Introduction to Fuzzy Sets Arithmetic & Logic Prof. Niladri Chetterjee Department of Mathematics Indian Institute of Technology – Delhi

Lecture – 28 Fuzzy Sets Arithmetic & Logic

Welcome students to MOOCS lecture on Fuzzy Sets Arithmetic and Logic.

This is lecture number 28. And I discussed earlier that in today's class, we shall discuss Mamdani style Fuzzy inferencing.

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Mandani Shype Fuzzy interencity So for ashat we studied if given a conditional re unconditional proposition 2 some information about Antecedent variable X ashat can be deduced for Y.

If you recall, so far what we studied is given a conditional or unconditional proposition and some information about antecedent variable X, what can be deduced for Y.

And we obtained a fuzzy set B' for corresponding consequent variable Y.

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In torse the one need a specific value for y not the fussy set. suppose a attaling machine meets to infor about of ashich is the speed of TOTAKON. suppose speed depends upon areight, quality A the fabric etc.

In practice, we need a specific value for *Y*, not the fuzzy set.

For example: Suppose a washing machine needs to infer about *Y*, which is the speed of rotation. And suppose it depends upon several factors such as weight of cloth, quality of the fabric, etc. (**Refer Slide Time: 03:57**)

If y comin on a Fuzzy set then how to decide about the referet of rotation? Mandani obje scheme helps us to identify the value of of course these are other Fuzzy Inferencing Schemer

If *Y* comes as a fuzzy set then, how to decide about the speed of rotation.

So, you understand the problem we got from the influencing a fuzzy set B', which should give the membership for different values of Y, but we need to choose one.

How to choose that?

So, Mamdani style scheme helps us to identify the value of *Y*.

Of course, there are other Fuzzy Inferencing schemes.

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For example : Sugeno Pont in Sugaro's method the ontput comes in the form of a function of the input remark establisher Pont in Mandani's scheme use get one openific lealine of through a technique of "Graphical Methods"

For example, Sugeno's methods is very well known one but, in Sugeno's method the output comes in the form of a function of the input variable values.

But in Mamdani's scheme we get one specific value of Y through a technique of graphical methods.

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Alloo another point is three may be multiple parallel mees 2 each rule has its own frazz set for the conservent Jariable. .: While inforcering are need to consider all the relevant rules to come a conclusion

Also, another point is there may be multiple parallel rules and each rule has its own fuzzy set for the consequent variable.

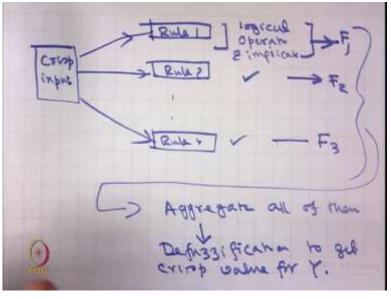
Therefore, while inferencing we need to consider all the relevant rules to come up to a conclusion.

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So typically the input will be crisp values for me maa En 10 variabl consegnent

So, typically the input will be the crisp values for the antecedent variables. Based on them we need to infer about the consequent variable.

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Pictorially, we have crisp inputs there may be rule 1 rule 2... rule n and in each rule we need logical operator and implication. Same is here, same is for them.

Based on that we get some fuzzy membership for some set F_1 , this give for some set F_2 and this gives for some set F_n .

Then we need to aggregate all of them and from there we use defuzzification to get it crisp value for that consequent variable *Y*.

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Illustration. Suppose we have two antecedent noariables : Temperature : 0°-30° Windropeed : O - 40 KMPH Ontput Gariable is "Comfortability" - artificial versable 0-12.

So, let us illustrate with an example.

Suppose we have two antecedent variables one is the *temperature* which we assume is in the range, $0 - 30^{\circ}$ C, another is *wind speed* and which can set between 0 - 40 Kmph and output variable is what we call *comfortability* of the weather which is a artificial variable, taking values in the range 0 - 12

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Suppose we have the following three only: Rule 1: If temperature in Low or coind speed in strong the weather in chilly Rule 2: If temperature is fair h Low and wind open is gentle them weather 10 consiferable

Suppose, we have the following three rules:

- Rule 1: If temperature is low or wind speed is strong then, weather is chilly.
- Rule 2: If temperature is fairly low and wind speed is gentle then weather is comfortable.

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Rule . 3: If Temperature in Not low and wind repead in eight them weather in shift; Suppose the input is Temperature in 15°C wind opend in 8 KMPH need to infor about comfretatoliky index = [0, 12]

• Rule 3: *If temperature is not low and wind speed is light then weather is stuffy.*

Suppose the given input is temperature is 15°C, wind speed is 8 *Kmph* and we need to infer about the comfortability index $\in [0, 12]$. So, I hope the problem is clear.

We have the input temperature 15°C and wind speed is 8 Kmph.

The problem is we have three different rules.

Each rule has something to say about temperature and wind speed and each rule gives a value for the consequent variable which is the weather comfortability index and that comes in different rules in different ways.

Therefore, question comes, how to infer for that given input?

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Let us first consider the Comfortatoility Index. Defined on the interval [0,12] And we have defined 3 Fuzzy sets on it.

So, let us first consider *comfortability index*. It is defined on the interval [0, 12] and we have defined three Fuzzy sets on it.

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These 3 sets are: Chilly, Comfortable & Sth O Chilly : A (x) = OT X Thus it has the following rohape: TRFN 0026

These 3 sets are *chilly*, *comfortable* and *stuffy*.

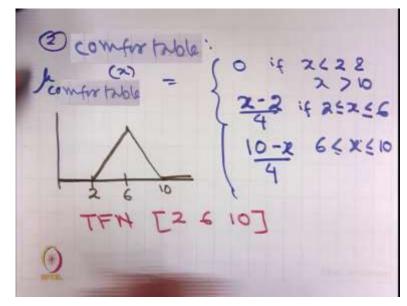
So, let us define

1. *chilly*

$$\mu_{chilly}(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 6\\ 1 & \text{if } 0 \le x \le 2\\ \frac{6-x}{4} & \text{if } 2 \le x \le 6 \end{cases}$$

We shall call it *half TrFN* and we may denote it as $\begin{bmatrix} 0 & 0 & 2 & 6 \end{bmatrix}$.

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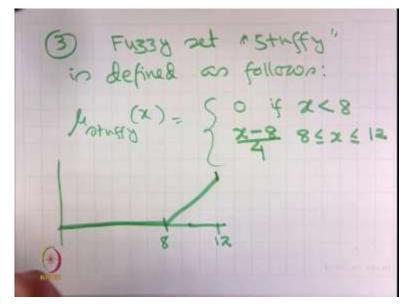
In a similar way,

2. comfortable

$$\mu_{comfortable}(x) = \begin{cases} 0 & \text{if } x < 2 \text{ or } x > 10\\ \frac{x-2}{4} & \text{if } 2 \le x \le 6\\ \frac{10-x}{4} & \text{if } 6 \le x \le 10 \end{cases}$$

So, this is going to be $TFN[2 \ 6 \ 10]$

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3. The fuzzy set *stuffy* is defined as follows:

$$\mu_{stuffy}(x) = \begin{cases} 0 & \text{if } x < 8\\ \frac{x-8}{4} & 8 \le x \le 12 \end{cases}$$

It is going to be linearly increasing in the interval [8, 12]. Therefore, it has the shape something like this.

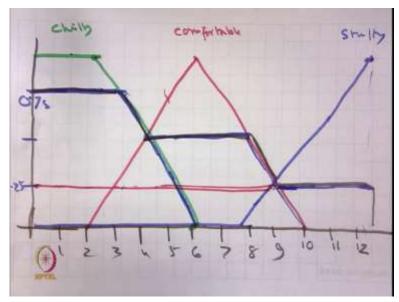
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Note that : i) These membership functions for the fussy sets defined on the consequent variable are very important. 2) These will be used in final decision making we will draw all three curves superimposed with each other in the following way:

Note that:

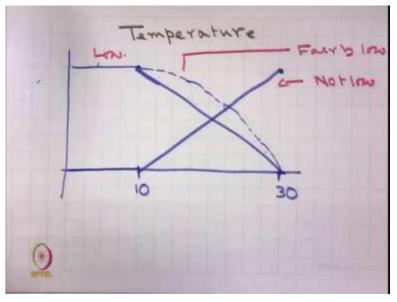
- i. These membership functions for the fuzzy sets defined on the consequent variable are very important
- ii. These will be used in the final decision making.
- iii. We will draw all three curves super imposed with each other in the following way:

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So, let us now draw the three curves.

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Let us now look at temperature and suppose we have the following:

Between 0 to 10 for low it is 1 and from 10 to 30 it is linear.

Therefore, fairly low as we know, we can use the square root and we may get something like this and not low this line.

So this is not low, this is fairly low and this is low.

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Let us now consider the other antecedent variably " Wind speed " 1. Its range is [0, 40] 2. Here too we have (KMPH) 3 Fuzzy subs defined on it () Light (i) gentle (ii) shong.

Let us now consider, the other antecedent variable, wind speed

- 1) its range is [0, 40]. Basically they are kilometres per hour.
- 2) Here, too we have 3 fuzzy sets defined on it which are
 - a. light
 - b. gentle
 - c. strong.

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Light: $M(x) = \begin{bmatrix} 1 & z \neq z = 0 \end{bmatrix}$ A_{light} Southle: $M_{gentla} = \begin{cases} 0 & if & z \leq s \\ 0 & x > 1s \\ 0 & x > 1s \\ 0 & x > 1s \\ 0 & if & z \leq s \end{cases}$ $A_{f} = \begin{bmatrix} x & y = [s, y] \\ 1 & x \in [y, ys] \\ 20 - x & if & x \in [y, 20] \\ 0 & if & x > 20 \end{cases}$

Let us now, look at this suppose, *light* is defined as follows.

$$\mu_{light}(x) = \begin{cases} 0 & if \ x < 0 \ or \ x > 15\\ 1 & if \ 0 \le x \le 10\\ \frac{x-10}{5} & if \ 10 \le x \le 15 \end{cases}$$

Similarly, gentle is defined as follows

$$\mu_{gentle}(x) = \begin{cases} 0 & if \ x \le 5 \ or \ x > 20 \\ \frac{x-5}{6} & if \ 5 \le x \le 11 \\ 1 & if \ 11 \le x \le 15 \\ \frac{20-x}{5} & if \ 15 \le x \le 20 \end{cases}$$

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$$\mu_{strong}(x) = \begin{cases} 0 & \text{if } x \le 15\\ 0 & \text{if } x \ge 40\\ \frac{x-15}{25} & \text{if } 15 \le x \le 40 \end{cases}$$

Now, our given input value is 8*Kmph*. Therefore, we need to calculate its membership to all the three fuzzy sets we have defined above.

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height (8) = 1 Agentle (8) = 1 Agentle (8) = 2 Anormag (8) = 0 In a rolimilar aroug are find out the membership balles for the other antecedent bariable 93 Temperature to the three fuzzz

And we have

$$\mu_{light}(8) = 1$$
$$\mu_{gentle}(8) = \frac{1}{2}$$
$$\mu_{strong}(8) = 0$$

In a similar way, we find out the membership values for the antecedent variable namely *temperature* to the three fuzzy sets.

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 $\int_{100}^{(15)} = 0.75^{-1} \\
 \int_{100}^{(15)} = \sqrt{0.75^{-1}} \\
 \approx 0.8 \\
 \int_{100}^{(15)} = 1 - \int_{100}^{(15)} \\
 = 0.25^{-1}$ 0.86

And we have the following

$$\mu_{low}(15) = 0.75$$
$$\mu_{fairly\ low}(15) = \sqrt{0.75} \approx 0.86$$
$$\mu_{not\ low}(15) = 1 - \mu_{low}(15) = 0.25$$

Now, we use these values for the 3 rules that we have in the domain and let us see how we apply Mamdani's scheme for coming into a decision.

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For Mandani inferencing we deviate from Lukaniewig Infact for Mandani Informeris Q->b has bruturalus min(a,b) In this approach we use the above

For, Mamdani inferencing we deviate from Lukasiewicz.

In fact, for Mamdani Inferencing

 $a \rightarrow b$ has truth value min(a, b)

So, in this approach we use the above.

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Let us new look at inforcening from the rules Rule 1: If temperature is Low Object is strong -> weather is chilly Mong (8) = 0.75 Z max (0.75,0) Mong (8) = 0 J - 0.75-

Let us down look at inferencing from the rules.

So, what is the Rule 1?

Rule 1 says that, *If temperature is low OR wind speed is strong* \rightarrow *weather is chilly*. Now, $\mu_{low}(15^{\circ}\text{C}) = 0.75$, $\mu_{strong}(8) = 0$

Now, we have OR therefore we take maximum of them max(0.75, 0) = 0.75

And therefore, when we apply the Mamdani inferencing. We get from the first rule that the original membership is clipped like this.

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Rule-2 Temperature in fairs wind speak is gently weather is comfortable s) = 0.8 = 0.86 > for a =

Let us down look at the second rule which suggests that

If temperature is fairly low AND wind speed is gentle THEN, weather is comfortable. Now, we obtained that $\mu_{faily low}(15^{\circ}\text{C}) = 0.86$, $\mu_{gentle}(8) = 0.5$.

Since, we have a conjunction and so, we combine them with the minimum. Therefore, the value coming as an *a* from the antecedent variables is 0.5. Therefore, now we apply implication for a = 0.5 for comfortable.

Temperature in Mot 1020 AND - word open in light Them weather in light (8) = 0.7 informe rule. ages

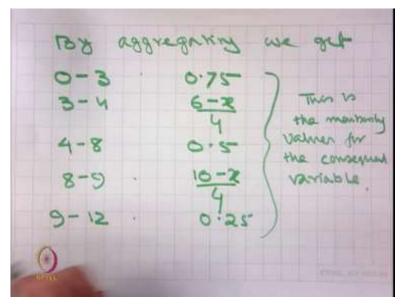
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Let us now look at rule 3 which says that

If temperature is not low AND wind speed is light THEN weather is stuffy. As before, $\mu_{not \ low}(15^{\circ}\text{C}) = 0.25$, $\mu_{light}(8) = 0.7$ Since, it is a conjunction, we use minimum and therefore, the output coming out from these two is 0.25. Now, we again apply inference rule.

Therefore, everything together after all the clipping, we get these to be the fuzzy set. So this is the fuzzy set that we obtained by using 3 rules. So we summarize this finding as follows.

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By aggregating, we get

0 - 3	0.75
3 – 4	$\frac{6-x}{4}$
4 – 8	0.5
8 – 9	$\frac{10-x}{4}$
9 - 12	0.25

So, this is the membership values for the consequent variable.

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The advantage of Mamdani's ocheme : () It is simple to understand (iii) Sol" does not require conving complicated mathe tii) The visualization method allown us to see the effect of each rule on the consequent variable

The advantage of Mamdani's scheme are the following:

- 1) It is simple to understand,
- 2) Solution does not require solving complicated mathematical equations.
- 3) The visualization method allows us to see the effect of each rule on the consequent variable.

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None we need to use Defizzification for obtaining or crisp value for "Comfortatoill'5 Index" Defuzzification can be done in many army - Centre of Maxima A Centroid Method

Now, we need to use Defuzzification for obtaining a crisp value for *comfortability index*.

Now, Defuzzification can be done in many ways

We discuss to what is called

- centre of maxima
- centroid method.

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Centre of Maxima i) theight of the fuggy set m = inf(h(s) = height)the onsport M = oup(h(s) = height) m = second (h(s) = height).. By defussification we take average of m, M $i.e = \frac{m + M}{2} = \frac{0+3}{2} = 1.5$

Centre of Maxima

Here, first we consider the height of the fuzzy set then we look at

$$m = \inf_{y}(\mu(y) = height)$$
$$M = \sup_{y}(\mu(y) = height)$$

We know that the height of the clipped fuzzy set is 0.75 and the infimum is 0 and the supremum is 3. Therefore, by defuzzification we take average of m and M that is

$$\frac{m+M}{2} = \frac{0+3}{2} = 1.5$$

Therefore, if we use center of maxima, then the output is 1.5.

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Port Controld method counseling The entire member ship curve of the clipped set obtained on the membership for consequent sariable " comfortatoilits index"

But, centroid method considers the entire membership curve of the clipped set obtained as the membership for consequent variable which in our case is the *comfortability index*.

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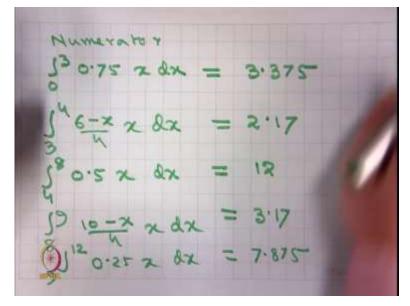
in for centroid method n Junan), x dx A Junan dr comp u

The formula for centroid method is on the entire fuzzy set *A* we take the weighted average

$$\frac{\int_A \mu_A(x) x dx}{\int_A \mu_A(x) dx}$$

So, let us compute the defuzzified value for the above problem. So, this is the membership function, we are going to use it for the defuzzification.

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So, numerator is

$$\int_{0}^{3} 0.75 \, x \, dx = 3.375$$
$$\int_{3}^{4} \frac{6 - x}{4} \, x \, dx = 2.17$$

$$\int_{4}^{8} 0.5 \, x dx = 12$$
$$\int_{8}^{9} \frac{10 - x}{4} \, x dx = 3.17$$
$$\int_{9}^{12} 0.25 \, x dx = 7.875$$

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: Numerator =

$$3:375 + 2:17 + 12 + 3:17$$

 $+7:875^{-} = 28.59$
Denominativ
 $33:75 + 2 = 2:25$
 $3:3.75 + 2 = 2:25$
 $3:6-x + 2 = 0:625$
 $3:6-x + 2 = 0:625$
 $3:6-x + 2 = 0:625$
 $3:6-x + 2 = 0:375$
 $3:6-x + 2 = 0:375$
 $3:6-x + 2 = 0:375$

Therefore, Numerator = 3.375 + 2.17 + 12 + 3.17 + 7.875 = 28.59

In a similar way, we calculate the denominator which is

$$\int_{0}^{3} 0.75 \, dx = 2.25$$
$$\int_{3}^{4} \frac{6-x}{4} \, dx = 0.625$$
$$\int_{4}^{8} 0.5 \, dx = 2$$
$$\int_{8}^{9} \frac{10-x}{4} \, dx = 0.375$$
$$\int_{9}^{12} 0.25 \, dx = 0.75$$

So together, when we sum them up, we get the value is equal to 6.

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The defussified value of the competatorility Forder in 28.59 = 4.76

Therefore, that defuzzified value of the comfortability index is

$$\frac{28.59}{6} = 4.76$$

As you can while understand that the centroid method is more computationally intensive in comparison with the centre of maximum method, but this gives a much more comprehensive value for the consequent variable.

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Note that : () There are many Implication function available in the literature. Klein-Dienes : I(a,b)
Reicher - bach
: I(A,b)
: I(A,b)
: are many
: 1-atab Mamdani's ft is ea

Note that

- There are many implication functions available in the literature as I have discussed before but still I give you a few name
 - Kleene-Dienes : $\mathfrak{T}(a, b) = \max(1 a, b)$
 - Reichen-bach: $\mathfrak{T}(a, b) = 1 a + ab$

Similarly, there are many but Mamdani's function is easy for visualization.

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we have used "choping" (2)for aggregation of rules. there is another method aling ashich main him the shape values but reduco the the output she the truth fron application The

Another important thing is that

- 2) We have used clipping for aggregation of rules. But, there is another method called scaling
 - a. which maintains the shape
 - b. but reduces the values on the basis of the output of the truth value coming from the application of the rules.

Okay friends. I stop here today. With this I conclude my topics on fuzzy logic in the next class. I shall start the concept of belief and possibility. Thank you so much.