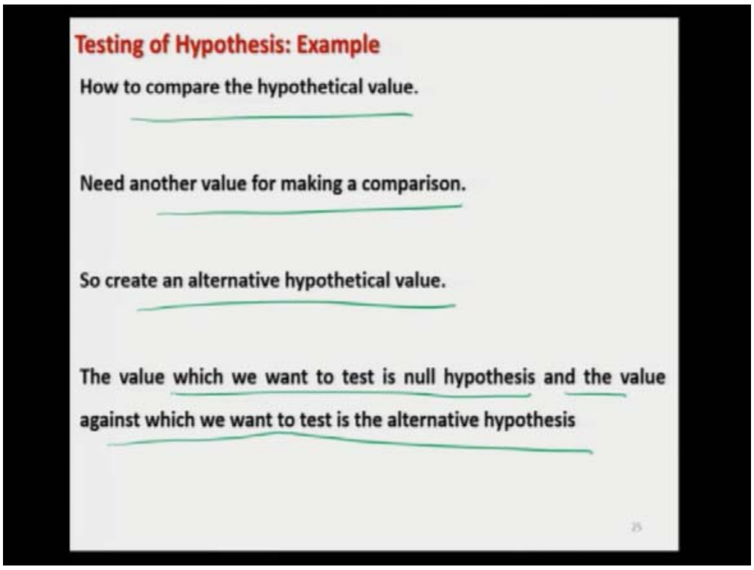

Now, the question is this now you have created one hypothesis which is which you have called as null but definitely you are trying to compare with something. So, the issue is how to compare the hypothetical value or how to compare this value with another value. So, we need one more value for making a comparison, so we try to create an alternative hypothetical value and the value which we want to test is the null hypothesis and the value against which we want to test is the alternative hypothesis. So, by this example I have explained you the concept of null hypothesis and alternative hypothesis.

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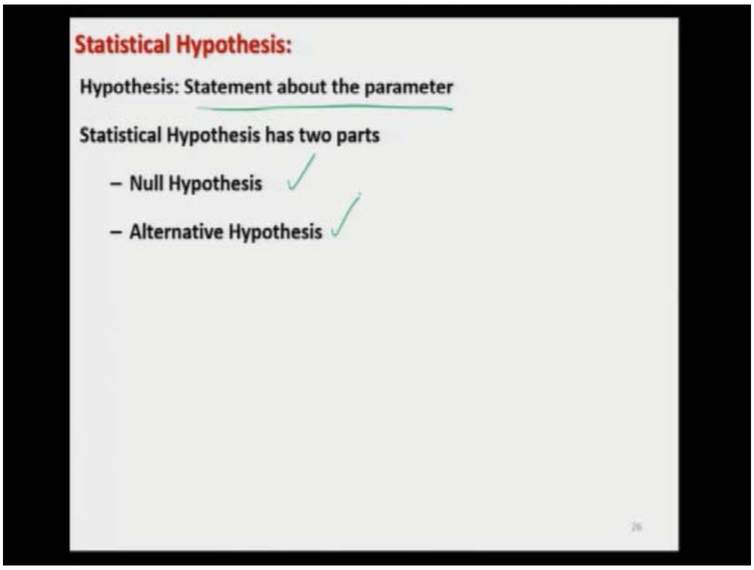

But now I will try to give them in more detail. So, the hypothesis is a statement about the parameter and it has two parts, null hypothesis and alternative hypothesis.

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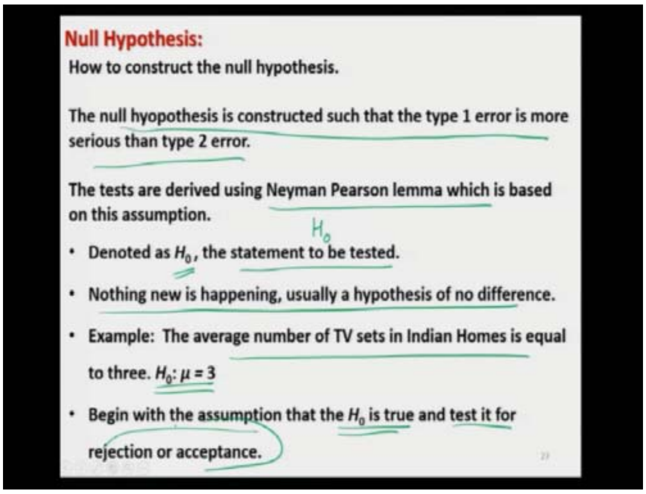

So, the question here is that how to construct the null hypothesis? So, the rule is the null hypothesis is constructs has is constructed such that the Type One Error is more serious than the Type Two Error and these tests are going to be found from Neyman Pearson lemma which is based on this assumption, that we will try to see later on.

So, this null hypothesis this is usually indicated by H_0. So, H means hypothesis and 0 in the subscript and this is a statement that we want to test and in a general common language we can say that this is a hypothesis of no different, that mean nothing new is happening. For example, the average number of TV sets in Indian homes is equal to 3.

So, I can write down here H_0 : $\mu = 3$ and we start the test procedure by assuming that H_0 is true. That means the average number of TV sets in the Indian homes is equal to 3 and then we try to test the statement on the basis of the decision rule and we want to know whether this H_0 is rejected or accepted.

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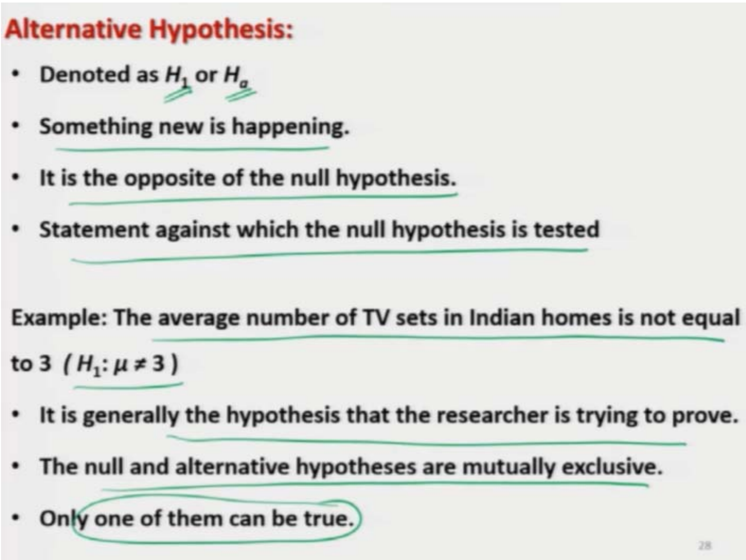

And similarly for the alternative hypothesis, this is indicated by H_1. So, this is like H_1, 1 in the subscript or H_a, in some books you will find this notation also a means alternative. So, this is a hypothesis about something new is happening and it is just opposite to the null hypothesis. And this is a statement against which the null hypothesis is tested, for example in the earlier statement I can now make an alternative hypothesis like this the average number of TV sets in Indian home is not equal to 3. So, I can write it down as H_1 : $\mu \neq 3$.

So, it is generally the hypothesis that the researcher is trying to prove and remember one thing that null and alternative hypothesis are mutually exclusive only one of them can be true at a given time, this cannot happen that on the basis of given sample of data both the null as well as alternative hypothesis can be true. If one is accepted then other is rejected and vice versa.

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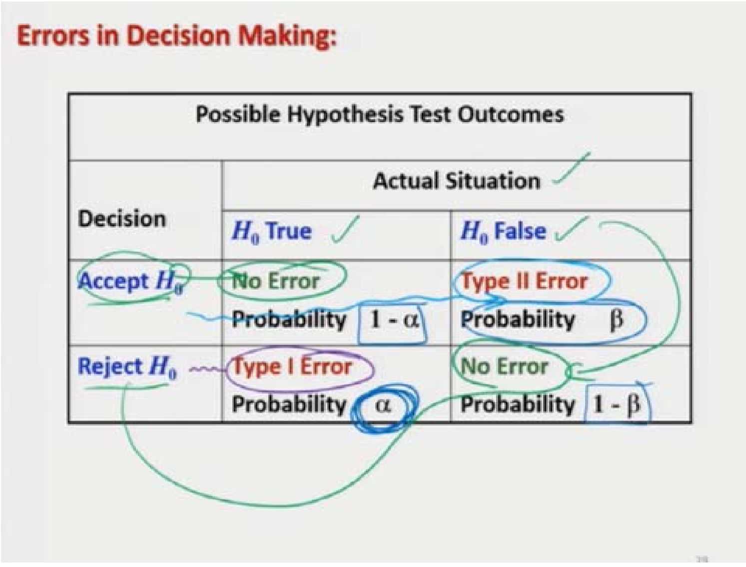

Now, the same what I shown you here in this table right, these two types of Error can be now defined in a more formal way. So, on the basis of this hypothesis that there is an actual situation related to the H_0 that H_0 is true or H_0 is false and there is a decision can be accept H_0 or reject H_0. So, when we are trying to accept H_0, when it is true then there is no Error and similarly when we are trying to reject H_0 when H_0 is false then also there is no Error.

We are happy but when we are trying to accept H_0 when H_0 is false, then there is an Error and this Error is called as type-2 Error and similarly when we are trying to reject H_0 when H_0 is true this is called as Type One Error and we are going to make a statement that Type One Error is assumed to be more serious than the Type Two Error. So, in a given situation try to find out what is the Error and then out of the two errors try to designate the more serious Error as Type One Error.

Now, in case if you try to find out the happening or non-happening of the event in terms of this Type One Error and Type Two Error, so we can find out the probability of Type One Error and this is indicated by α and the probability of Type Two Error that is indicated by β. So, obviously this probability that accept H_0 when it is true that will become 1 minus α and the probability that H_0 is rejected when it is false this will become here 1 minus β.

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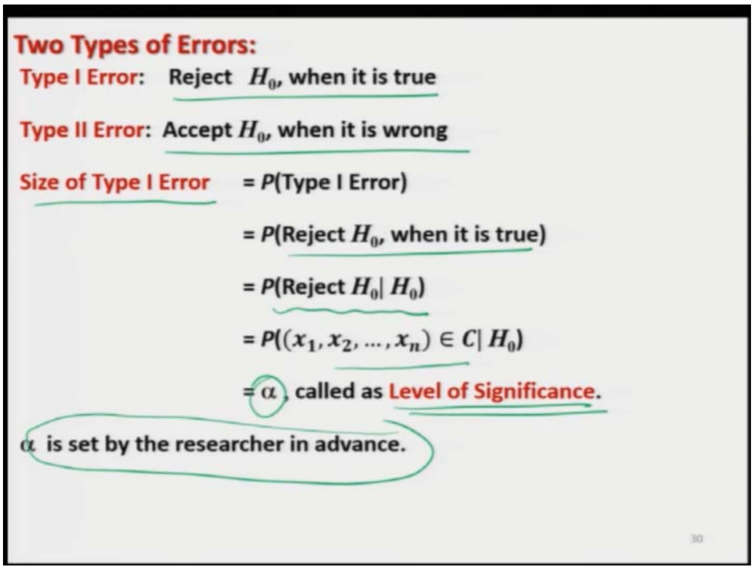

So, now we try to define these statements in a more formal way. So, Type One Error is reject H_0 when it is true, Type Two Error is accept H_0 when it is wrong and the probability of Type One Error is α and this is called actually as the size of Type One Error. So, the probability of Type One Error is reject H_0 when it is true and it is indicated by this symbol, the probability of reject H_0 and this given that H_0 is true.

So, that is the interpretation and in case if you are trying to write down the same thing in terms of your C and C* that you defined earlier as critical region, so where the regions the regions where you are going to accept or reject the hypothesis then you can write down here probability that X_1, X_2,…, X_n belongs to C when H_0 is true and this is indicated by α or this is also called as level of significance and this level of significance is set by the researchers in advance why, I will try to explain you after couple of minutes.

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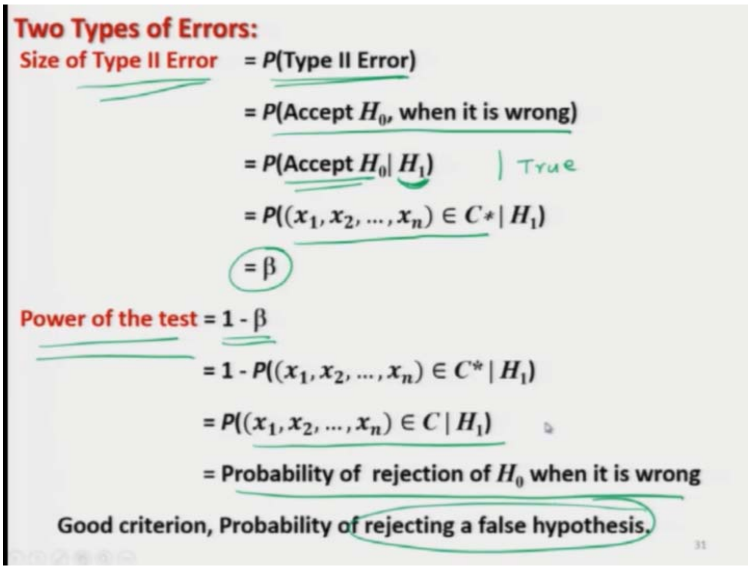

So, similarly we can also define the size of Type Two Error which is the probability of Type Two Error which is a probability of accept H_0 when it is wrong, so this can be written as probability of accept H_0 and one H_1 is correct, that is the standard language that after this symbol given we try to write down whatever the hypothesis is true. So, when H_0 is wrong that means H_1 is true and in terms of your C and C*, we can write down here probability that when X_1, X_2,…, X_n belongs to C* when H_1 is true and this probability is given by here β that is indicated by the symbol β.

That is a standard symbol and when we try to consider the term 1 minus β, this is defined as the power of the test because this is the probability that X_1, X_2,…, X_n belongs to C* when H1 is true. So, this is actually the probability that X_1, X_2,…, X_n are belonging to C. And this is probability of rejecting H_0 when it is wrong, so this is a good criterion the probability of rejecting a false hypothesis. So, that is why this is called as power of the test and we always want a test which has the more power.

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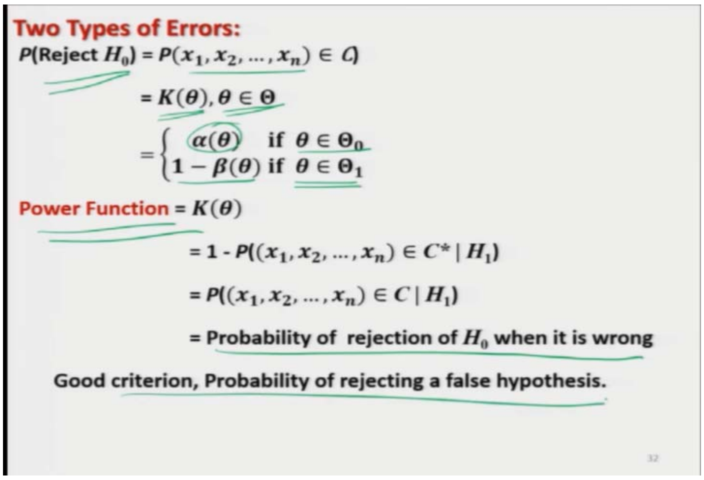

And in case if I try to combine these two things, I can write down in general here as a probability of rejecting H_0 is probability that $X_1, X_2, \ldots, X_n$ belongs to C and this probability I can express as a function of θ as K(θ) here, θ belong to Θ which takes value α or I am trying to represent here to just to indicate that it is depending on the θ.

So, I am writing here as say α(θ) when $\theta \in \Theta_0$ and this is equal to 1 minus β or 1 minus β(θ) if $\theta \in \Theta_1$ and this K(θ) is called as a power function. So, this is simply the probability of rejection of H_0 when it is wrong, so this is a good criteria probability of rejecting a false hypothesis.

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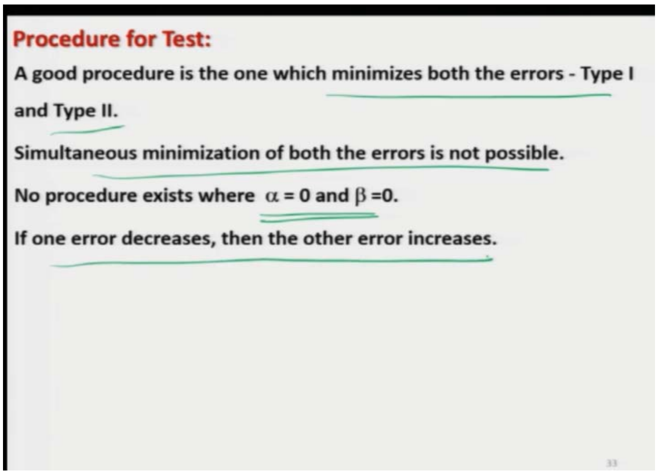

So, definitely a good procedure is the one to get the decision rule which minimizes both the errors Type One and Type Two but simultaneous minimization of both the errors is not possible, there does not exist any procedure where both the errors can be minimized. So, actually there does not exist any procedure where both this α and β can be equal to 0. So, what is really happening that if one Error decreases, then the other increases. For example, if this is my α and this is my here β, in case if I try to minimize α then β increases and if I try to minimize β then α increases. So, the simultaneous minimization that both α and β are going to be 0, it is not possible.

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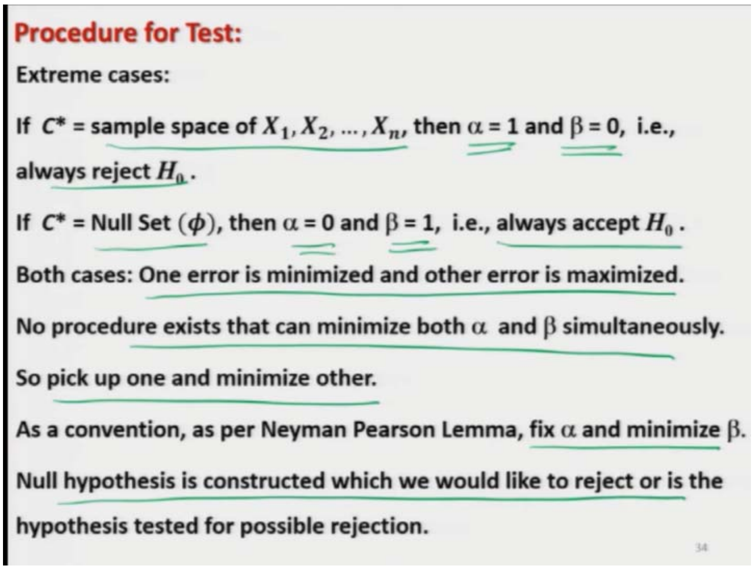

So, now what we try to do here that just to give you an example here, the two extreme cases will be suppose C* is the entire sample space, then in that case α will become 1 and β will become 0, that is, you will say that always reject H_0 and if I try to say that C* is a null set then α will become 0 and β will become 1, that is always accept H_0. So, these are two extreme decision and but in both the cases one Error is completely minimized and other is completely maximized. So, that is why there does not exist any procedure which can minimize both α and β simultaneously.

So, we have only one option, so we try to pick up one that we that means we try to fix one and then we try to minimize the other. So, it will be something like that if you try to say here fix one Error here and then you try to minimize the second Error. So, now obviously the Error which you want to fix, this can be which of the Error obviously the Error which is more serious that can be fixed because we cannot take a risk beyond that and then we try to minimize the other risk. This is the basic idea.

So, as a convention and according to the Neyman Pearson lemma which is used to develop such tests, we try to fix α and we try to minimize β and that is why the null hypothesis is constructed such that the Type One Error is more serious than the Type Two Error and this is also the requirement of the Neyman Pearson lemma.

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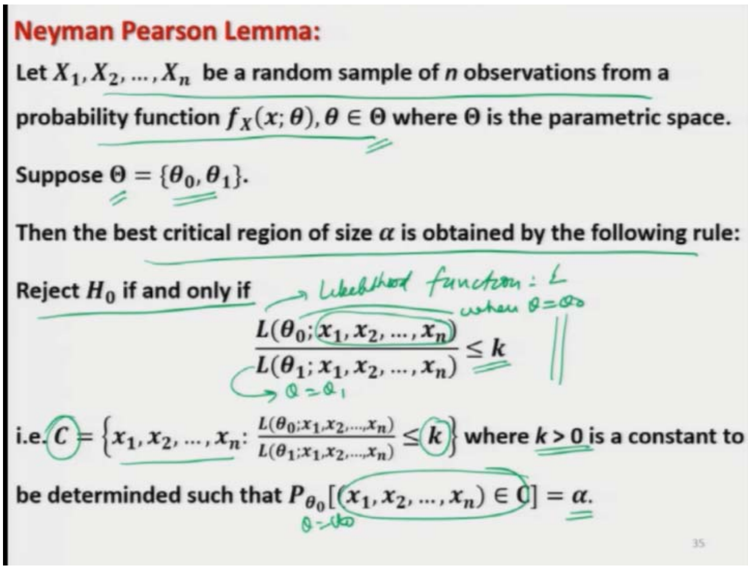

Now, what is this Neyman Pearson lemma? So, let me try to give you the statement of this lemma, I am not going to give you the mathematical details. So, this lemma helps us in getting such decision rule well that is mathematical but it is very simple to use. I will try to show you here, so let $X_1, X_2, \dots, X_n$ be a random sample from a probability function $f_X(x;\theta), \theta \in \Theta$. Θ is the parametric space and suppose this Θ has only two points θ_0 and θ_1.

Now, the best critical region of size α is obtained by the following rule that reject H_0 if and only if this condition hold true and if you try to read this condition what is this L is here, likelihood function. So, you are trying to find out the value of the likelihood function on the basis of given sample of data $X_1, X_2, \dots, X_n$ and when $\Theta = \theta_0$ or θ_0 and you try to find out another likelihood on the basis of given set sample of data when $\Theta = \theta_1$ and then you try to take the ratio and if this ratio is coming out to be smaller than some constant K, where K is greater than 0, then we can determine the best decision rule that this is the critical region that $X_1, X_2, \ldots, X_n$ means they will be exposed to this rule and if this comes out to be this ratio comes out to be smaller than K then we have the decision rule.

And this probability that $X_1, X_2, \ldots, X_n$ belongs to C under $\Theta - \theta_0$ = here α and α is the Type One Error. so this is what we are trying to do that we are trying to fix our α and then we are trying to use this expression to find out the decision rule.

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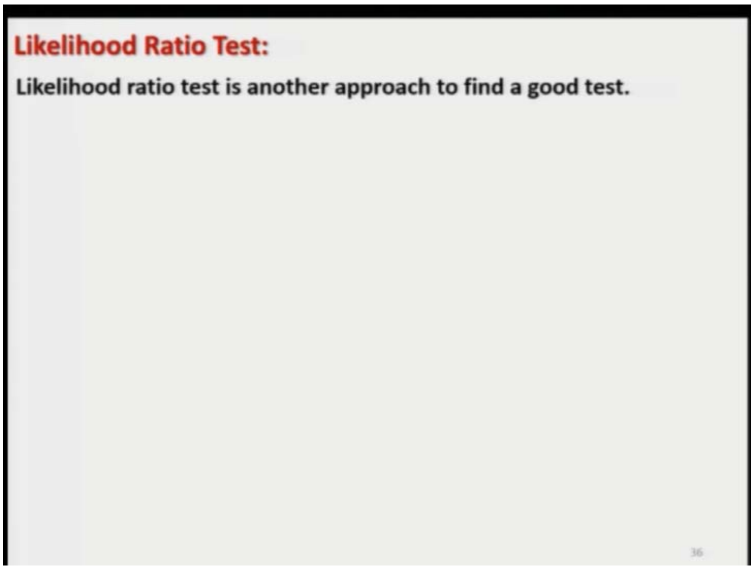

And similarly, we also have one more criteria that is one more rule that is called as Likelihood Ratio Test but definitely I will not give you here the details of the Likelihood Ratio Test but definitely Likelihood Ratio Test is a very general criteria that can give you the very good test what we call as the uniformly most powerful test from the on the basis of given sample of data and depending on the probability function.

So, now you can see here, now we have defined the rule by which we can obtain the best decision rule. Best decision rule where the power of the test is going to be maximum and you know means anybody who has got more power, he is supposed to be better than the person who has got less power.

So, now in this long lecture I have tried my best to give you all the concepts at a single place. I would request you that you try to revise it, try to put all this smaller concept inside your brain and try to interconnect them and you will see that these basic concepts are needed whenever we are trying to develop a decision rule and my problem was that if I try to break this lecture into two parts possibly you may forget something and I may also forget something, so that is why I have tried my best to give it in a single lecture. So, you try to revise it try to understand this concept and try to definitely read from the book and I will see you in the next lecture. Bye.