Introduction to Fuzzy Sets Arithmetic & Logic
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Lecture – 26
Fuzzy Sets Arithmetic & Logic

Welcome students to the MOOCs lecture on Fuzzy Sets Arithmetic and Logic. And, this is lecture number 26.

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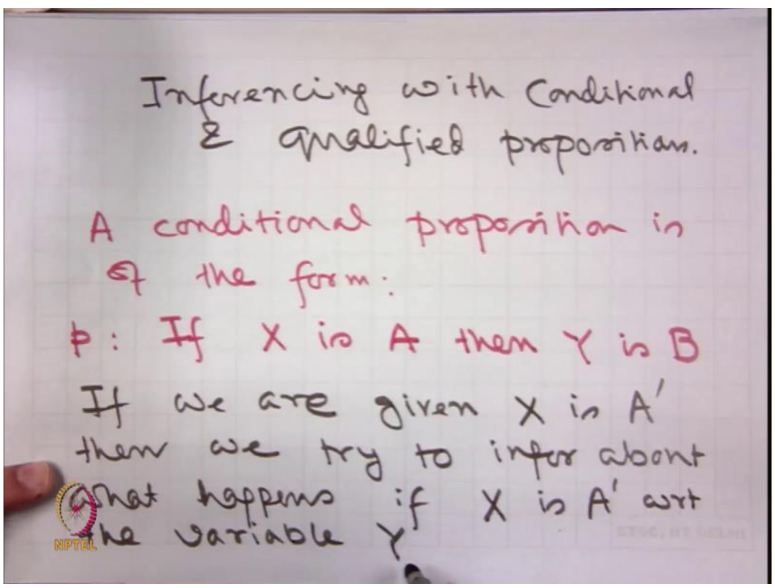

As I said at the end of the last class that today we shall discuss inferencing with conditional and qualified propositions.

We know a conditional proposition is of the form.

$p: If\ X\ is\ A, then\ Y\ is\ B$ and we have seen that,

If we are given X is A' which is another fuzzy set then, we try to infer about what happens if X is A' with respect to the variable Y.

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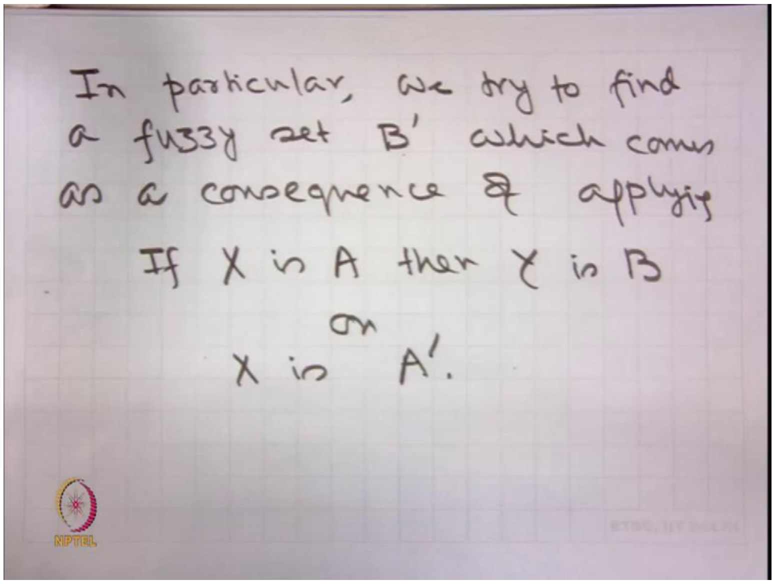

In particular, we try to find a fuzzy set B' which comes as a consequence of applying

$If\ X\ is\ A, then\ Y\ is\ B$ on $X\ is\ A'$

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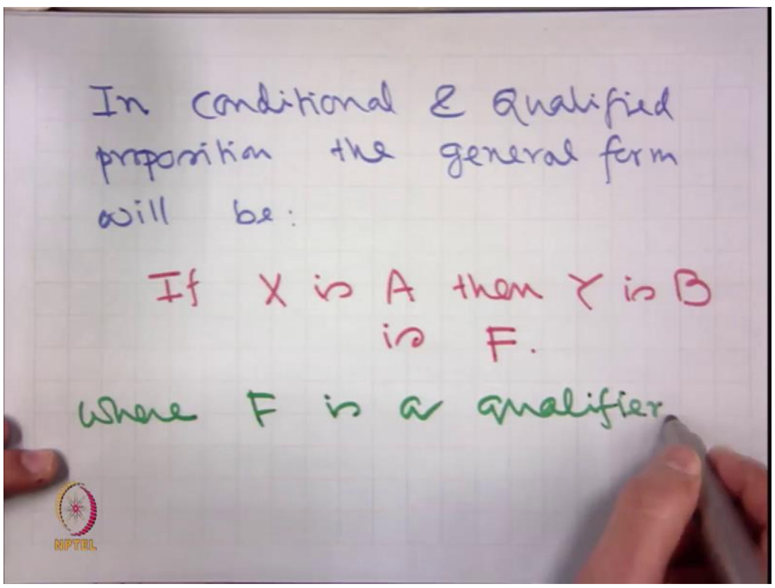

In conditional and qualified proposition, the general form will be

$$If\ X\ is\ A, then\ Y\ is\ B\ is\ F$$

where F is a qualifier.

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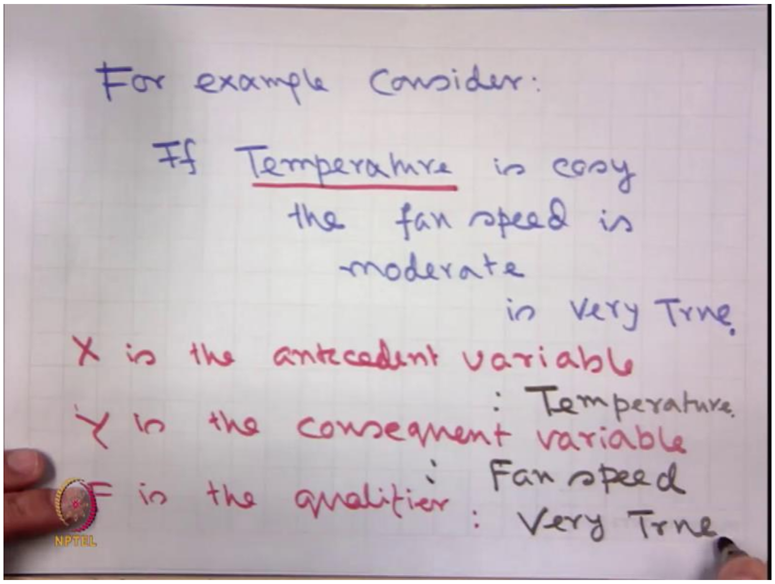

For example consider:

$$If\ Temperature\ is\ Cosy\ then, Fan\ Speed\ is\ Moderate\ is\ Very\ True.$$

So, here X is the antecedent variable. And in case of this example, which is $Temperature$.

Y is the consequent variable which in this case is $Fan\ Speed$ and F is the qualifier and which in our case is $Very\ True$.

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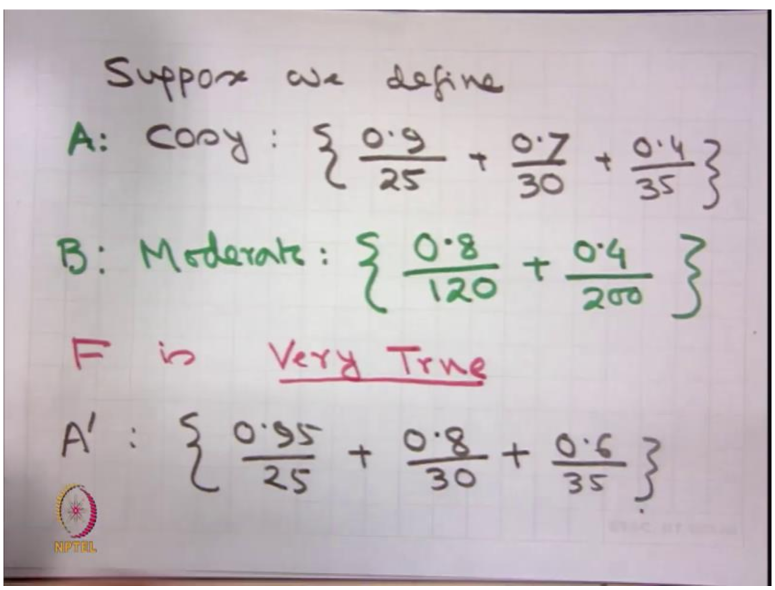

And, suppose we define

$$A: Cosy = \left\{\frac{0.9}{25} + \frac{0.7}{30} + \frac{0.4}{35}\right\}$$

$$B: Moderate = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$$

Since F is given to be $Very\ True$. Thus, it is differing from an unqualified proposition.

Suppose $A' = \left\{\frac{0.95}{25} + \frac{0.8}{30} + \frac{0.6}{35}\right\}$

So, this is the problem, we are given that

$$If\ Temperature\ is\ Cosy\ then, Fan\ Speed\ is\ Moderate\ is\ Very\ True.$$

and we need to infer about when it is given that the $Temperature$ comes from the set A' with the given membership.

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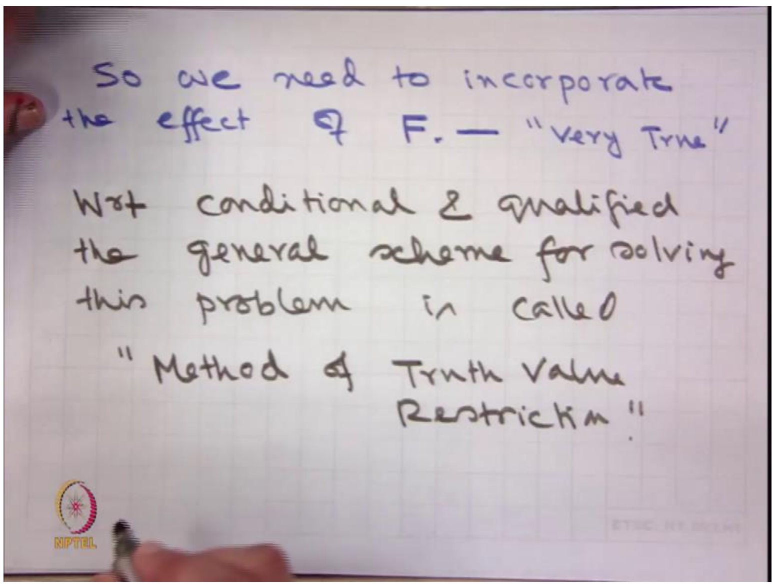

So, we need to incorporate the effect of F which in our case is $Very\ True$.

With respect to conditional and qualified, the general scheme for solving this problem is called '*Method Of Truth Value Restriction'* and it works as follows.

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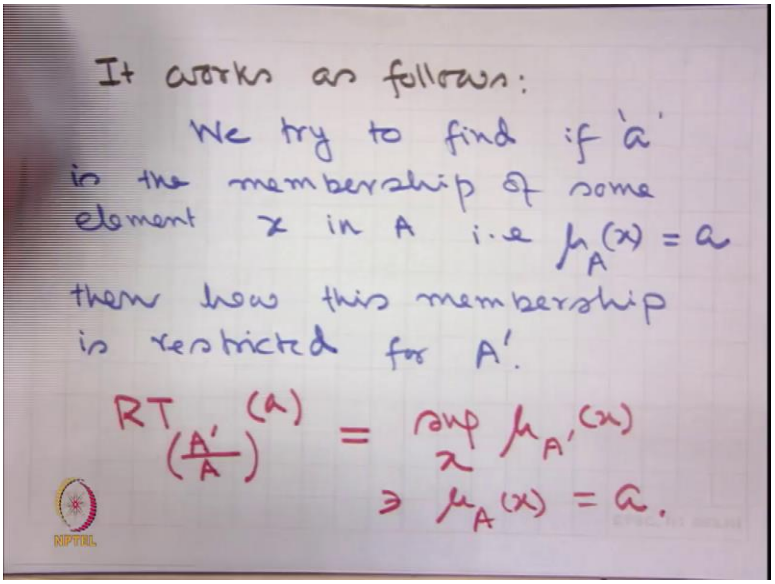

We look at or we try to find if a is the membership of some element x in A that is $\mu_A(x) = a$ then, how this membership is restricted for A'

The notation for this is $RT_{\left(\frac{A'}{A}\right)}(a) = \sup_{x \ni \mu_A(x)=a} \mu_{A'}(x)$

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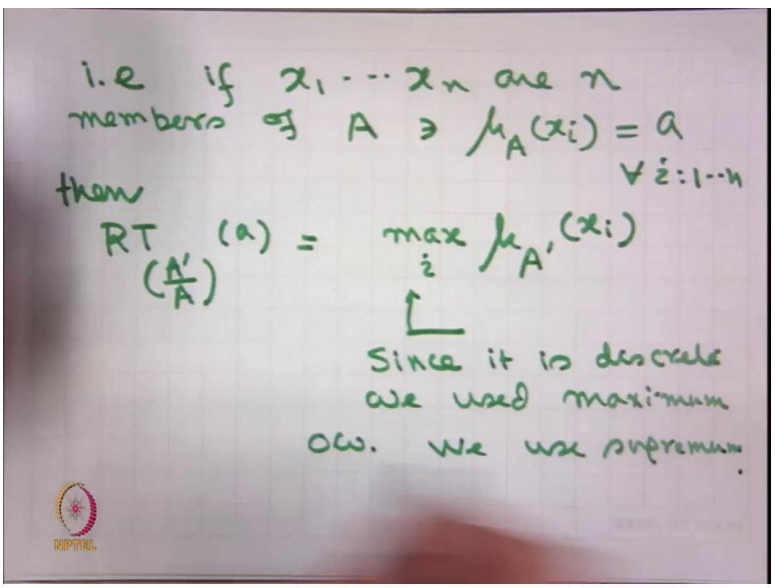

That is, if $x_1, x_2 \dots x_n$ are n members of A such that $\mu_A(x_i) = a$ for all $i = 1, 2, \dots, n$ then, the restricted to the value of a

$$RT_{\left(\frac{A'}{A}\right)}(a) = \max_{i} \mu_{A'}(x_i)$$

Note that since it is discrete, we used maximum otherwise, we use supremum.

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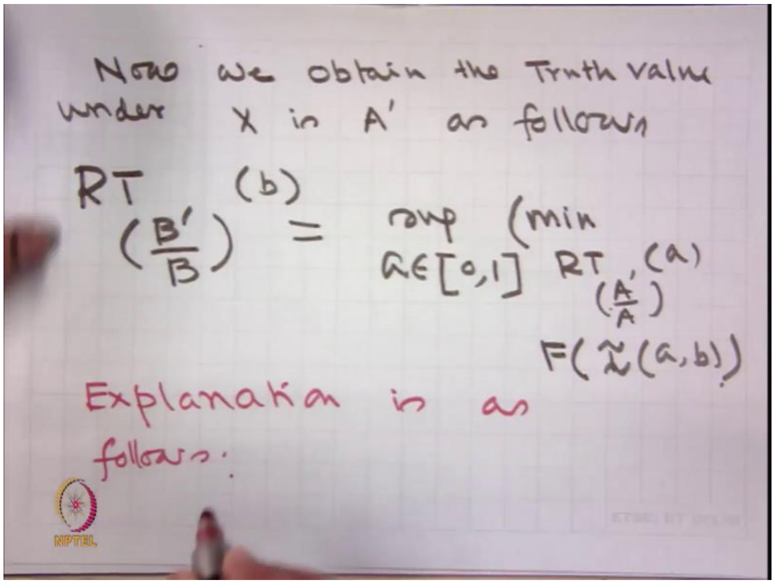

Now, we obtain the truth value under X is A' as follows:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \sup_{a\in[0,1]}\left(\min\left\{RT_{\left(\frac{A'}{A}\right)}(a), F\big(\mathfrak{I}(a,b)\big)\right\}\right)$$

This is a fairly complicated notation. So, let me explain that.

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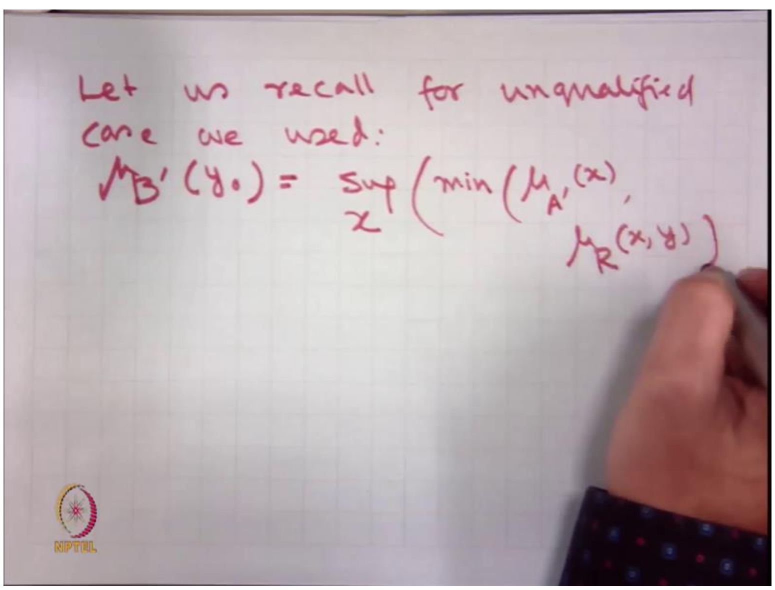

Let us recall that for unqualified case we used:

$$\mu_{B'}(y_0) = \sup_x\big(\min\big(\mu_{A'}(x), \mu_R(x,y)\big)\big)$$

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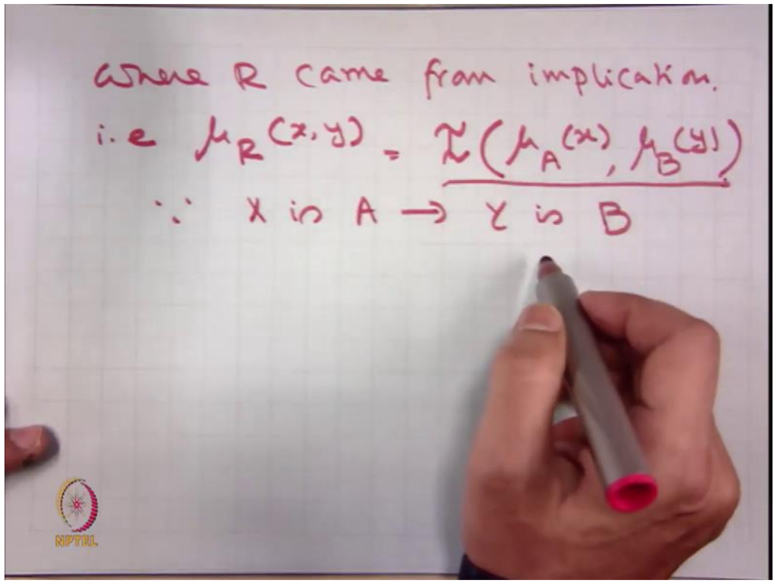

Where R came from implication that is

$$\mu_R(x, y) = \mathfrak{I}\big(\mu_A(x), \mu_B(y)\big)$$

That is, since X is $A \rightarrow Y$ is B, this implication is captured through the function $\mathfrak{I}$.

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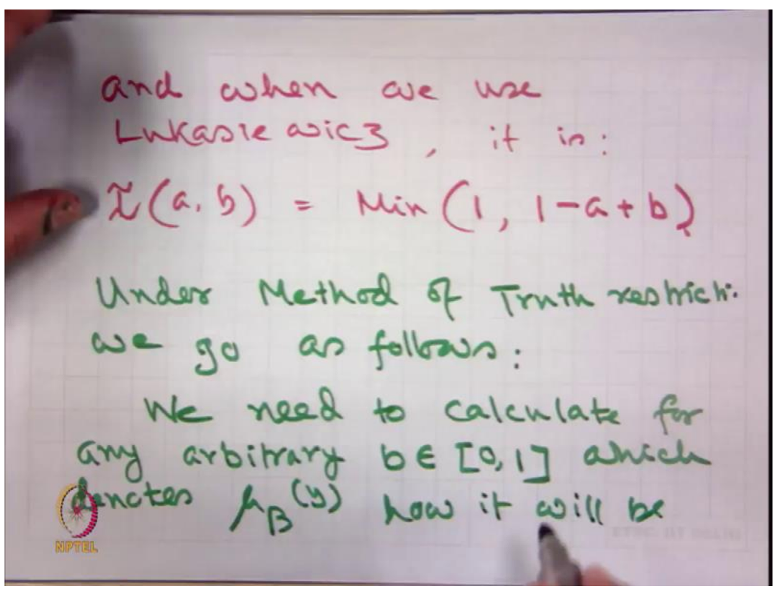

And when we use Lukasiewicz it is

$$\mathfrak{I}(a, b) = \min(1, 1 - a + b)$$

Under method of truth restriction, we go as follows.

We need to calculate for any arbitrary $b \in [0, 1]$ which denotes $\mu_B(y)$, how it will change or how it will be restricted when $X = A'$

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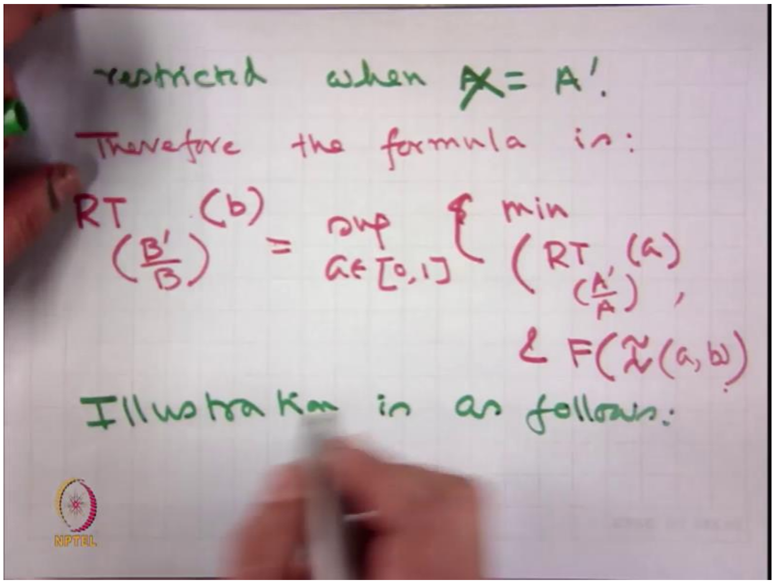

Therefore, the formula is:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \sup_{a\in[0,1]}\left(\min\left\{ RT_{\left(\frac{A'}{A}\right)}(a), F\big(\mathfrak{I}(a,b)\big) \right\}\right)$$

So, let me illustrate with an example. So the problem will be clear.

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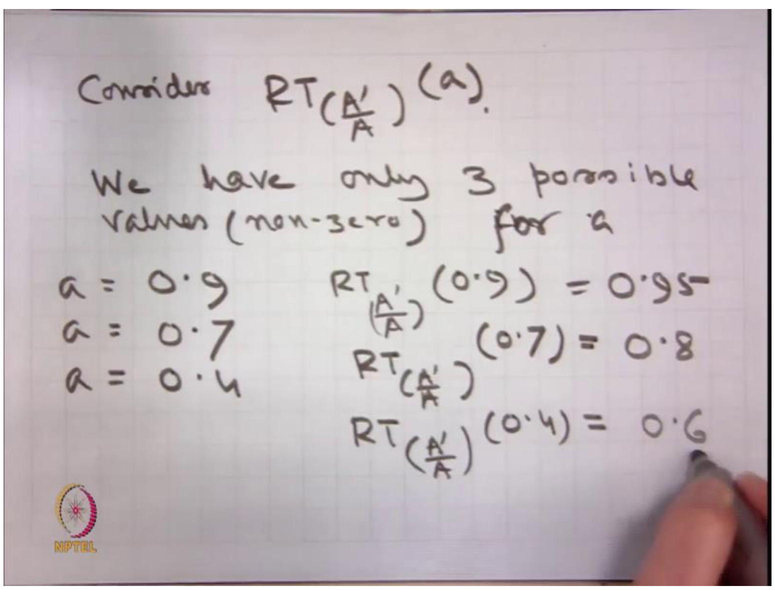

So, consider the problem that I have stated earlier that

$$A{:}\,Cosy = \left\{\frac{0.9}{25} + \frac{0.7}{30} + \frac{0.4}{35}\right\}$$

$$B: Moderate = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$$

Since F is given to be $Very\ True$. What is going to be the membership values under the B'?

So, consider $RT_{\left(\frac{A'}{A}\right)}(a)$ for any a.

We have only 3 possible values, I mean none zero values in A for a

$a = 0.9$ $\qquad RT_{\left(\frac{A'}{A}\right)}(0.9) = 0.95$

$a = 0.7$ $\qquad RT_{\left(\frac{A'}{A}\right)}(0.7) = 0.8$

$a = 0.4$ $\qquad RT_{\left(\frac{A'}{A}\right)}(0.4) = 0.6$

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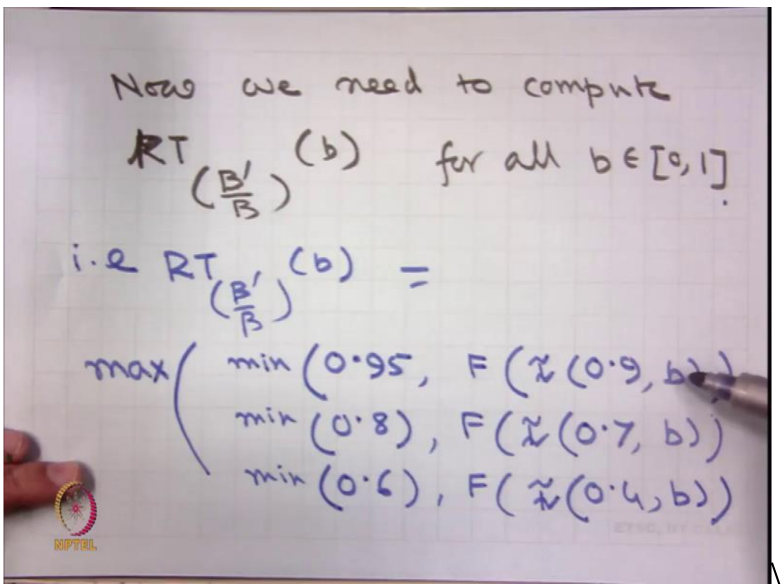

Now, we need to compute $RT_{\left(\frac{B'}{B}\right)}(b)$ for all $b \in [0, 1]$

That is, $RT_{\left(\frac{B'}{B}\right)}(b) = \max \begin{pmatrix} \min\left(0.95, F\left(\mathfrak{T}(0.9, b)\right)\right) \\ \min\left(0.8, F\left(\mathfrak{T}(0.7, b)\right)\right) \\ \min\left(0.6, F\left(\mathfrak{T}(0.4, b)\right)\right) \end{pmatrix}$

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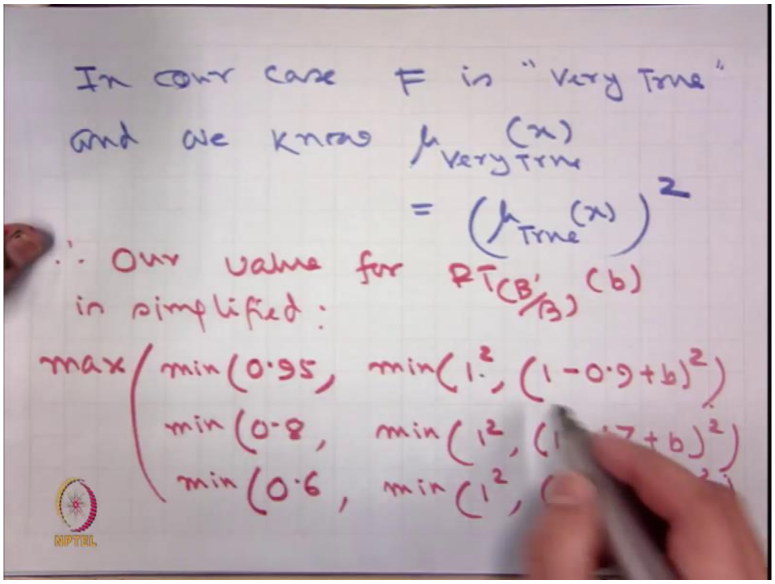

In our case F is $Very\ True$ and we know $\mu_{Very\ True}(x) = \left(\mu_{True}(x)\right)^2$

Therefore, our value for $RT_{\left(\frac{B'}{B}\right)}(b)$ is simplified as follows:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \max\begin{pmatrix} \min(0.95, (\min(1, 1 - 0.9 + b))^2) \\ \min(0.8, (\min(1, 1 - 0.7 + b))^2) \\ \min(0.6, (\min(1, 1 - 0.4 + b))^2) \end{pmatrix}$$

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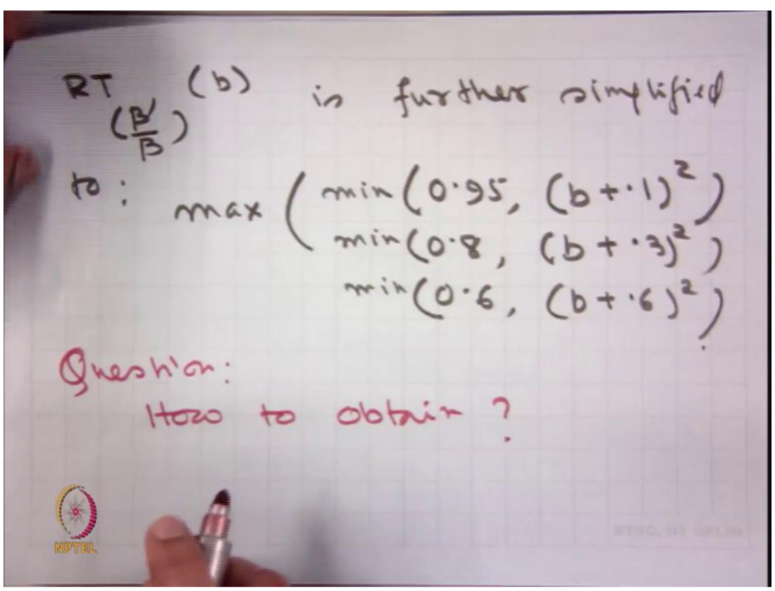

Therefore, $RT_{\left(\frac{B'}{B}\right)}(b)$ is further simplified to:

$$\max\begin{pmatrix} \min(0.95, (b+0.1)^2) \\ \min(0.8, (b+0.3)^2) \\ \min(0.6, (b+0.6)^2) \end{pmatrix}$$

Question is how to obtain this?

So, we go as follows we use a graphical method to solve this problem.

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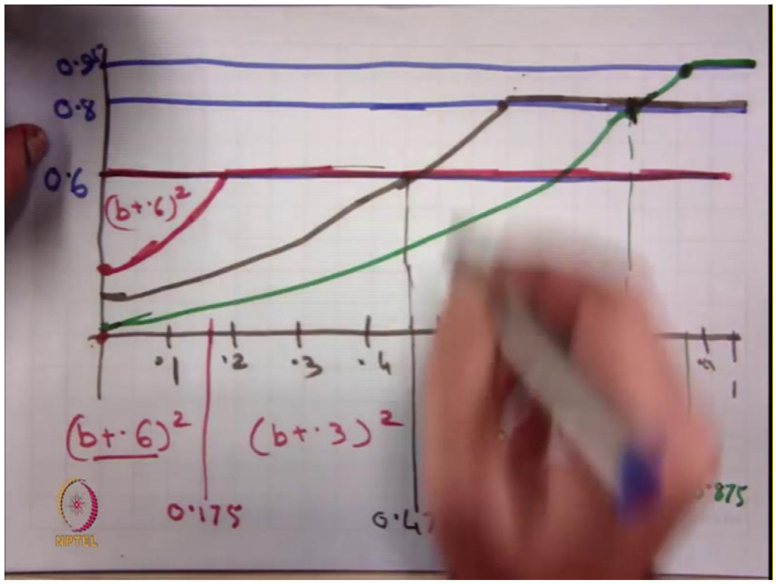

So, using the graph we see that, we can get different intervals in which different equations will serve as that maximum. And we can summarize this in the following way.

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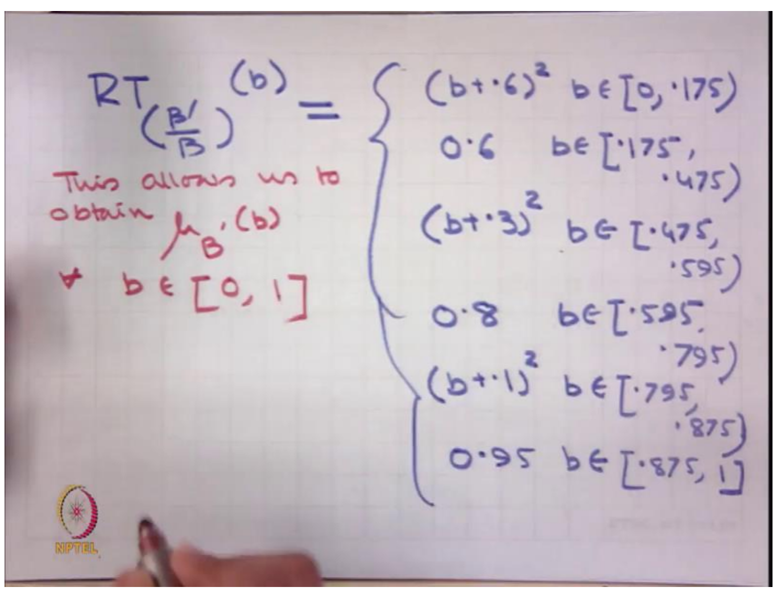

$$RT_{\left(\frac{B'}{B}\right)}(b) = \begin{cases} (b+0.6)^2 & b \in [0, 0.175) \\ 0.6 & b \in [0.175, 0.475) \\ (b+0.3)^2 & b \in [0.475, 0.595) \\ 0.8 & b \in [0.595, 0.795) \\ (b+0.1)^2 & b \in [0.795, 0.875) \\ 0.95 & b \in [0.875, 1] \end{cases}$$

So, this allows us to obtain $\mu_{B'}(b)$ for all values of b

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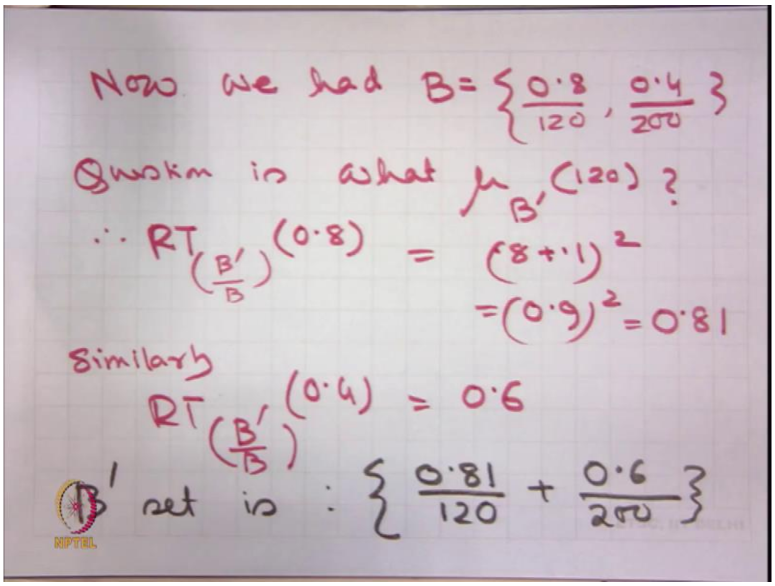

Now we had $B = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$

Question is, what is $\mu_{B'}(120)$?

$\therefore RT_{\left(\frac{B'}{B}\right)}(0.8) = (0.8 + 0.1)^2 = 0.9^2 = 0.81$

Similarly, $RT_{\left(\frac{B'}{B}\right)}(0.4) = 0.6$

Therefore, the conclusion is that the B'set is $\left\{\frac{0.81}{120} + \frac{0.6}{200}\right\}$

Like that, we could infer about the consequent variable with the help of the given implication and that the input is X is A'.

Okay. So, this is how we infer about conditional and qualified propositions..

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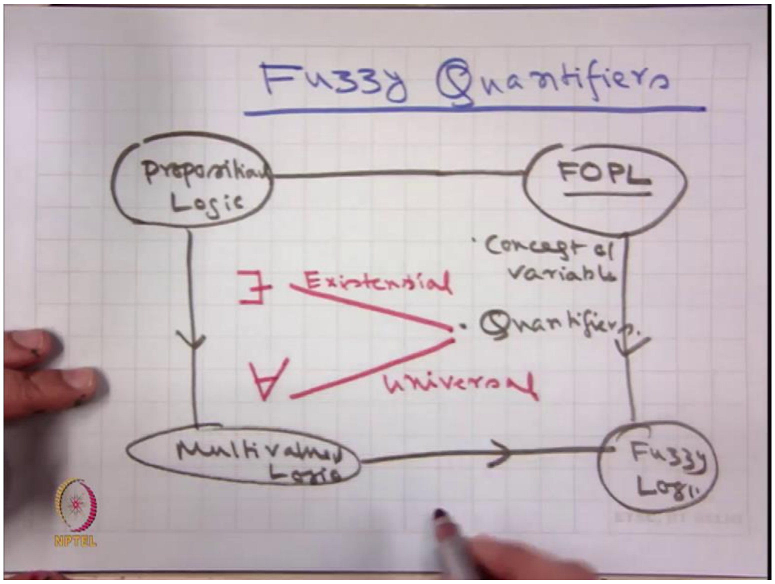

Next we want to study Fuzzy quantifiers

If you remember we started that when we have propositional logic we have expanded in two directions.

One is multi valued logic which we have been using for last few lectures and another way of extending it was through first order predicate logic and that is also contributing to fuzzy.

In fact, first order predicate logic gave us one is the concept of variables. And it has given concept of quantifiers.

In particular, we have used two types of quantifier one is ∃ which is existential quantifier and the other one is ∀ which is universal quantifier.

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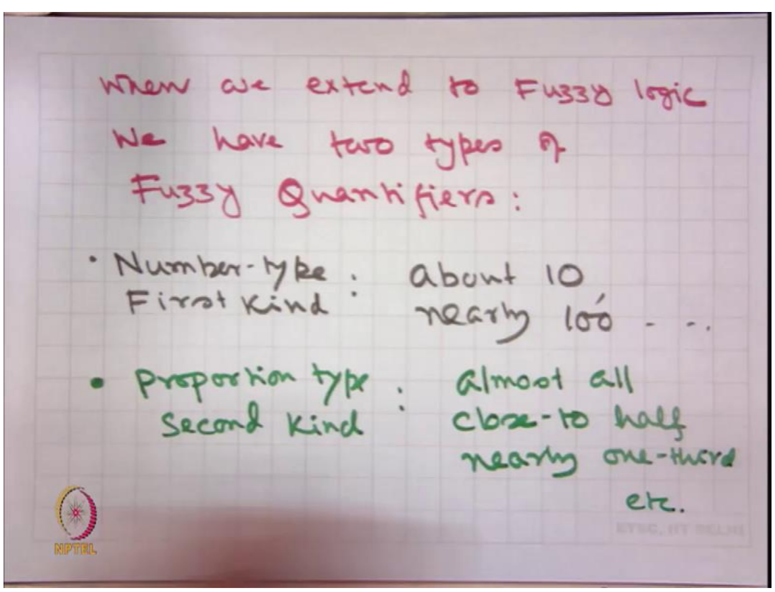

Similarly, when we extend it to fuzzy logic we have 2 types of Fuzzy Quantifiers:

1. Number type are often it is called First kind.

   That is here, we look at some fuzzy numbers.

   For example, $about$ 10, $nearly$ 100 etc.

2. Proportion type or second kind.

   Here, we look at instead of absolute values of Fuzzy way of describing the proportion.

   Example, $almost\ all$, $close\ to\ half$, $nearly\ one\ third$ etc.

Actually in our daily life, we use such fuzzy quantifiers

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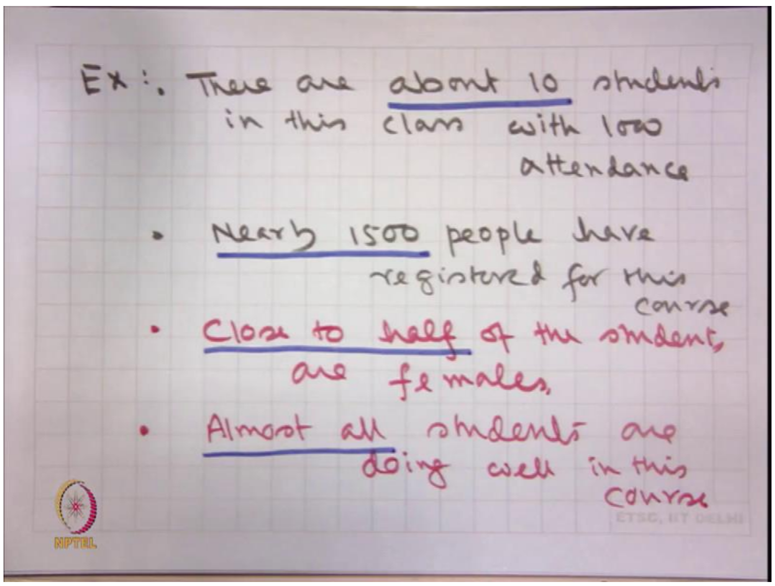

For example:

- There are $about$ 10 students in this class with low attendance or
- $Nearly$ 1500 people have registered for this course.

So, these are examples of fuzzy quantifier of first kind.

- $Close\ to\ half$ of the students are females.
- $Almost\ all$ students are doing well in this course.

These are fuzzy quantifier of second kind.

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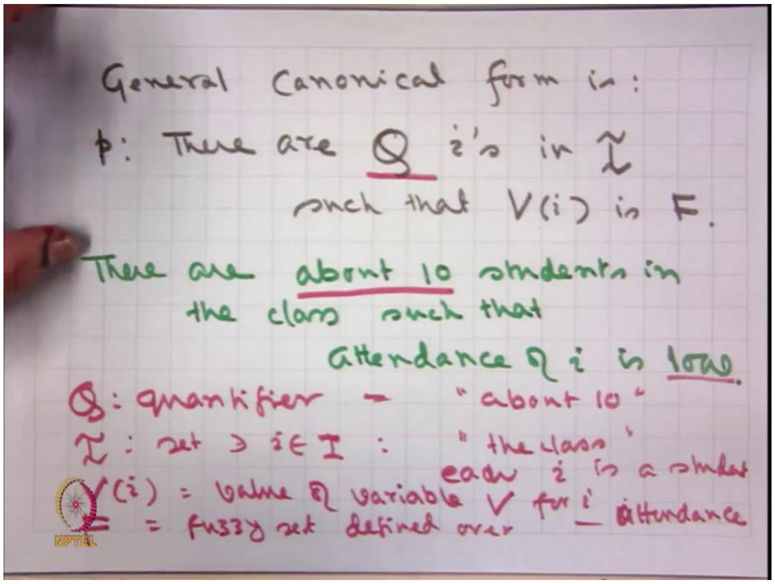

General canonical form is:

$$p\text{: There are } Q\ i\text{'s in } \mathfrak{T} \text{ such that } V(i) \text{ is } F$$

Say for example, consider

There are *about* 10 students in the class such that *attendance* of i is *low*.

Now, if we compare we can see that

Q: quantifier   *about* 10

$\mathfrak{T}$: the set such that $i \in \mathfrak{T}$. And in our case, $\mathfrak{T}$ is the *class* and each i is a *student*,

$V(i)$: value of variable V for i which in this case is *attendance* of i

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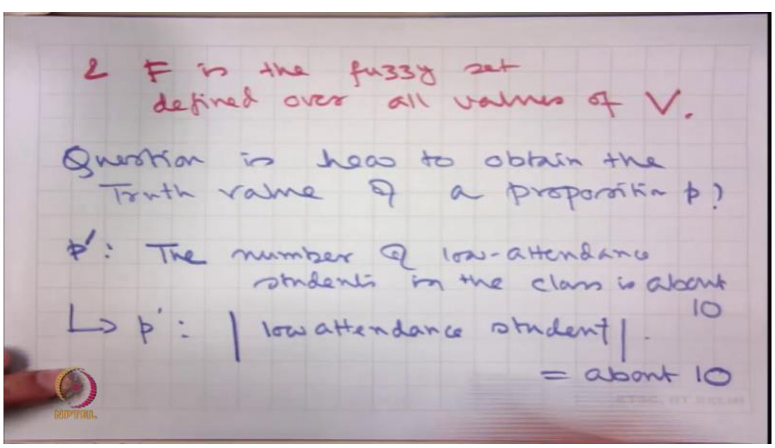

And, F is the fuzzy set defined over all values of V

Question is How to obtain the truth value of a proposition p?

So, from the above p we write p' as follows:

p': *The number of low attendance students in the class is about* 10

that is $p' = |low\ attendance\ student| = about\ 10$

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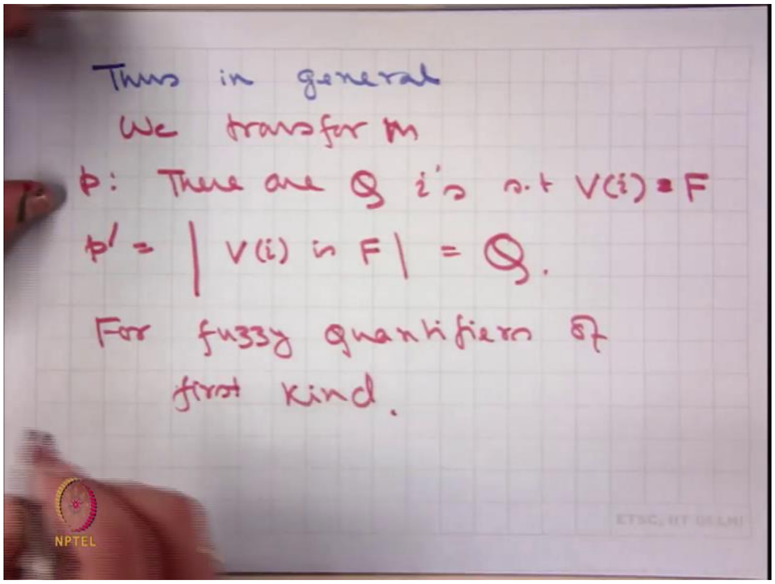

Thus in general, we transform

p: *There are* Q *i's such that* $V(i) = F$.

And this we transform to $p' = |V(i) \text{ is } F| = Q$

And this we do for fuzzy quantifiers of first kind.

So, how to solve this problem?

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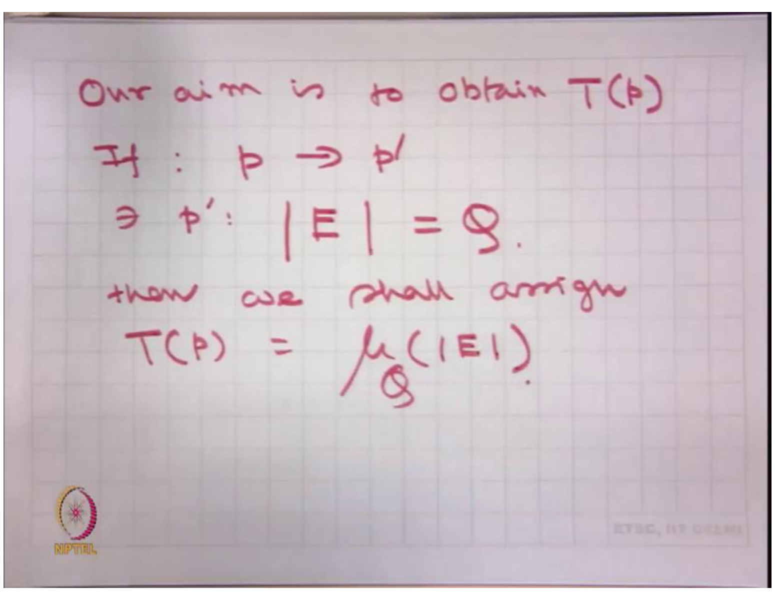

Finally, we will.

Our aim is to obtain truth value of $T(p)$

For which we will see if p goes to p' such that p': $|E| = Q$ then we shall assign $T(p) = \mu_Q(|E|)$

So, let me illustrate with an example.

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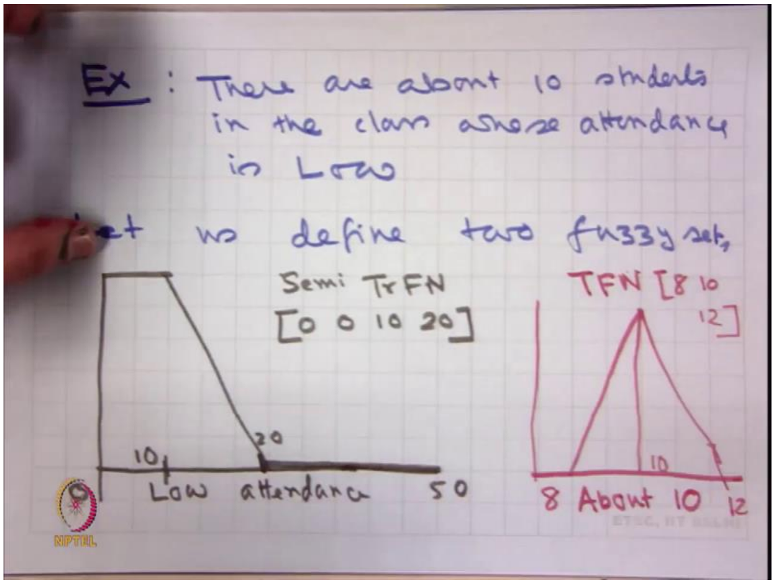

Example: $There\ are\ about\ 10\ students\ in\ the\ class\ whose\ attendance\ is\ Low.$

So, let us define two fuzzy sets as follows:

1. Fuzzy set $Low\ Attendance$ is defined over the set 0 to 50.

   If there are 50 lectures such that anything between 0 to 10 is low and between 10 to 20 it decreases like a straight line, above 20, the membership is zero. Therefore, it is a semi trapezoidal fuzzy number. $Low\ Attendance = [0 \quad 0 \quad 10 \quad 20]$

2. The second fuzzy sets is $about$ 10. And suppose we define it as follows. At 10 it is 1 and 8 to 12 it gives us triangular fuzzy number $[8 \quad 10 \quad 12]$

So, our aim is given a fact what is the true value of the statement the fact is as follows.

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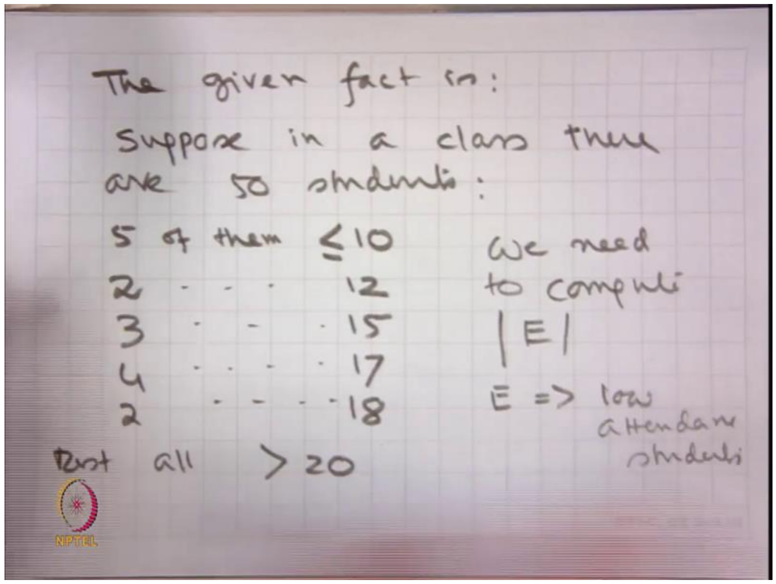

Suppose, in a class there are 50 students:

- 5 of them have attendance ≤ 10
- 2 of them have attendance ≤ 12
- 3 of them have attendance ≤ 15
- 5 of them have attendance ≤ 17
- 2 of them have attendance ≤ 18
- Rest all > 20.

Therefore, we need to compute the $|E|$ where E is low attendance students.

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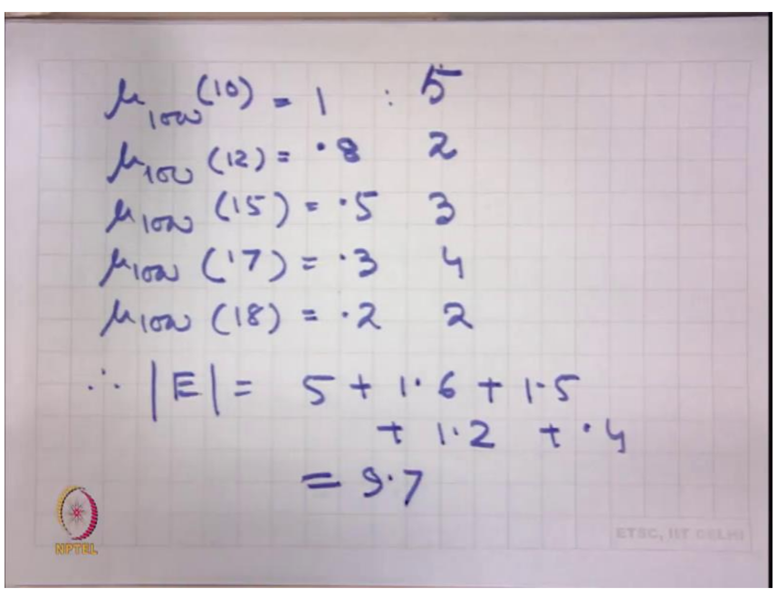

We know

- $\mu_{Low}(10) = 1$ and number of students is 5
- $\mu_{Low}(12) = 0.8$ and number of students is 2
- $\mu_{Low}(15) = 0.5$ and number of students is 3
- $\mu_{Low}(17) = 0.3$ and number of students is 4
- $\mu_{Low}(18) = 0.2$ and number of students is 2

Therefore, $|E| = 5 + 1.6 + 1.5 + 1.2 + 0.4 = 9.7$

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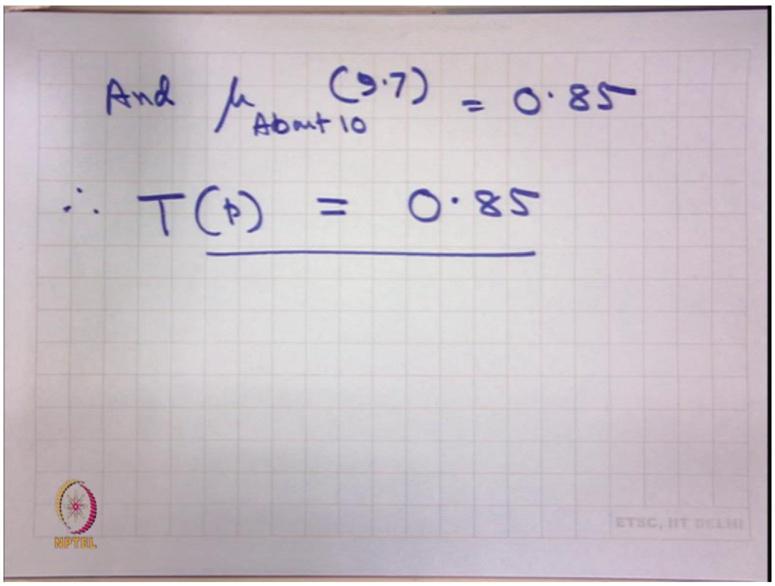

And, $\mu_{About\ 10}(9.7) = 0.85$.

$$\therefore T(p) = 0.85$$

Okay students. I stop here today. In the next class, I shall continue with fuzzy quantifiers and also we shall see how we can infer from quantified prepositions. Thank you.