Basic Calculus for Engineers, Scientists and Economists
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Lecture - 30
Multiple Integrals – 3

We will now go by, and try to solve what we had spoken about this Surface Integral.

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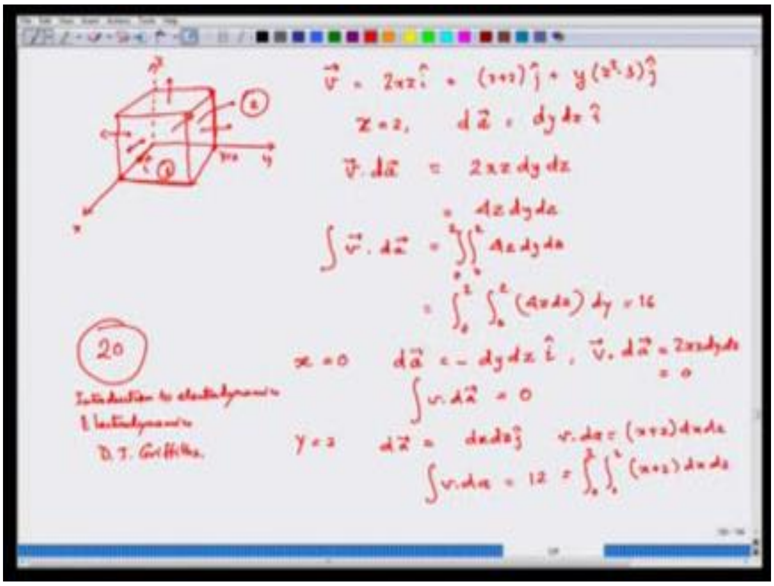

Just note that I am expecting that you to know what is i vector j vector and k vector look up any book on vector analysis for this, or the book of calculus that we have just tried, these our last class. So, we have do thinks quite fast. I will explain on or 2 points and then go on doing the rest.

Here we start with the first surface which is exactly facing us, the 1 here where we are essentially bothered about you observe there is no change in the vectors x, but the change is essentially in the vectors y and z. So, your x remains to be equal to 2 in the first case. This is a first surface, the 1 this is the surface one. Then d of a, this vector is d y d z along i vector the v dot d a assuming that x is equal to 2 stress, stress is 0. So, x is equal to 2 keeping that in mind. So, my v dot da would become 2 x z d y, d z of course, you can ask what about here we are doing a dock products. So, we are taking whatever coefficients have with i and whatever coefficient we have in with v with i and we are multiplying them.

Now, x is anywhere 2. It will become 4 z d y, d z. In this case v dot d a we are basically having a double integral though 4 z d d y d z we which we can of course, said d z d y y if you (Refer Time: 02:28) and it will ultimately come out to be 0, 16 in this case similarly will go to the next one, they are just opposite 1 to 1; which is the louder one, this one they will call at the second one. There you can observe that x is equal to 0 and d a vector is d y by d y into d z by that direction is not in the direction of i vector, but the direction is opposite to the i vector. There would be a minus sign. So, again v dot da is equal to the same thing 2 x z d y d z what here because it is 0 this is 0 because x is 0. Integral v dot da of this part on this particular part of the surface is 0.

Similarly you will go on doing the other cast, now you take the case say y equal to 2 then here you will have y is equal to 2 and you will have d a vector is d x d z vector along the j vector, and then v dot d a would be equal to x plus 2 d x d z and then you just if you integrate in the same way, using double integrals and then your answer is 12. This is nothing, but 0 to 2; 0 to 2 x plus 2 d x d z and that is 12. So, you can go on doing the doing it for all the other surfaces and then add up if you want to check the answer the answer here is 2, 0; 20.

These example is from the book called electrodynamics introduction to electrodynamics rather your introduction to electrodynamics what did your purposefully I took it from physics books because that gives you much better fast introduction when I introduction. The thing introduction to electrodynamics is a famous book by David J Griffiths is a very very famous book. So, once we are through with this let us go and see in the board.

I took another example from Griffiths to explain to you, how things can occur, for volume integrals what is the volume integrals, in volume integrals we only taking volume integrals for scalar functions. We do not usually talk about volume integrals for vector function, but then you can do then you have to separately walk on the scalar points. Here is the function of x y z and d tau is the volume element, if there is a volume like this.

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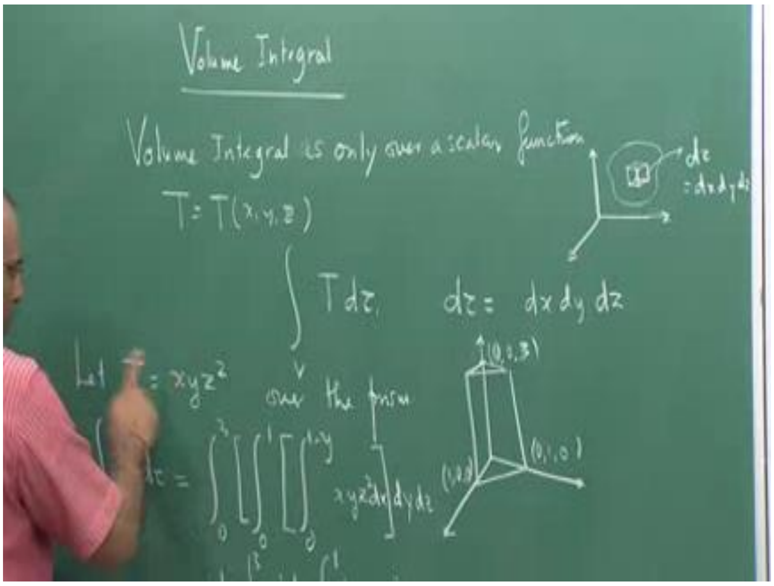

A body and d tau is a volume element which is nothing, but d x into d y into d z. This is the volume element whose volume d v or d tau is nothing, but d x into d y into d z the element of volume you can understand this is. If you move along this direction the x direction it is d x if you move along the y direction it is d y if you move along the z direction it is d z.

So, you draw a small cube basically you take the body of into small cubes and find the volume of all those cubes and then when you multiply those volume with the functions add them up and then take the limit by making the volume smaller and smaller and that is exactly what is going to give me the volume integral. This is the last part, I said from mathematicians, now we who are actually doing some math or interest.

Say I am going to take this one, this 1 over this presium. This is a presium which is cutting which has what is as at 0 3; 0 0 3, 1 0 0; 0 1 0, coordinates and the point 0 0 0 the origin. Now you see here the line joining 1 1 0 and 0 0 1 is actually the line in 2 dimensions which is given by the equations x plus y equal to 1 this line is nothing, but x plus y is equal to one. So, when y is varying from 0 to 1 m y x is varying from 0 to 1 minus y. If I want to integrate, I will first integrate with x varying from 0 to 1 minus y and then I will integrate from y varying from 0 to 1 then z varying from 0 to 3 and by doing this I get the answer 3 by x. I hope this is all pretty clear to you if you have, if you are not so.

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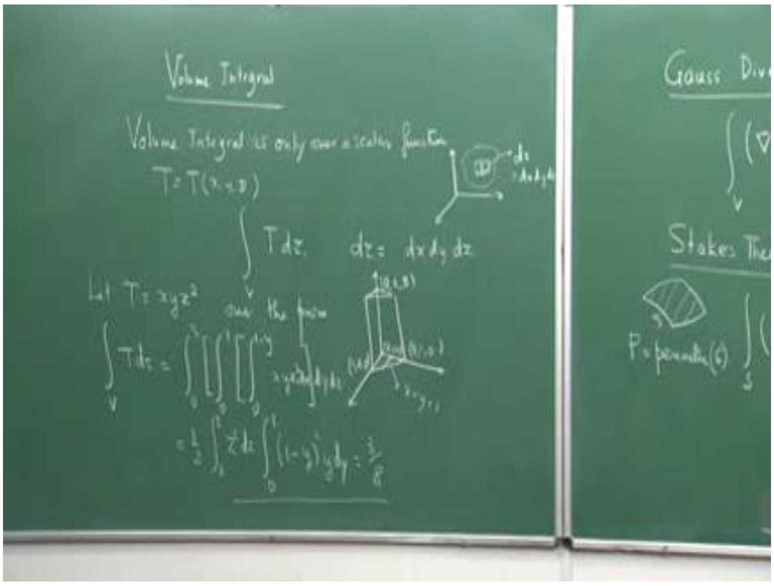

Here just look at the diagram once again lo say see that this is x plus y is equal to 1 and this is. I am first varying x, but when I am varying x and i am varying y. So, my what I am doing when I am varying z from 0 to 3, y has to then vary from 0 to 1 when I am varying y from 0 to 1, 0 - I am varying, but while I am varying z from 0 to 3, y get us varied from 0 to one, but while I am varying y from 0 to 1 x is getting varied from 0 from 0 point x is getting varied to this point 1 minus y. This has to be this sort of thing has to be very carefully done and that would finally, give you the integral.

See what you are essentially doing? By trying to find the some sort of a volume you are essentially trying to do what you are trying to basically, if get this area out and then multiplying by the height that is all. Now, I come to some important theorems here.

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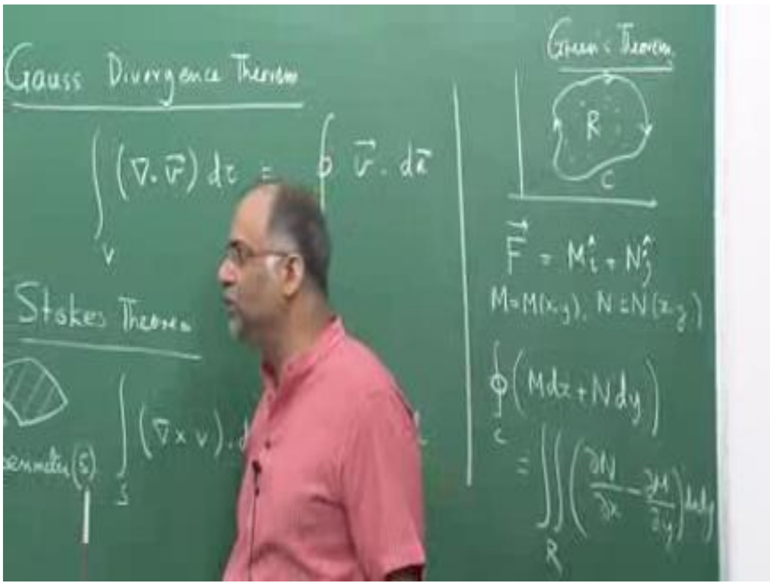

I will first mention the gauss divergence theorem which relates the volume integral and a surface integral, it tells you that, if you are trying to cut compute the volume of the divergence of a vector and then if s is the. This is the volume. This is of the volume this is the volume of an object these are 3 dimensional objects. Like this. There split a little make a 3 dimensional. Basically now I am taking the volume integral of a vector, the divergence of a vector over this, so how it is moving out from the volume. So, may be that the how say the how say of fluid is moving out form a given source. Light is moving out from a given source.

But, if I want to compute the volume integral, I can also converted into a surface integral if I consider the surface of this body the same body use volume I am considering.

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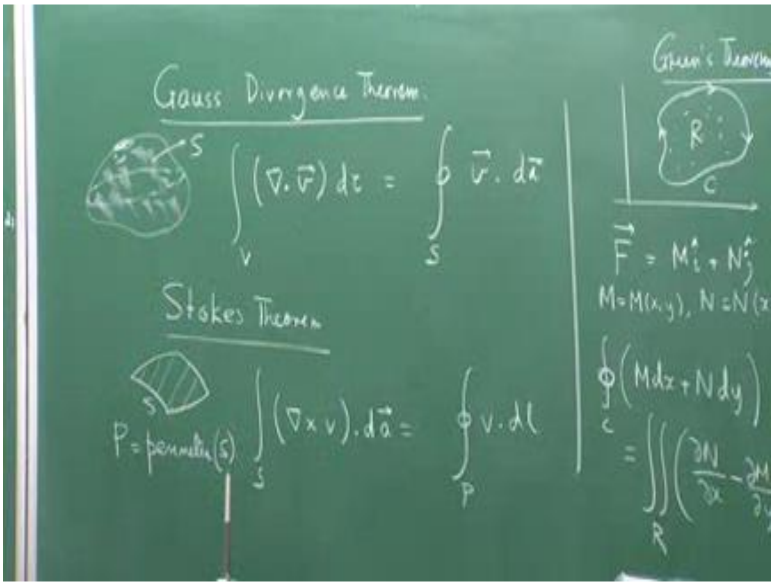

If I consider the surface then if I take the surface integral over the whole surface, why i have put this circulation sign surface integral then, that is same as the volume. Basically what it is trying to say that the spreading out of say a fluid from a given point is same as its flux across the surface area this is the flux it is the change of volume per unit area the change of volume per unit area is given by the flux and then over the small elemental volume it is v into da.

So, for unit area this is the volume per unit area and this is the change of the volume per unit area. So, want the fluid that crosses the unit area in a given unit time that is a flux and this is done over each small elemental area. So, what the total is this, this is given over the surface integral. It tells you that the total flux gives you the total divergence out of the volume the amount of fluid which moves out of the volume is same as the total flux over the region which is physically the case of course, then there is a stokes theorem what does the stokes theorem do it tells you that if you want to take the surface integral of the curl of a given vector v then if you take the perimeter of the same surface.

Perimeter of the patch of this surface this sort of surface, if you take the perimeter of that surface, surface patch, I cannot accord of close surface then to be difficult. So, of course, you can then define a perimeter of the circumference sort of thing if you have tail take the circumference.

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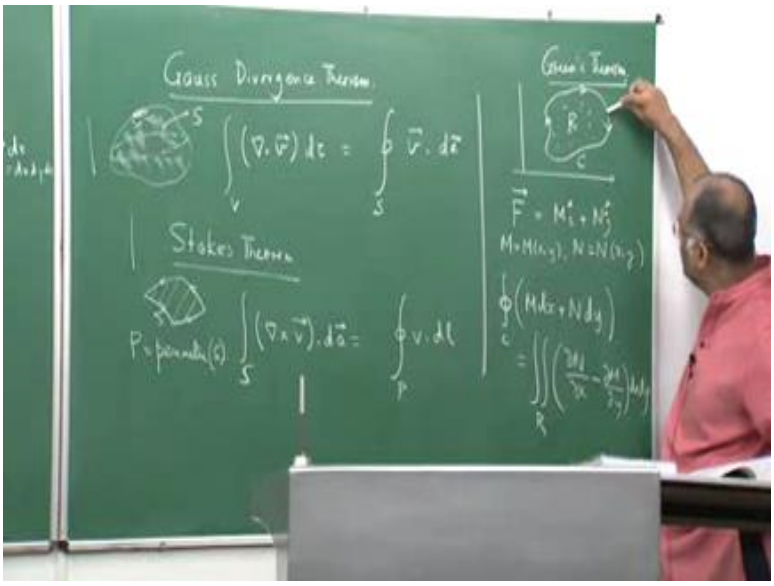

See if a surface area basically. Basically if you take the surface integral over this patch a small patch of area like this, then stokes theorem say this is same as taking a line integral over the perimeter line the boundary line integral over this whole close boundary that is what the stokes theorem says, but is not. So, easy to immediately realize and understand these 2 results is very short time to really speak about them, then there is a greens integral which is essential are two-dimensional thing which says that if you have a vector function which is given in terms of m i vector plus n j vector in terms of 2 vector function, which are themselves as a function it can written as functions of x y then if I move along.

I have this circular closed curve which is enclosing the region r and if I move to counter clockwise moving counter clockwise means if I move along the curve the inner part or the region of the curve should always be on my right if I move along this it will always on my right hand. If I am moving along this part in counter clockwise fashion then integral of m d x plus n d y is same as integral of del n del x minus del m del y into d x d

y. If you have worried how this thing has come this is nothing, but the curl or sorry this is nothing, but this vector it in a product with k vector.

This quantity is this. Those deep physical reasons are not. So, easy to figure out immediately, but what we are going to do is trying to try to show you something of how to compute because you have just seen how to compute surface integral you can just check out the gauss divergence. I will just give you a problem which can be of use to check out the gauss divergence theorem which of a great significance in many many things. The for example, you take a vector v.

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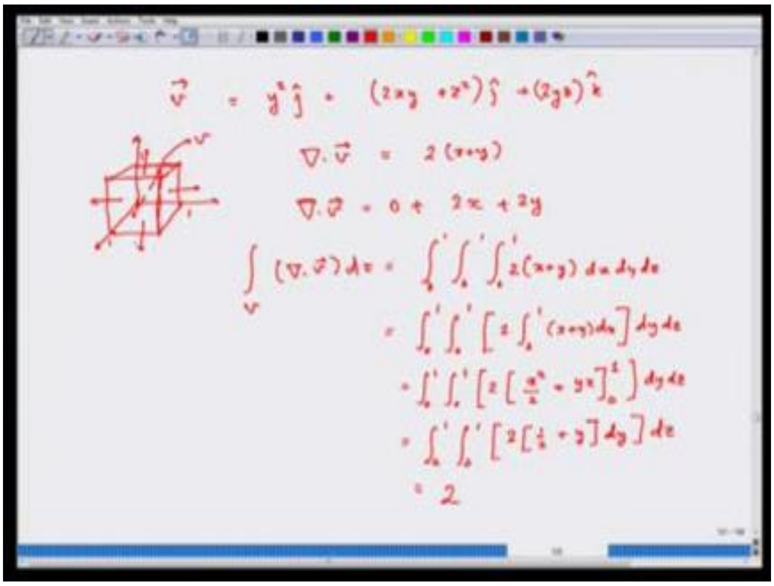

I am giving the example of from review the again which has lovely examples. So, you do it over the unit cube means now you have a take what my region the volume. Now, I have a unit cube of length breadth height 1 1, centimeter 1 meter whatever you want 1, 1, 1. So, what is in this case? This will turn out to become you take the full divergence will turn out to become 2 x plus y.

Let us see how. Basically you take the del instead of going too much detail for flux to take the derivative with respect to x here first case it is 0 plus derivative with respect to y. It is 2 x here it is derivative with respect to z. It is 2 y it is 2 x plus y that is the answer now you take this volume integral. Is this is my required volume basically v. If we take the volume integral that is nothing, but 0 to 1 0 to 1 0 to 1 2 x plus y d x d y d z.

If you just compute out, so first you compute out 0 1 0 1 2 into 0 1 x plus y d x. It will be 0 1 0 1 2 into you know what this will come out to be this will become x square by 2 plus y x that is from 0 to 1 d y d z if we just do it 0 1 0 one. So, x square by 2 is now this will give me to this will give me half 2 into half plus y. So, we just go on doing this your answer would be 2.

Now, surface integral you know here I am expecting over the whole surface. Total surface I am looking a total circulation. I am expecting over this 1 over the this 1 outer 1 this 1, I have 1 and also the 1 below.

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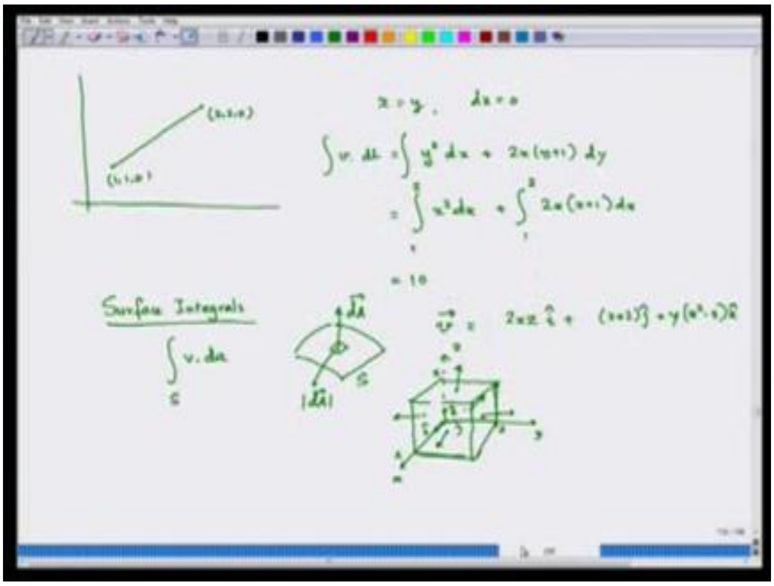

Just like the 1 we had discussed earlier. Here we are not taken the 1 we have just kept 5 surfaces just for it here we have will take we are we are taking here we have taking the five surfaces.

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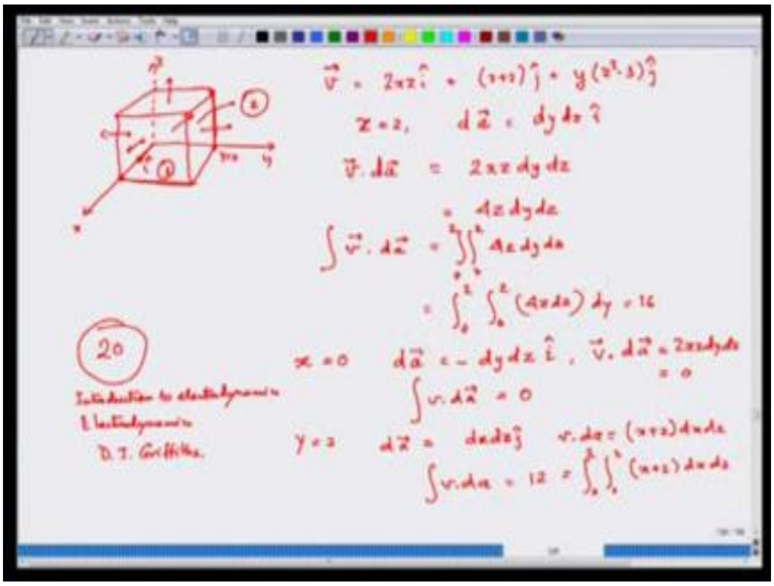

Here we are going to take all the surfaces even the last one. Here you really have to make a check yourself and then show that this will be. Let me take the first case where x is equal to 1 then and you have d d y d z. The first case, this first, first this one, this one; in this case v dot da would be equal to 0 1 0 1 y square d y d z.

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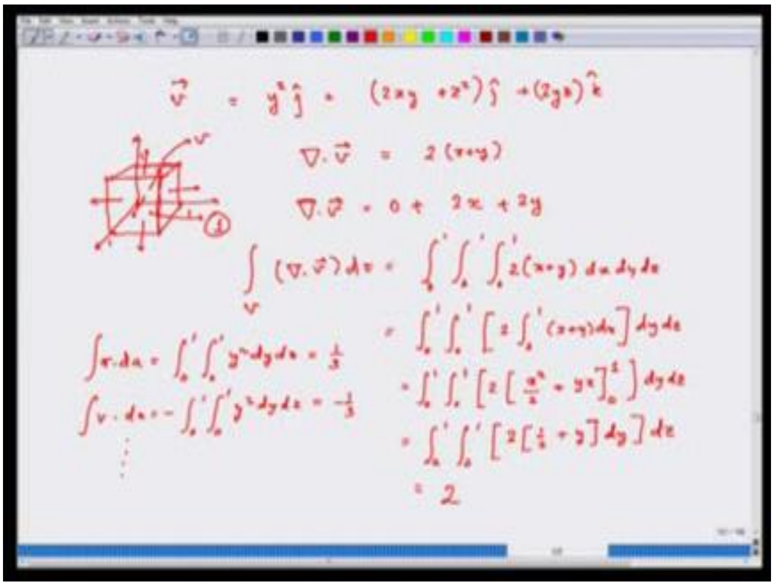

Similarly, if you take the same thing on the other side that is this 1 the last just the side the 1 opposite to the first side the facing side here you will have a minus sign and as v 4. In that way we will continue to do it and you verify that these are equal. So, hence we have finished the course the last, but are to be hurried. So, we had try give you as much examples as in possibly it is not possible to do just is to multiple integral in just 3 lectures of 20 minutes each.

I have tried to exceed in some of the cases, but i hope you have enjoyed the course I try to learn how to teach a very compact course in very short time. I learnt a lot in the sense that I now know what to do, what not to do, how to put things in order. This is actually a first experiment for me also hoping that I will do better if I get a next chance. If there any questions please feel free to write on the form and my TS would get back to you, if they cannot I will definitely answer back to you.

Thank you very much and hope you have enjoyed the course. Thank you once again for listening to me with all your concentration.

Thank you.