Applied Multivariate Analysis

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Lecture No. # 03

Multivariate Normal Distribution - I

After the sessions on the introductory concepts of multivariate analysis, we begin today's session with an example of a very important univariate, multivariate distribution namely the multivariate normal distribution. Before we formally define the multivariate normal distribution.

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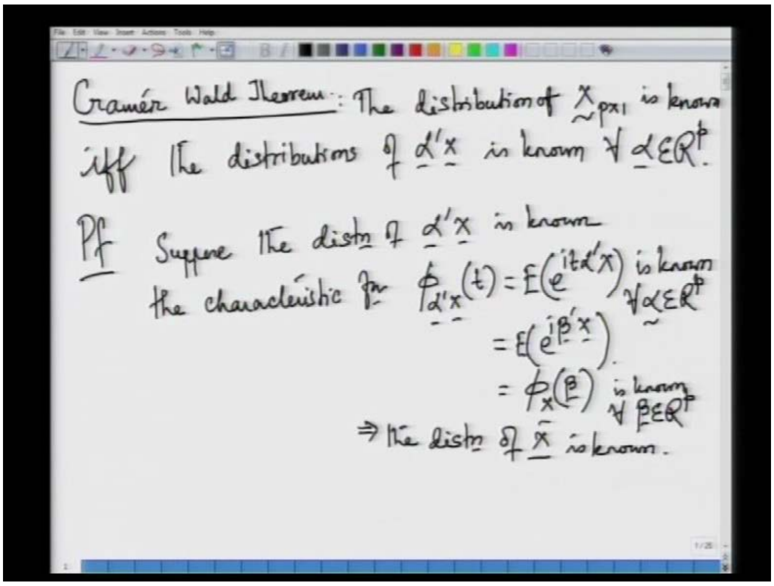

We talk about the Crame'r Wald theorem, which very generally states that this is, note that this is the theorem, which does not make any reference to any particular distribution, but it very generally states that the distribution of the random vector. This is our p-dimensional random vector X is known, if and only if the distributions of alpha prime X is known for all alpha has to belong to the p-dimensional space. Note that while X is a random vector or alpha prime X is a scalar, it is a uni dimensional random variable, and we are talking about different linear combinations of this random vector X. So, very generally the Crame'r Wald theorem tells us that the distribution of the multidimensional random vector X is uniquely determined by the distributions of the linear combinations of type alpha prime X. For the proof of the theorem, we will just a very simple proof, we will use the concept of characteristic function to which you have been already introduced.

For the first part, say for the if part, we take that suppose- the distribution of alpha prime X is known. So, we are essentially proving the sufficiency or the if part of the theorem here we assume that the distribution of alpha prime X is known. If it is, we can say the characteristic function of alpha prime X at t so we are using the notation phi of alpha prime X at t by definition this is nothing but expectation of e power i t alpha prime X. Since we make a claim that the distribution of alpha prime X is known, the characteristic function has to be known, is known for all alpha in R p. Note that these alphas are also vectors.

Now, if this is known, I can simply write that this is what, expectation of e power i some beta prime X, where I am taking t alpha prime as beta prime or on the other way to say it beta is nothing but alpha t. Now, what is this expectation of e power i beta prime X, this is nothing but the characteristic function of… I can simply write that this is the characteristic function of the random vector X at beta now. Since this is the random vector, this also has to be a vector; now, we have a beta and this is also known for all beta and where does beta belong to, it belongs to R p. So, if this is known for all beta this implies that the distribution of X is known. So, we prove one part of the theorem that if distributions of alpha prime X is known for all alpha the distribution of X is also known.

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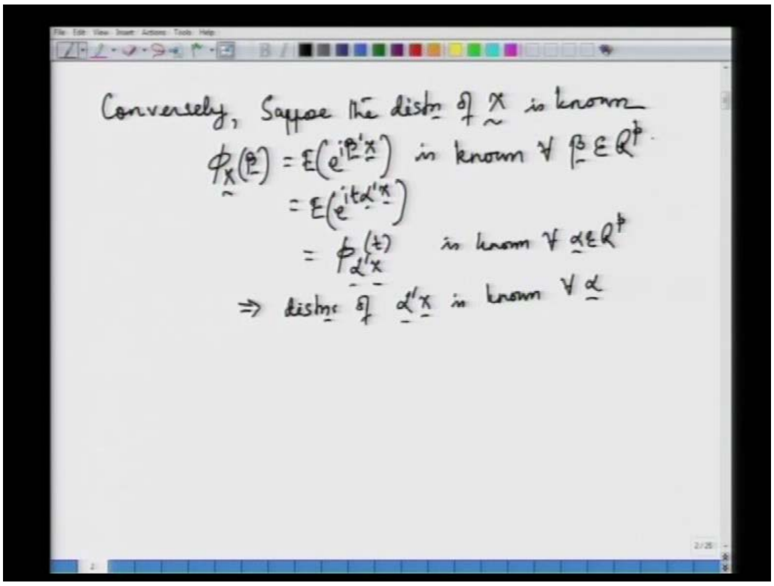

For the other part that is the converse part which is the only if part where we assume that the distribution of the random vector X is known, suppose the distribution of X is now known ,if it is. So, then I make a claim that the characteristic function phi X at beta which is nothing but expectation of e power i beta X is known for all beta belonging to R p. Similarly, I can write that this is nothing but e power i t alpha prime X is known for all beta or now for all alpha, because beta prime is nothing but t alpha prime and this is nothing but the characteristic function of alpha prime X at t, and this is known for all alpha now implying that distributions of alpha prime X is known, for all alpha thereby completing the proof this is the Crame’r Wald theorem with the Crame’r Wald theorem at the background ,we will now formally define the multivariate normal distribution.

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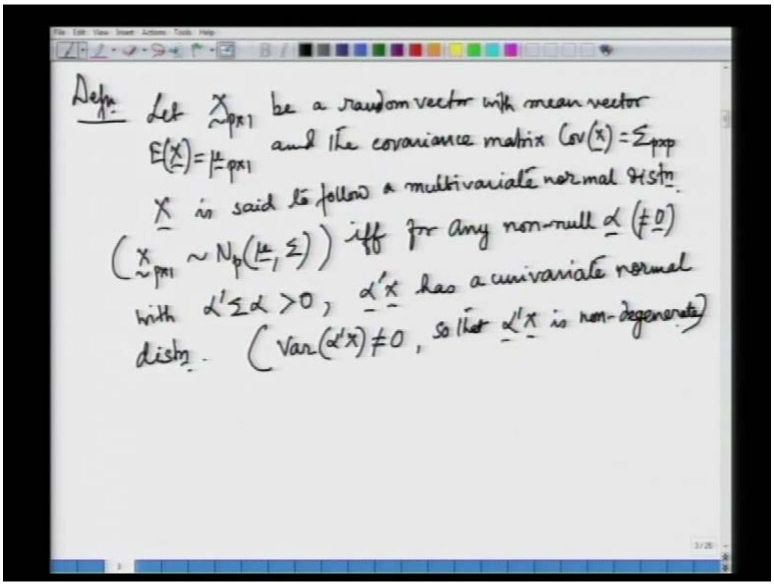

So, this is the definition let X be a random vector with mean vector denoted by mu this is the expectation of X which is another vector mu and the covariance matrix sigma which is a p by p square matrix X is said to follow a multivariate normal distribution. For the notation we say that X the p-dimensional random vector is following a p variate normal distribution. So, we use this p as the subscript here with the parameters mu and sigma mu being the mean vector and sigma being the covariance matrix. So, this is called notation X following N p mu sigma if and only if for any non-null alpha is not a null vector with the quadratic from alpha prime sigma alpha strictly greater than zero alpha prime X has a univariate normal distribution.

Now, you can probably realize why this restriction has been put in place that alpha prime sigma alpha is strictly greater than zero this is required. So, that obviously variance of alpha X we would not like this to be equal to zero. So, that this is not equal to zero. So, that the random variable that is alpha prime X is non- degenerate. If the variance is equal to zero we are going to have a degenerate random variable and we would like to avoid a situation like that. So, this is the definition of the multivariate normal distribution which we are going to tackle with various univariate normal distributions some results will automatically follow from the definition and we are going to state them one by one.

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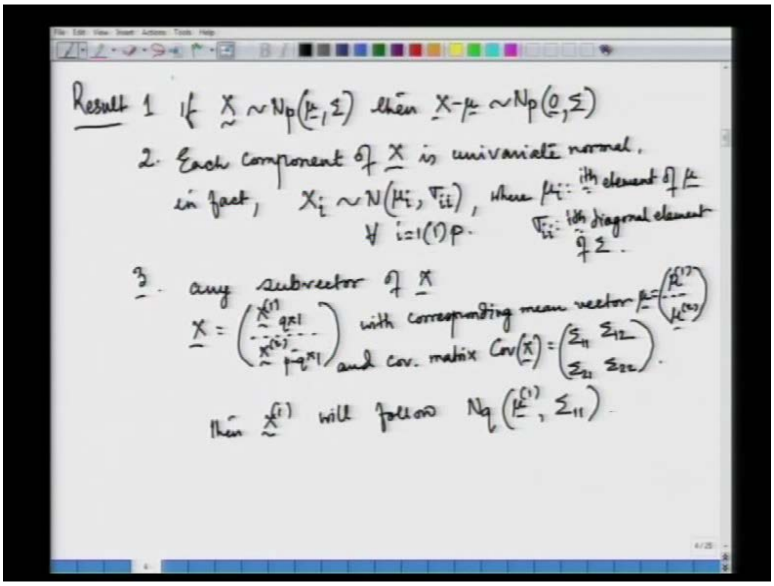

The first result is if X follows a p variate multivariate normal with mu sigma then X minus mu is also going to follow a p-dimensional multivariate normal distribution, now with the change in the mean vector this being a null vector, now the covariance matrix remains the same namely sigma. Now, it is not difficult to see why this is going to follow on multivariate normal distribution all we have to consider is linear combination of X minus mu just like we had considered for the X random vector. So, if I consider alpha prime X minus mu, we can easily see that the stochastic part of this variable that is alpha prime X is following a multivariate normal distribution and hence the whole thing is also going to follow multivariate normal distribution with the adjustment being made in the mean vector and the covariance matrix.

So, then this we have and the next result that we are going to state is each component of X this vector is in fact, univariate normal in fact, I can say that X i this is going to follow univariate normal with mean mu i and variance sigma i i say where of course, mu i is nothing but the i th element of the mean vector mu and sigma i i is the i i th diagonal element or the i i th element of sigma. Note that here also we are talking about a very special form of alpha prime X for X i if I take alpha as a vector which has zero in all places except in the i th position where we need a one then all i have is alpha prime X that should follow a univariate normal distribution and with the choice of alpha this is exactly what is happening and this is true for all i from one to p as many components as there are in the random vector.

The third one we have talked about partitioning of the random vector. So, we consider such a partitions suppose we partition. So, any sub vector of X suppose I consider a partitioning of the p-dimensional vector into two components the first one having q components and the second one having p minus q components. I can make up this X 1 and X 2 in whichever way I like picking up any components from the whole vector X and I can form X 1 and X 2. So, this is a sub vector of X, but what is important the corresponding mean and covariance matrix they should be formed accordingly with corresponding mean vector mu also partitioned in the similar manner mu 1 and mu 2 and covariance matrix covariance of X taking a picture like sigma 1 1 sigma 1 2 sigma 2 1 and sigma 2 2. If I consider then X 1 for example, we will follow a q-dimensional multivariate normal distribution. This is going to follow normal q with mean mu 1 and the covariance matrix sigma 1 1. Now, all these results that we have stated till now, they can be summed up in a general result and we are now going to state that.

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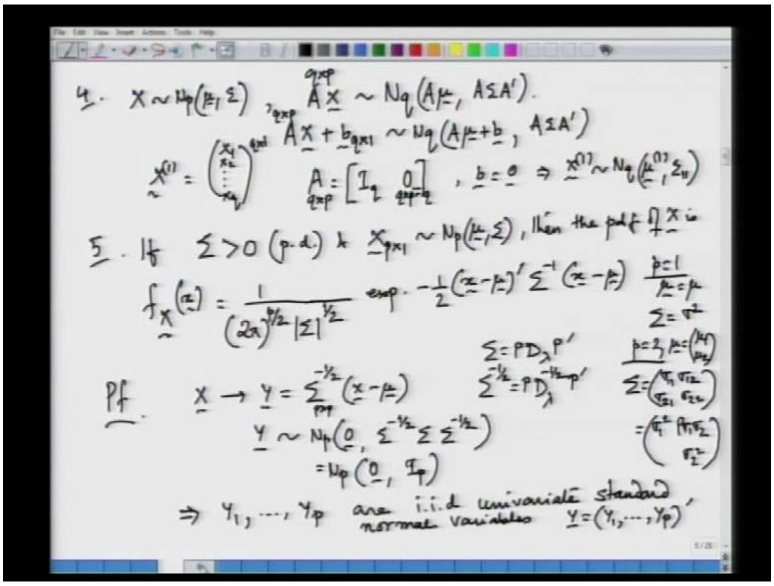

This says that if X follows a p variate multivariate normal distribution with mean vector mu and sigma and we consider any rectangular matrix q by p A matrix of constants then A X is going to follow N q variate multivariate normal distribution with mean A mu and covariance matrix A sigma A transpose, further I can also have A is a rectangular matrix A X. I can also add a vector here which is q-dimensional vector. So, this will again follow normal a q variate multivariate normal with A mu plus b as the mean vector covariance matrix however, does not change, because this is just a shift in the location. Now, again it is a difficult to see that why A X or A X plus b will follow multivariate normal distribution again if we consider some linear combination of this alpha prime X, now alpha will be long to R q. So, it is from there we will straight away go to another linear combination of X.

So, it is some beta prime X there beta will belong to, now be R p and since X is multivariate normal that is also the linear combinations will be univariate normal and hence the linear combinations of A X are also univariate normal giving us that the whole random vector A X will be multivariate normal. For example if X 1 as we have talked in the earlier result if this X 1 is comprised of the first q elements of the X vector. So, these are X 1 X 2 to X q the q dimensional sub vector that I have taken out from X the whole random vector then… So, if I take A as I q and then the null matrix obviously, this being q by p this is q by p minus q and if I take the other part that is the vector is a null vector well I can see that X 1 this will give me that X 1 is going to follow a q variate normal distribution with mean mu 1 and covariance matrix sigma 1 1.

So, till now ,we have been talking about the multivariate normal distribution without actually referring to the probability density function of the distribution. At this point, we are going to talk about the pdf of the multivariate normal distribution. So, this is our next result if sigma is well by this notation, we are going to mean that sigma is a positive definite matrix and X follows normal p mu sigma then the pdf of X is f of X at the point X is 1 by 2 pi to the power p by 2 determinant of sigma to the power half and then we have an exponent term which is minus half X minus mu transpose sigma inverse which is defined, because we have taken sigma as positive definite matrix minus mu. Note that if you take p equal to one you should get the pdf of the univariate of the normal distribution in that case what is a mu vector for p equal to one mu vector is nothing but the scalar mu the constant mu for the sigma matrix generally first element is taken as sigma square.

So, that we have a univariate normal distribution with mean mu and variance sigma square and we can easily get back the univariate normal pdf, if p is equal to two now, we have a mean vector comprised of mu 1 and mu 2 the covariance matrix well by the general notation it is sigma 1 1 sigma 1 2 sigma 2 1 sigma 2 2, but usually what we take is sigma 1 square rho sigma 1 sigma 2 and sigma 2 square the rho is the correlation coefficient. So, this is a general notation that is followed for the bivariate normal distribution. So, again with this mu and this sigma if we write down the pdf we will get the pdf of the bivariate normal distribution.

Now, we talk about how we get this pdf the first place at the very beginning we take a transformation. So, we consider a transformation from X to Y and Y is nothing but sigma this matrix the inverse square root matrix of this sigma 2 power minus half and then I take x minus mu. Note that defining this matrix is not a problem again we have taken sigma as a positive definite matrix and we all we use is the spectral decomposition of the sigma matrix writing sigma is nothing but P D lambda p prime. So, this D lambda is nothing but the matrix with the diagonal elements as the eigen values of sigma it is a diagonal matrix and the p matrix is the matrix whose problems are the corresponding orthonormal eigen vectors. So, once we have this decomposition it is very easy to write what is the inverse square root matrix it is nothing but P D lamda this to the power minus half prime, all it means is the same diagonal matrix now, the eigen values lamda in place of that we are going to write 1 by root lamda. So, with this transformation what can I see will this is in fact, a very special form of the matrix A and this is again a very special form of the vector b.

By the previous result I immediately have that now, this being a p by p matrix. So, this by follows a p variate normal obviously, now with mean as the null vector and the covariance matrix we have seen is A sigma A prime with our A as this it is going to be well it is going to be sigma minus half sigma sigma minus half this is the diagonal matrix which is symmetric. So, taking the transpose basically means the same matrix and this is nothing, but p variate normal with mean null and covariance matrix as the identity matrixes of order p. Now, once we say this we can also say by a very definition that this means Y 1 to Y p are nothing, but i. i. d univariate standard normal variables of course, I want to write p well these are components of Y. So, this is Y 1 to Y p. So, writing the pdf of the random vector Y is basically now, writing the joint pdf of the p i. i d univariate standard normal variables which we know which we can handle very easily.

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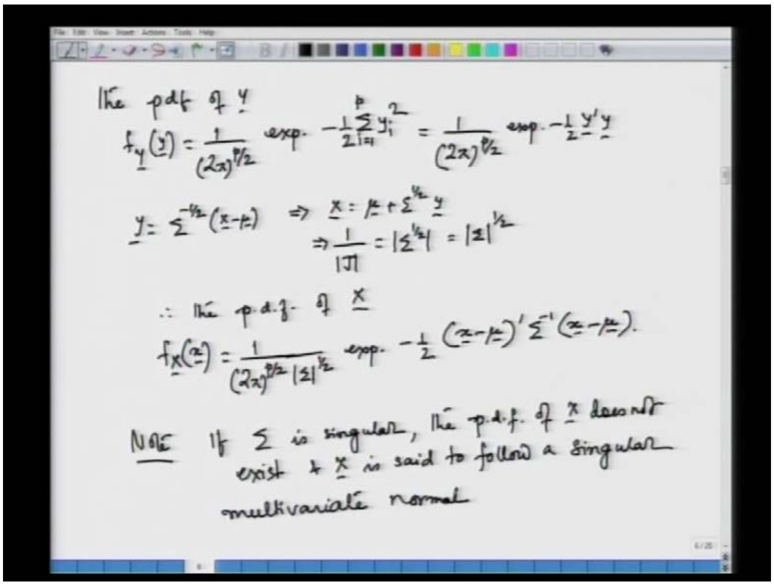

Hence we are going to write the pdf of Y vector is nothing but it is just the product of p univariate standard normal distributions which I can simply write as 1 by 2 pi to power p by 2 and the exponent power is nothing but minus half summation y i square i from p well I can be write this in the matrix notation this is 1 by 2 Pi 2 power p by 2 and X component part is minus half with y transpose y. Note that we have used the transformations we have to talk about the jacobian of the transformation, before we get back to the pdf of the original variable that is X. Now, for the jacobian of the transformation again note that the transformation that we have used is y is nothing but sigma minus half x minus mu let us write it in terms of X. So, that we have X is actually equal to Mu plus the square root matrix of sigma and y, what we get here if we consider the differentiation in this form, what we get is not the jacobian, but the inverse of the jacobian which now, implies that one by this jacobian is determinant of this matrix which is again nothing but determinant of sigma to the power half, it is basically square root of the determinant of sigma.

So, with this in the background I can write therefore, the pdf of our original random vector that is X is f of x well the constant term remain a such 2 pi to power p by 2 I have to consider reciprocal of this jacobian. So, it comes in the denominator with the same power determinant of sigma to the power half and in the exponent part. I will now, write what I am getting in terms of instead of y transpose y, which is y is nothing but this which I am going to use and I am simply getting x minus mu transpose sigma inverse x minus mu. So, this is the pdf of the multivariate normal distribution p variate normal distribution with mean vector mu and covariance matrix sigma again for p equal to one you will get the pdf of the univariate normal random variable with p equal to two you are going to get the pdf of the bivariate normal distribution make a small note here that.

Note that we had started with sigma being a positive definite matrix. If sigma is singular covariance variance matrix strictly speaking can be positive semi definite. If this is singular, if sigma is singular the pdf of X does not exist very naturally, because sigma inverse does not exist in that case and X is say to follow a singular multivariate normal distribution ,well now we are going to talk about the characteristic function of X. So, we have already defined, what is the characteristic function of a random vector?

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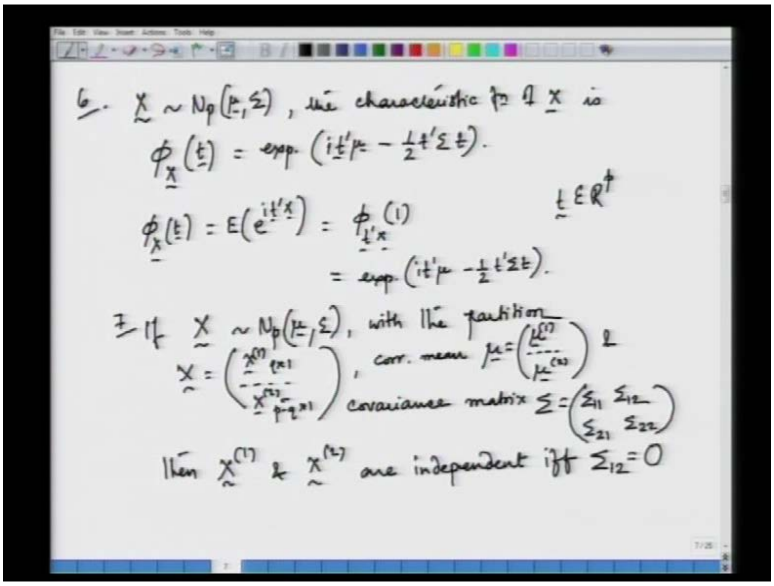

This is our next result number six, if X follows a p-dimensional multivariate normal distribution with mean vector mu and dispersion matrix sigma, the characteristic function of X is given by phi of X at we use the same notation of t, but now t is a vector, because this is a random vector now, and this is given by exponent of i t prime mu minus half t prime sigma t. How we get this? Well, we can talk about the characteristic function of a random variable very easily; and that random variable is nothing but the linear combination of this X. So, we talk about phi of X at t is by definition expectation of i t prime X, but this can also be the characteristic function of the variable t prime X at 1, t belonging to R p, then what is the distribution of t prime X? This is nothing but univariate normal with mean whatever t prime mu and variance as t prime sigma t. So, this is nothing and we need know, what is the characteristic function of a univariate normal distribution is, we have to be careful that now this is at 1.

So, with this we can easily write that this is nothing but exponent of i t prime mu, because this is now the mean of t prime X and minus half t prime sigma t ,this is a scalar quantity it is a variance of t prime X it is half t half sigma square t being equal to one here. So, this is half t prime sigma t and which is nothing but the characteristic function of the random vector X again for p equal to one you can easily check this you get the characteristic function of the univariate normal distribution. Now, our next result is we have talked about while portioning a random vector we have talked about the independence of the constituents part when do we have independence. Similarly in this set up also we are going to talk about the independence of the constituent parts of the random vector.

So, the results states that if X is multivariate normal with mu sigma with the partition X is the first component is a q-dimensional subvector the second component is a p minus q the residual part with corresponding mean vector mu similarly partitioned into mu 1 and mu 2 and covariance matrix sigma. Now, comprising of block matrices, four such blocks; we have the constituent part that is X 1 and X 2 are independent if and only if the off diagonal block sigma 1 2 is a null matrix. So, this if and only if part this is important in the case of the multivariate normal distribution they are independent if and also conversely if this is a null matrix X 1 and X 2 are will be independent.

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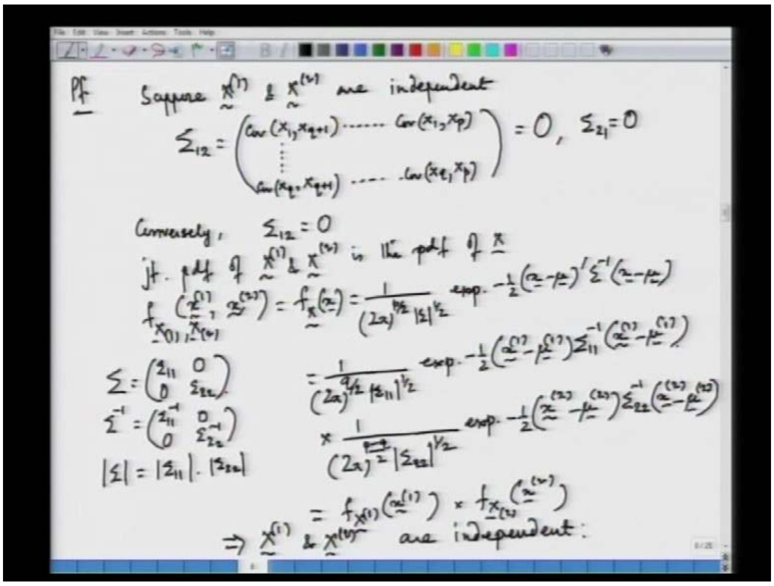

So, we are going to the proof of this result ,in the first part we take the only if part or the necessary part and we say that suppose- X 1 and X 2 are independent if they are independent whether they are multivariate normal or not what we have is that any component of X 1 and any component of X 2 we consider the covariance between them it will be always equal to zero, that we have the sigma 1 2 matrix to call that this is nothing but the first element will be our X covariance of X 1 with X q plus 1 and the last element in the first row is going to be covariance between X 1 and X p. Similarly, we continue till the q th component. So, this is covariance between X q and X q plus 1 and then we have covariance between X q and X p.

Now, since we have X 1 is comprising of X 1 to X q and X 2 is comprising of X q plus 1 to X p and it is given that they are independent we necessarily have all these elements equal to zero giving us a null matrix here and of course, sigma 2 1 which is simply the transpose of sigma 1 2 is also a null matrix then… So, the first part is proved, conversely sigma 1 2 is a null matrix is this generally true well no, because we know that if this is if the covariances equal to zero it does not imply that the variables are independent. Now, we have this as a special situation for the multivariate normal distribution and we have a situation we are assuming that the half diagonal blocks they are null matrices and assuming this we go on proving the other part. So, now, the joint pdf of X1 and X 2 is nothing but the pdf of X.

So, what I have is f of X 1 X 2 at x 1 it is better to use the same notation here x 1 x 2 this is nothing but f of X at x and now, that we know the pdf of the multivariate normal distribution I simply write this as 2 pi to the power p by 2 sigma to the power half exponent of minus half x minus mu transpose sigma inverse x minus mu, but note that this sigma has a very special feature what is that sigma is sigma 1 1 null null and sigma 2 2 giving you two things that we need actually sigma inverse will be nothing but the inverses of the blocks and determinant of sigma is the product of the determinants of the blocks. So, once we use this and we distribute this constant part also not equally to the corresponding way what we get is nothing but the first I take q out of this and I take this one part of the product. So, this is sigma 1 1 to power half and I also take right this part as the two constituent parts and the sigma inverse as whatever in the way that we have written here and then combine the two quadratic forms and then I can easily see that this is nothing but exponent of minus half x 1 minus mu 1 note that mu is also partitioned in the same manner as mu 1 and mu 2 and here I am going to have sigma 1 1 in inverse with x 1 and mu 1 the other part is nothing but the 2 pi with rest of it which is p minus q by 2 determinant of sigma 2 2 norm with minus half and the other part the residual path from the original quadratic form is coming here which comprises of x 2 vector.

So, I have x 2 minus mu 2 sigma 2 2 inverse x 2 minus mu 2 which is nothing but f of X 1 at x 1 times f of X 2 at x 2.Giving me the joint pdf of X 1 and X 2 is the product of the marginal p d fs implying that X 1 and X 2 are independent. Again a very special feature of the multivariate normal distribution. Let us now talk about the conditional distribution of one part of the random vector given the other part.

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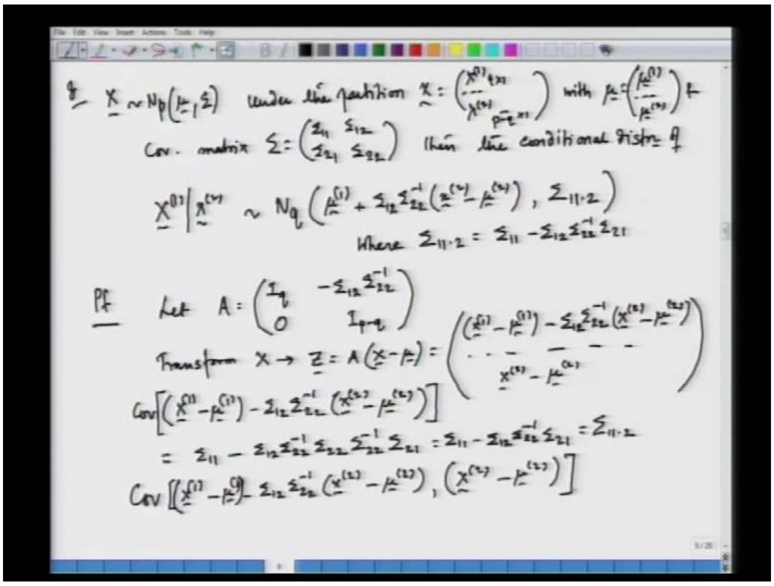

Our next result is on the conditional distribution, we have X following p variate multivariate normal with mu sigma under the partition x into the sub vectors X 1 and X 2 the first being a q-dimensional and the second one is p minus q with corresponding mean vector mu similarly partitioned into mu 1 and mu 2 and covariance matrix sigma as this blocks sigma 1 1 sigma 1 2 sigma 2 1 and sigma 2 2 then the conditional distribution of any one part say X 1 given X 2 this is also a multivariate normal distribution ,this follows a q variate normal distribution with mean mu 1 plus sigma 1 2 sigma 2 2 1 inverse x 2 that is the given value of the sub vector random vector X 2 it is denoted by x 2 here x 2 minus corresponding mean vector mu 2 and covariance matrix as sigma 1 1 dot 2 this is special notation we use, where sigma 1 1 dot 2 matrix is nothing but sigma 1 1 minus sigma 1 2 sigma 2 2 inverse sigma 2 1.

Now, note that other part that is X 2 given X 1 can be obtained by symmetry and we will just replace 1 by 2 here and we get p minus q variable normal distribution with the proper changes in the parameters for the proof of this theorem we just use a very special form of the A matrix we have to take A as I q that is the q-dimensional identity matrix then the next block is minus sigma 1 2 with sigma 2 2 inverse the block over here is the null matrix and then we have a p minus q order identity matrix. So, this is the very special choice of a specific choice of the A matrix which we require to prove the conditional distribution part and then we take a transformation, the transformation is X to Z say Z being equal to this A times X minus mu. See if I want to write the corresponding partition of this whole matrix it is taking a picture like somewhat like this we have x 1 minus mu 1 minus the matrix A coming into the picture here sigma 1 2 sigma 2 2 inverse and then we have x 2 minus mu 2 the other part however, is simply x 2 minus mu 2.

So, this is actually the partition of Z into Z 1 and Z 2 say where Z1 is this and Z 2 is this one ,from here let us now check what is the distribution of this Z matrix of the Z random vector for the mean part we can say that this is going to be the null vector very easily let us have a look at the covariance matrices what is happening to the covariance. So, firstly we consider for the first block we consider covariance of X 1 minus mu 1 the first part minus sigma 1 to sigma 2 2 inverse X 2 minus mu 2. So, a covariance this is equal to first is this variables. So, we take covariance of this is nothing, but sigma 1 1 minus sigma 1 2 sigma 2 2 inverse covariance of this part is sigma 2 2 and we have to take transpose of this constant matrices here.

So, this is now going to be sigma 2 2 inverse transpose of this same thing, because this is a symmetric matrix and transpose of sigma 1 2 being sigma 2 1, so that now what we have here is sigma 1 1 minus sigma 1 2 sigma 2 2 inverse 1 sigma 2 2 inverse gets cancelled and we have a sigma 2 1 here for which we are using the new notation sigma 1 1 dot 2. Now, we look into the covariance between these two vectors now, so that is covariance of x 1 minus mu 1 minus sigma 1 2 Sigma 2 2 inverse x 2 minus mu 2 with let us put the bracket here with x 2 minus mu 2.

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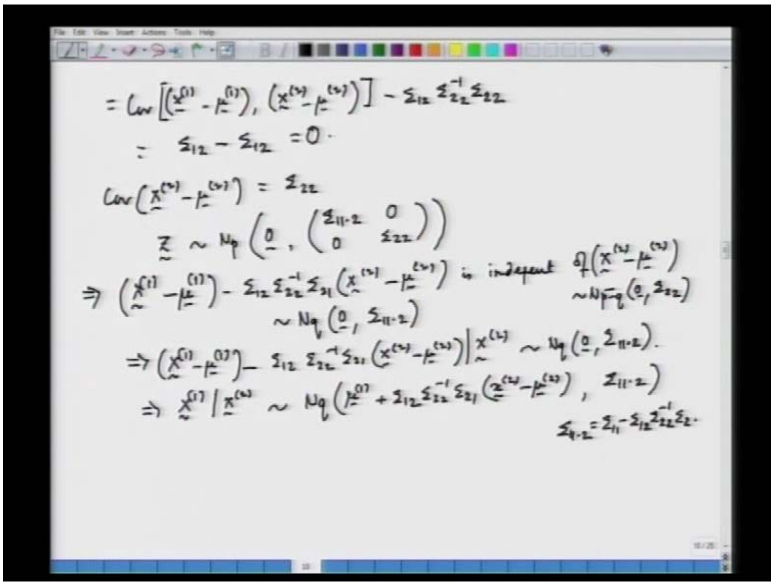

Now, this covariances going to be equal to first we consider the first part and we get this is nothing but covariance of X 1 minus mu 1 with X 2 minus mu 2 and we have a residual part here which is nothing but sigma 1 2 sigma 2 2 inverse and sigma 2 1,so this first part is giving me sigma 1 2 and this is going to be sigma 2 2 and hence I have sigma 1 2 minus sigma 1 2 giving me an null matrix and the last the forth block. So, the third block is also going to be the same thing a transpose of the null matrix which is the null matrix itself ,and for the fourth block we are going to consider covariance of x 2 minus mu 2 which is nothing, but sigma 2 2, this gives me Z following a p variate normal with mean vector null and the covariance matrix as sigma 1 1 dot 2 null null and sigma 2 2. Now, Z being a multivariate normal distribution with the covariance matrix which has it is half diagonal blocks as null matrices, we can easily say that the constituent parts of Z vector that is x 1 minus. So, this directly implies that the constituent parts of Z that is x 1 minus mu 1 minus sigma 1 2 sigma 2 2 inverse sigma 2 1 x 2 minus mu 2 this is independent of the other part that is x 2 minus mu 2.

So, these two are independently distributed. So, this we are going to write the vector in the random vector notation. So, this now being independently distributed we can also see what are the distributions of these separately this is going to follow a q variate normal with mean null vector and the covariance matrix as we have seen is nothing but sigma 1 1 dot 2 and this follows a p minus q variant normal distribution with mean null and the covariance matrix as sigma 2 2. Now, this being independent of X 2 the unconditional and the conditional distributions remains same. So, I can say that the conditional distribution of X 1 minus mu 1 minus sigma 1 2 sigma 2 2 inverse sigma 2 1 X 2 minus mu 2 given X 2 that is the conditional distribution this is the same as the unconditional distribution and this to follows are normal q with mean zero and covariance matrix as sigma 1 1 dot 2, and now I take consider the shift of the locations. So, this is nothing but if I consider the conditional of X 1 given x 2 from here all I have is the change in the mean vector, because for given X 2 this part is known non- stochastic and this is going to follow a q variant normal distribution with mean mu 1 this part is going now to the mean part and we have the rest of it that is plus sigma 1 2 sigma 2 2 inverse sigma 2 1 for given x 2 at the x 2 the non-random part now, and there is no change in the covariance matrix it remains as it is.

So, this is now the conditional distribution of X 1 for given X 2 the mean is taking this form and the variance covariance matrix sigma 1 1 dot 2 once again is said as max 1 1 dot 2 is nothing but sigma 1 1 minus sigma 1 2 sigma 2 2 inverse sigma 2 1. Similarly we can have the conditional distribution of X 2 given X 1.

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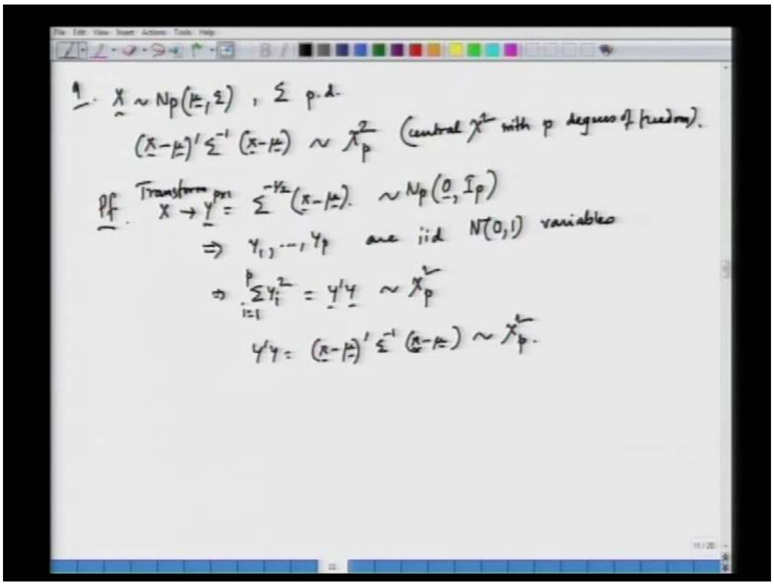

Now, this was a ninth result. So, we go to the next one that was eighth and this is our ninth result which says that for X following a p variate normal with mean mu and covariance matrix sigma, sigma positive definite, let us call at this point again what we are considering is now, a quadratic form in x minus mu transpose the associated matrix is sigma inverse this completes quadratic form. So, x minus mu transpose sigma inverse x minus mu, this follows a central chi square distribution with p degrees of freedom a central chi square with p degrees of freedom. To prove this again we consider the same transformation that we have considered earlier from X to Y transform where Y is the square root inverse of the sigma matrix we have been particular here we are written that sigma is definite.

So, no problem in defining the inverse of the square of matrix and then we have X minus mu with this in position we have already seen that this Y follows p variate normal with mean as null vector and variance covariance matrix as the identity matrix of order p which again implies that Y 1 to Y p the components of this p- dimensional Y random vector these are i. i. d standard normal variables. If that is so, that we know that sum of this square of this p random variables i from 1 to p summation Y i square which is actually in matrix notation Y transpose Y this follows a chi square with pdfs of freedom. So, we have Y transpose Y which is nothing but our x minus mu transpose sigma inverse x minus mu this follows a central chi square with p degrees of freedom simple. Now, we consider a different quadratic form involving X and the associated matrix sigma.

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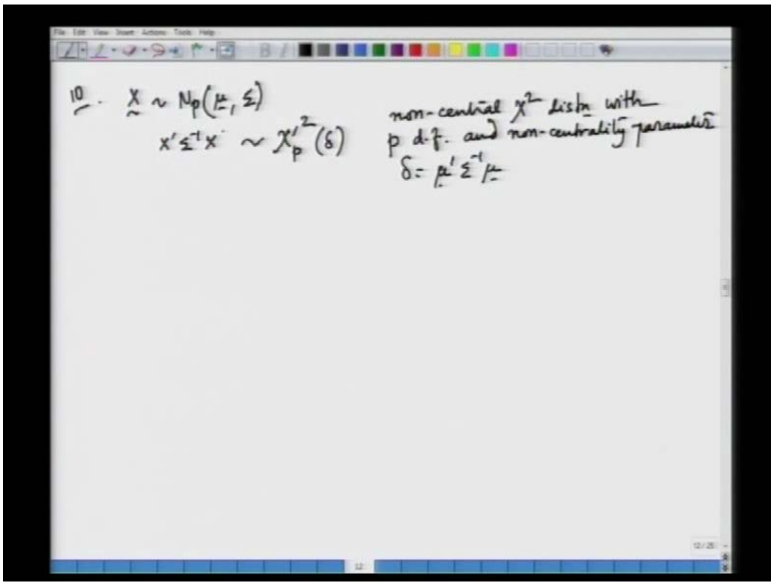

Our tenth result, which says that X following a multivariate normal with mean mu and variance covariance matrix sigma, we have X transpose sigma inverse X. Note that we do not make location shift here, and we say simply that X transpose sigma inverse x is following and non-central chi square with p degrees of freedom and the non-centrality parameter delta. So, this is a non-central chi square distribution with p degrees of freedom and non-centrality parameter delta, this is equal to mu transpose sigma inverse mu. Since we have not made the location change, we are landing up with a non-central chi square distribution. Now, we consider the quadratic form of the type X transpose sigma inverse X without making any location shift with the mean vector mu. We are going to start our next session with the proof of this result.