Applied Multivariate Analysis
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Lecture No. # 20
Manova - III

Now that we have discussed the theoretical aspects of one way Manova model; we are now, looking into the practical example, with real life data, we have at hand the US data of hospitals, its hospital data, and these hospitals have been grouped, according to two factors. One is ownership and the other is the certification level; both the factors having three level each.

And then what we do is, we are testing the effectiveness of ownership that is one of the factors at the moment, because we are considering one way manova model. So, we are testing the effectiveness of ownership of factor of the hospitals, on the different cost aspects, cost to the hospitals. So, since this is the multidimensional case, we have we are taking more than one cost, but then again to restrict the data dimension, we are considering about four costs to the hospitals, so that we are keeping the data dimensionality to four.

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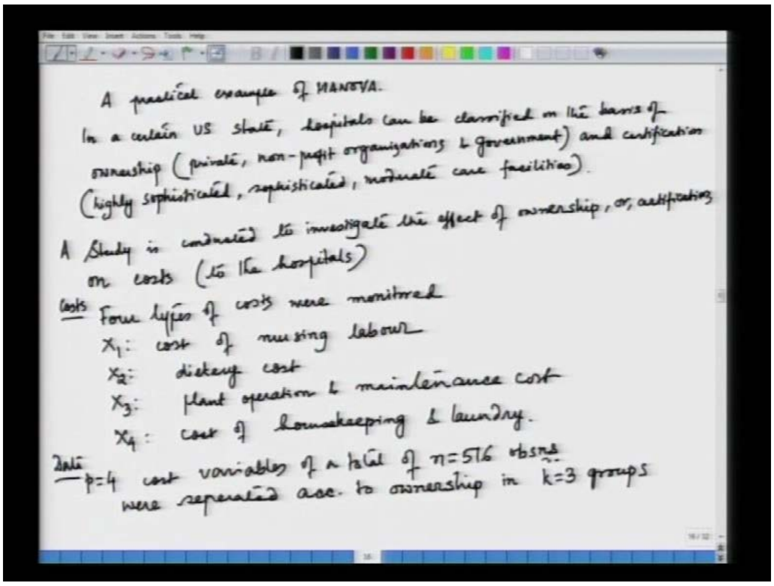

So, what we had already it said in the last session, that we have in a certain US state hospitals are classified on the bases of ownership, and certification, there may be more factors as I have mentioned. But let us say that at the moment we have data classified in these two groups, out of which, only one we can test at the moment, because we have the knowledge of one way manova. And this study is basically conducted to investigate the effect of ownership on costs to the hospitals, so I said I had introduced the cost factors.

So, we have the first one, so this is these are the different costs; first one is, so basically four costs were, four types of costs were monitored, and data were collected on them. The first one is the cost of nursing labour; the second one is the dietary cost, how much each hospital is spending on these aspects. Then we have x 3, the third one the plant operation and maintenance cost plant essentially the hospital, plant operation and maintenance cost and finally, we have the cost of housekeeping and laundry right.

So, we would like to see whether, the ownership factor and this cost the whether, what is the level of effect of this cost on the ownership factor, the ownership factors mean private non-profit organizations and government. So, from this all I know is that, I have p equal to 4 that is the data dimensionality, so about the data what can I say that I have p equal to 4, four cost variables of a total of n that is 516 observations.

So, this value gives me the total number of hospitals, that have been covered we still do not know, how many are there in each level of the factor. So, now, we have these were separated, these observations were separated according to ownership again to stress upon which factor we are considering at the moment; and in k or 3 groups obviously, because because its ownership we have 3 groups, k equal to 3 groups.

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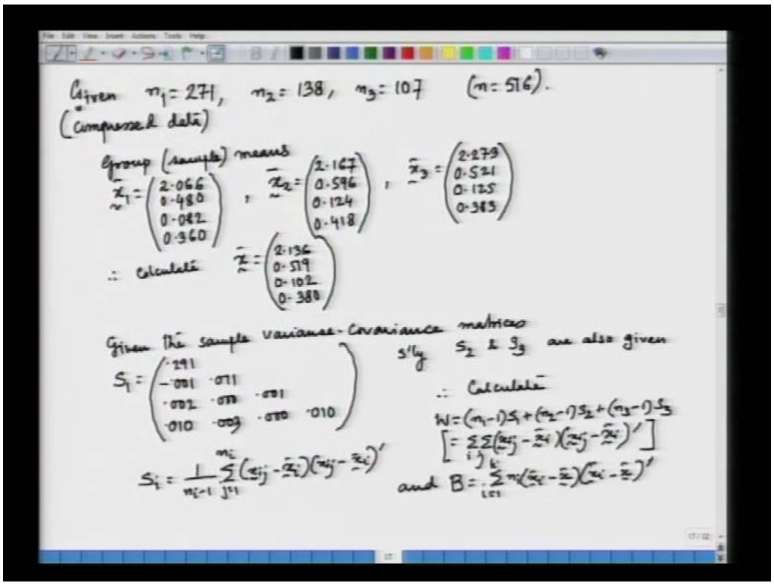

And some more information are given to me, I have given that n 1 that is under the first type of ownership I have 271 observations, and the second one I have 138 and the third one I have 107 observations giving me n equal to 516. So, I have this is the data from, number of data points from each category of ownership; now we have the, so basically what is given is not the data in totality, we have here the compressed data.

Some compressed data, which is obliviously good enough for me to do the analysis, this is common in situation where the dimensionality is high, we need not write the or take help of the whole data at every point, at every level of calculations. Some compressed information is good enough for my analysis, sometimes this will become mandatory, if say the data are little sensitive, say I have included one variable which is the little bit sensitive, which includes some some such information which are not to be disclosed at the vary ground level.

So, what I have is the is compressed data of this type, so I need to know the group means. So, the group sample means are given to me, group sample means I have to have these obliviously without which I cannot proceed further. So, the first one x 1 bar is given by 2.066, then I have 0.480 four-dimensional, let me make it, then I have 0.082

and 0.360. So, basically here, I am getting the first one say 2.066 this is giving me the sample mean, under all cost the first cost x 1 in the first coming coming in the first group, that is the first group is probably the private ones. Second is second element is that of the same group this is x 1 bar, but this is pertaining to the second type of cost. So, similarly, I have the for the second type of ownership, I have x 2 bar and these values for the four variables for the four cost variables are 2.167, 0.596, 0.124 and 0.418.

And then I have the third group sample means, these are 2.273 then I have 521 125 and 383, since I know the n i's and I also know the x i bars, I can calculate even if this is not given therefore, calculate the sample mean the overall sample mean x bar has 2.136 and then you have 519 102 and 380. I also have the given the sample variance covariance matrices. So, again instead of the individual data observation, I have been given some compressed information, the sample sizes in each category, the group sample means and then I am I have been given the sample variance, covariance matrixes.

So, the first one, I am using the notation S 1 for this is a 4 by 4 symmetric matrix with elements like 0.291, then I have minus 0.001, then next one is 0.001 0.002, this is 0 being kept three places of decimal 001, and then I have 0, this is 0 1 0 followed by 003 and the last element is 0.010. And similarly, I have S 2 and S 3, these are also given otherwise, again I cannot proceed, so I have been given this three sample variance, covariance matrices, I can see see very I know that S I, if you say it for the i th group then this is S i is nothing but, 1 by n i minus 1.

And these is how, I defined the sample variance covariance matrix, so x i j minus x i bar prime sum over j from 1 to n i. So, what I have mean after this just as after these group means, I have calculated the overall sample mean similarly, I need to calculate my within matrix for which I need the each of these individual sample covariance matrices. So, that I am able to calculate the within matrix, within variability matrix which we denoted by W.

So, what is that it is not nothing but, therefore, calculate again we can calculate this matrix W, I am interested in W and B, because these are coming these two are the matrices, where my interest lie because, these two are coming in that lightly wood ratio criterion lambda star. So, basically I have to arrive at these 2 B I can see, I will get it with these means information for W, I have to calculate it with the help of sample variance, covariance matrices and this is nothing but, n minus 1 S 1 plus n 2 minus 1 S 2 and n 3 minus 1 S 3. So, this is since, well you know that this is nothing but, the x i j minus x i bar x i j minus x i bar transpose this matrix. So, this is equal to this and with each of the S i's having been defined in this way (Refer Slide Time: 11:56).

W matrix is nothing but, this matrix which we can be calculated with the help of the n i's and S i's and what about B, well B is also it can be calculated with whatever information I have, there with the compressed data this is nothing but, n i times x i minus x bar x i bar minus x bar transpose, i goes from 1 to k in this case it is 3.

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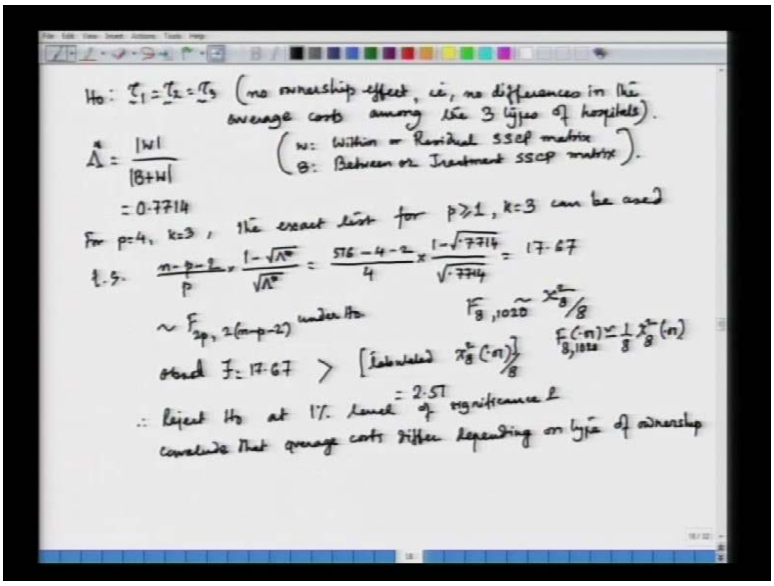

So, now we test the hypothesis, we are interested in the hypothesis that tau 1 is equal to tau 2 is equal to tau 3, inverse I say that there is no ownership effect, that is no differences in the average costs among the three types of hospitals. So, my assumption is this whether it is a private hospital or a non-profit organization or a government hospital, there is basically no difference as far as average cost, pertaining to those four factors are considered.

So, that is what my null hypothesis is, that there is no ownership effect and on these cost which I have considered, among the 3 types of hospitals right. I know that the test statistic criterion etcetera, for this is the Wilks's lambda, if you recall is given by determinant of W by determinant of B plus W. Now, in the manova (()) again W is nothing but, the within or residual sum of squares cross cross product matrix, and B is the between or the treatment sum of squares and cross product matrix. So, if you wish you can this keep it here that, W is the within or residual sum of squares cross product matrix, and B is the between or the treatment in this case is ownership factor, the treatment sum of square cross product matrix. So, this is the Wilks's lambda criterion given by determinant of W by determinant of B plus W and since, W and B they they depend only on the data, I have all these values to me, I have the matrix with me and this value comes out to be 0.7714, I have to all I have to do is calculate the determinant of to 4 by 4 matrices.

Now, here for p equal 2 this is for p equals 2, because we have not, p is p is the data dimensionality is 4 and k the number of groups is 3, can we use in a exact test yes, we can the exact test for we have listed the situations, where we can use exact test. And we can use the exact test for the situation, where I have p is greater than equal to 1, because I have p equals 4 and k equals 3, this case can be used and what is this test statistics for that.

So, can be used and the test statistic here is n minus p minus 2 by p this with 1 root lambda star by root lambda star, and this gives me, I have n has total number of observations that is 516 minus the dimension minus 2 by the dimension, and then I have I have calculated the Wilks's lambda this is root of 0.7714 divided by this giving me the value 17.67. That is the observed value of the test statistic, I have from the data and this test statistic is following and F distribution which degrees of freedom twice p twice n minus p minus 2 under H naught.

So, this has to be compare with I have to be compare this with F 2 p is 4, and then the second degrees of freedom times out to be quite high, because n is 16 516, so this is 1020. Now, you will hardly have any table where this has been tabulated, what we do is we use that this is x approximated by the chi square distribution, central chi square distribution with 4, not 4 we have two times p, so this I have 8 F 8 the first one. So, this is chi square central chi square with 8 degrees of freedom, but divided by that 2 p, so this is getting divided by 8.

So, this observed the tabulated value, observed F is 17.67 and the tabulated chi square 8 at 01 same. So, this I will have this F 8 1020 at 01 this will be approximately equal to 1 by 8 chi square 8 at the same level. So, this tabulated value is equal to given by, the whole thing is given by divided by this is given by 22.51, so I am calculating I am comparing 17.67 with tabulated this by 8. And this is obliviously, greater than or equal greater than this 2.51, telling me that I can reject therefore, reject H 0 at 1 percent level of significance. And conclude that, average costs at least cost is that we have considered the four types of cost, they do different average costs differ, according depending on type of ownership; which is somewhat logical depending on type of ownership.

So, you may say that, why are we insisting on there are exact test, because here n is sufficiently large, n is equal to 516, so obviously, we can use the plane Wilks's criterion or with the Bartlett's correction. So, let us also do that, so that we have the plane directly we are going to the chi square test, without going to the chi square via the F test.

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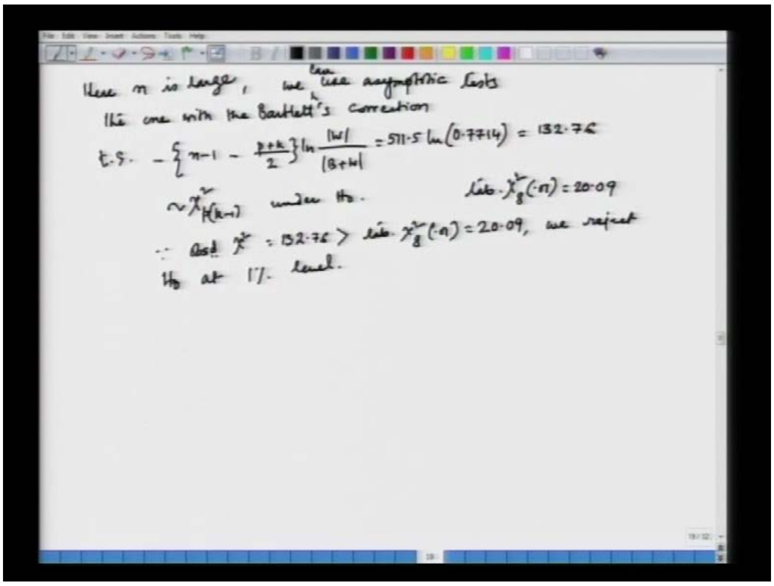

So, let us check what happens then, let us calculate a fresh the test statistics, now since here n is large, so we can simply use asymptotic test (No audio from: 20:44 to 20:53) and let us say the one with the Bartlett's correction, the test statistic is nothing but, we have minus of n minus 1 and then, the factor that come is p plus k minus of p plus k by 2 and then it is log of determinant W by determinant B plus W. And in this situation, this is the constant term is 511.5 with log of you already know that factor is 0.7714, and this is having the value 132.76.

Now, this chi square test, this test statistic follows a chi asymptotically follows the chi square distribution, with p times k minus 1 degrees of freedom under H naught and we have to see that tabulated chi square p is 4 k minus 1 is 2. So, chi square 8 at 0.01 and this is nothing but, 20.09. So, we have already use this value in the earlier satiation, and since observed chi square that is 132.76 is greater than the tabulated value, tabulated chi square 8 at 0.01 that is equal to 20.09, the asymptotic test also reject, we reject H naught at 1 percent level.

So, we say that is, this is not a big deal we though, we had some information about the exact test, so we have we found the situation which matches with the situation that we had. So, where we can take help of the fab that p is greater than equal to 1 and k is exactly equal to 3. So, we took the help of that, to go to the exact test we came to the f statistic, but there also we use some approximation, because the second degrees of freedom was too high.

So, that was again approximated by the chi square, and we reached are the conclusion of rejection of H naught. Now, without going that way, without going through the exact test, we came to our approximate test, because n is clearly large here, we went by the Bartlett's correction and saw that in this way also, if we use the asymptotic test also the hypothesis is rejected. And there is definitely ownership effect on this average costs.

Now, we had mentioned that, we have data which have been separated according to two factors, that is ownership as well as certification. So, though we had data we could not use this information at that level, because we only knew how to handle one way manova. I said that if somebody is interested in knowing that, what is the what is the effect of ownership and certification and obliviously an interact action effect of the two factors, then, we have to go to the two way manova.

So, let us go into the theoretical aspects of two way manova, and then with the help of the data that is the extra data that we need here, the data separated due to the certification of the different hospitals. If we have that information also, we can complete the two way manova analysis also.

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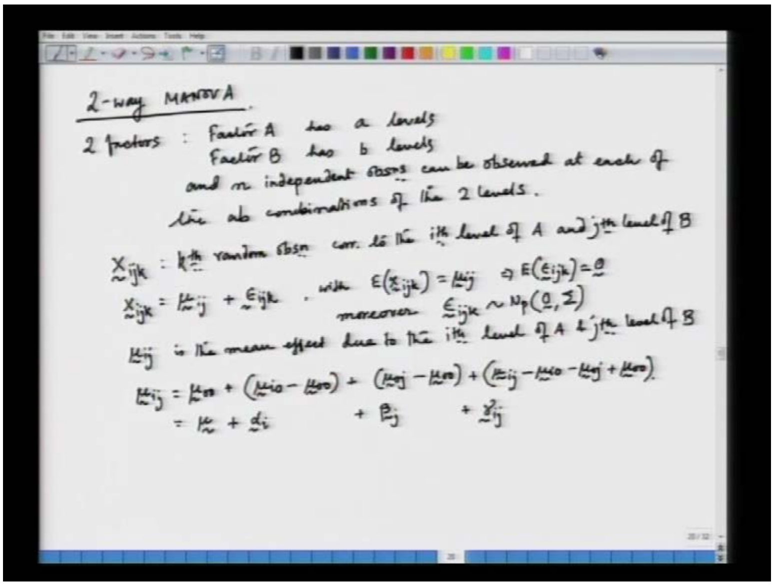

So, let us go to two way manova model right here, so now, we know that here we must have two factors right, so a factor A, a factor ones, a factor A with has a level, factor B has B levels say. So, if we try to do draw parallel with the with our with our real life data example, what you can say is say factor A is the ownership factor, then it has how many levels well a is equal to 3 here, because we talked of three types of ownerships. Factor B say is the certification factor, this also has three levels this is not necessary that A and B can take different values.

But, this is coincidentally this is B is also equal to 3 here, because we have talked about three types of certifications, but one thing here we consider is that, we have N independent at least initially, we have N independent observations, can be observed or can be can be collected or can be monitored at each of the (()). So, it is a two way classification and there is a b such types of classes.

So, I have at each of the a b combinations (No audio from 26:32 to 26:40) (Refer Slide Time: 26:32) of the two levels a b combinations of the two levels right. So, my data vector x i j and k I need a third subscript, and this is nothing but, the k th random vector or random observation corresponding to the i th level of factor A, i th level of, now I will simply write i th level of A and j th level of B. So, that is the situation I have and that is why they are three subscripts i j and k, the k th random observation corresponding to the i th level of A j th level of B.

So, I know that I goes from 1 to A, because I have said that there are A levels of factor A, then j goes from 1 to B, because there are B levels of the second factor B, and k goes from 1 to n, because for each of the a b combinations I have n independent observations. Again I assume a very simple model for the manova, this analysis and I have x i j k this is mu i j plus the, this is the mean effect part and the error part right.

So, with expectation of x i j k is mu i j and hence implying, I have this error having mean vector is the null vector more over for the inference part. I have some distributional assumption; I use the assumption that this is following a p variate, normal distribution with mean as the null vector, and the variance covariance matrices sigma.

So, mu i j is, let us concentrate on this part and try to split it up into the different factors, so this mu i j is the mean effect due to the i th level of A, and j th level of B. And let us see how we handle this, so as before I bring in and overall effect or the grand mean part, that is mu and denote it by mu null null. And then we have, this is how the adjustment goes, I have mu i naught minus this grand mean effect, then I again put a mu naught j again with the minus of overall effect.

Now, since I have already brought these factors, what is the adjustment that I have to do finally, I have to consider a mu i j here, then I have to take out a mu i naught I, have to take out mu naught j and since, mu the overall effect is has come twice with the negative sign, so I must put a plus mu naught naught here. I use some special notations for this mu naught naught is replaced by the usual mu, as in the one way manova case, the overall have a grand mean effect.

Now, there we had only mu i minus mu, which was the tau I, here it is mu i naught minus mu naught naught, and this is say now alpha i. And for this I have beta j, and this is giving me the, let us use some another notation gamma and depending on both i j (Refer Slide Time: 31:02). And this, basically this gamma i j part has the interpretation of the interact action effect of A and B, factor A and factor B.

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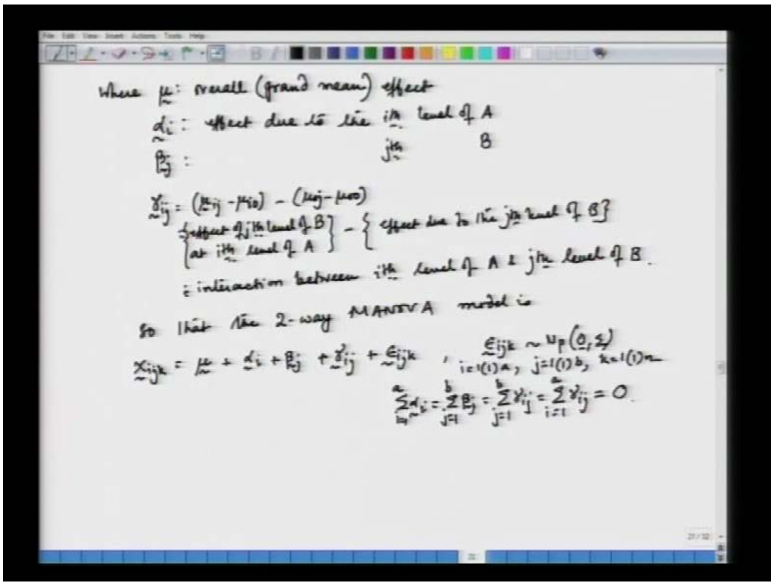

So, let us just describe the all these term that parameters that we have introduced, so where mu as before is the overall or grand mean effect, then I have alpha i which is effect due to the i th level of A beta j is the effect due to the j th level of d and gamma i j gamma i j is nothing but, if you have a look at it is mu i j minus mu i naught.

Let us go back it and see, what is this is gamma i j mu i j minus mu i j minus mu i naught minus mu naught j plus this. So, this we have already introduced as the effect due to the j th level of B and what is the rest of it well this is nothing but, so I have this, let us also write it in the parameter form, it will be easier for us this is mu i naught minus, this comes in the subscript, so mu i naught minus mu naught naught.

And then it is mu naught j minus mu naught naught is the is this sorry this is going to be mu i j, the first part this is not alpha i, what I have written here is alpha i this is not the first part of gamma i j, it is mu i j minus mu i naught and then it is mu naught j minus mu naught naught that is ok.

So, this is here you see here i is fixed and I am taking out this mean effect from the j th, from this mu i j, so basically it is what remains is the effect of j th level of B, but not overall at for a fixed i. So, this is at i th level of A, i th level of (Refer Slide Time: 33:48) and then this is nothing but, the effect due to the j th level of B; and this is essentially we denote this as the interaction effect between i th level of A and j th level of B.

Somehow I can write, so that the two way manova model is you have x i j k, again the k th observation due to i th level of A, j th level of B this is equal to mu plus alpha i plus beta j plus gamma i j and finally, the error term E i j k with the very important assumption, that this is a multivariate normal. And you have i from 1 to a, j from 1 to b, k from 1 to n and what is more we have also those constraints, because we have defined this alpha i is nothing but, if you recall it is mu i naught minus mu naught naught.

Similarly, for beta j and gamma i j also we had something in the background, so all those factors are giving me some constraints, which are just like we had in the case of the one way manova here, we will have more such constraints. So, these are sum of alpha i over i from 1 to a this sum is equal to sum of the beta j sum over j from 1 to b, this is also equal to sum of gamma i j keep i fixed take sum over j from 1 to b or consider the other type of sum gamma i j by the definition, if you check this very easy to do.

So, you take the summation keep j fixed, and consider the sum over i from 1 to a all of these are equal to 0. So, this completely describes my two way manova model, and then what we do is we splits the data to get the sum of squares and cross product matrices.

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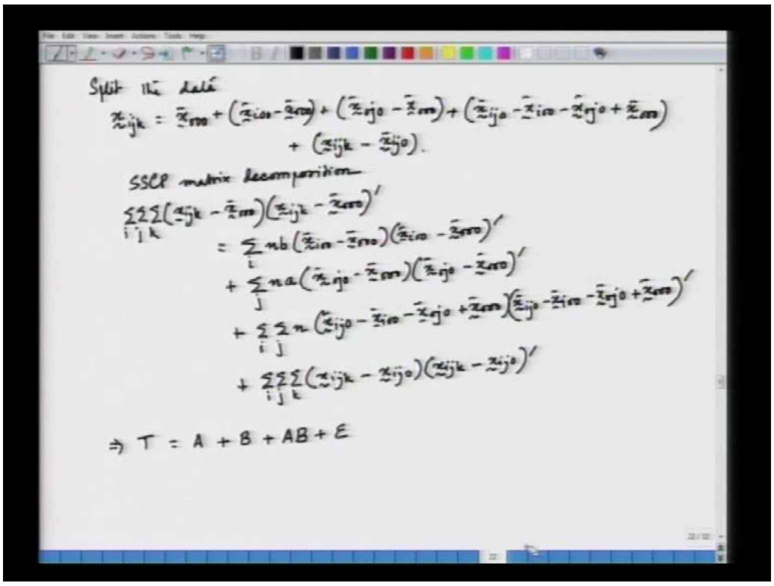

So, this is split the data from the random vector, we come to the data, so this is small x i j k equivalently, we write it as x naught naught bar, then x i naught bar minus the overall sample mean, we have we have three subscripts here. So, this is going to be x i naught naught, here also 3, and then we have x naught j naught bar minus this, and then we have we have to do some adjustments here, we have to write x i j naught bringing in this terms (Refer Slide Time: 37:48). And then because, we have already defined what our alpha i beta j and gamma i j are we also have a clue has to how to get these terms here, because these are essentially the estimates of those parameters.

So, gamma i j naught and then gamma i naught naught bar minus, then gamma naught j naught bar and I also put the overall sample mean, so that what remains is x i j k minus x i j naught. So, we proceed with calculation of the sum of squares cross product matrices and, so sum of squares cross product matrix decomposition, we start as in the one way case, we start with that within sorry the total variability of the data and that is now, we have sum over 3 such things x i j k minus the total variability.

So, I have the overall mean and x i j k minus x naught naught transpose, this is equal to the first curve, for the first one, let say I am keeping sum over i. So, the other two factors are coming n for k and b for j and what I have is x i naught 0. So, I have taken already summations over j and k, so these j and k have become have vanished now, and I have x i naught naught bar x overall and then the transpose of this.

Next I am considering leaving sum over j, which means I have taken care of sum over i and k. So, I have n and a here and then I have vectors x naught j naught and the triple zeros are coming this one. At the next step I consider sum over k, so that i and j are remaining here, and then I have for n and then x i j naught minus x i naught. So, x naught j naught, and the same terms let us write it x i j 0 naught minus x i naught naught minus x naught j naught plus the overall sample mean with this.

And finally, I have to have a triple summation; because I am what I am trying to do is trying to write it in these four factors. So, these have been deliberately, it has been split up into this form, because I already have an idea about the sample estimates of the parameters. And after bringing adding and subtracting all those extra factor, what remains in the end is x i j k minus x i j naught with x i j k minus x i j naught.

So, this splitting is, giving me let us denote these matrices by letters, so this is total the sum of squares total sum of squares total, sum of squares and cross product total matrix. So, this by T, then the first one in the right hand side by A for the factor A, second one is B for factor B and the third one this is the interaction effects.

So, let us write A B for this under last one is for the residual, so let us write E for this. And then we come from here, we can form the two way manova table to complete our analysis.

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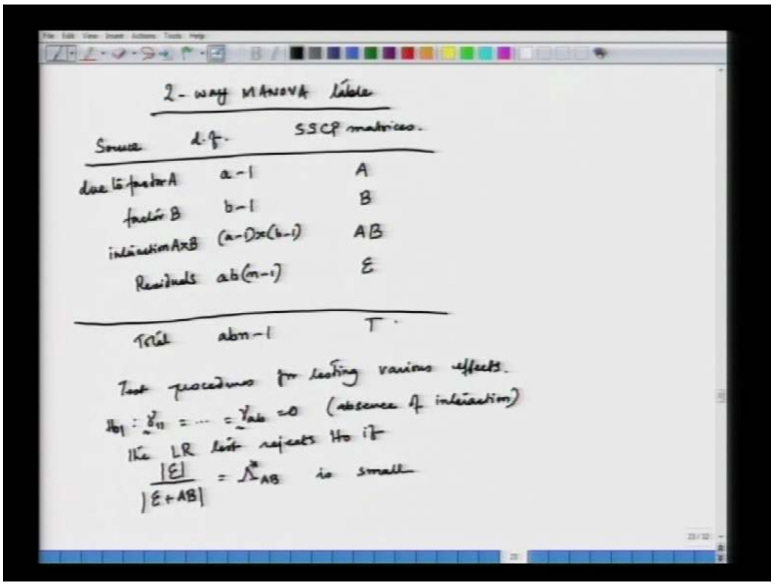

So, each of these matrices can be calculated without any problem, and hence we have the two way manova table, first is of course, the source of variation, then the degrees of freedom then the sum of squares cross product matrices. And next we will talk about the hypothesis test statistics and test procedures. So, first source is a due to factor A source of variation due to factor A, then factor B then the interaction of interaction A B, then we have the residuals, and all these or adding up to the total variability present in the data.

The corrected total the degrees of freedom, because we have the first constraints that sum of the alpha i that is equal to 0. So, I have loss of 1 degrees of freedom here giving me a minus 1 similarly, for factor B I have one loss of 1 degrees of freedom. So, this interaction has degrees of freedom a minus 1 product of these 2 degrees of freedom total I know has to be a b n minus 1 and hence the residual degrees of freedom is a b times n minus 1.

A sum of square cross product matrices are a by my notation, this is B this is A B, the residuals is E and then this is T, I make a test the test procedures for testing various effects, now the first hypothesis here, the first null hypothesis here quite obliviously quite logically is the, that we have the equality of the interaction effects and this is equal to 0 (Refer Slide Time: 44:37). So, this if that is equal to 0, this is accepted then we will move on to the next next group of hypothesis, where we can test for the equality of means of the two factors separately. So, at the first stage, we test that gamma 1 1, in this way I go to the last one that is gamma a b that is the last last of factor A, and last of factor B this these are all 0, this is absence of interaction of the two factors.

And for this the L R test rejects H naught, if this well I have the residual part that is the within matrix for me not using the bracket here, so let us just leave it at, E this is the test statistic here. And the between matrix here this role is getting played by the, because I am testing for the interaction effect, the within is always the residuals. So, sum of squares and cross product matrix residuals, but the between role is been taken over by the A B matrix.

So, this is getting divided by, so basically I have to write put a determine sign actually, and then we have E plus A B determinant of this, and this is let us denote this by lambda A B star this is small.

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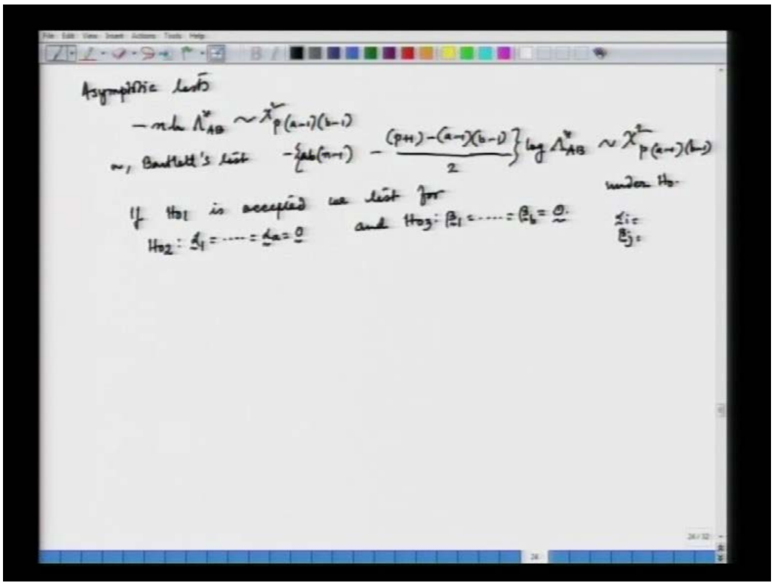

The asymptotic test can be we have minus n of log lambda A B star this converges to the chi square with p and corresponding degrees of freedom, that is a minus 1 and b minus 1 or the Bartlett's test, where I have a slight modification here. This is A B n minus 1 which is now the total number of observations, and then I have it is A B n minus 1 the total n minus 1. So, that is the degrees of that is the degrees of freedom for the total sum of square total, and then we have minus of p plus 1 a minus 1 b minus 1 this by 2. So, this is slightly different from what we have, in the one way analysis case the correction factor here is in fact, a b n minus 1. And this is with the log of lambda A B star this also asymptotically follows a central chi square with p a minus 1 b minus 1 under H naught.

So, we are going to find the value of this since, we know the matrices there is no problem in calculating this Wilks's lambda criterion, and then we go for the next level of test that is if H naught 1 is accepted, we all know the test procedures. Now, if this is accepted that is there is, in fact no interaction we test for H naught 2, that is the equality of the different levels of factor a and these are now tested separately beta 1, 2 to beta b.

Now in fact, I can say that this are equal and also equal to the null vector, because what we are essentially testing is mu i minus if you if you recall that how are these alpha i's and beta j's are defined right. And the constraints that we have, if these are all equal they must be equal to 0 similarly, here also we can mention this in our one way Anova as well. Now, if we have not mentioned it already there in the in the hypothesis part wherever we are writing the hypothesis, because we have the constraint it always enables me to write that this is equal to 0.

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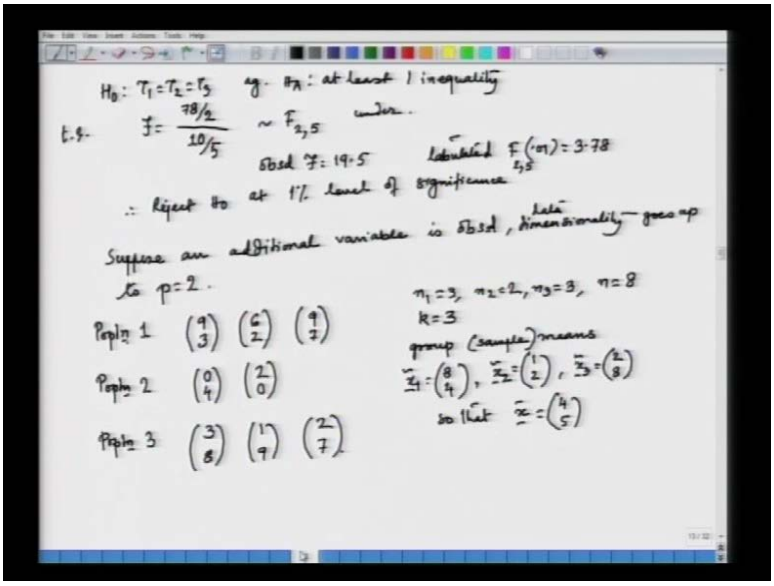

So, let us just go back a few pages, and this is for the manova, this is for the Anova case I have the constraint this right.

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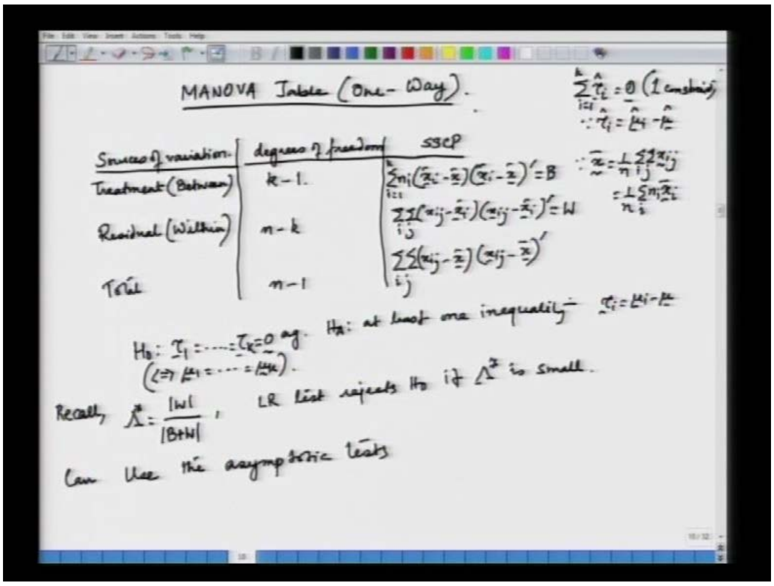

Here, we can mention these are for the equalities we are obviously, not going to write equal to the null vector, but since we have come to the constraint that, some of these are equal to the null vector we can always write this as 0. And at least 1 inequality, so this can be actually corrected by writing at least 1 is or inequality that is that is fine at least 1 inequality, not a problem right that is all.

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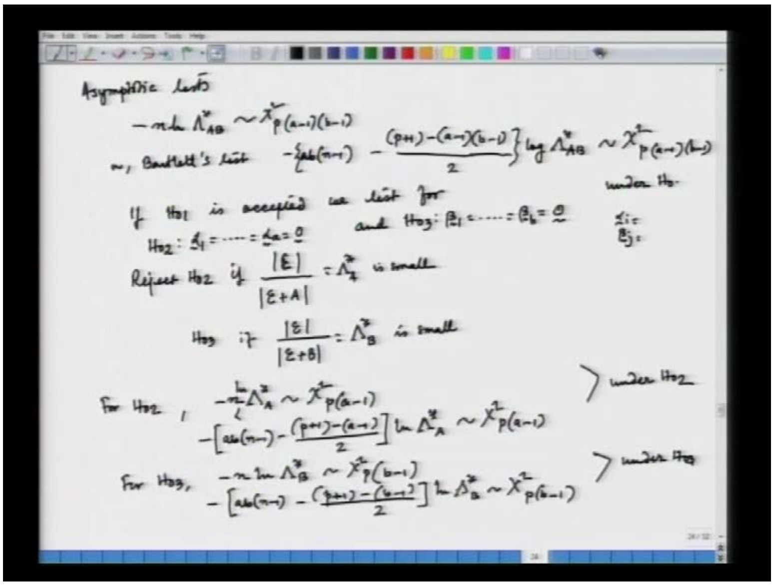

So, let us go to our last part, wherever we had stopped, because of the constraints we can write this and then for testing this hypothesis, the second hypothesis after the acceptance well, it needs to be mentioned here, if the third one if the first one is rejected, that there is some interaction effect, then there is no point and testing that these effects, are there is no effect and these are all equal to the null null vector. So, if that is accepted if H naught 1 is accepted, then when only we go for H naught 2 obviously, this has no bearing with the other group that is even if this is H naught 2 is accepted or rejected we will go and check for H naught 3.

So, this for the first part that is for H naught 2 we reject, so we can write this as reject H naught 2, if determinant of E the within matrix, and now the between matrix is nothing but, the A matrix this we denote by lambda A star is small and reject H naught 3, if determinant of E by determinant of E plus B that is lambda B star is small.

For the second one, that is for H naught 2, I can use that n lambda A star this follows asymptotically follows a central chi square with p times A minus 1 we have we have p sorry, a levels of the factor a or with Bartlett's correction we have A B n minus 1 plus p plus sorry this is minus, sign common minus here p plus 1 minus a minus 1, the correction factor by 2 this with log of (()) a log here minus n log lambda star A log of lambda A star, this follows the same chi square with the a minus 1 under H naught 2 (Refer Slide Time: 53:44).

And for H naught 3 the third one, I have minus n log lambda B star following chi square p b minus 1 all with the Bartlett's correction I can use a b and minus 1 minus p plus 1 with b minus 1 by 2, this with log lambda B star this follows a chi square p with b minus 1 under H naught 3.

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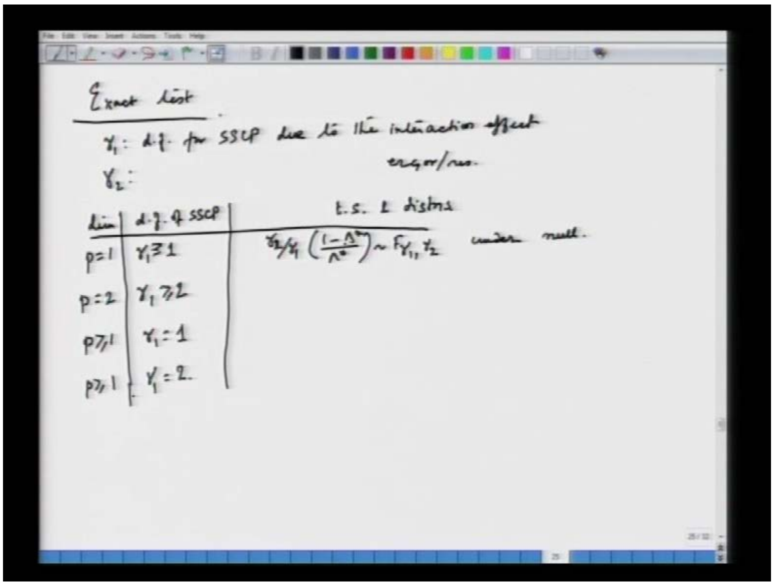

So, I can use I can find the values and immediately get the test procedures, I get my decisions, and next what I do is I list down the exact test situations that are available. So, let us say that the degrees of freedom say, let us define this as the degrees of freedom for the sum of squares cross product matrix, due to the interaction effect. And let gamma 2 be the error effect, on the residual effect and under the respective hypothesis, if we have situations like the dimension or the number of variables, the degrees of freedom of respective sum of squares cross product. Then the test statistic and their distribution will quickly write down this is p equals 1, p equals 2 greater than equal to 1 and greater than equal to 1 also. So, the situation here is gamma 1 is greater than or equal to 1, gamma 2 can be anything, gamma 1 is greater than equal to 2, gamma 1 is equal to 1 and gamma 2 is equal to 2. So, for any value of the other or this is also gamma 1; so this is for any value of gamma 2, and the test statistics are gamma 2 by gamma 1 1 minus lambda star by lambda star of F gamma 1 gamma 2 under the null hypothesis.

So, we are considering respective hypothesis null hypothesis and accordingly, we are forming the test statistics. And for the next one is what we have is again this note that this, lambdas are going to be respectively a b or a or b according to the null hypothesis. So, we are going to continue complete this exact test table; and if possible we will go to a data example or we will continue with the new topic hence.