Applied Multivariate Analysis

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Lecture No. # 06

Some Problems on Multivariate Distributions – I

(( )) for the last five lectures, we have been seeing some basic concepts in multivariate analysis, and also we have looked at multivariate normal distribution, and some basic results concerning multivariate normal distribution. What we will today the look at are some simple problems on the concepts that we have covered so far right.

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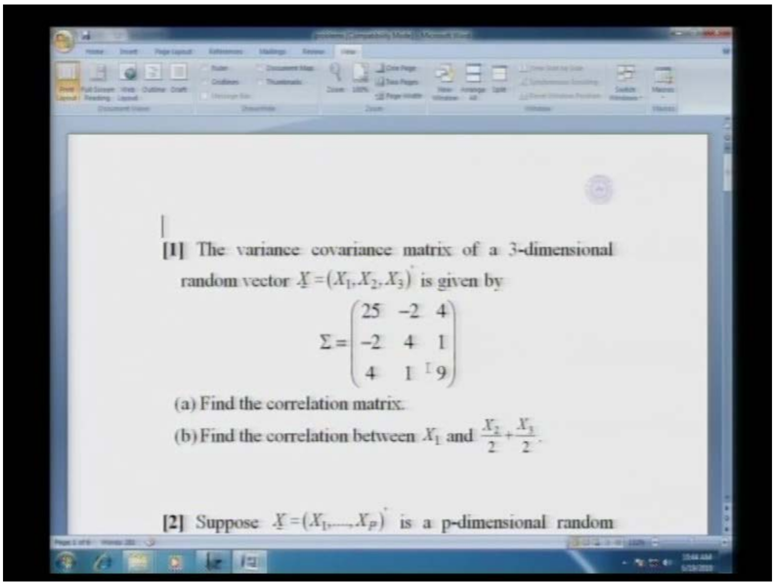

So, we look at some problems as I said, so this is the first problem what we have, the variance covariance matrix of a three-dimensional random vector X 1, X 2, X 3 is given by this sigma matrix. Now, we have to find the correlation matrix corresponding to this random vector, and also later on we will find the correlation between this random vector, random variable; and this random variable derived from the random vector.

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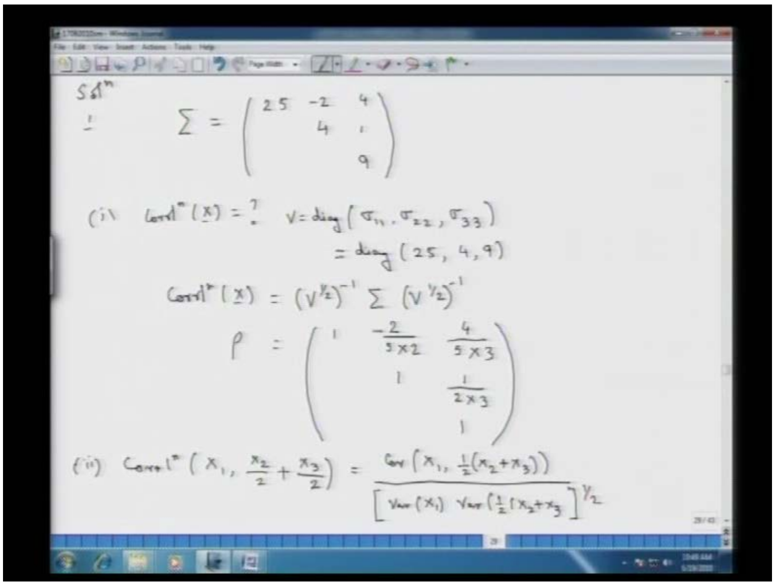

Let us see, how to look at this solution. So, we have this sigma matrix, so we are looking at the solutions one by one of the problems. So, the sigma matrix what we have is given by the following 25, 4, 9 on the diagonals; 1, 2 th element is minus 2; 1, 3 th element is 4, and this is 1 right. So, if we have this to be the sigma matrix, we are trying to find the correlation matrix. So, the correlation matrix of this X is what we are trying to find out right. Now, if we have sigma matrix given by this, then this basically is a variance of the first component, this is the variance of the second component, and this is the variance of the third component. This is the covariance between the first, and the second component.

This is the covariance between the first and the third component and this is the covariance between the second and the third component. Now, this is symmetric matrix. So, no need to write the lower diagonal elements out here. So, what we will first find out is this matrix which is going to hold. So, this is the first entry which is sigma 1 1; this is sigma 2 2; this is say sigma 3 3 and from this variance covariance matrix this is nothing but, this actually is the diagonal matrix. So, I will just write it as a diagonal matrix. So, that this v let us defined that to be the diagonal matrix, which is holding the variance terms.

So, this is the diagonal matrix with elements as 25 4 and 9 right. Now, the correlation matrix of x as we had seen in basic concepts of multivariate analysis, this is going to be v half inverse sigma v half inverse. So, this basically is going to be this sigma matrix multiplied pre multiplied by v half inverse and post multiplied by v half inverse. So, what do we get? We get from the sigma matrix the following. Now, this is the correlation matrix. So, the three diagonal entries would be correlation between X 1 and itself, X 2 and itself and X 3 and itself. So, all of them are 1's. Now, the correlation between X 1 and X 2 is going to be given by this element.

So, that is minus 2, which is the sigma 1 2 element out here; that divided by root over sigma 1 1, sigma 2 2. So, this is going to be 5 multiplied by 2, which is the standard deviation of the X 2 element. Similarly, the 1 3 th element is the correlation correlation between X 1 and X 3. So, that is going to be given by 5; this multiplied by the square root of 9. So, that is 3 right. Now, similarly this element what we are going to get is the correlation between X 2 and X 3. So, that is given by 1, which is the covariance what we have out here and then the standard deviations of X 2 and X 3, which is 2 in to 3 right.

So, this is our correlation matrix for this particular random vector X 1, X 2, X 3 right simple. Now, the second problem, second part of the problem was to find out. So, we have obtained this particular term here and then, we are trying to find out the correlation coefficient between X 1 and x half of X 1 plus X 2 plus X 3 right. So, let us see how to do that. So, we are trying to find out in the second part of the problem; this was the first part of the problem. So, the correlation between X 1 and X 2 by 2 plus X 3 by 2; So, this would be given by covariance between X 1 and half of X 2 plus X 3; this divided by under root of the respective variances. So, this is variance of X 1 and this is variance of half of X 2 plus X 3 right. So, we will actually compute this term; we will compute this term. This term ofcourse is given directly from the variance covariance matrix and we will derive what is this in the next slide here.

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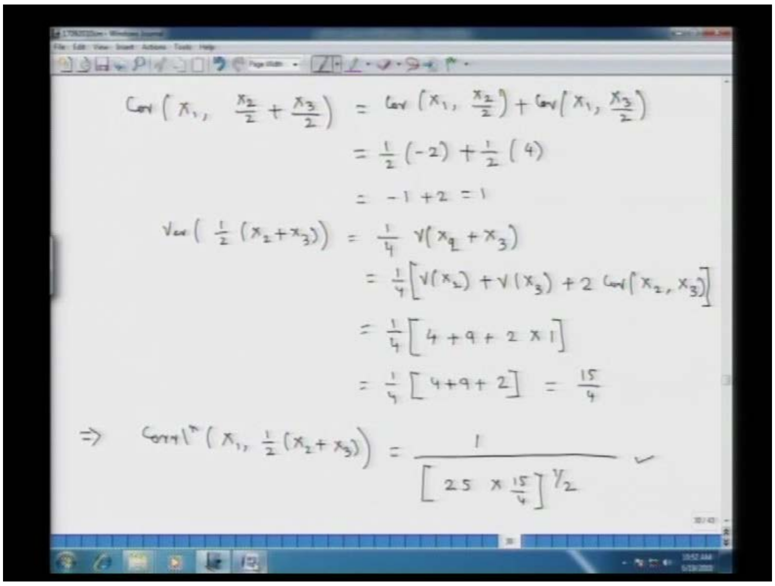

So, let us look at, what is covariance between X 1 and X 2 by 2 plus X 3 by 2? So, this is covariance between X 1 and X 2 by 2; this plus the covariance between X 1 and X 3 by 2. So, this is half of covariance between X 1 and X 2. Let us see, what is that covariance between X 1 and X 2? So, the covariance between X 1 and X 2 is this element here minus 2. So, we will have this as minus 2 then plus half of covariance between X 1 and X 3, what is that? So, covariance between X 1 and X 3 is this element 4. So, this is covariance between X 1 and X 3 and that is equal to 4 right.

So, whatever is that; so, this is minus 1 plus 2. So, this is equal to 1 right. Now, the next thing that we need to compute is variance of this particular element. So, that we are now looking at what is the variance of half of X 2 plus X 3. So, that is equal to 1 upon 4 variance of X 2 plus X 3. So, that is equal to variance of X 2 plus variance of X 3 plus twice covariance between X 2 and X 3 right. So, what are these entries? From the variance covariance matrix, variance of X 2 is what we see as 4 and variance of X 3 is what we see as 9. So, that this is 4 plus 9 plus 2 times covariance of X 2 X 3. What is that? Covariance between X 2 and X 3 is 1. So, that this is 2 into 1.

So, whatever that comes one fourth of this plus 2. So, this is 15 by 4. So, we have the constituent elements to compute this particular correlation term. So that, what we finally get; this would imply that, the correlation coefficient between X 1 and half of X 2 plus X 3; that is that is given by the covariance of this term, which has turned out to be 1. And then, we have variance of X 1 we have variance of X 1 that is given by 25. So, that this is 25 multiplied by 15 by 4 into half and that is what correlation coefficient between the two random variables derived from the random vector. So, that solves the first problem.

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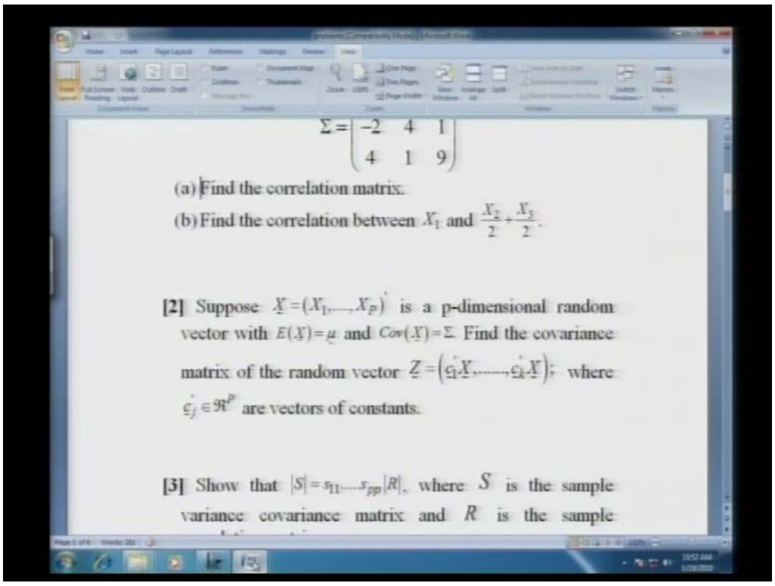

Let us now look at the second problem. So, the second problem is this that we have got X, the multivariate random vector which is p dimensional with a mean vector as mu and the covariance matrix as sigma. We are trying to find out, what is the covariance matrix of the random vector which is derived actually from this random vector p dimensional random vector X and Z is given by these k components. Each of this c j's belonging to R to the power p.

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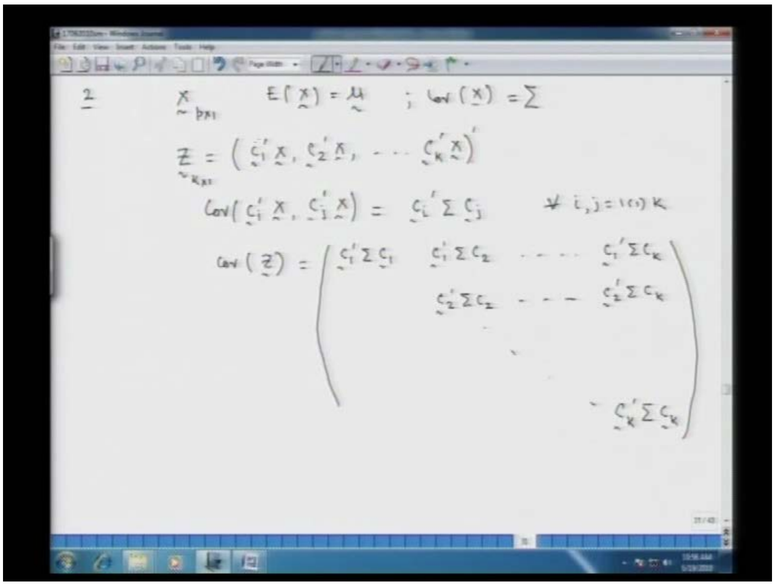

Let us see, how to find out this particular solution for this particular problem. So, we have X, the p dimensional random vector with expectation of X vector given by this mu vector and the covariance matrix of X given by sigma matrix. And we are looking at a new random vector Z, which is given by c 1 prime c 1 prime X, c 2 prime X and we had that as c k prime X. So, the last element entry is c k prime X. So, this is c k prime X. So, for this particular random vector, which is what we are now trying to find out? Now, we try to find out what is basically the covariance between any c i and c j.

So, let us try to find out covariance between c 1 prime or rather in general c i prime X and c j prime X right. So that, from the definition of covariance of these two quantities, it is going to be c i prime covariance matrix of X, which is sigma that multiplied by this c j right. So, we will have this for all i j into 1 to up to k right. So, the covariance matrix of this Z vector is going to be given by this would be the covariance between c 1 prime X and itself. So, that this is going to be c 1 prime sigma c 1 and this would be the covariance between c 1 prime X and c 2 prime X. So, that would be given by c 1 prime sigma c 2 and the last entry would be the covariance between c 1 prime X and c k prime x.

So, that would be given by c 1 prime sigma c k. This entry would be c 2 prime sigma c 2 and this would be the covariance between c 2 prime X and c k prime X. So that, this would be given by this term and the last entry would be the covariance between c k prime X and itself. So that, that would be given by c k prime sigma c k. So, that this basically is what we are trying to find out. So, this is the k dimensional vector; what we have because this is the first entry; this is the second and this is the k th entry and we have all these c i' s belonging to R to the power p confirming to this particular random vector, which is X right. So, this is what the covariance matrix is that, we wanted to derive for this new random vector q k dimensional.

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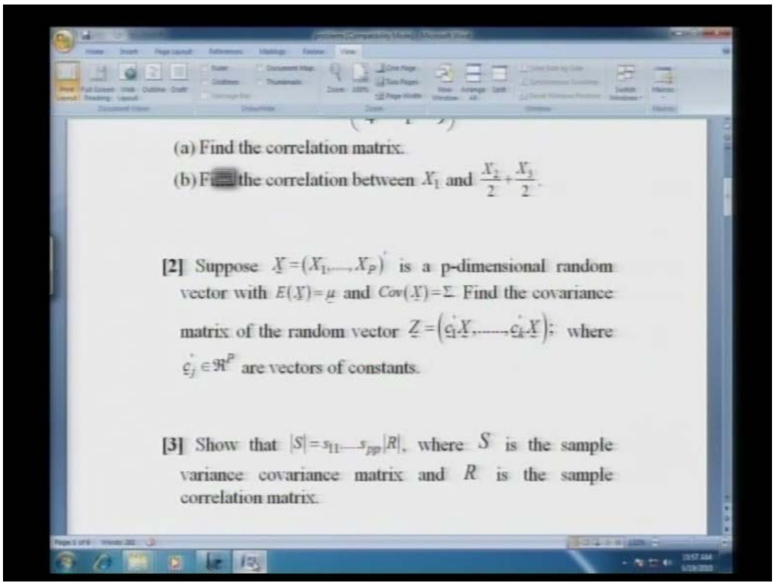

Let us now look at the third problem. The second problem is done. So, we are looking at third problem. We are trying to find out or rather establish the fact that, determinant of S is equal to the product of s i i; i equal to 1 to p and then, the determinant of R what are these? S is the sample variance covariance matrix and R is the sample correlation matrix. So, let us look at how to establish this particular fact.

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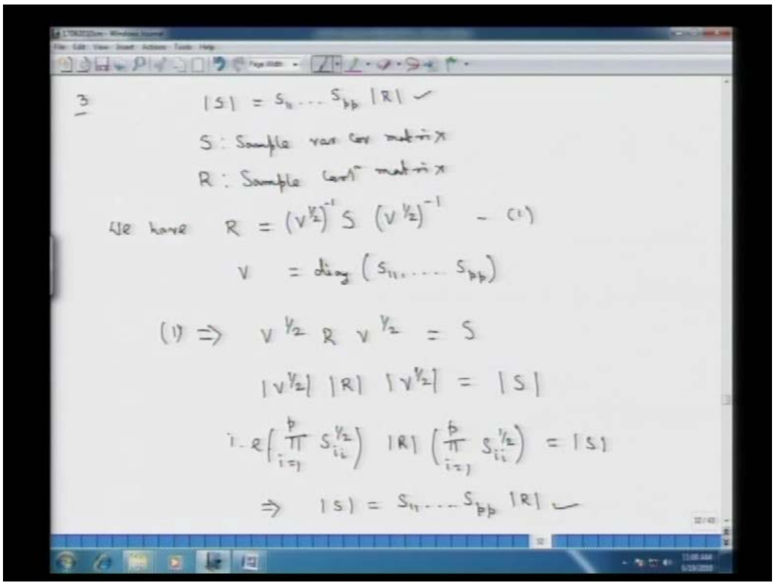

What we are trying to prove is the following that, S is equal to s 1 1, s p p into R; where S is the sample variance covariance matrix and R is what we have as the sample correlation matrix right. Now, what do you know about the relationship between the sample variance covariance matrix and the correlation matrix? This is what, we know. So, we have the relationship that R, the correlation matrix can be derived from the sample covariance matrix in the following way that; we take S, the sample variance covariance matrix and then, multiplied that with v half inverse and post multiplied by v half inverse. And what is this v half?

v half is the diagonal matrix holding the covariance variance terms. So, that is diagonal s 1 1, s 2 2, s p p right. So, this is the p dimensional diagonal matrix what we have as v half? Now, this step number 1; one would imply that, if we now this is a non singular matrix. So, what we have is multiplying by v half pre multiplying by v half and post multiplying by v half of equation number 1. We will have this to be given by that is equal to S right. Now taking determinant what we will be having is that determinant of v half into determinant of R into determinant of v half, that would be given by determinant of this S matrix.

That is, what we have now note that; determinant of this v half matrix I am sorry actually this is not v half yet; this is just v matrix. So, the v matrix is just the diagonal with s 1 1, s 2 2, s p p as its diagonal entries and then, we erase that to v to the power half. So, if we look at determinant of v to the power half, what will happen to this is s 1 1 to the power half, s p p to the power half. So, we will have this as a product i equal to 1 to p, s i i to the power half; that is what is determinant of v half and this is determinant of R. And then, this is once again the same as what we have already obtained.

So, this is product i equal to 1 to up to p, s i i to the power half; that is equal to determinant of S. So, this would imply that, determinant of S is equal to product or rather just the elements s 11, s 2 2, s p p that is product of s i i's simply; that multiplied by determinant of R as was to be proved right. So, we proved this particular relationship in this problem. Let us now move on to the fourth problem.

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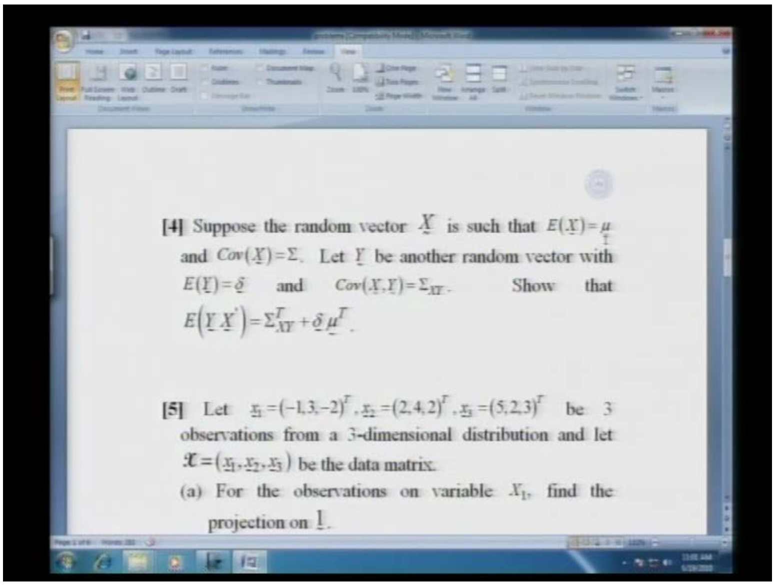

So, the fourth problem basically is this, that we have a random vector X, which has expectation vector as mu, covariance matrix as sigma. Y is another random vector with mean vector as delta, and covariance between X and Y given by sigma X Y. We are trying to show that, expectation of Y, X transpose that is equal to sigma X Y transpose plus delta times mu transpose. Let us see, how to prove that this particular result.

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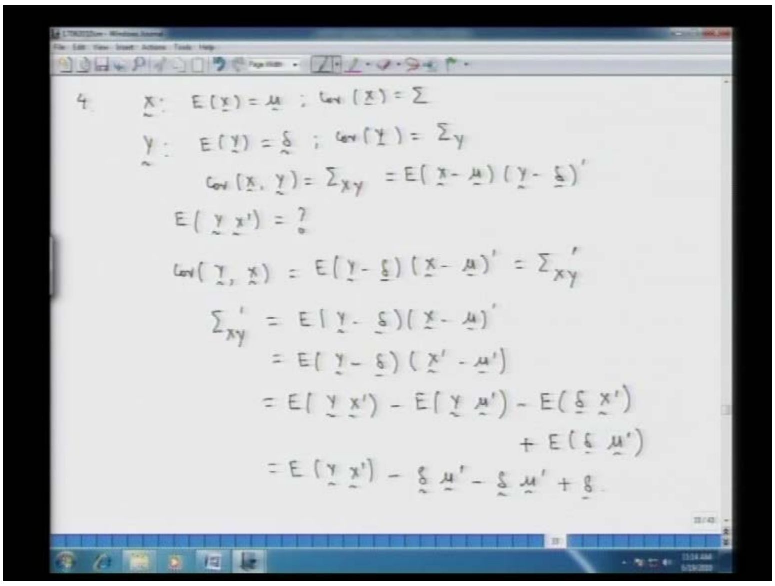

So, what we have is the following that, expectation of X vector is equal to mu vector and the covariance matrix of X vector is sigma matrix. So, this is for the random vector X. Let us have for the random vector Y as, expectation of Y equal to this delta and the covariance matrix of Y is say given by sigma Y and what we have also is expectation or rather the covariance between X and Y is denoted by sigma X Y. So, under these given conditions, what we are trying to establish is the relationship of expectation of Y X transpose and the given quantities. So, we are trying to have an expression for expectation of Y X transpose; that is what was the problem.

So, we were trying to get to this. So, this is equal to what? That is the question. Now, if we have this particular term to be given. So, this is what? This is equal to expectation of X minus mu into Y minus delta transpose. Now, if we write covariance between Y and X vector. So that, that would be given by from the definition of covariance between two random vectors, that is going to be given by Y minus delta into X minus mu transpose. So, this covariance between Y and X, which is given by this, is nothing but just the transpose of this particular quantity and thus, this is equal to sigma X Y transpose.

So, let us keep this sigma X Y transpose as it is. Sigma X Y transpose; that is given by Y minus delta into X minus mu transpose. Now, let us now open up this particular thing here and take expectation term by term. So that, this is Y minus delta X transpose minus mu transpose. So, we will have four terms here, which is expectation of Y X transpose;

this minus expectation of Y mu transpose; this minus expectation of delta times X transpose; this plus expectation of delta mu transpose right. So, we keep the first term as it is, expectation of Y X transpose; this minus this is a non stochastic part.

So, this will just be expectation of Y into mu transpose. So, that is delta into mu transpose; this minus by the similar logic this is delta times mu transpose; this plus this is non stochastic completely. So, this is delta times mu transpose. So, what we have is one of these terms cancelling out and this thus is equal to expectation of Y X transpose minus delta times mu transpose. So, this would imply this would imply that, expectation of Y X transpose is sigma X Y transpose; this plus the term that we had here, which is delta times mu transpose. And this is precisely the result that we were trying to prove. So, we have this particular result been proved.

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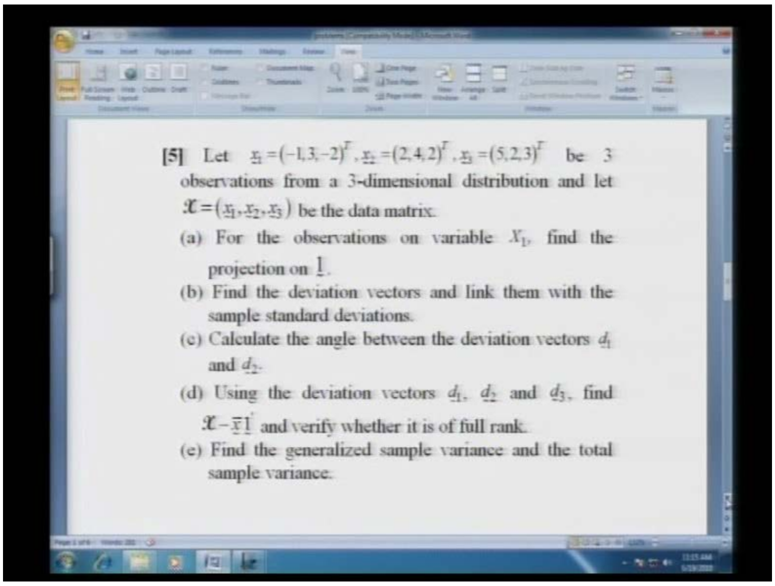

Let us now move on to the next problem, which is problem number five, this here. So, we have these three random vectors X 1, X 2, X 3. These are three observations from a three dimensional distribution. And we denote by this script X, the data matrix holding the three as the three column vectors and then, we will be deriving all these things. So, let us first write what we have here. So, let us do the solution of problem number five.

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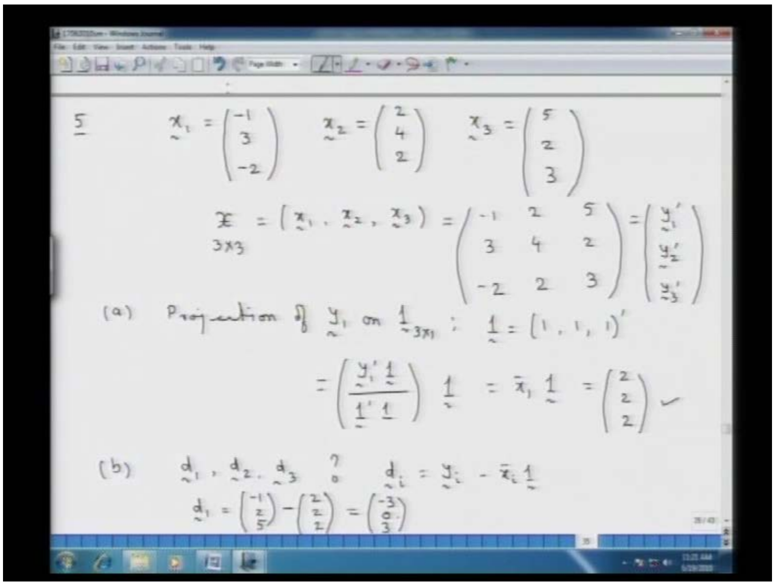

So, we have this X 1 vector given by minus 1, 3, minus 2 minus 1 3 minus 2; this X 2 vector, I will just see what these elements are; so, this X 2 vectors 2 4 2. So, this is 2 4 2. The third observation vector is what we have here is 5 2 3 5 2 3. And then, using these three as the three observation vector, we form the data matrix; which is a 3 by 3 data matrix, which is holding this X 1, X 2 and X 3 vectors. So, this is what? This is this minus 1 3 minus 2; 2 4 2; 5 2 3 right. Now, given this particular information, let us see what the problem is asking? For the observations, for there is the first part of the problem; for the observations on variable X 1, find the projection on 1. 1, the column vector three dimensions each element being 1.

So, that this is what we are trying to get in the first. Now, this can also be written in the following way that, this is Y 1 transpose, Y 2 transpose, Y 3 transpose. What is Y 1 transpose? Y 1 transpose is having these three as the three entries and what are these three? These three are the three observations corresponding to the first component in the three observations. So, this is the first component; first component value in the first observation X 1; this is the first components value in the second observation and this is the third value of the first component in the third observation vector.

So, this minus 1 2 5 are the three observations that we that is what, we have corresponding to the first variable. And we are trying to find out, this is the problem. To find out the projection for the observations on variable X 1; that is the first row of that script X matrix. We are trying to find the projection of that on 1 vector. So, we are trying to find out this projection of this Y 1 on this 1 vector which is 3 by 1. So, where this 1 vector is nothing but, it is holding the three entries as 1. So that, that would be given by Y 1 prime 1 divided by 1 prime 1; this multiplied by this 1 vector and this is what, this is going to give us X 1 bar.

Because 1 transpose 1 is 3 and this Y 1 transpose 1 is going to give us the sum of all these three observations corresponding to the first variable. And thus, this is nothing but X 1 bar 1 and for the given problem X 1 bar is 5 plus 2 plus minus 1 divided by 3. So, that is the mean. So, 76 divided by 3. So, that is two. So that, this is a vector containing all 2’s out here right. So, this is what, is the projection of Y 1 on the column vector 1. Let us see, what is the second part of the problem? Find the deviation vectors? And link them with the standard sample standard deviations. So, what is that we are trying to find out? We are trying to deviation vector d 1, d 2 and d 3. What are these quantities? Now, what is this d i?

d i is going to be given by Y i minus X i bar times 1. So, it is basically calculating the deviation of each of the observations from the corresponding mean of the variable; that is collected from the three observations, that is what we have. So, let us look at what those quantities are. So, this d 1 is going to be given by Y 1; Y 1 is this vector. So, it is minus 1 2 5; that minus the mean vector, which is what we have obtained here; which is 2 2 and 2. So, that is the first deviation vector. So that, the elements are this is minus 3; this is 0 and this is 3 right. So, this is what we have as the first deviation vector. In a similar way, one can obtain what is the second deviation vector, third.

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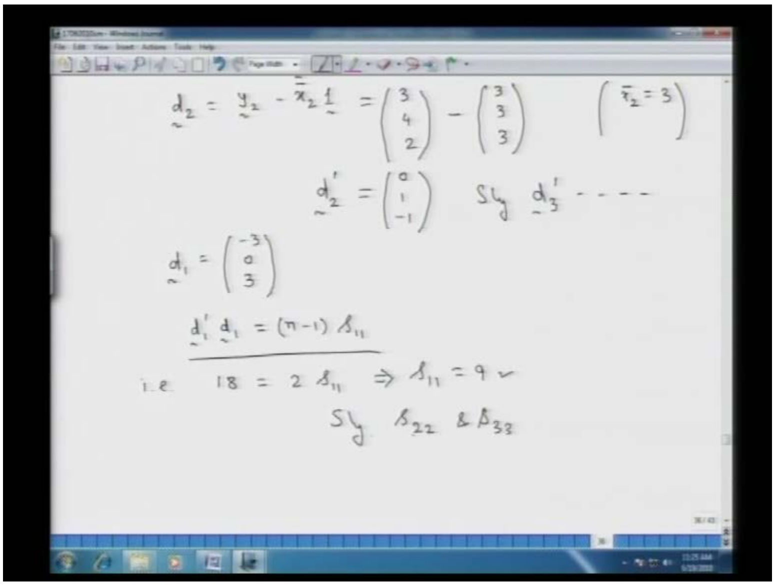

So that, this d 2 will be given by this Y 2 vector; that minus X 2 bar X 2 bar times this 1 vector and what would that be equal to? From the data matrix, this Y 2 prime is 3, 4, 2. This is 3, 4, 2; this minus X 2 bar. X 2 bar will be computed from these three observations. So that, this is 6 plus 3 9 9 divided by 3 is 3. So, what we have is X 2 bar. So that, what we have is X 2 bar is equal to 3. Using that, we will have this X 2 bar 1 vector given by this. So, that the d 2 deviation vector is given by 0 1 minus 1 right. Similarly, one can also obtain this d 3 in the similar way and as what we have obtained for d 1 and d 2 right.

Now, that is what a computation of the deviation vector is. Now, the second second part of this particular problem was to link these deviation vector with that of the sample variances. So, what is that? Let us illustrate that, using this first deviation vector. The first deviation vector is minus 3 0 3. So, this d 1 vector minus 3 0 3 minus 3 0 3; Now what we know about the square of the norm of this these deviation vectors is the following that, d 1 prime d 1. Now, these are the deviation vectors, that is x i j minus x i bar. Those are the entries for the deviations.

Now, when we are looking at d i prime d i, then we are basically looking at the sum of squares of these entries and that is what is going to lead us to n minus 1 s i i, the sample variance corresponding to the i th component right. So, that if we use this particular thing for this deviation vector, what we what we are getting is d 1 prime d 1 is going to be given by 9 plus 9 that is 18. So, it is the square the sum of square of the entries for this d 1 deviation vector and that is going to be given by n minus 1. What is n? Here, n is the number of observations. So, that is 3 minus 1 2 that times s i i.

So, this would imply that, the sample variance I am sorry this would be s 1 1; because what we are looking at is corresponding to the first observation, first variable rather. So, that this is our s 1 1. So, that what we have is this s 1 1. s 1 1 is going to be given by 9 right. Similarly, one can obtain s 2 2 and s 3 3 can be obtained using the deviation for s 2 2 will be using the deviation vector d 2 and for the s 3 3 element, we would be using the d 3 deviation vector right. So, this is not the transpose. This is just the vector itself.

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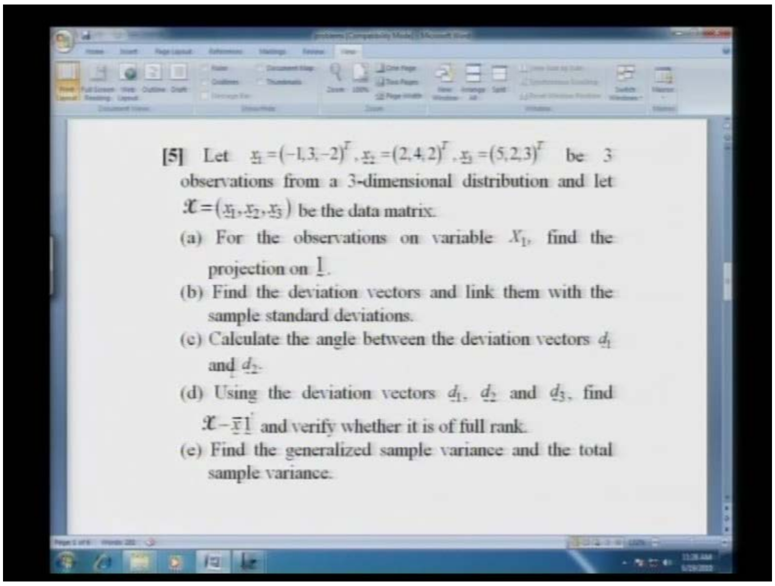

So, that completes the second part of this problem. In the third part of the problem, we are trying to calculate the angle between the deviation vectors d 1 and d 2.

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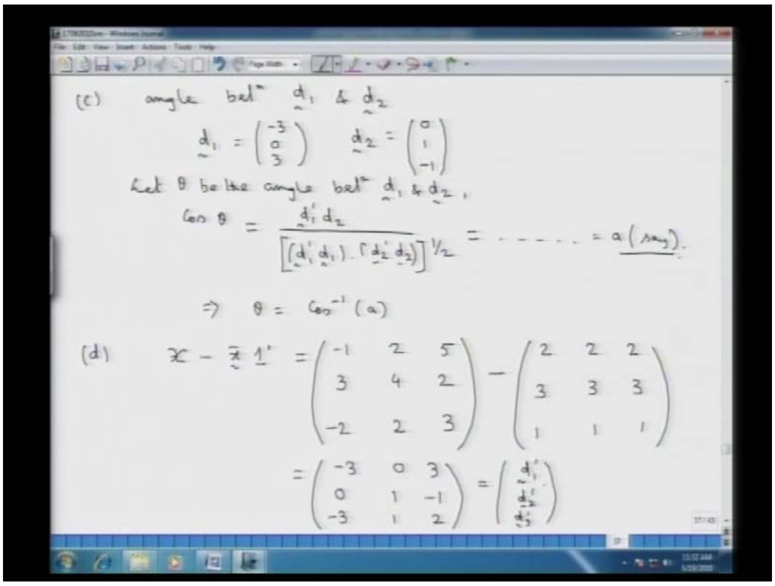

What we are now trying to find out? Then this c is the angle between d 1 and d 2. Now, what are these vectors d 1 and d 2? d 1 was given by this minus 3 0 3 minus 3 0 3 and d 2, the deviation vector was given by 0 1 minus 1 0 1 minus 1 0 1 minus 1 right. So, we are trying to find out what is the angle between d 1 and d 2. So, let theta be the angle between d 1 and d 2, then what we have is cosine of that angle is given by d 1 prime d 2; that divided by d 1 prime d 1 into this d 2 prime d 2 that under root right.

Now, you can compute what is d 1 prime d 2. Using d 1 prime d 2 can be computed from this d 1 and d 2. d 1 prime d 1 we have already computed that, that if I remember correctly was 18 and d 2 prime d 2 similarly can be computed from out here and then, whatever it comes is the cosine of this. Suppose this turns out to be a; so, this would imply that theta, the angle between the two deviation vectors d 1 and d 2 is cosine inverse of a, say what? is the numerical value here; that is what is going to come using this d 1 and d 2 vectors. Let us now move on to the next part of the problem. Let us see, what is that we are trying to find out here?

So, using the deviation vectors we are trying to find out this. This basically is the deviation from respective observations. So, script X minus X bar 1 transpose and we are we would also like to check whether it is a full rank. The reason why we are trying to check it is full rank would be obvious. Let us we have already computed d 1, d 2 and d 3 similarly. So, we are trying to find out this X minus X bar 1 transpose. So, this X X matrix is the data matrix, which actually what we have was minus 1 2 5; this is 4 3 4 2; minus 2 2 and 3 right; this minus x bar 1 transpose.

Now, X bar is holding the three means of the three variables. So, the first variables mean as what we have computed was 2, the second variables mean; this is sum of these divided by 3. So, that was equal to 3 and the mean of the third variable is equal to 1. So, that this X bar 1 transpose matrix is going to be given by 2; this is the mean for a first variable. Then, the mean for the second variable and then the mean for the third variable, which is all 1's out here right. So, that this is going to be given by the difference of these two. So that, this is I will just write it what is this.

So, this is minus 3 0 3; this is a first deviation vector d 1 and this is a second deviation vector as what we had computed. So, you see that it is basically going to hold deviation vectors in the three rows and similarly, the third deviation vector is minus 3 1 2. So, this is what we have at one stroke the all the three deviation vectors d 1 prime d 2 prime and d 3 prime right. Now, in order to compute the sample variance covariance matrix, we can simply use this particular matrix that is what we have computed. Because the sample variance covariance matrix is going to be formed from these deviation vectors d 1, d 2, d 3.

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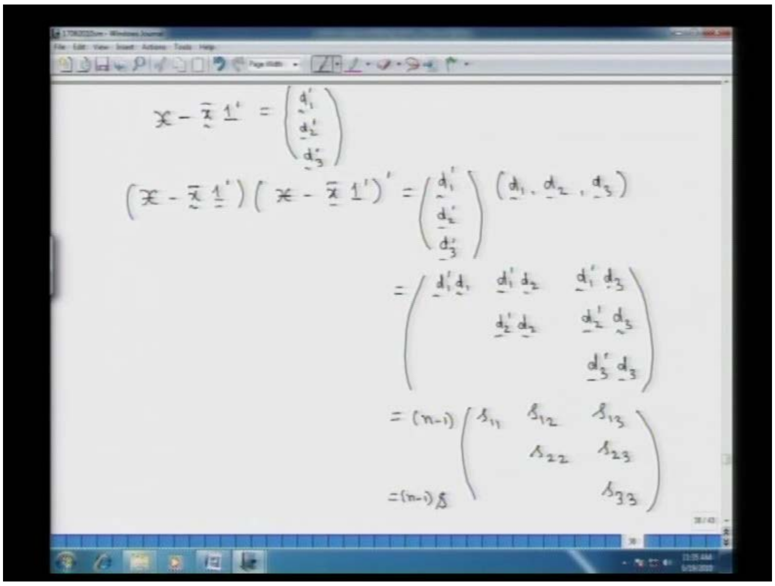

What is that? If we have this X minus X bar 1 transpose given by this d 1 transpose, d 2 transpose and d 3 transpose, then what is this matrix X minus X bar 1 transpose times X minus X bar 1 transpose transpose. So, this is this multiplied by the transpose of this. So, that, what now we have is d 1, d 2, d 3 transpose and then, the transpose of this; which is d 1 d 2 and d 3. What is this going lead us to? This is going to lead us to d 1 prime d 1 d 1 prime d 1 is n minus 1 times s 1 1; this is d 1 times d 2; this is d 1 times d 3; this is d 2 prime d 2; this is d 2 prime d 3 and this is d 3 prime d 3 right.

Now, this is n minus 1 times s 1 1; this is n minus 1 times s 1 2 and so on. So, that is what we have is n minus 1 times this matrix, which is s 1 1, s 1 2, s 1 3; this is s 2 2, s 2 3, and this is s 3 3. Now, both these matrices are symmetric. So, no need to write the lower diagonal part of this particular matrix. So, this is what we have. Now, what is this? This is the sample variance covariance matrix that is, this is n minus 1 times this s matrix right. Now, whether this S matrix is going to be singular or non singular will depend on whether this matrix, what we have computed is singular or non singular and that is basically the point.

Why we are trying to see whether this matrix X minus X bar 1 transpose, which is holding d 1 prime d 2 prime and d d 3 prime as a three row vectors; whether that is of full rank or not. Let us see if we can see that from here, this s matrix is going to be given by is given already by this particular term out here. And if you look at these three columns here, clearly it is rank deficient. Because this this one, this third column plus the first column is going to lead us to… let me just write it fresh here.

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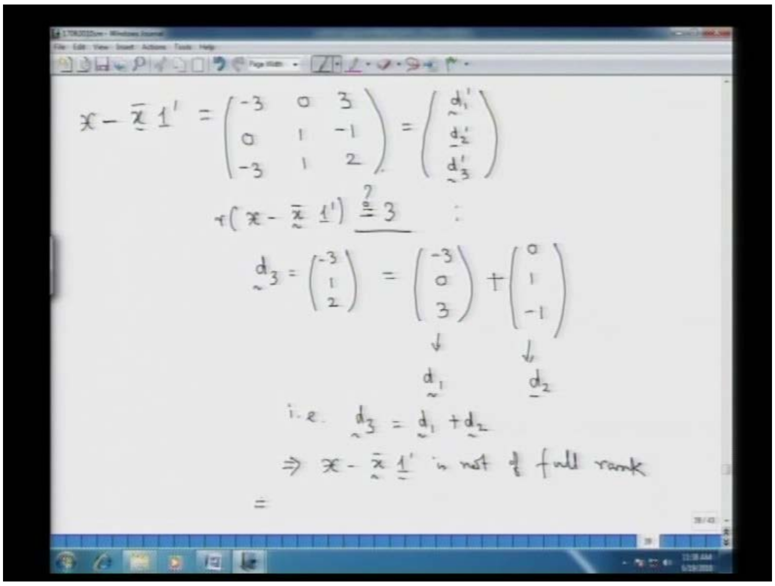

So, that what we have is this X minus X bar 1 transpose matrix is minus 3 0 3 0 1 minus 1 minus 3 1 2 right where this had, this as d 1 prime, this as d 2 prime and this as d 3 prime. So, we have these three vectors. Now, if we look at this matrix and try to find out rank of X minus X bar 1 transpose, whether that is 3 or not? Whether that is full rank or not? this is what is the question. So, as we clearly see from here, that this d d 3 vector which is minus 3 1 2; that is equal to say minus 3 0 3 this which is d 1 and then, d 2 is 0 1 minus 1.

So, this is minus 3; this is 1 and this is 2. So, what is this? This is my d 1 vector and what is this? This is my d 2 vector and hence, we have d 3 vector; this is just d 1 vector and d 2 vector, not their transposes. So, this is d 1 vector plus this d 2 vector. So, if we have in this particular matrix one row being actually redundant. So, that is linearly dependent on these two rows. So, this would imply that, X minus X bar 1 transpose is not of full rank. So, if this is not of full rank; so, will be this S matrix and S matrix will not be a full rank hence.

So, this would further imply that, S is not of full rank. So, this S is going to be a positive semi definite matrix, because there will be one Eigen value which is 0. Let us look at the last part of this particular problem to find out the generalized sample variance and the total sample variance. So, what are those quantities that would be derived from the S matrix, where the S matrix is given by this particular term here n minus 1 times that. So, using the deviation vector, the S matrices can be computed.

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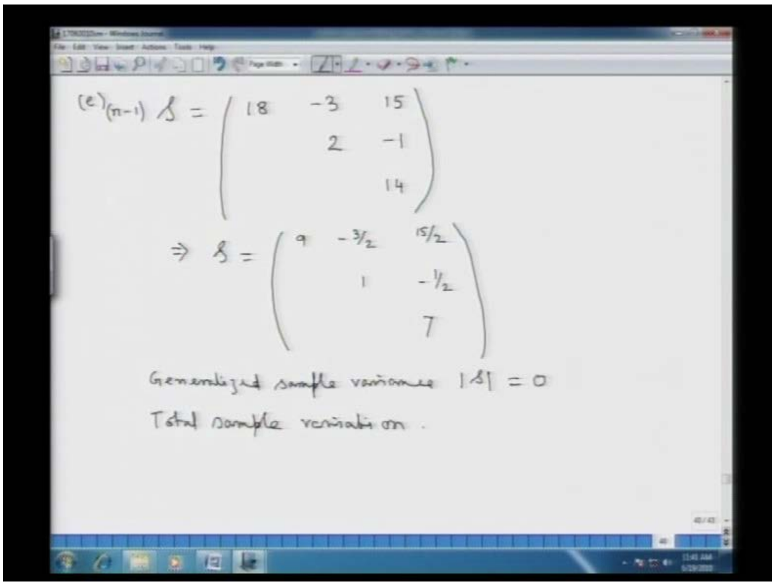

The S matrix after taking the products, the d i prime d j would turn out to be the following. I will just write that here this is this comes out to be 18 minus 3 15 2 minus 1 14. This is just the deviation terms here. So, this is what we have n minus 1 times S. So, this is d 1 prime d 1 and this is d 2 prime d 2 and so on; this is d 1 prime d 2; this is d 1 prime d 3; this is d 2 prime d 3 and this is d 3 prime d 3; n minus 1 is 2. So, this would imply that, this S matrix is this divided by 2. So, this is 9; this is 3 by 2; this is 15 by 2; this is 1; this is 7 and this is minus half right. Now, we are trying to find out what is the generalized inverse and the total sample variance.

So, generalized sample variance is nothing but determinant of this. Now, we do not need to compute explicitly the determinant of this; because what we have already proved is that, S is not of full rank. So, the determinant is going to be 0. And then, we are also trying trying to get the total sample variation. This is what, we had earlier defined in our theory classes that, the total sample variation is going to be trace of this S and that is, 9 plus 1 plus 7 equal to whatever. So, we have the generalized sample variance from this and also the total sample variance from this. That completes this problem number five.

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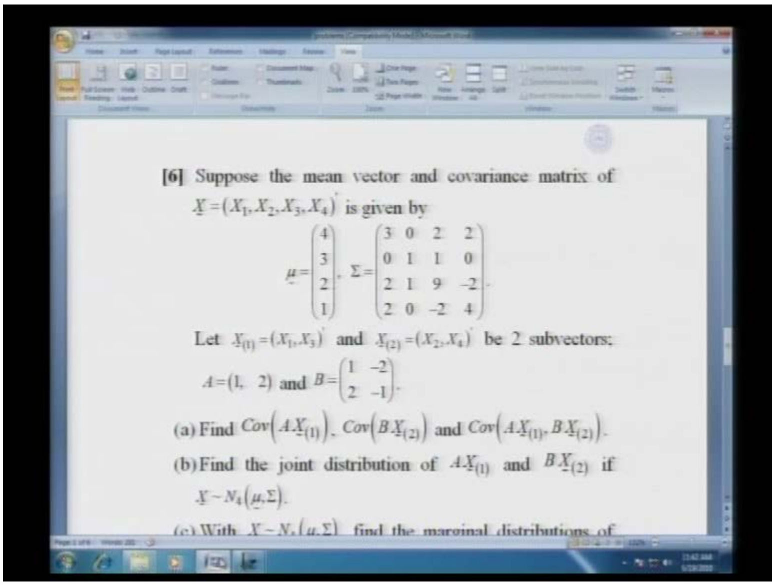

Let us now move on to the next problem. Problem number six is this. So, we have in problem number six that, X is having the following quantities. This is the mean vector corresponding to X. This is the variance covariance matrix of this 4 dimensional random vector. We make 2 sub vectors X 1 as X 1, X 3 and the second sub vector as X 2, X 4 and we consider a vector A, which is 1, 2 and this as another matrix. We are trying to find out covariance between A X 1 and covariance of A X 1, covariance of B X 2, covariance between A X 1 and B X 2; that is a first part of the problem. Let us do the problems one by one.

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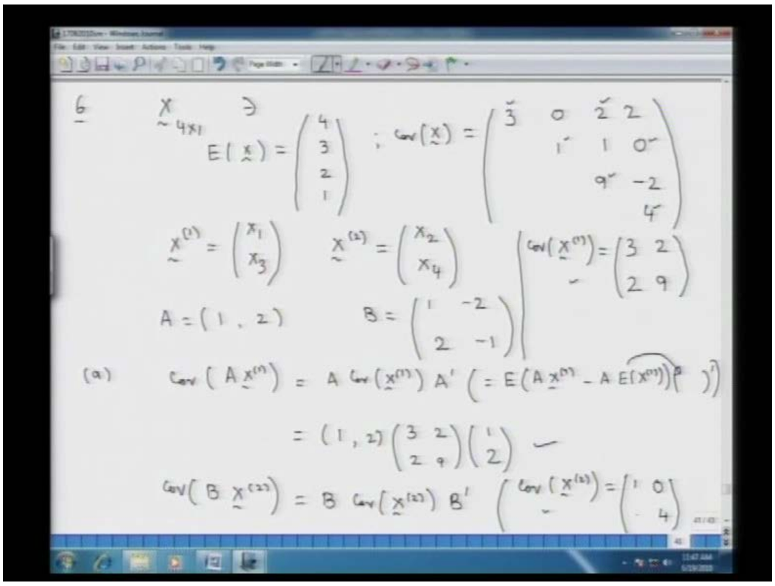

Let us move on to this this problem number six. We have X, a 4 dimensional random vector such that, expectation of X is given by that mean vector. I will have to see, what is what are the entries there, 4 3 2 1; this is 4 3 2 1 and we have the covariance matrix of this X, a 4 by 4 matrix. So, the entry is I will have to just see what these entries are. I have to write these entries. So, these are the entries here. I will just write it from here. These are the entries; this is 3 0 2 2; this is 1 1 0; this is 9 minus 2; this is 4. The rest of the elements can be obtained by symmetric. So, this is what we have.

Now, under this these conditions, we had defined the first sub vector X 1, which is X 1 and X 3 and the second sub vector X 2 as X 2 and X 4 right. And we had defined A to be a row vector having the entries 1 and 2 and B the 2 by 2 matrix, asymmetric; 1 minus 2 and 2 minus 1. Now, under these are the given conditions. So, under these conditions, we are first trying to get what is covariance of A X 1. So, covariance of A X 1 is going to be given by A covariance of this X 1 sub vector times A transpose, that is simple. Why is that equal to that? Because this is expectation of A X 1 minus A times expectation of X 1.

Then, the transpose of this particular element here and this is what, we will be getting finally. So, we need to find out what is covariance of this A covariance of this X 1 sub vector. So, X 1 sub vector is given by X 1 X 3. So that, what we can actually see is the following. Let us try to see, what is covariance of this X 1 sub vector? Now, what are the entries that, this covariance of X 1 is going to hold? It is going to be variance of X 1. So, from here, the variance of X 1 is 3 and then the 2 2 th element with the variance of X 3 say, variance of X 3 is 9 and then, the half diagonal elements of this covariance matrix would be given by the covariance between X 1 and X 3.

What is that? This is this entry. So, covariance between X 1 and X 3 is 2. So that, the covariance between X 3 and X 1 would also be equal to 2. So, this is the covariance matrix of X 1 sub vector. So, what will be having this as 1 2 3 2 2 9 1 2 right. So, this is what the covariance of A X 1 is. Now, the second part was to find out the covariance of B X 2 vector here. That would be B times covariance of X 2 sub vector; that multiplied by B transpose. Now, this would require covariance of this X 2 sub vector. The way that we had obtained this covariance of X 1, we can similarly obtain this. What is this going to be? X 2 is the sub vector which has X 2 and X 4 as the two two variables.

So, that the first entry would be variance of X 2 which is 1, then variance of X 4 is 4. So, that this is 4. And then, the covariance between X 2 and X 3 will be the half diagonal element. So, covariance between X 2 and X 4 is 0. So, that this is what is going to be the covariance matrix of the X 2 sub vector. So, we can actually plug in that and get what is covariance of this. So, this B matrix is written somewhere here this. So, that is 1 minus 2 2 minus 1 and then, the covariance matrix of this X 2 sub vector is 1 0 0 4 and then, this is the transpose of this. So, that this is 1 2 minus 2 minus 1. So, this is what the covariance of B X 2 component. Let us move on to the next part. What was the next part, to find out the covariance between A X 1 and B X 2?

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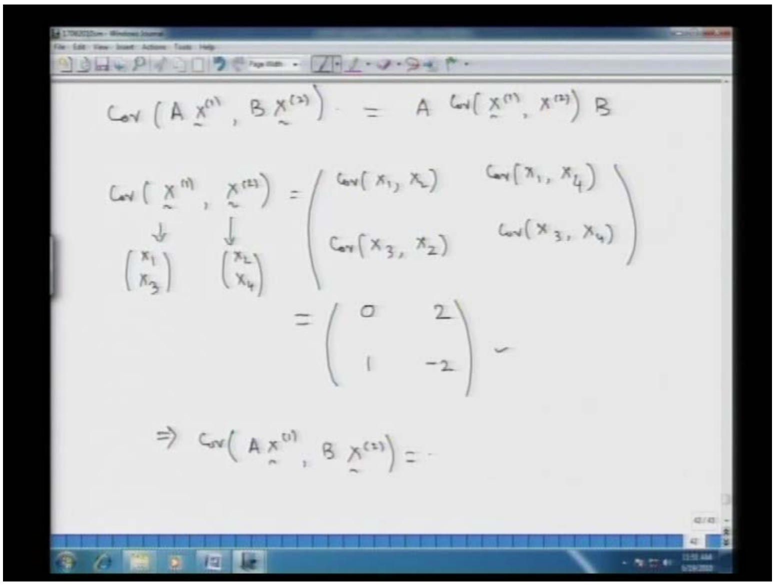

So, this is the covariance between A X 1 and B X 2. So, the covariance between A X 1 and B X 2 is going to be given by A covariance between X 1 and X 2 times this B right. Now A, B are known things. So, what we need to find out is covariance of this X 1 and X 2 sub vector. Now, what would that be given by; now remember that, this X 1 has entries X 1 and X 3 and this X 2 sub vector has entries X 2 and X 3. So, the covariance matrix of this X 1 and X 2 would be given by the following that, this is the first entry would be covariance of X 1 with X 2.

The second would be covariance of X 1 with X 3 and this is covariance of X 3 with X 2 and this is covariance of… this entry is not X 3; this is X 4. If I remember correctly, this is yeah X 1 is X 1 X 3; X 2 is X 2 X 4. So, that this is X 2 and X 4. So, that this would be actually X 1 X 1, X 2; X 1, X 4. So, this is X 3 X 2 and this is X 3 X 4. From the sigma matrix, we will actually look at what is covariance between these components. So, if we look at the sigma matrix, all these entries; this is covariance between X 1 and X 2. So that, we will have the first element here as 0.

Then, covariance between X 1 and X 4 X 1 and X 4 would be 2 here. Covariance between X 1 and X 4 is 2; covariance between X 3 and X 2 X 3 and X 2 would be same as covariance between X 2 and X 3. So, that is 1 here. So, the next entry is 1 here and covariance between X 3 and X 4 can also be computed. X 3 and X 4 would be same as X 4 X 3. So, that this is minus 2. So, this is minus 2. So, this is what, is the covariance matrix between X 1 and X 2. So, we can use this particular term here; as you can see, that this covariance between the two did not be actually symmetric.

So, this would imply the desired result that, this is the covariance between these two terms that is equal to now A was given to be 1, 2, and then the covariance matrix is what we required out here, which is 0, 2, 1 minus 2 that multiplied by the transpose. I am sorry this is the transpose out here, this covariance of these two, so B transpose. So, B was given by this, and hence its transpose is 1, 2 minus 2 and minus 1. So, this is what is now solving this covariance between these two vectors.