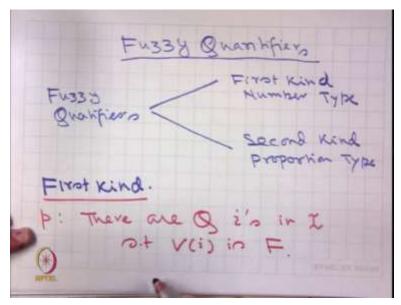
# Introduction to Fuzzy Sets Arithmetic & Logic Prof. Niladri Chatterjee Department of Mathematics Indian Institute of Technology – Delhi

# Lecture – 27 Fuzzy Sets Arithmetic & Logic

Welcome students to the MOOCs is goes on Fuzzy Sets Arithmetic and Logic. This is lecture number 27. If you remember, in the last class we started Fuzzy Quantifiers and in today's class, I shall continue with that.

### (Refer Slide Time: 00:37)



We know that fuzzy quantifiers are of two types,

one is a first kind that is number type

and other is the second kind which is proportion type.

And we were discussing first kind and we have looked at propositions such as:

p: There are Q *i's* in some set I such that V(i) is F where, F is a fuzzy set and Q is a fuzzy quantifier of first kind.

# (Refer Slide Time: 02:20)

are about 10 smalents in this class with 1000 attend The statement may THORE complicated These are about 20 students in this class "whose attendance 102 but Performance

And the example that we studied was:

There are about 10 students in this class with low attendance.

In practice this variable or say, this statement may be more complicated.

Say, for example, suppose we write

p: There are about 20 students in this class whose attendance is low but, performance is good.

So, it is a more general version we have one fuzzy quantifier.

We are looking at essentially two different fuzzy sets, which give attributes to the students.

One is attendance is low and the other one is performance is good.

### (Refer Slide Time: 04:30)

To obtain Frath value of ; use can carite p an follows: There are about 20 1000- attendance and good - performance students in the clam Again as before we need to find scalar translinality of the set E

So, to obtain to the truth value of p; we can write p as follows.

Let me call it p':

There are about 20 low attendance and good performance student in the class. And therefore, again as before we need to find scalar cardinality of a set *E*. (**Refer Slide Time: 06:05**)

here The set 35 cu ith Since both look Rt ave

Where, *E* is the set of students with *low attendance and good performance*. Since, both are fuzzy sets we look at

 $\mu_{low \ attendance}(x)$  and  $\mu_{good \ performance}(x)$  for all x in the class.

# (Refer Slide Time: 07:28)

ICW & CNI (x) in obtained

Therefore,

$$|E| = \Sigma_x \left( \mu_{low \ attendance}(x) \land \mu_{good \ performance}(x) \right)$$
$$= \Sigma_x \min \left( \mu_{low \ attendance}(x), \mu_{good \ performance}(x) \right)$$

Therefore, if we can define these two sets and we can compute the membership value for each student to both the classes by using this summation, we can get |E|.

And once |E| is obtained  $T(p) = \mu_{about \ 20}(|E|)$ , this part is very similar to our earlier example. So, I hope that you understood how to solve a problem of this type with that background.

### (Refer Slide Time: 09:24)

FU33y Quantifiers Et 2rd kind. Ex: About a quarter of the sudents of this class are B. Tech students a class description Given ose can obtain Trut spirmen the above

Let us now focus on fuzzy quantifiers of 2nd kind.

Example:

About a quarter of the students of this class are B. Tech students.

Given a class description, we can obtain truth value of the above statement *p*.

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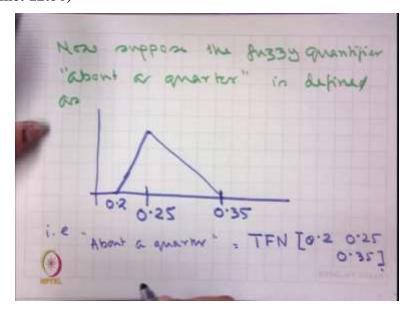
Suppose the class has 100 shall 28 are B. Tech student, we de fine or all x & class cut follows

Suppose the class has 100 students of which 28 are B. Tech students.

Therefore, for all  $x \in$  class. We define a set *B*.*Tech* as follows:

 $\mu_{B.Tech} = \begin{cases} 1 & \text{if } x \text{ is studying B. Tech} \\ 0 & \text{otherwise} \end{cases}$ 

Therefore, |E| where  $E \Rightarrow B$ . Tech student set  $= \sum_{x=1}^{100} \mu_{B.tech}(x) = 28$ (**Refer Slide Time: 12:50**)



Now, suppose the fuzzy quantifier 'about a quarter' is defined as this set that is,

About a quarter =  $TFN[0.2 \quad 0.25 \quad 0.35]$ 

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.: T(+) = / (28/100) Gbont - a - q/ nav tw = (0.28) about - e - guarter And by our definition M (0.28) = 0.7 Malout - anguarter = T(P) = 0.7

Therefore,  $T(p) = \mu_{about\ a\ quarter}\left(\frac{28}{100}\right) = \mu_{about\ a\ quarter}(0.28)$  $\frac{28}{10}$  is the proportion of student who are studying B. Tech. And by the definition that we have given

 $\mu_{about\ a\ quarter}(0.28) = 0.7$ 

Therefore, truth value of the above statement can be determined as T(p) = 0.7

#### (Refer Slide Time: 15:30)

A more difficult scenario. p: Almost all the young students of this class are performing well We asont T(D) given certain facto of the clam

Let us consider slightly more difficult scenario.

Suppose the proposition is

p: Almost all the young students of this class are performing well.

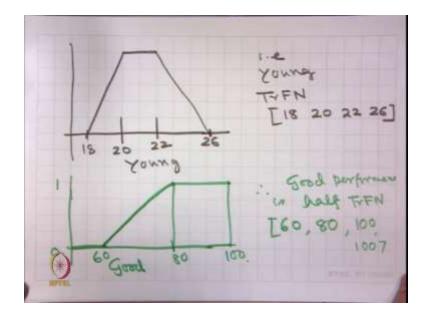
And we want T(p) given certain facts of the class and suppose we have the following fact. (Refer Slide Time: 16:42)

(muril (2) Marko MIN Sh.lut Age 900 rowny シュ ×1 24 80 Y2 85 ł 22 X2 0 75 3/4 32 73 Y4 68 Y4 0.4 25 Na 0.6 72 0.6 23 3/4 XS 1 90 22 7.6 80 1/2 24 42 2. 0.6 0 72 28 0 XY 0.4 0'4 3/4 68 23 0 4 30 90 0

Student	Age	$\mu_{Young}(x)$	Marks	$\mu_{Good}(marks)$	Min
<i>x</i> <sub>1</sub>	24		80		
<i>x</i> <sub>2</sub>	22		85		
<i>x</i> <sub>3</sub>	32		75		
<i>x</i> <sub>4</sub>	25		68		
<i>x</i> <sub>5</sub>	23		72		
<i>x</i> <sub>6</sub>	22		90		
<i>x</i> <sub>7</sub>	24		80		
<i>x</i> <sub>8</sub>	28		72		
<i>x</i> <sub>9</sub>	23		68		
<i>x</i> <sub>10</sub>	30		90		

Now, to obtain  $\mu_{Young}(x)$  and  $\mu_{Good}(marks)$ , we have to define the fuzzy set *young* and the fuzzy set *good performance*. Suppose we define them as follows.

(Refer Slide Time: 19:10)



*Young* is a trapezoidal fuzzy number *TrFN* [18 20 22 26] and *good performance* is a half *TrFN* [60 80 100 100]

Student	Age	$\mu_{Young}(x)$	Marks	$\mu_{Good}(marks)$	Mi
<i>x</i> <sub>1</sub>	24	$\frac{1}{2}$	80	1	
<i>x</i> <sub>2</sub>	22	1	85	1	-
<i>x</i> <sub>3</sub>	32	0	75	$\frac{3}{4}$	(
<i>x</i> <sub>4</sub>	25	$\frac{1}{4}$	68	0.4	-
<i>x</i> <sub>5</sub>	23	$\frac{3}{4}$	72	0.6	0
<i>x</i> <sub>6</sub>	22	1	90	1	
<i>x</i> <sub>7</sub>	24	$\frac{1}{2}$	80	1	-
<i>x</i> <sub>8</sub>	28	0	72	0.6	(
<i>x</i> 9	23	$\frac{3}{4}$	68	0.4	0
<i>x</i> <sub>10</sub>	30	0	90	1	

With these two fuzzy sets define like that, let us now fill in the above table.

Therefore, now we need to look at the minimum because we are looking at both *young* as well as *good performance*. Therefore, we look at the conjunction of this and therefore, by standard intersection or by standard fuzzy t-norm, we are using the minimum.

(Refer Slide Time: 23:40)

Scalar Cardinality | E | = 4.25 None are need to obta the proportion of young & good purforming students of the clam.

Therefore,  $|E| = \frac{1}{2} + 1 + 0 + \frac{1}{4} + 0.6 + 1 + \frac{1}{2} + 0 + 0.4 + 0 = 4.25$ 

Now, we need to obtain the proportion of young and good performing student of the class.

### (Refer Slide Time: 25:12)

So, if we define |E| = W then

$$W = \frac{|Young \cap Good \, performance|}{|Young|} = \frac{4.25}{|young|} = \frac{4.25}{4.75} = \frac{17}{19} = 0.895$$

Therefore, given the fact of the class, the truth value of the statement that, *almost the all young students of the class are performing good* is 0.895.

#### (Refer Slide Time: 27:12)

Thus for the fussy Quantifiers of 2nd kind we have the fellowing Canonicol form: p: Among the US in V Attribut. D.t. A.(U) in Fis - age Three are Q U'S Az(u) in

Thus, for the fuzzy Quantifiers of second kind, we have the following canonical form.

p: Among the v's in V such that  $A_1(v)$  is  $F_1$  there are Q v's such that  $A_2(v)$  is  $F_2$ . This is apparently complicated, but let us compare with the example just I have given. V is the set of students,  $A_1(v)$  is that attribute age,  $F_1$  is the fuzzy set young, Q is the quantifier almost all and  $A_2(v)$  is that tribute performance or marks and  $F_2$  is the fuzzy set good. (Refer Slide Time: 29:43)

p': Q Ein and Ein i.e almost all young sindents are good-performing romulades ... E, = young (age (us)) E2 = good (marks (u)) Ne conclude p: W in

So, when we have such a canonical form, we convert it into a form like this:

 $p': Q E_1$ 's are  $E_2$ 's

that is almost all young students are good performing students. Therefore,

$$E_{1} = Young(age(v))$$
$$E_{2} = Good(marks(v))$$

And we conclude p': W is Q

(Refer Slide Time: 31:07)

the meaning amere 00 ronorethed E2 in E, Total class studen Hours EZ how many

Where W is the measure of subsethood of  $E_2$  in  $E_1$ .

Graphically suppose, this is the total class this is  $E_1$  the set of students who are young and  $E_2$  is how many of them are good.

(Refer Slide Time: 32:31)

Here EIL = Z Lyong (age (us)) NE21 = Z min (Lyong (etc) 'Mgood [E10E2] = we general an follows

Here

$$|E_1| = \Sigma_v \mu_{Young} (age(v))$$
$$|E_1 \cap E_2| = \Sigma_v \min \left( \mu_{Young} (age(v)), \mu_{Good} (marks(v)) \right)$$

Therefore, we can generalize as follows.

## (Refer Slide Time: 33:43)

$$|E_i| = \sum_{i} f_{i} (A_i(u))$$

$$|E_i \cap E_2| = \sum_{i} \min_{i} (f_{i} (A_i(u)))$$

$$\int_{i} (A_i(u))$$

$$\int_{i} (A_i(u)$$

 $|E_1| = \Sigma_{\nu} \mu_{F_1} \big( A_1(\nu) \big)$ 

$$|E_{1} \cap E_{2}| = \Sigma_{v} \min\left(\mu_{F_{1}}(A_{1}(v)), \mu_{F_{2}}(A_{2}(v))\right)$$
$$\therefore W = \frac{|E_{1} \cap E_{2}|}{|E_{1}|}$$

This is the ratio that one can use to obtain the truth value for p.

Obviously,  $0 \le W \le 1$ 

### (Refer Slide Time: 35:24)

Inference from Quantifiel proportitions All quantified propositions can be written as Wing  $W = |E| - W_{P} = \frac{|E| \cap E_{2}}{|E|}$ where

Now let us look at inference from quantified propositions.

We have seen that all quantified propositions can be written as

*p*: *W* is *Q* 

where W = |E| for first kind that is number type of fuzzy sets or  $W = \frac{|E_1 \cap E_2|}{|E_1|}$  for second kind of quantifiers.

(Refer Slide Time: 37:00)

et purpose The inform city in that aman lified giv erm propaniki what can deduce then 1 from each W b :

Now the purpose of inferencing is that, given n quantified propositions, what we can infer or what can be deduced from them.

That is, if we have

$$p_1: W_1 \text{ is } Q_1$$

$$p_2: W_2 \text{ is } Q_2$$

$$\vdots$$

$$p_n: W_n \text{ is } Q_n$$

And our aim is to find a statement p: W is Q and we try to obtain it truth value.

Here. Each  $Q_i$  is a quantifier and each  $W_i$  is the cardinality of an appropriate fuzzy set.

# (Refer Slide Time: 39:11)

The generalized version 51 informers in that If these exist functions  $f(\omega, \dots, \omega_n) \leq \omega \leq g(\omega, \dots, \omega_n)$ Them one may conclude P: Wing if g is at least  $f(g_1 - Q_n)$ 2 at most  $g(g_1 - g_n)$ 

Now, the generalized version of inference is that:

If there exists functions  $f(W_1, W_2 \dots W_n) \le W \le g(W_1, W_2 \dots W_n)$ 

Then one may conclude p: W is Q

if Q is at least  $f(Q_1, ..., Q_n)$  and at most  $g(Q_1, ..., Q_n)$ . Therefore, if we could obtain such functions, then we can conclude W is Q.

(Refer Slide Time: 41:00)

In practice we use a simpler versin: If there exists a function  $f(\omega_1, \cdots, \omega_n) = \omega_0$ E F(Q1 - QW) - Q them we can infor prov the proposition: P. ... Pr.

In practice we use a simpler version, which says that

If there exists a function f such that  $f(W_1, ..., W_n) = W$  and  $f(Q_1, ..., Q_n) = Q$ . Then we can infer that p: W is Q from the propositions  $p_1, p_2, ..., p_n$ .

(Refer Slide Time: 42:24)

Il worran m preparoi him. rains on about to inter

For illustration, considered these two propositions:

 $p_1$ : There are about 100 days in a year when it rains.

So, this is a statement with a fuzzy quantifier of first kind *about* 100.

 $p_2$ : On about half of the rainy days there is traffic jam

So, this is a fuzzy quantifier of the second kind.

So, we want to infer *p*: *There are Q traffic jam days in a year*.

I think the idea is clear. We have some fuzzy quantifier related statement about on how many days there is rain and a proportional statement on how many days there is a traffic jam. So, we want to infer about on a how many days in a year there is a traffic jam. We want to have an idea of this fuzzy quantifier Q.

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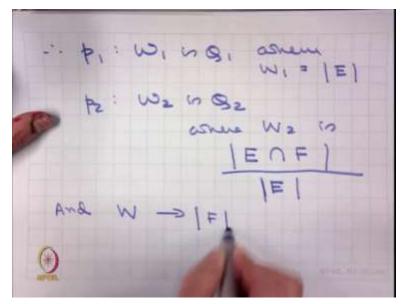
genrabize : Q1 = About 100 - Jumps Q2 = About half - Humps et E & F be the set of days in a year. athen E as taimy days F as majfic sam days

So to generalize this we have

$$Q_1 = About \ 100 \ - \text{from } p_1$$
  
 $Q_2 = About \ half \ - \text{from } p_2$ 

And let *E* and *F* be the set of days in a year when *E* corresponds to rainy days and *F* corresponds to traffic jam days.

(Refer Slide Time: 46:24)



Therefore, we have  $p_1: W_1$  is  $Q_1$  where  $W_1 = |E|$ and  $p_2: W_2$  is  $Q_2$  where,  $W_2$  is the proportion  $\frac{|E \cap F|}{|E|}$  Assume that traffic jam causes only with rainy days. So,  $\frac{|E \cap F|}{|E|}$  gives us the proportion on how many of the rainy days there is going to be traffic jam. And *W* is going to be the cardinality of *F*. (**Refer Slide Time: 47: 42**)

Typically are consider f(a,b) = ab  $f(\omega, \omega_2) = \omega_1, \omega_2$ = |E| . [ENF] multiplication

We can consider typically, we consider f(a, b) = ab

If you remember, we have stated this that from the given statements, these are the quantifiers, we are trying to get F and  $W_1, W_2 \dots W_n$  are the different fuzzy sets defined on the set of elements in our case the set of days.

Therefore,  $f(W_1, W_2) = W_1 W_2 = |E| \frac{|E \cap F|}{|E|} = |E \cap F| = W$ 

That is, this is the number of traffic jam days among the rainy days. So, what we do fuzzy multiplication.

(Refer Slide Time: 49:30)

Suppose we have Eon an Del HAL Q2 = About half TEN [.4.5.6]

Suppose, we have

$$Q_1 = About \ 100 : TFN[90 \ 100 \ 110]$$
  
 $Q_2 = About \ half : TFN[0.4 \ 0.5 \ 0.6]$ 

(Refer Slide Time: 50:29)

TE Obtain & are TIMIKPY TEN [90 100 110] By the TFN [0:4 0:5 06] > nearb 100 By ashich is amanifier of first kind of second ind

To obtain Q we multiply the  $TFN[90\ 100\ 110]$  by the  $TFN[0.4\ 0.5\ 0.6]$ .  $Q_1 = nearly\ 100$  which is a quantifier of first kind and  $Q_2 = about\ half$  is of second kind. (Refer Slide Time: 51:51)

$$f = [90 + 0.4, 10 + 0.6]$$

$$= [35, 66]$$

$$I = [100 + 0.5] = 50$$

$$In general$$

$$d = [90 + 10d, 10 - 10d]$$

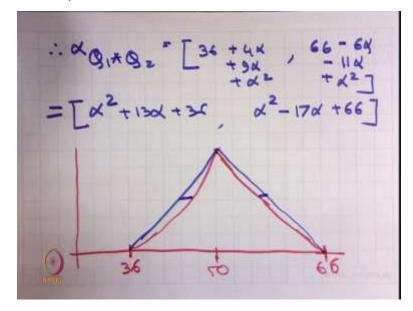
$$d = [0.4 + 0.1k, 0.6 - 0.1k]$$

Therefore,  ${}^{0+}Q = [90 * 0.4, 110 * 0.6] = [36, 66]$  ${}^{1}Q = [100 * 0.5] = 50$ 

In general

$${}^{\alpha}Q_1 = [90 + 10\alpha, 110 - 10\alpha]$$
$${}^{\alpha}Q_2 = [0.4 + 0.1\alpha, 0.6 - 0.1\alpha]$$

(Refer Slide Time: 53:09)



Therefore,

$${}^{\alpha}(Q_1 * Q_2) = [36 + 4\alpha + 9\alpha + \alpha^2, 66 - 6\alpha - 11\alpha + \alpha^2]$$
$$= [\alpha^2 + 13\alpha + 36, \alpha^2 - 17\alpha + 66]$$

Therefore, if we plot it, it appears like a triangular fuzzy number, but we know that it is not. In fact, it is going to be quadratic equation. So, we can expect a shape like this. And, we have already studied that such type of shapes can be approximated by triangular fuzzy number provided the maximum gap is within a threshold. So like that we can estimate the value of Q.

### (Refer Slide Time: 55:01)

From fuzzy quantifiers can be further extended an fullows: product pyllogiom. Pi: Q, E's are F's P2: Q2 (En & Fin) are Q's P: GIQ2 E's an Fizch

The concept of inferencing from fuzzy quantifiers can be further extended as follows. One of them is called product syllogism that is, suppose we have proposition

 $p_1: Q_1 E's are F's$  $p_2: Q_2 (E's and F's)are G's$ 

From that to want to conclude by a similar way is  $p: Q_1Q_2 E's$  are F's and G's

(Refer Slide Time: 56:38)

6 We asknot to infor ROF out of E. Win Q1 propersion 2 W2 in Q2 propersion about ENF

We want to infer about  $G \cap F$  out of *E*. Therefore, we can write it as

 $p_1: W_1 \text{ is } Q_1 \text{ proportion } \frac{|E \cap F|}{|E|}$  $p_2: W_2 \text{ is } Q_2 \text{ proportion } \frac{|E \cap F \cap G|}{|E \cap F|}$ 

(Refer Slide Time: 58:10)

. We infor: W = WIW2 = ENF X ENFOR = <u>lenenaj</u> lei brokon in Dre dranklige Fri

Therefore, we infer

$$W = W_1 W_2 = \frac{|E \cap F|}{|E|} \times \frac{|E \cap F \cap G|}{|E \cap F|} = \frac{|E \cap F \cap G|}{|E|}$$

. That is, it gives the proportion type quantifier for Q E's are F's and G's.

(Refer Slide Time: 59:18)

The other type of information in Consoment conjunction syllogiam p: Bi En are Fo
p: B En are Fo ≥ a

The other type of inferencing is called consequent conjunction syllogism where

p<sub>1</sub>: Q<sub>1</sub> E's are F's
p<sub>2</sub>: Q<sub>2</sub> E's are G's

and we want to infer

p: Q E's are F's and G's.

(Refer Slide Time: 1:00:29)

Horo many me F's 2 G

We want to infer about how many of the *E*'s are *F*'s and *G*'s.

(Refer Slide Time: 1:00:59)

Here & in given my [ > MAX (0, 9,+92-1)] NIE MIN (Q. Q.)] conce we have defined MIN and MAX earlier

So, here Q is given by

 $[\ge MAX(0, Q_1 + Q_2 - 1)] \cap [\le MIN(Q_1, Q_2)]$ 

where we have defined MIN and MAX earlier.

(Refer Slide Time: 1:01:59)

 $\int_{M}^{(3)} (3)$ =  $\log q$  min  $(h_{A}^{(x)}, h_{B}^{(y)})$ =  $\log q$  min  $(h_{A}^{(x)}, h_{B}^{(y)})$   $g = \min(x, y)$  more  $(h_{A}^{(h)})$   $\int_{M}^{(3)} (3) = \log \max(h_{A}^{(h)})$   $\int_{M}^{(3)} (x, y) = 3 = \max(x, y)$   $h_{B}^{(y)})$ 

For your recollection,

$$\mu_{MIN(A,B)}(z) = \sup_{z=\min(x,y)} \min(\mu_A(x), \mu_B(y))$$
$$\mu_{MAX(A,B)}(z) = \sup_{z=\max(x,y)} \min(\mu_A(x), \mu_B(y))$$

Like that, when we are given different proportion type quantifiers we can infer about some other statement or other proposition p, when we are given  $p_1, p_2 \dots p_n$ , that is n different quantifier propositions that are quantifier or the proportion.

Okay students, with that I stop fuzzy quantifier. In the next class, I shall look into very interesting type of fuzzy inference system, which is called Mamdani scheme for inferencing given a set of propositions or fuzzy propositions and an input variable. Till then thank you.