Introduction to Fuzzy Sets, Arithmetic and Logic
Prof. Niladri Chatterjee
Department of Mathematics
Indian Institute of Technology - Delhi

Module - 3
Lecture - 8
Fuzzy Sets, Arithmetic and Logic

Welcome students to the eighth lecture of fuzzy sets arithmetic and logic. In the last class, I have introduced you to the concept of fuzzy numbers.

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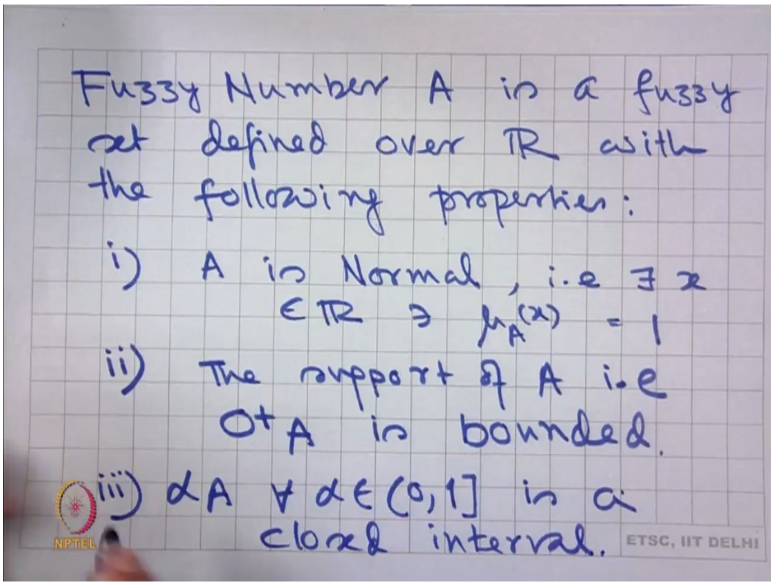

So, for a quick recall,

A fuzzy number is a fuzzy set defined over with the following properties.

i. is normal. That is, there exists , such that
ii. The support of A. That is, is bounded.
iii. for all is a closed interval.

And, we have stated a theorem which says that;

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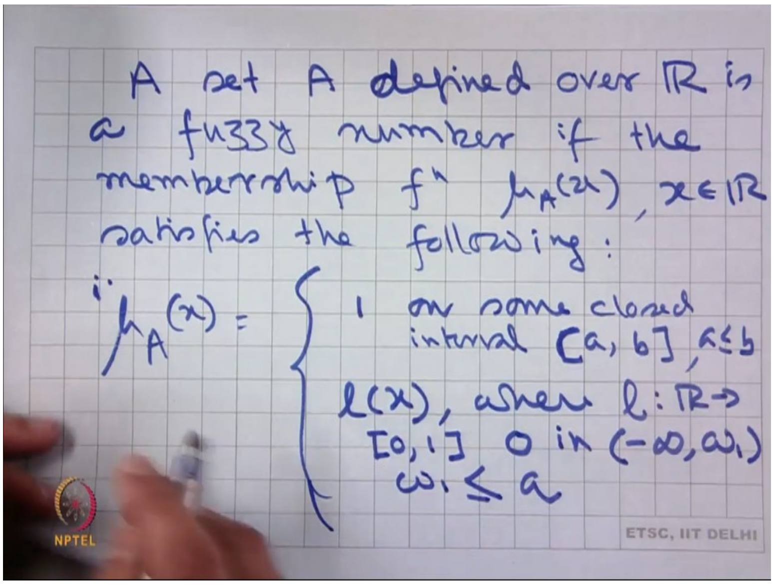

A set defined over is a fuzzy number if the membership function satisfies the following.

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where and for , where and

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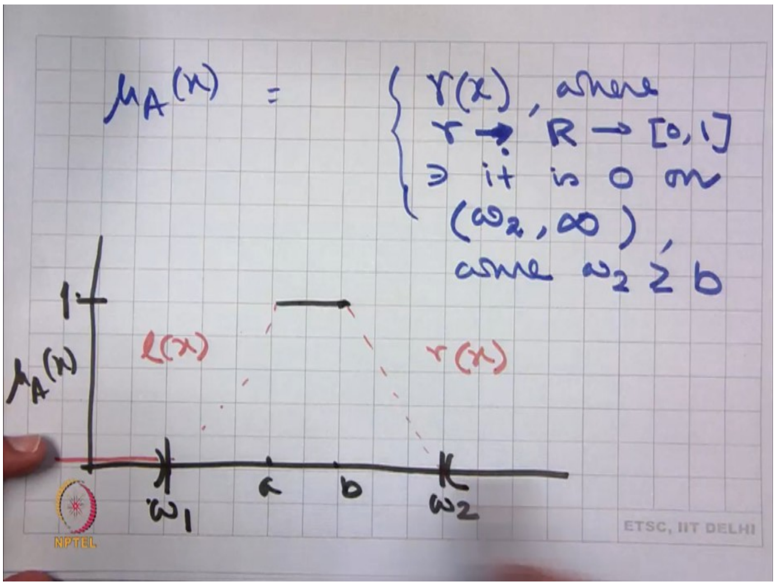

.

Or pictorially, if we look at it, what I am saying that looks like this. On an interval, say , it is . There exists some, such that it is there. There exists a , such that there. Or in other words, the will look like this. from here to here, it is called From here to , it is So that, beyond it is before it is

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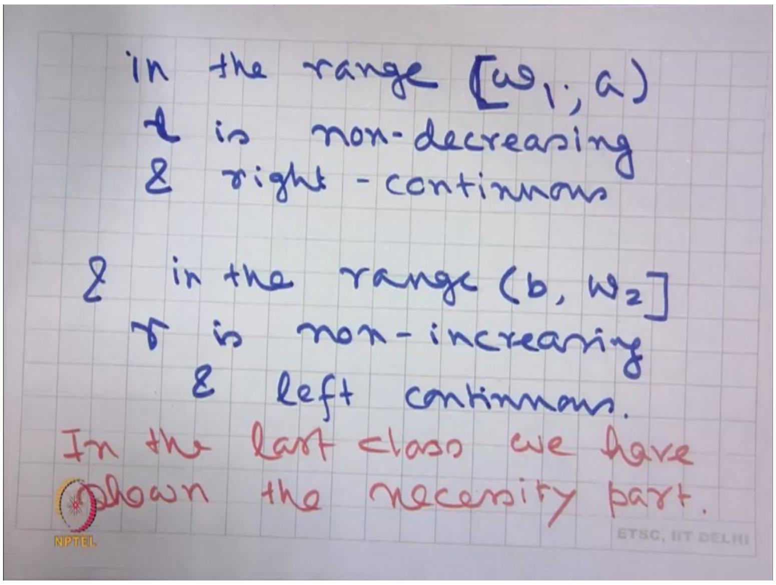

And in the range to , is non-decreasing and right continuous.

And in the range to , is non-increasing and left continuous.

In the last class, we have shown the necessity part.

Now, I want to prove the sufficiency part.

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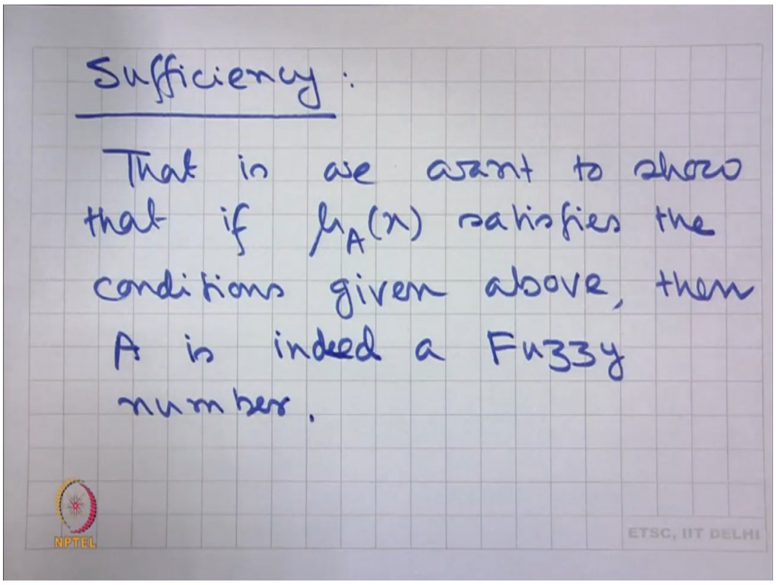

That is, we want to show that, if satisfies the conditions given above, then is indeed a fuzzy number.

That is, given these properties of the membership function, we want to establish that will follow the three properties of a fuzzy number.

And that proof is as follows.

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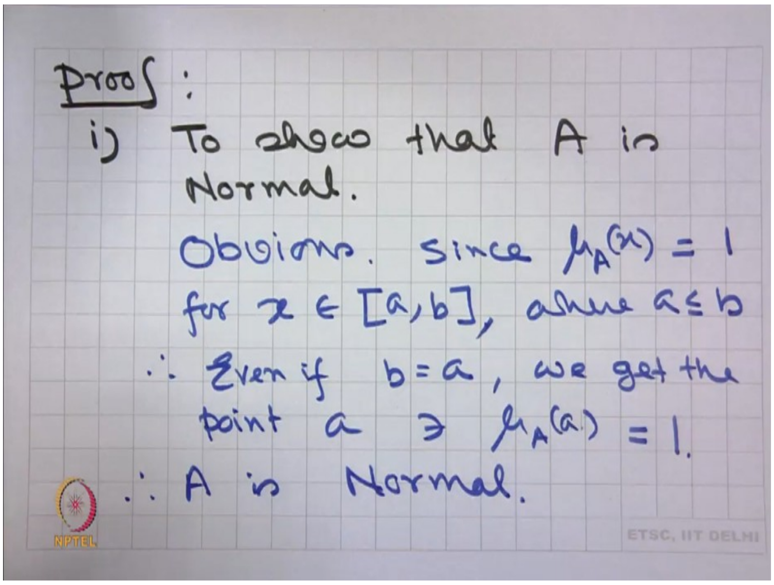

i. We want to show that is normal. This is obvious.

Since for , where .

Therefore, even if , we get the point such that

Therefore, A is normal.

In other cases, when b is actually greater , obviously the entire interval will have the membership value .

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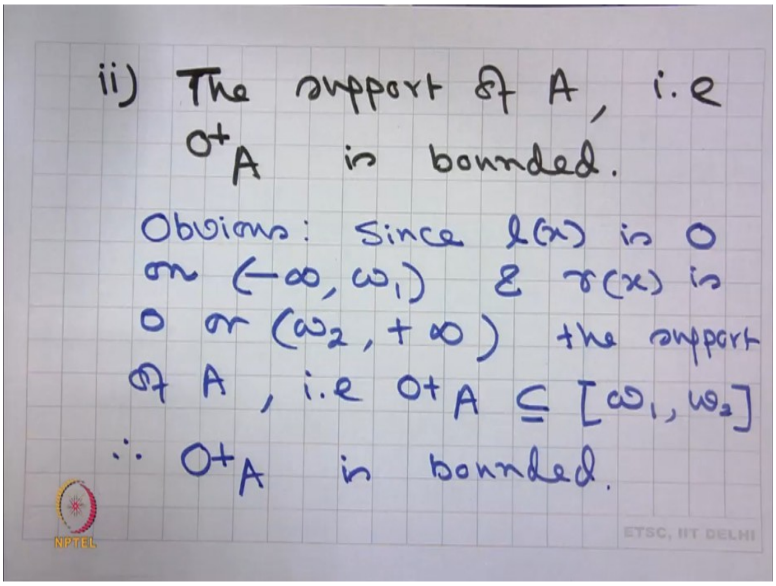

ii. We want to show that the support of A, that is is bounded.

Proof: It is obvious.

Since is on and is on . The support of A, that is

Therefore, is bounded.

Now, we need to prove the property 3.

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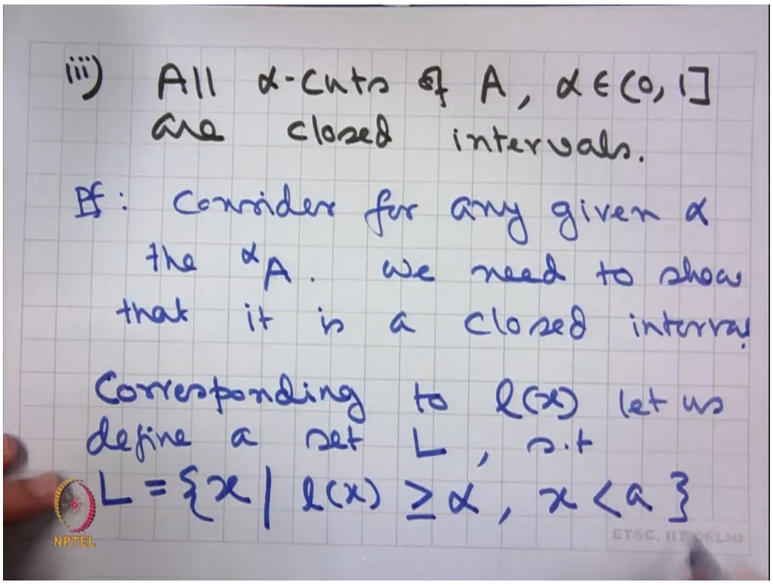

iii. All -cuts of A, are closed intervals.

We prove it in the following way. Consider, for any given , we need to show that is a closed interval.

Corresponding to , let us define a set such that

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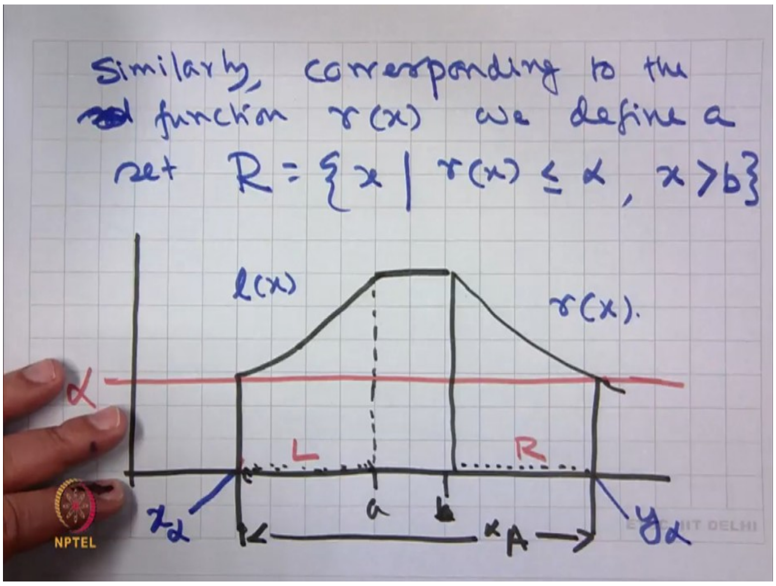

Similarly, corresponding to the function we define the set such that

Let me explain it pictorially. Suppose this is the picture.

This is and this is . So, we consider to be this line.

Therefore, the -cut of A is this interval. And I am now looking at the set of which are less than . That means, I am looking at points in this region and

So effectively, I am looking at this interval for choosing the set In a similar way, I am looking at points in this interval for the set So, this is going to be my set ; and this is going to be my set

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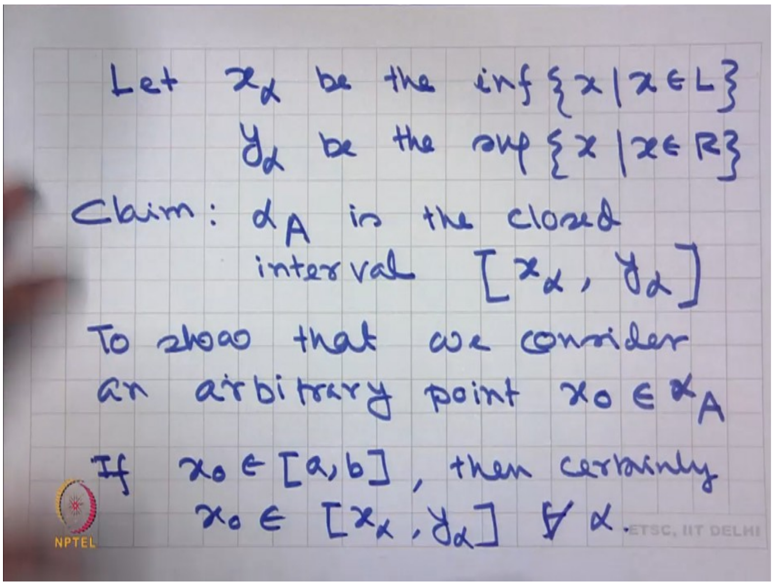

Let be the and be the

Claim: is the closed interval

To show that, we consider an arbitrary point

 has 3 possibilities.

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-
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i. If then certainly, for all

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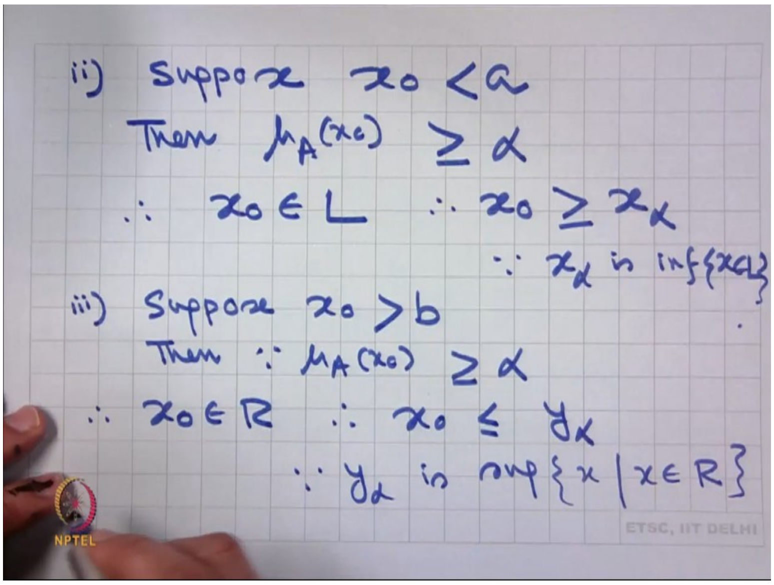

ii. Suppose , then .

We know because we have chosen from alpha-cut.

Therefore, , because ; and .

Therefore, , since is .

iii. Suppose , then since and ; therefore, . Therefore, , since is the .

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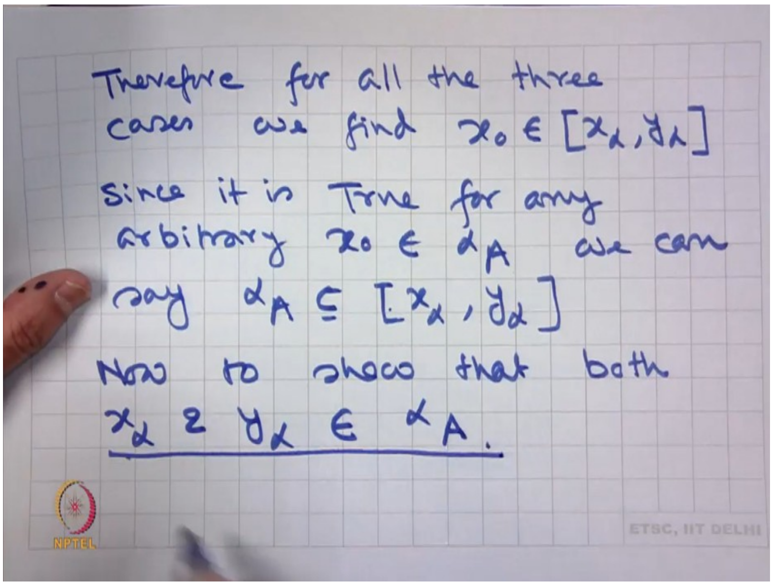

Therefore, for all the three cases, we find

Since it is true, for any arbitrary , we can say .

Now, to show that, both and

That will ensure that the two boundary points and also belong to the -cut. And therefore, the -cut of is actually the closed interval .

So, now we want to show that and belonging to -cut of A.

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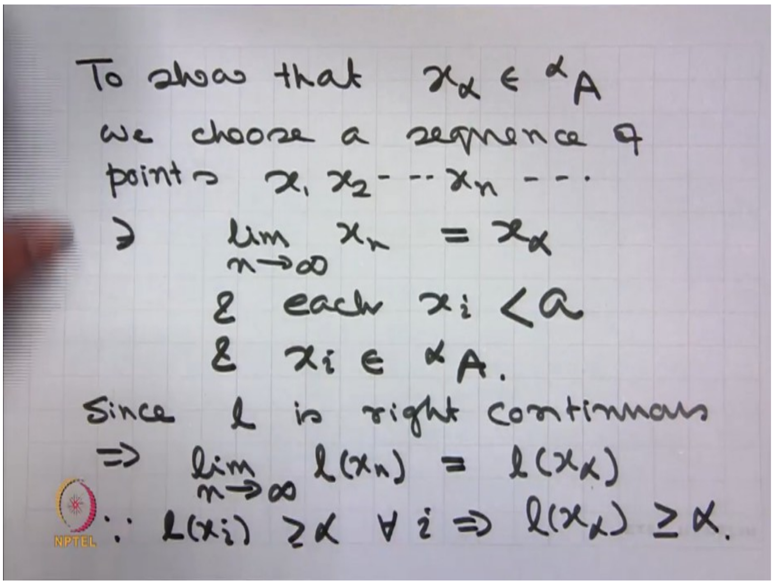

To show that , we choose a sequence of points , such that and each . and .

If we look at that diagram again, I am choosing a sequence of points such that and they belong to . That means, I am restricting myself to this set of points.

Since, is right continuous.

.

Since each ; therefore, since for all

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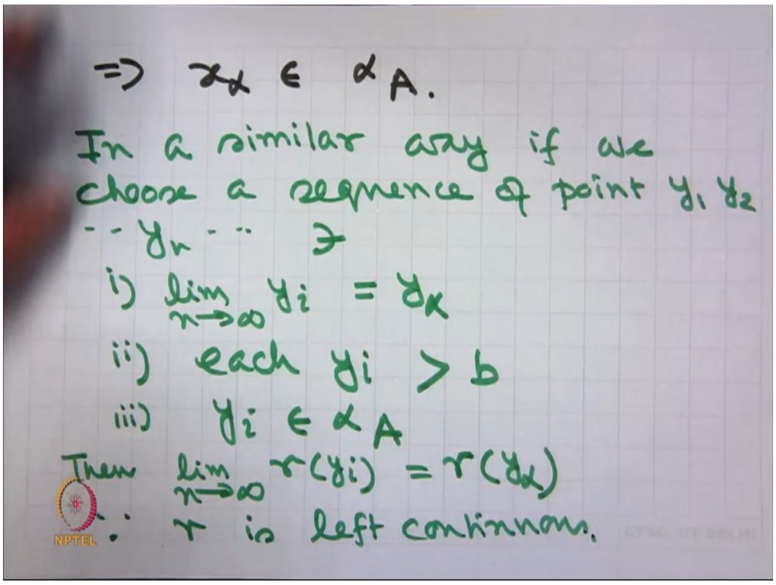

In a similar way, if we choose a sequence of points such that i.

ii. each

iii.

That is, with respect to the diagram I am restricting a sequence of points , choosing from this interval whose limiting value is .

Then, l, since is left continuous.

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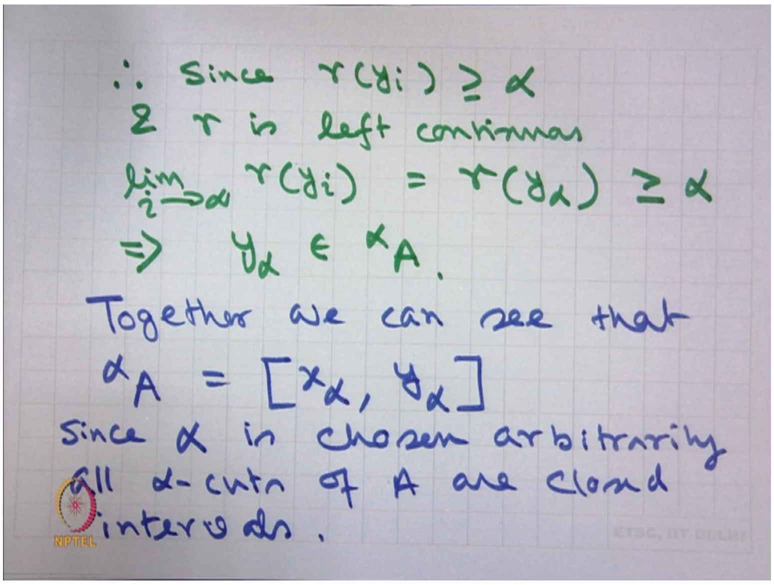

Therefore, since and is left continuous, .

Therefore, together, we can see that

Since is chosen arbitrarily, all -cuts of are closed intervals.

So, we proved that, if the membership function of satisfies the property as stated earlier, that is,

,

where and for , where , is monotonically non-decreasing and right continuous and , is monotonically non-increasing and left continuous.

Then, we find that the corresponding fuzzy set satisfies all the properties of a fuzzy number. And therefore, any fuzzy set over having the membership function satisfying all these 3 conditions is also going to give us a fuzzy number.

In other words, these 2 definitions are equivalent.

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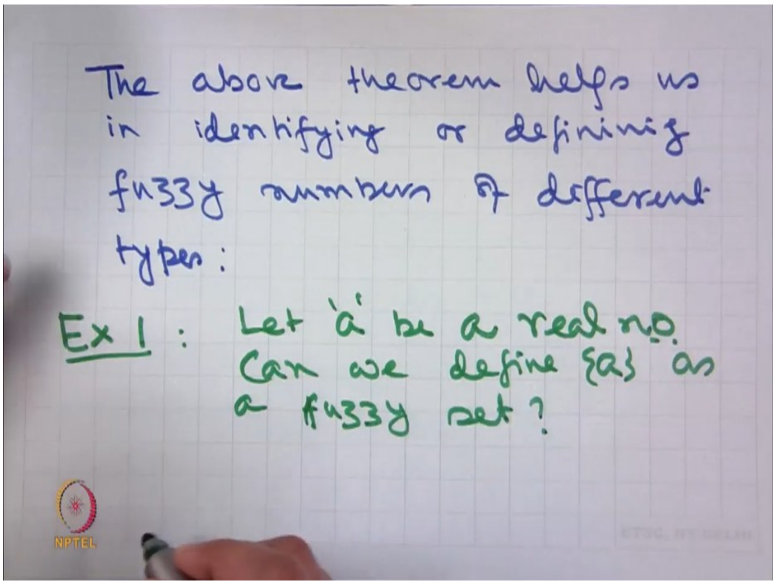

The above theorem helps us in identifying or defining fuzzy numbers of different types. Example 1: Let be a real number. Can we define as a fuzzy set?

Yes.

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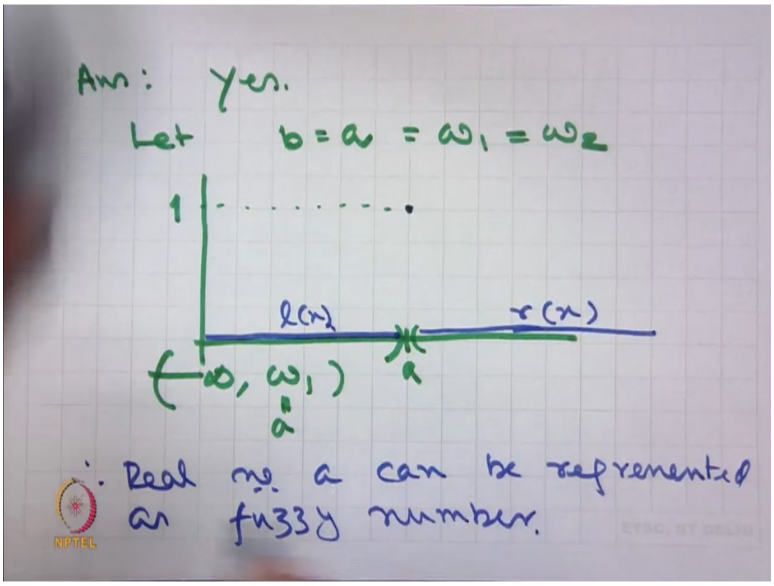

Let

Therefore, what we are getting?

If this is , then in the closed interval, which is nothing but the singleton , the value is

In this interval , where , .

And similarly, in the interval ,

And thus, we get only a singleton point in the support of the set , which has full membership to the set. Therefore, the real number can be represented as a fuzzy number.

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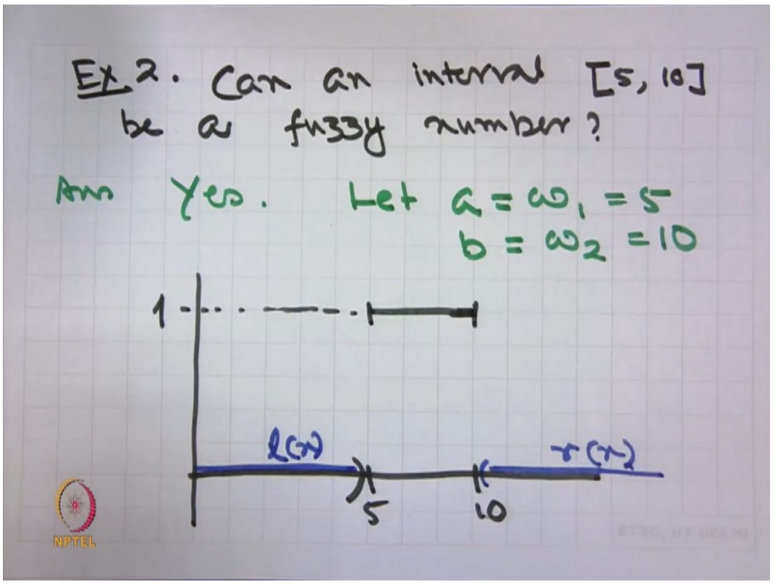

Example 2:

Can an interval, say be a fuzzy number?

Answer is yes.

Let and.

Therefore, what we have?

In the interval , we have the membership value . Since ; in the interval the membership is , which is in the interval , . And therefore, only the points belonging to the interval have membership That is, all the real numbers between to in the closed interval have full membership to the fuzzy set. And therefore, this is going to be a fuzzy number.

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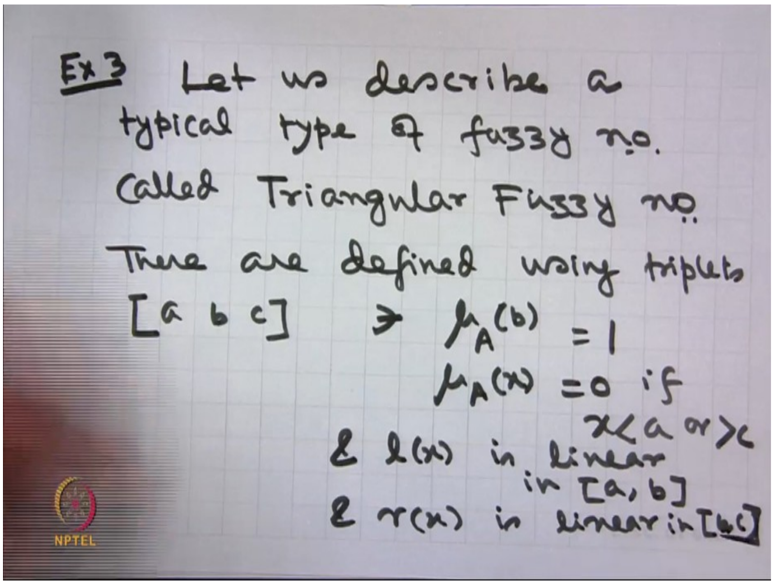

Example 3:

Let us describe a typical type of fuzzy number called triangular fuzzy number. Typically, they are defined using triplets such that ; if or and is linear in ; and is linear in . So, let me use a diagram.

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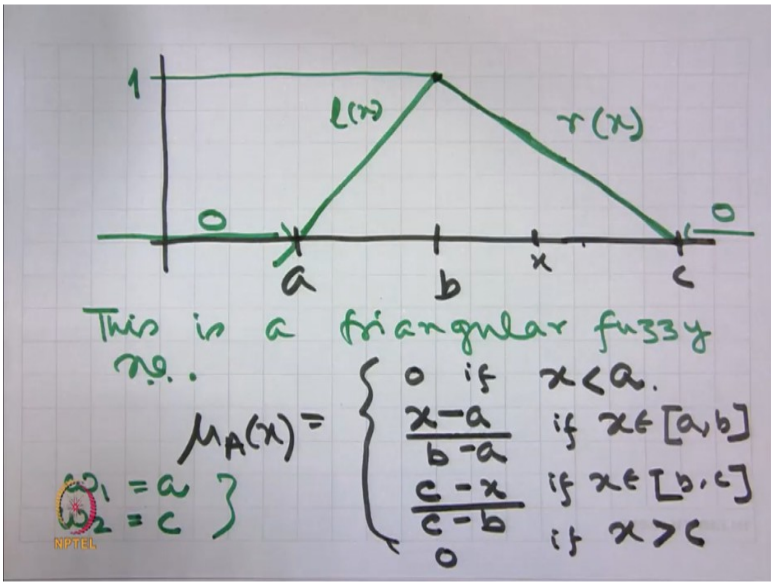

Therefore, once and are given, we get a unique fuzzy number defined as follows. This is a triangular fuzzy number. Its membership is going to be as follows:

So, you notice that, and .

Therefore, we can say that and . And this, along with the above definition, we get a triangular fuzzy number.

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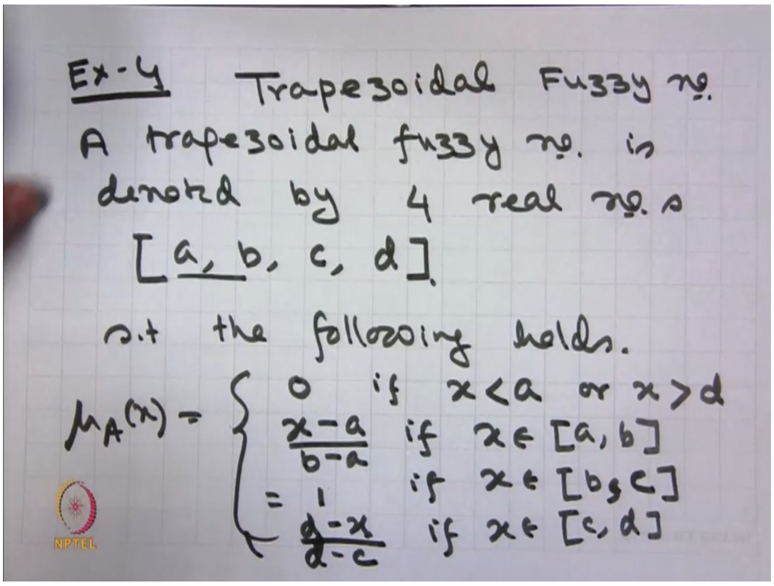

Example 4: Trapezoidal fuzzy number.

A trapezoidal fuzzy number is denoted by four real numbers, say , such that the following holds.

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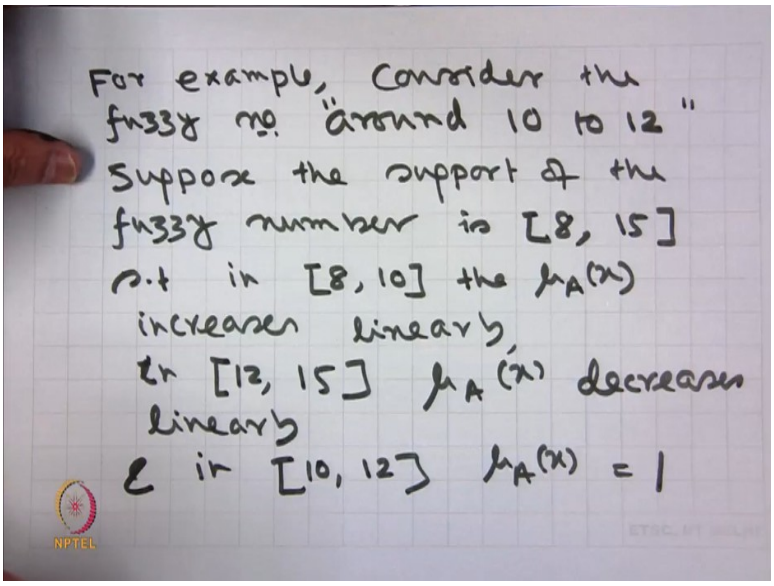

For example:

Consider the fuzzy number, say . And suppose the support of the fuzzy number is , such that in , the increases linearly; in , decreases linearly; and in ,

In that case, we shall get a trapezoidal fuzzy number as follows.

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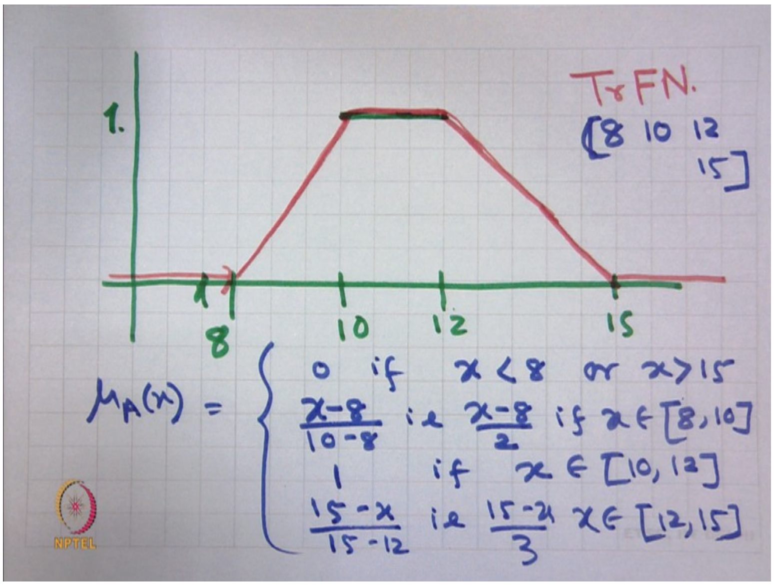

If we look at the shape of the membership function, it looks like a trapezium. Therefore, it is a trapezoidal fuzzy number denoted by

Also, we can write it as the 4-tuple: . What is going to be the membership function?

So, this gives us a trapezoidal fuzzy number.

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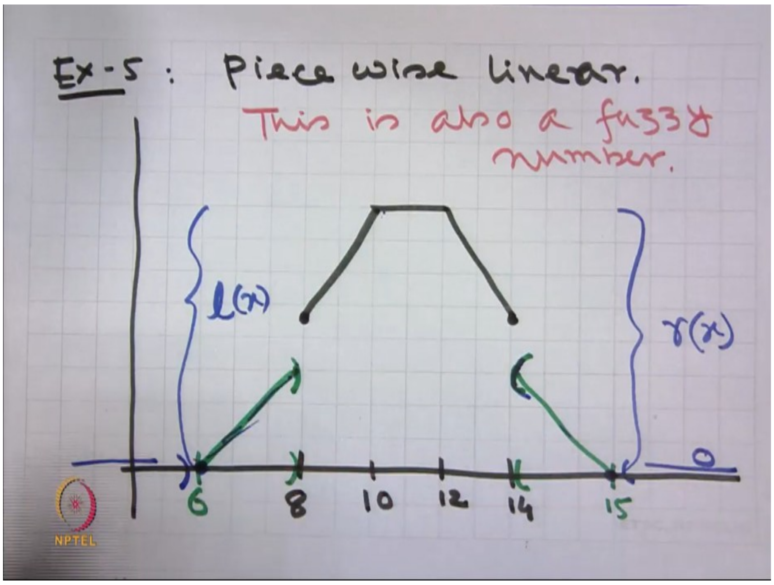

Example 5: Piecewise linear. Consider the following diagram.

Suppose in this interval, let me call it again, say to, the value of membership is . At and at , the membership is half. As it is right continuous, these will be included there. Between to , the membership is to . And this is piecewise linear. And suppose, here between to , it is piecewise linear. And it goes up to to 0.

And this is my ; and this is my ; which are, say for example; is monotonically non-decreasing. Here it is monotonically non-increasing. It is right continuous. This is left continuous. And below it is ; and above it is Therefore, and . In between, this is not a continuous line, but it is piecewise linear. Therefore, this is also a fuzzy number.

I like you to work out on these functions, because this equation of this line is coming from; that between to , it is growing from to . Between to , this line is going from to 1. Similarly, this is from to . And this is from to 0. Therefore, we can easily find the equations of the 4 line segments. And here the value is anyway. So, that gives us the definition of a piecewise linear fuzzy number.

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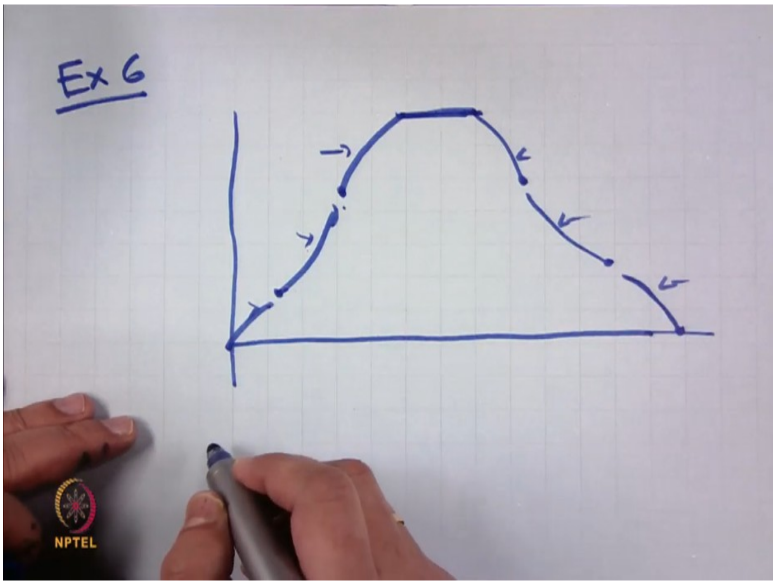

Example 6:

It need not be that and have to be linear all the time. A more general scenario can be like this, where these are piecewise continuous. And that gives a shape of a general fuzzy number where and are satisfying the properties given earlier. Therefore, by appropriately defining the small segments of curves, we can always get a fuzzy number. But, for this course, because of the ease of handling linear functions, we will restrict ourselves to fuzzy numbers where memberships are given by linear functions. And also, mostly we shall concentrate on continuous fuzzy numbers.

Okay students, I stop here for this class.

In this class, we have understood what is a fuzzy number; what are the different types of fuzzy numbers and their shapes.

In the next class, I shall start with interval arithmetic. And to give you the basic nuances of interval arithmetic, so that, when you want to do arithmetic operations of fuzzy numbers, we can take out alpha-cuts and then apply the arithmetic operations on those alpha-cuts to generate the alpha-cut of the final answer.

We shall investigate in next few classes, how all these arithmetic operations are conducted. Till then, thank you very much.