Applied Multivariate Analysis

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Lecture No. #19

Manova – II

We continue our discussion, with manova, the multi-dimensional analog of anova. Anova as we all know is the practice of partitioning the total variability in in the data into different components, which are due to the sources of variation. So, we have already seen, that this is done with the help of a simple model, as part of the estimation procedure is considered, and then for the testing purposes, we need some distributional assumptions as well.

So if you recall, we had started, we had just started the discussion with the one way manova model; one way in the sense, that there is one only one factor. So, we are going to split the variability into this factor, and obviously the part that comes along with it is the residual part. So, together we we we are going to have two sources of variation in a one way manova model - one is the soul of factor, which is present, and the other is due to the residual.

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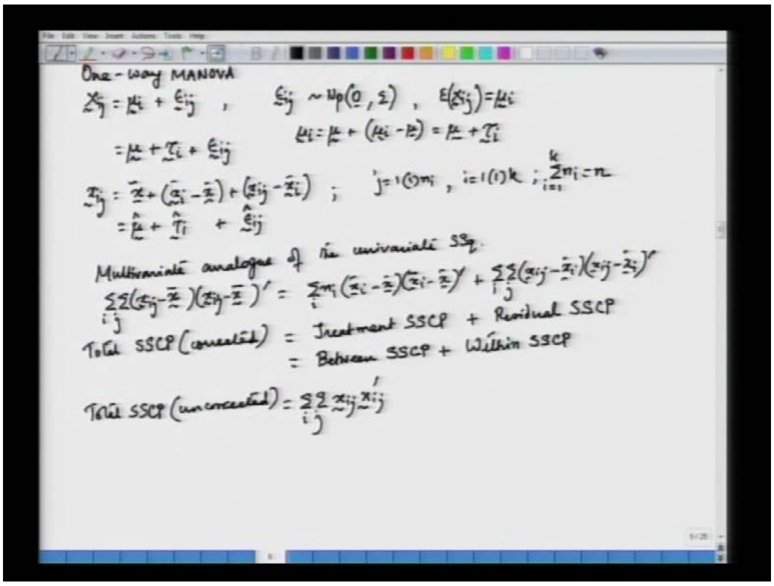

So, let us just recall the model once again, we had taken that x i j, that is the j th observation pertaining to the i th group or population is basically made up of its mean effect mu I, which is the mean effect in the i th group, and then the error present in the that is the j th error in the i th group. And for the distributional assumption part, we had assume that these errors are nothing, but p dimensional normal with mean 0 vector and variance covariance matrix sigma.

Now, this mean is obvious from the fact, that we have expectation of x i j is nothing, but the mu i, which is the mean effect; Now this mean effect was further broken up into the grand mean or the overall effect, and then the rest of it, the residue which remains is the mu i minus the overall effect, and this part in fact, was considered as the treatment effect or the factor effect, the only factor which is present now. So, we denote we have denoted by tau i.

So that the model, the one way manova. Let us put the headings, the one way manova that we are considered, and this is nothing, but the mean effect or the grand mean effect in fact, or the overall effect, then the treatment effect, and obviously, the error part. With the error part following, we have the assumption on this part, and hence an assumption on the random observation x i js also.

Now we had also seen, that if we can split a data, let us take a particular data vector x i j, and then this is analogously splitted up as, this is the x bar, the overall sample mean. And then, we have the group mean with the difference with the sample mean, and then the rest of it, which is x i j minus the group means x i bar. So, this gives me, an estimate of the overall effect. This giving me, the estimate of the treatment effect T i, and the last part is giving me the residual, and we are denoting this by E i j hat.

So next, our next task is to get the sum of squares due to the different sources. So, we are considering the multivariate analog of the univariate sum of squares. Let us see, how we do that. So, here we will have, not just sum of squares, but sum of squares as well as cross product matrices instead of scalar quantities, which are simply sum of squares. We are going to have matrices, and which will pertaining to the sum of squares as well as the sum of cross products.

So let us see, first we consider what we have been what we have been calling as the within matrix, within essentially means, this within comes from the fact, that this is representing the within population or within group variability. So, that we have x i j minus x x i bar, the group means, and then the transpose over this, sums are over. We may mention here, that within a group, say for the i th group, j is going from 1 to n i, which means that, it does not have to be a balanced design all the time, we need not have a constant n for all the group, we can have for the i th population or i th group or sample size of n i. And then, we have i from 1 to k, meaning that we have k populations, and for simplicity sake, we were denoting that, the total number of samples, that is when each of these n i es are sum over the k groups, we have a total of n samples.

So this can very easily, be seen as a sum of terms like n i with x i bar minus the overall mean x i bar minus the overall mean transpose sum over I, and then the other part. Now this is, in fact the corrected sum of squares cross products. So, in fact, what we are considering here, is the observation minus the sample means, the overall sample means. So what we were considering here, is the double summation over x i j, the observations minus the sample, the overall sample mean, and the transpose of this, which can be shown to be a sum of the matrices like n i summation of n i x i minus the group mean minus the overall mean, and what remains is the observation x i j minus the group means, a term like this.

Now, this can be easily established, by adding and subtracting this group mean x i bar from the in this expression, and it can be easily separated. So the term that we have in the left hand side is called the total sum of squares cross product corrected, and this is now equal to the treatment sum of squares cross product plus the residual sum of squares and cross product. Now, this treatment is essentially in the anova manova purlins. Otherwise, what we had seen, that this is nothing, but our between sum of squares cross product, which means the variability between the different groups and or populations, and residual is actually the within sum of squares cross products. That is, within a particular population. And this is of the corrected total sum of squares cross product, if we consider the uncorrected total sum of squares cross product, it means that we are not going to consider the deviation from the overall mean at it is. So, we can write it here, that the total sum of squares cross product uncorrected is nothing, but the sum of squares of the observations, which x ij x ij transpose sum over both i and j.

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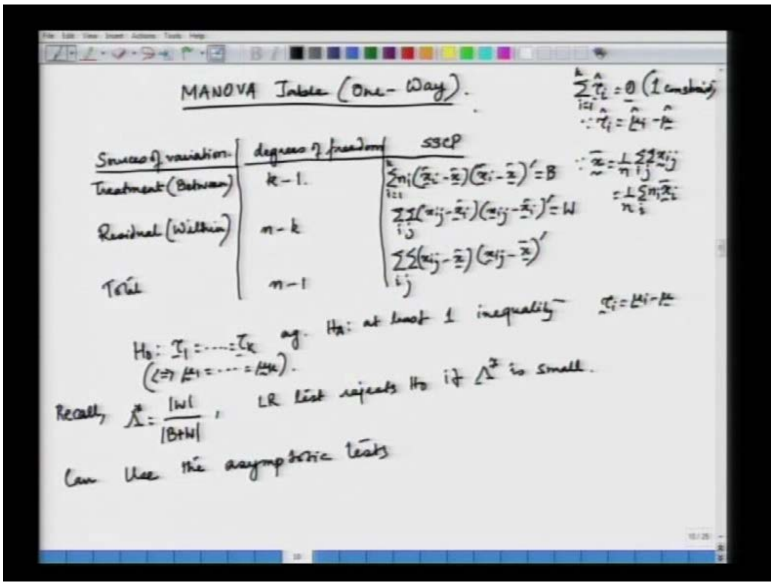

So with this, we can now form our manova table, and see the table is going to include. So this is the manova table, for the one way analysis. And we are going to have the first column is the sources of variation, that we had mentioned in the in the beginning, that we are partitioning the variability into the different sources of variation.

Then we have, what is known as degrees of freedom, and next we have the sum of squares cross product. So, the first one is, due to the factor, here we are calling it as treatment. So, that is the treatment, no point in saying treatment 1, 2, because we have just one factor, and this is actually the between sum of squares cross product. And then, we have the total, total by default is going to mean the corrected total, and the rest of it is then automatically residual, which is the within sum of squares cross product.

Degrees of freedom is for the treatment, we have how many populations? We have k groups, and minus 1. Now this minus 1, is coming from the, we are losing one degrees of freedom, because of the one constraint. Recall, what is the constraint, that tau i hat i from 1 to k. This has got to be null, since our tau i hats are nothing, but mu i hat minus mu hat.

So, this constraint, this one constraint is resulting in the loss of one degrees of freedom, and the total degrees of freedom, next we will look at it, is n minus 1. Total is, total sample size from n, and here the loss of one degrees of freedom is essentially, because of the taking of deviation. So, that we have, since we have a constraint like since x bar is nothing, but 1 by n. Some of the observations i j, or the weighted means of the observations you have, is 1 by n x i bar.

So this factor, coming into the into the factor of total, that is corrected sum of squares cross product, we are losing one degrees of freedom here, and then the rest can be calculated easily, and it is n minus k degrees of freedom for the residual. For the sum of squares cross product term, we have, we know, what we have is n i x i bar minus x bar x i bar minus x bar transpose, we denote this by the B matrix, the total is will be not going to use it directly, but any way we are going to write B plus W, for the total matrix, and this is nothing, but the corrected sum of squares cross product total, the corrected one. And the residual is x i j minus x i bar x i j minus x i bar transpose i and j, and this is W matrix.

So, the total is nothing, but the B plus W matrix. So, this is our anova table, and we are going to test the hypothesis, that the null hypothesis, that we test here. That the mean effects due to the treatment are same, there is no treatment effect as such. So, these are all equal, against the alternative at least one inequality is there, and recall.

Now, this is nothing, but if you see that, this is if you recall, what our tau I hat are? In fact, tau i is that, the parameters. These are mu i minus mu bar. So, this is basically, equivalent to testing the k population means, which are mu 1 to mu k. And we have already done, the ground work for this, if you recall, we have tested. We have already seen, how to test k population means, when we are considering the normal populations. So this is exactly, what is happening here. By our distributional assumption, we have normal population, and we are testing the equality of k population means, but now in terms of the treatment factors.

So, the test procedure remains essentially the same, if you recall our wilks lambda criterion, it was lambda star. And it was given by. So, recall that our wilks lambda, was nothing, but determinant of W by determinant of B plus W. And the test procedure was, that this rejects H naught, if the L R test, that this is now, this is the basis, this is coming from our L R test. So, we can write the L R test rejects H naught, if lambda star is small.

So this is essentially the exact test, which we most of the time try to manage with the asymptotic test, if you recall, because we have a very nice result of the convergence of this criterion to the chi square distributions. So what we had was, that we know the asymptotic test. So let us just write, we can use can use the asymptotic test. Let us say test, because we have one as it is the lambda test, and another was the bartlett's test.

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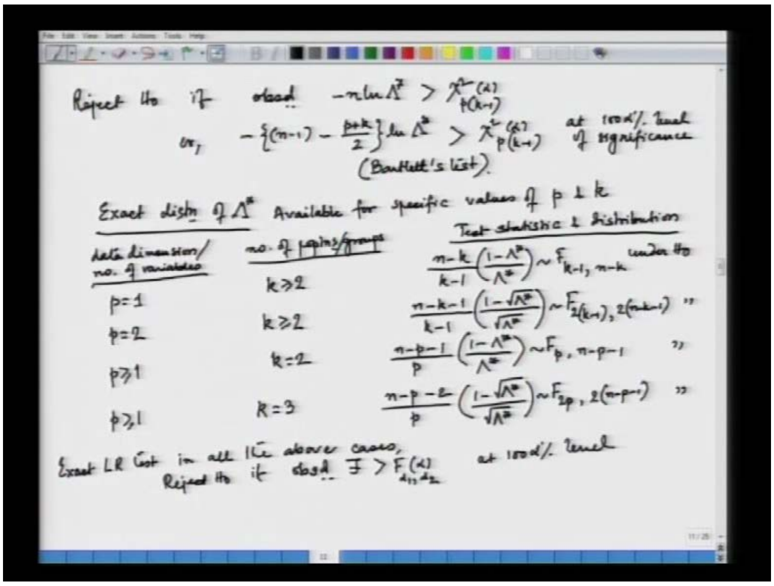

So, the tests are given by, reject the null hypothesis, if observed not the lambda star, but a slight variation, that is n log lambda star following sorry this is under the distributional assumption, and so we are giving the test criteria. So, this is greater than, the observed one is greater than chi square p k minus 1, this was the degrees of freedom. And the bartlett's test or you can write the bartlett's test has slightly different constant term, that is n minus 1 minus p plus k by 2, and then we have log of lambda star, this greater than p k minus 1. Well, you may also write that at alpha level, so this is at 100 alpha percent level of significance. This is the upper alpha cut off point of the distribution with the relevant relevant degrees of freedom. This is the the second one is actually the bartlett's test. It is better to use, because it gives faster convergence. So, it is better approximation to the asymptotic distribution that is the chi square distribution.

Now, we will list down the exact distribution in some of the cases. So, we have the exact distribution of the wilks lambda available for some of the cases, available for specific values of p and k, specific values of p, that is the dimension, and k that is the number of groups or population. So, these are. So, first it is the data dimension or the number of variables. Very plainly speaking, that we are considering, and then we have the number of populations or groups, whatever you call it, and then the test statistic and distribution.

Now, these are available again for very specific values, very small values of p and k, that is low data dimensionality, and fewer population groups. So, the first one is for p equals 1, and k is greater than equal to 2. If this is the situation, well we are talking of the usual anova, then because p equal to 1 means, we do not have a multidimensional data, what we essentially have is uni dimensional data. And then, the test statistic is n minus k, note that n is nothing, but the total sample size, which is sum over n i es, and this is divided by k minus 1, the product with 1 minus lambda star by lambda star, so obviously, since the rejection rule is for small lambda, here it will be for large lambda, and this follows f k minus 1 n minus k under H naught.

The second one is for two dimensional data. So, that when we have p equal to 2, when we actually have the manova, and then the same k for k greater than equal to 2, that is two or more groups, then the test statistic is in the form of n minus k minus 1 by k minus 1, the product with 1 minus root of lambda star by root of lambda star. And this follows an F distribution, with twice k minus 1 twice n minus k minus 1 under H naught is obviously there.

The next one, is giving a bound for this dimensionality. So, p is greater than or equal to 1, and giving a specific value. So, just the opposite situation of the first one, k is equal to 2, and p is greater than or equal to 1. We can use the test statistic, n minus p minus 1 by p. The product with 1 minus lambda star by lambda star, this is going to follow an F distribution with p n minus minus 1 under H naught. And then, there is a last case where p is greater than equal to 1, which basically means we can use it for the uni dimensional or the multidimensional case. There is an equality here, and k is particularly equal to 3, that is for a small number of groups or populations.

So, earlier case was k equal to 2, here it is k equal to 3. and the test statistic looks like, n minus p minus 2 by p. Just a small change here, and here we have a 1 minus root of lambda star by root of lambda star. This is following an F distribution with twice p, twice of n minus p minus 1 under H naught.

Now the test procedure, the L R test, the exact likelihood ratio test, we may add here, in all the above cases, is reject H naught. If the observed F is greater than F at alpha with the relevant degrees of freedom. So, let us say that, it is d 1 and d 2, whatever be the case at hundred alpha percent level. Because as we said earlier, if the test is reject lambda, when lambda star, wilk's lambda is small. Whenever we are defining the statistic in this form, it has to be it reject when it is large, hence the test procedure that we give in terms of the F statistic is the observed F is greater than the upper alpha percent cut off, upper alpha cut off point of the F distribution, with the corresponding degrees of freedom.

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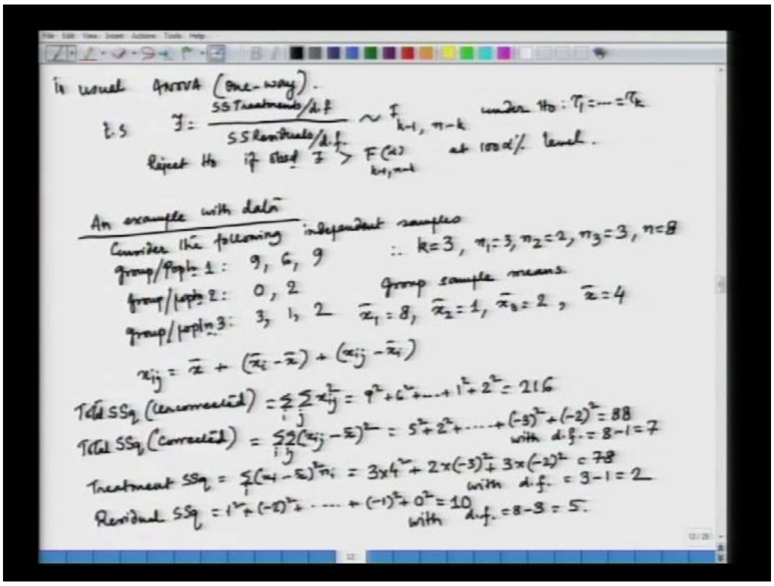

Now, if you recall the usual anova, where we have the uni dimensional data. So, usual anova, what happens in usual anova, one way say we are still restricted to the one way situation. If you recall the test statistic, comes in the form of test statistic is the F, which is the sum of squares treatments, divided by its degrees of freedom, and divided by sum of squares residuals, divided by its own degrees of freedom. So, this follows, the F distribution with the respective degrees of freedom for treatments, it is k minus 1 for residuals, it is n minus k under H naught. Well, H naught remains the same, that is you have equality of k treatment effect. These are no longer vector valued, but these are scalars now, because we are talking of the usual anova, and the F statistic is always in this form. And the test procedure is, we reject H naught, if observed F is greater than F alpha at k minus 1 degrees of freedom. So, this is at 100 alpha percent level.

Let us look at a data example, an example or an example with data. Let us, let us start with one way anova, the the data example, note that we are going to start with the one way anova, that is basically we are going to start with a random observations, which will be one dimensional. And at the second stage, to explain the manova, we will add one more data dimension to which. So, from p equal to 1 will more higher to p equal to 2. And then, to explain how we get the manova analysis statistics explicitly.

So, the first the first case is, for the univariate data. And we consider, the consider so the following independent populations, following independent observations from populations. So, let us say, the independence samples, and we have from group or population 1, we have say 3 observation, 9, 6, and 9, a very simple example. Then from the second group, group or population 2, we have two observations, say 0 and 2, these are pertaining to some variable, some factor. We are just laying stress on the data itself, later on we can take up a practical, a more practical example, which we will have better interpretation to the data. And the third population or third group, have 3 it has 3 observations 3 1 and 2.

So therefore, I can say few things from here, that k number of groups is 3, we have 1, 2, and 3. I can say, what are my n is. So, n 1 is 3, n 2 is 2, and n 3 is again 3, 3 to 3 observations. So, that total observations is 3 plus 2 plus 3 that is 8.

I can also calculate the group means. So, the group sample means are once we have the observations given in totality, there is no problem at all. So, groups sample means, if I try to calculate, the first one is x 1 bar, and it is nothing, but the mean of the sample mean of the first three observation, or the three observations from the first group, and that is 8. Similarly I have x 2 bar which is 1, x 3 bar is nothing, but 2 giving me the overall sample mean x bar is 4. I have univariate data. So, the group sample means and group over and the overall means, these are also scalar valued number scalar valued value variables, you can see very easily, that these are uni dimensional the values are 8, 1, 2, and 4.

Now, we go to the sum of squares splitting. So, we do not have the cross product term here, because we are dealing with uni dimensional data. So, sum of squares splitting is sum of squares total or total sum of squares uncorrected. So before that, if you realize that what my observation is are. So, I can how can I look at, a single observation x i j, that is the j th observation from the i th group. Well I look at it as, made up of x bar. And then, I have x i bar minus x bar , and then this is x i j minus x i bar. And then it becomes easier for me, to consider the splitting. So, I have sum of squares, the total sum of squares uncorrected is nothing, but sum of squares of x i j square double summation over i and j, and this is nothing, but 9 square plus 6 square, i go on taking the square of the observations 1 square plus 2 square, which gives me 216. I am not exactly interested in this, I have, but instead I am interested in total sum of squares corrected, corrected means that from each of the observations, I have to consider it is deviation from the overall sample mean, and then consider they are sum of squares.

So, this is x i j minus x bar whole square, and this is if I take out 4 from the first one, I have 5 square plus 2 square. In this way I go up to, 1 minus 4 that is giving me minus 3 square and 2 minus 4 that is minus 2 square, and this is coming to 88. And if I write with d f, well d f is nothing, but n minus 1, which is 8 minus 1 equal to 7.

Now, I consider the treatment or the between sum of squares. Treatment sum of squares, if you recall is nothing, but it is just summation of tau y hat square, which is nothing, but summation of x i minus x bar square. So, I have x i minus x bar square, but obviously, I cannot forget this n i. So, this is giving me, I have thrice, I have the, this is same for the first i th population. So, for the first population, note that x i bar is 8. whereas, x bar is 4.

So, I am going to have 3 times 4 square, and then I am going to have 2 times x i for the second x i bar for the second population is 1. So 1 minus 4, that is 2 times minus 3 square, and then again 3 times 2 minus 4, that is minus 2 square. And this is giving me 78, with degrees of freedom equal to k minus 1, that is 3 minus 1 not 2. And similarly, well there is strictly speaking, there is no need for me to calculate the residual sum of squares.

Now, I can simply consider the difference between the total corrected total sum of squares and treatment sum of squares to get it. But for verification sake, you can also do that. And what you have to consider is actually sum of these sum of squares of this deviations x i j minus x i bar. So, what you can do is, from each observation, take out it from the first observations from the first group, consider deviation, that is 8 in all the 3 cases. So, first you have 1 square, and then you have minus 2 square, and for the last group you have this observation. From each of these observations, you are going to consider the deviation from x 3 bar, that is 2, and you are going to have the last two observations, I am writing as minus 1 square and 0 square.

So, that is giving you, 10. It has to give you because 88 minus 78 is 10, with degrees of freedom is equal to n minus k. So let us write, that in numbers n is 8, and k is equal to 3, which is giving me 5. So, we have split up the sum of squares due to the treatments, due to the residuals, and due to total corrected. And we have also obtained the corresponding degrees of freedom.

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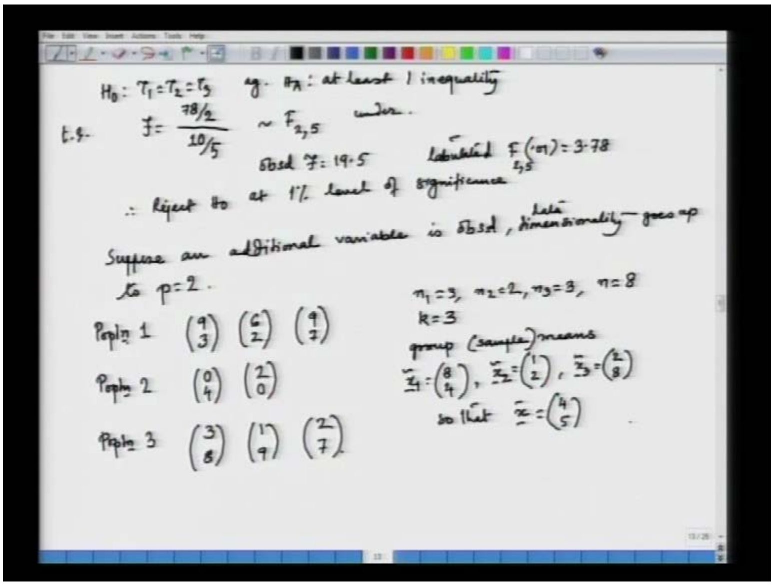

Now, we know what is the hypothesis , that we are testing. The null hypothesis that we are testing is nothing, but tau 1 equal to tau 2 equal to tau 3 against the alternative, that at least 1 in equality is there. And this is the usual anova. So, we can just simply consider the f statistic the statistic, that is used for this purpose, and which is sum of squares treatment, which is 78 minus it is degrees of divided by it is degrees of freedom, which is to the whole divided by the sum of squares residual, which is 10, and divided by its own device of freedom which is 5. So, this is the test statistic, and this follows the F k minus 1 that is 2, n minus 1 that is 5, under H naught. So, our test procedure is rejects H naught, if observed.

Now, let us see what is observed, F here. So, observed F here is 19.5, it comes out to be 19.5. And the tabulated F at 0.01with 2 and 5 degrees of freedom is having the value 3.78. Therefore, under the light of the given sample, reject H naught the null hypothesis at 1 percent level of significance. So, we are rejecting the null hypothesis that, there is no treatment effect. The means coming from the treatments are all equal, we are rejecting the null hypothesis.

Now, what we are going to do is, we are going to extend the dimensionality of the data. Going to make it two dimensional, just add one more variable. This is plainly speaking, this is just adding one more variable on which data are compiled, or on which data are collected. To make it a two dimensional case. So, that we can apply at the manova technique here.

So, we had recall, we had our data. As now this is the extension, suppose an additional variable is observed. So now, the data dimensionality goes up to 2 p equal to 2. So, let us look at the data first. Now if you recall, we had 3 populations, everything else will remain same. The first population or group, it had the observations. If you recall as 9, 6, and 9, scalar valued observations.

Now we are saying, that we are making it vector valued. So, we have the information on one more variable. And hence the first observation, from the first population is now a vector 9, 3. similarly I am adding the information on the same data, to all the observations of the first populations. So, I have, this completes my observations from population one.

So, next is population two. If you recall, the observations were 0 and 2 in the uni dimensional case. I am including information on one more variable. Hence my observations, the complete observations are now becoming two dimensional 0 4 and 2 0. And the third one is if you recall, the observations were originally 3, 1 and 2. So, that I am adding one more line them, making the complete observations 3 8, 1 9, and 2 7.

Now let us see, what are the information that I have from here. So, as before my n 1 remains 3, then n 2 remains 2. This makes a complete observation 0 4, and 2 0 makes another complete observation. So, that n 2 is 2, and n 3 is 3 again. And So, that the total sample size is 8. Number of group or populations, well I have stuck to 3 populations or 3 groups. So, I have k equals 3. And now the group means, group sample means, which are essentially x 1 bar x 2 bar and x 3 bar, but unlike in the earlier case, these are now going to be vector, because I have x 1 bar is nothing, but the mean of these 3 observations. So, this is giving me 8, 8 was already there. We had already obtained 8 for from our earlier calculation.

Now, we are calculating only for the second line, second element of these observations, and this average is giving me 4. Similarly for x 2 bar, the first element I already know, that was 1. And now 4 and 0, i consider the mean of this, and I am getting 2. And x 3 bar is giving me 2, which was already there. And the next entry is 8. So, that x bar, the overall sample mean is 4, again it is there. I can directly use it from my anova calculations, because I have not changed the first line of the data anywhere. So, all these values remain the same and this is the situation that I have.

So as before, we are going to.. I will repeat the sum of squares operation for the second variable, that we have now considered, the newly considered variable.

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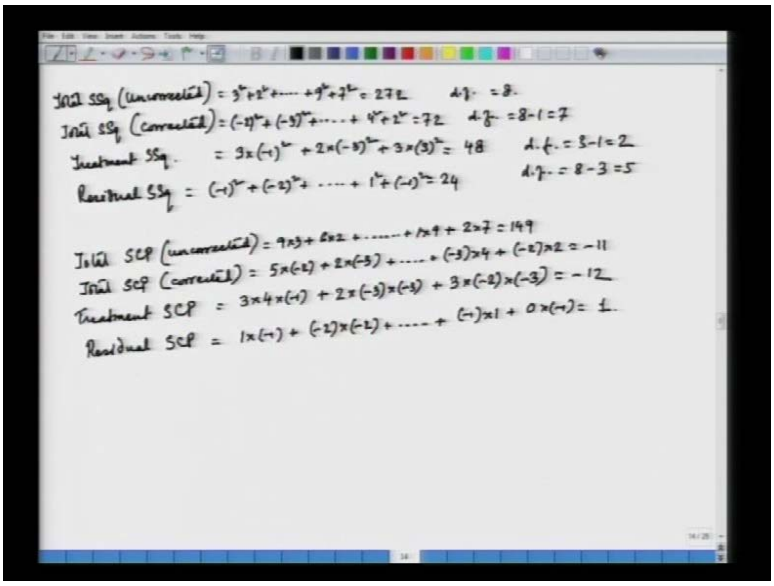

So, we have sum of squares total uncorrected exactly the same operation repeated for the second line of data, and this is going to give me 3 square plus 2 square up to 9 square plus 7 square. And this is adding up to 2 7 to 2. By the way, the degrees of freedom for this is full n, it is with degrees of freedom equal to 8. But we are really not using this, again total sum of squares corrected is, we have to take the deviation of the sample over all sample mean from the observation, the sample mean now, which is which is coming in as the second entry, of the vector of the sample mean vector. And this is going to be minus 2 square minus 3 square, this is going up to 4 square plus 2 square giving me 72, with degrees of freedom equal to 8 minus 1, that is 7. And then, we have the sum of squares treatment, sum of squares or the between sum of squares.

This is now simply 3 times, as we have done the earlier case three times minus 1 square plus 2 times minus 3 square plus 3 times 3 square, and this gives me 48 with d f as k minus 1, which is 3 minus 1 or equal to 2, and then the residual sum of squares or within sum of squares. And this again is as before we have minus 1 square plus minus 2 square, this is essentially again for cross checking, because once we have got total square correct, sum of squares corrected, and treatment sum of squares, we did not calculate this. And this is minus 1 square giving me 24. So, 24 plus 48 adds up to 72 and degrees of freedom is n minus k that is 8 minus 3, which is 5.

Now, in order to complete the manova table, now this is this is not anova, that we are using testing it for uni dimensional data, we have now a multidimensional two dimensional data. So, to complete the manova table, it is not enough that we have got the sum of squares values, but we also need the sum of cross product value to complete the data matrix.

So let us see, how do we consider the sum of cross product calculation. So, what we are going to consider is now, the sum of cross product. So, just like the sum of squares total etcetera, we have total sum of cross product sum of cross products. So, just one is enough, uncorrected. What we do is essentially we consider the product between the first and the second, and then we get, what we get is 9 times 3 plus 6 times 2 and this goes up to 1 cross 9 plus the last 1 is 2 cross 7, and this gives me 149, total sum of cross product corrected.

Now, we have to consider the deviation, whatever the values that we have, after considering the deviation, consider them and then consider their products. So, it is like 5 times minus 2 plus 2 times minus 3. In this way, minus 3 times 4 plus minus 2 times two giving me minus 11.

So, sum of squares as long as they were, we did not have any minus or any negative terms, but these being cross product terms, we can have negative results well. And then we have the treatment or between sum of cross products, treatment sum of cross product this will be 3, which is the number of observations, the common number of observations in the first group. So, that 3 times 4 into minus 1, for the second population 2 and then it is the common value minus 3 with and minus 3, then for the last population, it is 3 with minus 2 and minus 3. So, this is giving me minus 12. And then, the last one is the residual sum of cross product or within sum of cross product and that is 1 times minus 1 plus minus 2 times minus 2. So, this is going up to minus 1 with 1 and 0 with minus 1 giving me 1. So, that these two, add up to minus 11. And now, I can form the manova table.

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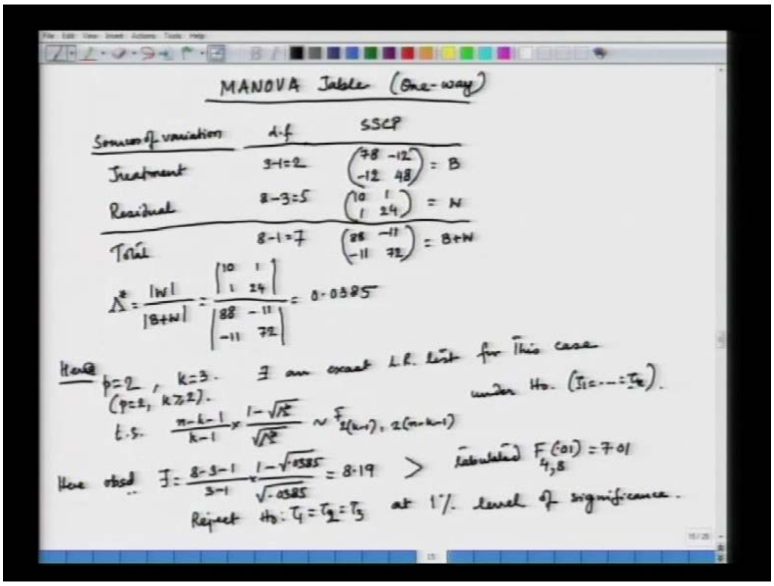


Let us go to the fresh page, to form my manova table. So, this is for one way, I have the sources of variation. The first column, then we have the sum of well we have the degrees of freedom, and then we have the sum of squares and cross product matrices now instead of a scalar.

So, of the first one is due to treatment, the treatment effect. 3 of them minus 1, so degrees of freedom is 3 minus 1 equals 2, then due to the residual is the error present in the model, and that has degrees of freedom n minus k that is equal to 5. And the total variability in the data, that is split up in to these two factors. So, total is n minus 1 that is 7, and let us look what does these matrices?

The first one is, well I have sum of squares treatment .For the first set of data, which was if you recall, which was 78, and sum of squares treatment for the second second variable which was 48. The off diagonals will be the sum of cross products that is minus 12, and this is a symmetric matrix, this is minus 12 again.

Similarly, the sum of squares cross product matrix for residual, if you consider the sum of squares residual from the first variable that was 88, for the second variable it was 72, and that I am sorry I am writing the total here. The residuals will be this is the complete matrix, this is our residual matrix. So, this is n sum of squares residual from the first variable, from the second variable 24, and the off diagonals that is sum of cross product is 1, and for total it is now 88, you can simply add up these 78 plus 10 giving me 88 48 plus 24 giving me 72, and these are giving me minus 11. And you can check it from your calculation.

So, we call this matrix as our, if you recall it is the between sum of squares cross product matrix. So, this is between, and this is the within sum of squares cross product matrix. So, this is our between plus within matrix. And then, recall what you're the likelihood ratio criteria and the lambda star, wilk's lambda is nothing, but determinant of W by determinant of B plus W. So, this is essentially, determinant of the matrix 10 1 1 and 24 divided by the determinant of the matrix 88 minus 11 minus 11 72, and then we get the value of this as 0385.

Now, what is the situation we have here? We have p equal to 2, because we have two dimensional data, and k equal to 3. So, we can use the exact test, one of the exact test that we had stated right. So, there exists an exact L R test, for this case. And just check, that the test statistic in that case is nothing, but what we are going to use is the test statistic for the case p equals 2 and k is greater than equal to 2, that case.

So, the test statistic for that was n minus k minus 1 by k minus 1, that is for p equal to 2 and k greater than equal to 2. And this is considered with 1 minus root of lambda star by root of lambda star. And this follows, an if distribution with degrees of freedom to twice k minus 1 twice n minus k minus 1, under null hypothesis.

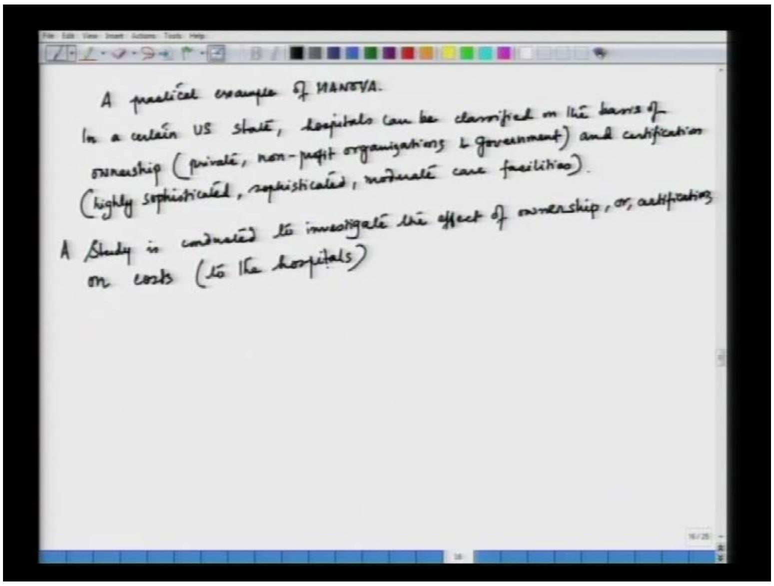
So, here it is equality of treatment. Now these are now vectors tau 1 to tau k. So, here observed the value of the test statistic f is nothing, but 8 minus 3 minus 1 by 3 minus 1. And then we have calculated, this wilk's lambda. So, this is nothing, but 1 minus root of 0385 by root of 0385, this is giving me the value 8.19. This I am going to compare with the tabulated f of degrees of freedom 4, that is twice k minus 1 that is 2. So, this is 4 and n minus k minus 1 is n minus k is nothing, but 5 and 5 minus 1, that is 8. At level one percent level, this value is equal to 7.01. So, this is greater than tabulated f at this. So, we reject H naught, that the 3 treatment effects are equal at one percent level of significance.

So after the anova test, we augmented the data dimensionality by 1. And we did a manova analysis, a one way analysis and we arrived up to our test criterion with the help of the wilk's lambda, will be could have used the asymptotic test in this case, because we have said that in most of the situations, where we use the likelihood ratio test, we go for the asymptotic convergence and use the chi square distribution. But since in this case particularly, we had that simple situation of p equal to 2 and k equal to 3, we choose one those four exact tests given to us provided to us, and we went we have gone by that test. And we have reached at that, we have reached our decision, that we are rejecting the null hypothesis of the equality of treatment effects at one percent level of significance .

Now, this is essentially a typical data example. I have not given you a proper interpretation of the data, we have to think of some variable, variable one which has values 9 3 etcetera for the first population. And then when it is augmented by one more variable. So, we have to think of two variables which have got values like this.

So, let us talk about a very practical example or real data, and see how this manova analysis, actually help us in getting some useful analysis from data.

(Refer Slide Time: 53:05)



So, this is a practical example (No audio from 53:06 to 53:13) of manova. The setup is something like this, that in a certain US state, you have hospitals or clinics. These can be, hospitals can be classified on the basis of ownership. So, these classifications are private, nonprofit organizations, and government. So, three different classifications of ownership of hospitals, and also they may be classified according to the certification that they have received. And certification that, some of them are certified to be highly sophisticated or just sophisticated or with moderate care facilities.

So, there are essentially two factors on which data have been collected about different hospitals in a particular US state. First factor is the ownership factor, there are 3 groups under this factor. One is the private ownership, the second nonprofit organizations, and the third is government ownership. And then we have a second factor also, that is the certification that these hospitals have received. And they have been certified as highly sophisticated, sophisticated, and moderate giving moderate care facilities.

Now, what we do is (( )) analysis the study is to, essentially a study is conducted to find is, a study is conducted to investigate the effect of ownership or certification or both. In fact… So this is, if we say if we say now the one thing that we can stress upon is that, these two factors that we have classify that is ownership and certification now, there can be more several other factors on which data can be compiled and collected can be separated. But at the moment, we are going to talk about these two factors. And then, we conduct the study, where we first investigate the effect of ownership or certification. This, we can handle with our one way anova one way manova, but if we go for the effect if we go for the effect of both, now this cannot be handled with the one way manova, we have to consider the two way manova for this.

So initially, we will see that we are going to investigate the effect of either this factor or that factor, it is either ownership or certification. But again, I say if I want to look at the effect of both or the combined effect, we have to go to two way manova, we which we have not yet handled.

So, a study is conducted to investigate the effect of ownership or certification. And on costs, that is costs to the hospitals. Now I am saying that, I am considering not an anova, but a Manova. So obviously, cost will have to be more than 1. So, we are actually actually be considering four cost types (( )) to the hospital. So, that the data dimensionality in this case is actually 4, and hence we are talking about a practical example of how to handle four-dimensional one way Manova.