# Introduction to Fuzzy Sets Arithmetic & Logic Prof. Niladri Chatterjee Department of Mathematics Indian Institute of Technology – Delhi

# Lecture – 26 Fuzzy Sets Arithmetic & Logic

Welcome students to the MOOCs lecture on Fuzzy Sets Arithmetic and Logic. And, this is lecture number 26.

## (Refer Slide Time: 00:34)

Informating with Conditional & qualified proportion. A conditional proposition is of the form : p: If X is A then Y is B If we are given X is A' them we try to infor about what happens if X is A' with

As I said at the end of the last class that today we shall discuss inferencing with conditional and qualified propositions.

We know a conditional proposition is of the form.

p: If X is A, then Y is B and we have seen that,

If we are given X is A' which is another fuzzy set then, we try to infer about what happens if X is A' with respect to the variable Y.

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In particular, we try to find a fuzzy set B' which comes as a consequence of applying If X is A then X is B X is A'.

In particular, we try to find a fuzzy set *B*' which comes as a consequence of applying *If X is A, then Y is B* on *X is A*' (**Refer Slide Time: 03:30**)

In conditional & Qualified proposition the general form will be: If X is A them Y is B is F. where F is a qualifier,

In conditional and qualified proposition, the general form will be

If X is A, then Y is B is F

where F is a qualifier.

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For example consider: If Temperature is cosy the fan speed is moderate in very True X is the antecedent variable the consequent variable the qualifier : Very E

For example consider:

If Temperature is Cosy then, Fan Speed is Moderate is Very True. So, here X is the antecedent variable. And in case of this example, which is Temperature. Y is the consequent variable which in this case is Fan Speed and F is the qualifier and which in our case is Very True.

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Suppose are define A: CODY: 2019 + 0.7 + 0.4 35 B: Mederate: \$ 0.8 + 0.4 } is Very True F  $5 \frac{5}{0.92} + \frac{30}{0.8} + \frac{32}{0.6}$ \*

And, suppose we define

$$A: Cosy = \left\{ \frac{0.9}{25} + \frac{0.7}{30} + \frac{0.4}{35} \right\}$$

$$B: Moderate = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$$

Since F is given to be Very True. Thus, it is differing from an unqualified proposition.

Suppose  $A' = \left\{ \frac{0.95}{25} + \frac{0.8}{30} + \frac{0.6}{35} \right\}$ 

So, this is the problem, we are given that

If Temperature is Cosy then, Fan Speed is Moderate is Very True.

and we need to infer about when it is given that the *Temperature* comes from the set A' with the given membership.

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so we need to incorporate a effect of F. - "Very True" Wat conditional & qualified the general scheme for solving this problem in called "Method of Truth Value Restrictim!"

So, we need to incorporate the effect of *F* which in our case is *Very True*.

With respect to conditional and qualified, the general scheme for solving this problem is called *Method Of Truth Value Restriction* and it works as follows.

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It works as follows: We try to find if  $\dot{a}'$ is the membership of some element  $\chi$  in A i.e  $\int_{A} (\chi) = a$ them how this membership is restricted for A!  $RT_{(A')} = R \mu_{A'}(x) = \Omega,$  $RT_{(A')} = R \mu_{A'}(x) = \Omega.$ 

We look at or we try to find if a is the membership of some element x in A that is  $\mu_A(x) = a$  then, how this membership is restricted for A'

The notation for this is  $RT_{\left(\frac{A'}{A}\right)}(a) = \sup_{x \ni \mu_A(x)=a} \mu_{A'}(x)$ 

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i.e if 
$$\chi_1 \cdots \chi_n$$
 are n  
members of  $A 
e f_A(\chi_i) = a$   
there  $\chi_{2:1\cdots n}$   
 $RT_{(A)} = \max_{i} f_{A_i}(\chi_i)$   
 $(A_i) \int_{\Sigma_i} \int_{\Delta_i} \int_{\Delta_i}$ 

That is, if  $x_1, x_2 \dots x_n$  are *n* members of *A* such that  $\mu_A(x_i) = a$  for all  $i = 1, 2, \dots, n$  then, the restricted to the value of *a* 

$$RT_{\left(\frac{A'}{A}\right)}(a) = \max_{i} \mu_{A'}(x_i)$$

Note that since it is discrete, we used maximum otherwise, we use supremum.

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Now we obtain the Truth Value moder X is A' as follows RT (b) = one (min)(B') = Re[0,1] RT (A)(A)F(C)(G,b)F(2(a,b)) Explanation is as follows.

Now, we obtain the truth value under X is A' as follows:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \sup_{a \in [0,1]} \left( \min\left\{ RT_{\left(\frac{A'}{A}\right)}(a), F(\mathfrak{T}(a,b)) \right\} \right)$$

This is a fairly complicated notation. So, let me explain that.

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Let us recall for unqualified case we used:  $MB'(y_0) = \sup_{X} \left( \min \left( \mu_A^{(X)} \right) \right)$ 1p(x, y)

Let us recall that for unqualified case we used:

$$\mu_{B'}(y_0) = \sup_{x} \left( \min\left(\mu_{A'}(x), \mu_R(x, y)\right) \right)$$

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are R came from implication. i.e  $\mu_{R}(x,y) = \mathcal{X}(\mu_{A}(x), \mu_{B}(y))$ : X in  $A \rightarrow Y$  in B

Where R came from implication that is

$$\mu_R(x,y) = \mathfrak{T}\big(\mu_A(x),\mu_B(y)\big)$$

That is, since *X* is  $A \rightarrow Y$  is *B*, this implication is captured through the function  $\mathfrak{T}$ . (**Refer Slide Time: 16:35**)

and when we use Lukasle asicz, it in:  $\mathcal{X}(a,b) = Min(1, 1-a+b)$ Unders Method of Truth restrict. we go as follows: We need to calculate for any arbitrary be [0,1] anch denotes AB(3) how it will be

And when we use Lukasiewicz it is

 $\mathfrak{T}(a,b) = \min(1,1-a+b)$ 

Under method of truth restriction, we go as follows.

We need to calculate for any arbitrary  $b \in [0, 1]$  which denotes  $\mu_B(y)$ , how it will change or how it will be restricted when X = A'

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restricted when  $\mathbf{x} = \mathbf{A}'$ . Therefore the formula is:  $\mathbf{RT} \quad (\mathbf{b}) = \sup_{\mathbf{a} \in [0,1]} \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$   $\mathbf{E} = \left( \begin{array}{c} \mathbf{RT} \quad (\mathbf{a}) \\ (\mathbf{A}) \end{array} \right)$ 

Therefore, the formula is:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \sup_{a \in [0,1]} \left( \min\left\{ RT_{\left(\frac{A'}{A}\right)}(a), F(\mathfrak{T}(a,b)) \right\} \right)$$

So, let me illustrate with an example. So the problem will be clear.

(Refer Slide Time: 19:32)

Consider 
$$PT(A'_{A})(A)$$
.  
We have only 3 possible  
values (non-sero) for a  
 $a = 0.9$   $PT_{A}(0.9) = 0.95^{2}$   
 $a = 0.7$   $PT_{A}(0.7) = 0.8$   
 $R = 0.4$   $PT_{A}(A')$   
 $RT_{A}(A')(0.4) = 0.6$ 

So, consider the problem that I have stated earlier that

$$A: Cosy = \left\{ \frac{0.9}{25} + \frac{0.7}{30} + \frac{0.4}{35} \right\}$$

$$B: Moderate = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$$

Since *F* is given to be *Very True*. What is going to be the membership values under the *B*'? So, consider  $RT_{\left(\frac{A'}{A}\right)}(a)$  for any *a*.

We have only 3 possible values, I mean none zero values in A for a

$$a = 0.9 RT_{\left(\frac{A'}{A}\right)}(0.9) = 0.95$$
  

$$a = 0.7 RT_{\left(\frac{A'}{A}\right)}(0.7) = 0.8$$
  

$$a = 0.4 RT_{\left(\frac{A'}{A}\right)}(0.4) = 0.6$$

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Now we need to compute RT (b) for all b [ [ ]. i.e  $RT_{(B')}(b) =$  max(min(0.95, F(x(0.9, b))) min(0.8), F(x(0.7, b)) min(0.6), F(x(0.4, b))

Now, we need to compute  $RT_{\left(\frac{B'}{B}\right)}(b)$  for all  $b \in [0, 1]$ That is,  $RT_{\left(\frac{B'}{B}\right)}(b) = \max \begin{pmatrix} \min\left(0.95, F(\mathfrak{T}(0.9, b))\right) \\ \min\left(0.8, F(\mathfrak{T}(0.7, b))\right) \\ \min\left(0.6, F(\mathfrak{T}(0.4, b))\right) \end{pmatrix}$ 

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In cour case  $\mp$  is "very true and we know  $\mu_{very true}$ =  $(\mu_{true})^2$ :. Our value for  $PT_{(B'_{B})}(b)$ in pimplified: max (min(0.95, min(12, (1-0.9+b)^2))^2) (min(0.8, min(12, (1-0.9+b)^2))^2)

In our case *F* is *Very True* and we know  $\mu_{Very True}(x) = (\mu_{True}(x))^2$ Therefore, our value for  $RT_{\left(\frac{B'}{B}\right)}(b)$  is simplified as follows:

$$RT_{\left(\frac{B'}{B}\right)}(b) = \max\left( \begin{array}{c} \min(0.95, (\min(1, 1 - 0.9 + b))^2) \\ \min(0.8, (\min(1, 1 - 0.7 + b))^2) \\ \min(0.6, (\min(1, 1 - 0.4 + b))^2) \end{array} \right)$$

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$$\begin{array}{c} \text{RT} (b) & \text{is further simplified} \\ \text{to} : & \text{max} \left( \min \left( 0.95, \left( b + \cdot 1 \right)^2 \right) \\ \min \left( 0.8, \left( b + \cdot 3 \right)^2 \right) \\ \min \left( 0.6, \left( b + \cdot 6 \right)^2 \right) \\ \text{Min} \left( 0.6, \left( b + \cdot 6 \right)^2 \right) \\ \text{Meshion:} \\ \text{Hozo to obtain ?} \\ \end{array}$$

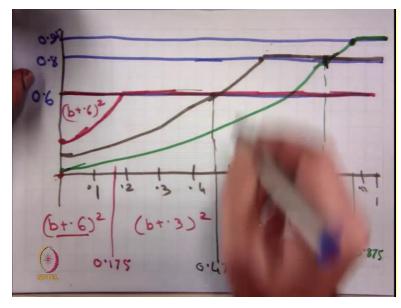
Therefore,  $RT_{\left(\frac{B'}{B}\right)}(b)$  is further simplified to:

$$\max \begin{pmatrix} \min(0.95, (b+0.1)^2) \\ \min(0.8, (b+0.3)^2) \\ \min(0.6, (b+0.6)^2) \end{pmatrix}$$

Question is how to obtain this?

So, we go as follows we use a graphical method to solve this problem.

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So, using the graph we see that, we can get different intervals in which different equations will serve as that maximum. And we can summarize this in the following way.

## (Refer Slide Time: 34:10)

 $RT_{(B')} = \begin{cases} (bt \cdot 6)^2 & b \in [0, \cdot 175] \\ (B') &= \\ 0 \cdot 6 & b \in [\cdot 175], \\ 0 \cdot 175], \\ 0 \cdot 6 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10 & b \in [\cdot 175], \\ 0 \cdot 10$ obtain 66T.595 be 0.95 PE

$$RT_{\binom{B'}{B}}(b) = \begin{cases} (b+0.6)^2 & b \in [0,0.175) \\ 0.6 & b \in [0.175,0.475) \\ (b+0.3)^2 & b \in [0.475,0.595) \\ 0.8 & b \in [0.595,0.795) \\ (b+0.1)^2 & b \in [0.795,0.875) \\ 0.95 & b \in [0.875,1] \end{cases}$$

So, this allows us to obtain  $\mu_{B'}(b)$  for all values of b

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Now we had B= <u>{0.8</u>, <u>0.4</u> } Question is what  $\mu_{B'}(120)$ ?  $\therefore RT_{(B')}(0.8) = (8+1)^{2}$   $= (0.9)^{2} = 0.81$ Similarly (0.4) = 0.6  $PT(B') = \frac{0.6}{5}$  B' set is :  $\frac{0.81}{120} + \frac{0.6}{200}$ 

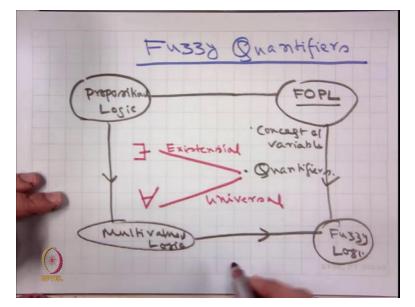
Now we had  $B = \left\{\frac{0.8}{120} + \frac{0.4}{200}\right\}$ Question is, what is  $\mu_{B'}(120)$ ?  $\therefore RT_{\left(\frac{B'}{B}\right)}(0.8) = (0.8 + 0.1)^2 = 0.9^2 = 0.81$ Similarly,  $RT_{\left(\frac{B'}{B}\right)}(0.4) = 0.6$ 

Therefore, the conclusion is that the *B*'set is  $\left\{\frac{0.81}{120} + \frac{0.6}{200}\right\}$ 

Like that, we could infer about the consequent variable with the help of the given implication and that the input is X is A'.

Okay. So, this is how we infer about conditional and qualified propositions..

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Next we want to study Fuzzy quantifiers

If you remember we started that when we have propositional logic we have expanded in two directions.

One is multi valued logic which we have been using for last few lectures and another way of extending it was through first order predicate logic and that is also contributing to fuzzy.

In fact, first order predicate logic gave us one is the concept of variables. And it has given concept of quantifiers.

In particular, we have used two types of quantifier one is  $\exists$  which is existential quantifier and the other one is  $\forall$  which is universal quantifier.

## (Refer Slide Time: 41:21)

when we extend to FUB38 logic ONE two Quantifiers : Number-ybe . about 10 First kind nearly 10 100 proposition type : Second Kind almost all ere

Similarly, when we extend it to fuzzy logic we have 2 types of Fuzzy Quantifiers:

- Number type are often it is called First kind.
   That is here, we look at some fuzzy numbers.
   For example, *about* 10, *nearly* 100 etc.
- 2. Proportion type or second kind.

Here, we look at instead of absolute values of Fuzzy way of describing the proportion. Example, *almost all, close to half, nearly one third* etc.

Actually in our daily life, we use such fuzzy quantifiers

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EX: There are about 10 students with 1000 attendance Nearb 1500 people registered femal Almost all ahoo

For example:

- There are *about* 10 students in this class with low attendance or
- *Nearly* 1500 people have registered for this course.

So, these are examples of fuzzy quantifier of first kind.

- *Close to half* of the students are females.
- *Almost all* students are doing well in this course.

These are fuzzy quantifier of second kind.

## (Refer Slide Time: 45:48)

General Canonical form in : p: There are Q i's in I such that V(i) is F. There are about 10 students in the class such that attendance q i is low. S: quantifier - "about 10" T: set 3 iEI: "the class" Y(i) = value & variable V for i attendance = fussy set defined over \_ attendance

General canonical form is:

*p*: There are Q *i*'s in  $\mathfrak{T}$  such that V(i) is *F* 

Say for example, consider

There are *about* 10 students in the class such that *attendance* of *i* is *low*.

- Now, if we compare we can see that
- Q: quantifier about 10

 $\mathfrak{T}$ : the set such that  $i \in \mathfrak{T}$ . And in our case,  $\mathfrak{T}$  is the *class* and each *i* is a *student*,

V(i): value of variable V for i which in this case is attendance of i

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2 F is the fussy set defined over all values of V. Querskin is head to obtain the Truth value of a proportion \$? >: The number of low-attendance students in the class is about 1000 attendance student !. = about 10

And, F is the fuzzy set defined over all values of V

Question is How to obtain the truth value of a proposition *p*?

So, from the above p we write p' as follows:

p': The number of low attendance students in the class is about 10 that is p' = |low attendance student| = about 10

#### (Refer Slide Time: 50:29)

Thus in general We transfor m p: There are Q i's not V(i)=F p' = |V(i) in F| = Q.For fussy quantifiers of first kind

Thus in general, we transform

p: There are Q i's such that V(i) = F. And this we transform to p' = |V(i) is F| = QAnd this we do for fuzzy quantifiers of first kind. So, how to solve this problem?

#### (Refer Slide Time: 51:43)

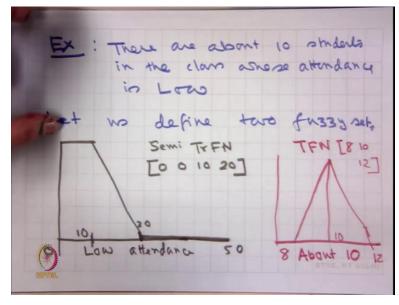
Our aim is to obtain T(b) H: p > p > +': |E| = 9. them are shall arrigh T(P) = M(IEI)

Finally, we will.

Our aim is to obtain truth value of T(p)

For which we will see if *p* goes to *p*' such that p': |E| = Q then we shall assign  $T(p) = \mu_Q(|E|)$ So, let me illustrate with an example.

#### (Refer Slide Time: 52:41)



Example: *There are about* 10 *students in the class whose attendance is Low*. So, let us define two fuzzy sets as follows:

1. Fuzzy set *Low Attendance* is defined over the set 0 to 50.

If there are 50 lectures such that anything between 0 to 10 is low and between 10 to 20 it decreases like a straight line, above 20, the membership is zero. Therefore, it is a semi trapezoidal fuzzy number. *Low Attendance* =  $\begin{bmatrix} 0 & 0 & 10 & 20 \end{bmatrix}$ 

 The second fuzzy sets is *about* 10. And suppose we define it as follows. At 10 it is 1 and 8 to 12 it gives us triangular fuzzy number [8 10 12]

So, our aim is given a fact what is the true value of the statement the fact is as follows.

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The given fact in: suppose in a class there are so smamle . 5 of them <10 We need 2 · · · 12 to compute 3 · · · 15 |E| 4 · · · 17 E=> 1020 attend attendary mdeli Post all >20

Suppose, in a class there are 50 students:

- 5 of them have attendance  $\leq 10$
- 2 of them have attendance  $\leq 12$
- 3 of them have attendance  $\leq 15$
- 5 of them have attendance  $\leq 17$
- 2 of them have attendance  $\leq 18$
- Rest all > 20.

Therefore, we need to compute the |E| where E is low attendance students.

### (Refer Slide Time: 56:29)

 $\begin{array}{c}
\mu_{100}(10) = 1 : 5 \\
\mu_{100}(12) = \cdot 8 & 2 \\
\mu_{100}(15) = \cdot 5 & 3 \\
\mu_{100}(17) = \cdot 3 & 4 \\
\end{array}$ MION (18) = ·2 2 · | E |= 5+1.6+1.5 + 1.2 + .4 = 9.7

We know

- $\mu_{Low}(10) = 1$  and number of students is 5
- $\mu_{Low}(12) = 0.8$  and number of students is 2
- $\mu_{Low}(15) = 0.5$  and number of students is 3
- $\mu_{Low}(17) = 0.3$  and number of students is 4
- $\mu_{Low}(18) = 0.2$  and number of students is 2

Therefore, |E| = 5 + 1.6 + 1.5 + 1.2 + 0.4 = 9.7

#### (Refer Slide Time: 57:57)

And Ju At	() mt 10	.7) = 0.85	
·· T(+)	=	0.82	
<u>@</u>			

And,  $\mu_{About \ 10}(9.7) = 0.85$ .

## $\therefore T(p) = 0.85$

Okay students. I stop here today. In the next class, I shall continue with fuzzy quantifiers and also we shall see how we can infer from quantified prepositions. Thank you.