### Introduction to Fuzzy Sets Arithmetic and Logic Prof. Niladri Chatterjee Department of Mathematics Indian Institute of Technology - Delhi

## Lecture-24 Fuzzy Sets Arithmetic and logic

Welcome students to the MOOCs lecture series on fuzzy sets arithmetic and logic this is lecture number 24.

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FUBBY Sets Arithmetic Logic Lecture - 24

And in this lecture we shall look at truth values of a Fuzzy Proposition. (**Refer Slide Time: 00:41**)

We discused "Multi-valued Logic" 2 are have seen that Lingwistic helges are aften used to alter the Truth value f & Fuzzy proportion Howevers, sometimes we may confine with the intended meaning

Recall that, in the last class we discussed *Multi-Valued Logic* and we have seen that *Linguistic Hedges* are often used to alter the truth value of a Fuzzy Proposition.

However, sometimes we may confuse with the intended meaning.

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Ex: Consider "Not Tall." Does it mean "short"? Pershaps not. In fact if somebody is considered "Not Tall" - by that we may mean "Rather Tall" or "Fairby Tall" need not be Thura S

For example:

Consider Not Tall.

Does it mean Short?

Perhaps not, in fact if somebody is considered *Not Tall* by that we may mean he is *Rather Tall* or *Fairly Tall*.

Thus  $\overline{S}$  need not be 1 - S(**Refer Slide Time: 03:53**)

This more clear if use wides "Not Very Tall" consider Does it mean Surely Not. Ideally we should have some standard linguistic hedge to describe all these qualified propositions. Front perhops Z any conserves.

This is clearer if we consider Not Very Tall.

Does it mean Very Short?

Surely not.

Ideally we should have some standard linguistic hedge to describe all these qualified propositions.

But perhaps there does not exist any consensus.

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Within the above limitation values of a Fussy pro In today's discussion we shall not consider Fuzzy Quantifier, Therefore we get the following four kinds of Fuzzy proposition is to obtain techniques on Truth values to them. Our aim

Within the above limitation we shall discuss truth values of a Fuzzy Proposition. In today's discussion we shall not consider Fuzzy Quantifiers. Therefore, we get the following 4 kinds of Fuzzy Propositions. Our aim is to obtain techniques to assign truth values to them.

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the four types of Unembitional

Now the 4 types of fuzzy propositions are:

1. Unconditional and Unqualified.

Example:

Today is hot,

*hot* is a fuzzy term. Therefore, this is a fuzzy proposition. It is unconditional because it is not constrained by any condition like if something happens then today is hot. Similarly, the statement is unqualified that means that we are not assigning any fuzzy truth value to the statement itself.

Similarly, we can have say something like

John is very tall,

This box is heavy.

These are all propositions of the type unconditional and unqualified.

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Unconditional & qualified. Today is hot is T John is very tall Irne S rather True This box is heavy Not Trne.

2. Unconditional and qualified.

Say for example:

Today is hot is True

John is very tall is rather True or

This box is heavy is not True.

Thus we can see that on one hand we have a fuzzy statement and then we have a qualifier which qualifies the truth value of the entire statement.

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Conditional 2 unqualified. If the course is difficult Them Recoults are poor. the temperature is hat then speed of-fan shanld be eigh.

3. Conditional and Unqualified.

Examples:

If the course is difficult then results are poor.

If the temperature is hot, then speed of fan should be high.

As you can understand that these are conditional because the overall truth value of the statement depends upon the condition given in the form of *if-then*.

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onditional 2 qualified. the course is defficult Then resoluts are poor is Very True tempera mr. them speed of should MOYA

4. Conditional and Qualified.

For example:

If the course is difficult then results are poor is very true.

If the temperature is high, then speed of fan should be high is more or less true.

So these are the statements of the type conditional and qualified.

The question is how do we assign truth values to different types of fuzzy propositions.

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Unconditional & unqualified. The general form of these atatements in P: V in F. astere V in a variable that taken values from a set.

1. Unconditional and Unqualified.

The general structure or the general form of these statements is

p: V is F,

where V is a variable that takes values from a set.

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For example: If V is Temperature them it may take values in the range say 0°-48°C in our context. 118 If V is theight take values say

For example:

If *V* is temperature then it may take values in the range 0 - 48°C in our context. Similarly, if *V* is height it may take values, say in the interval 5' - 6'6'' (**Refer Slide Time: 17:15**)

One point has to be noted. ashow we say "Johnis Tall" the variable is not John Rather we refor to the height of the person John.

Now, one point has to be noted.

When we say *John Is Tall*, the variable is not *John* rather we refer to the height of the person *John*. (**Refer Slide Time: 18:19**)

Similarly if the statement is Today is bot, are mean the temparature of today is Therefore athen we arent to arright the Truth value of & fussy proposition we need to cook at the membership of the value & of V in the set F.

Similarly, if the statement is *Today is Hot* we mean the temperature of today is high. Therefore, when we want to assign the truth value of a fuzzy proposition we need to look at the membership of the value v of V in the set F.

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So consider the statement p: John in Tall. We agent to arrige Frenth value to p, i.e we ason to compute Suppose height of John is 5'8". And suppose we define a Bh333 set Tall our the set of

So consider the statement

p: John is Tall

We want to assign truth value to p, that is we want to compute T(p) that is truth value of the statement or of the proposition p.

Now suppose height of John is 5'8"

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And suppose we define a fuzzy set *Tall* on the set of real numbers that give the height of human beings.

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So as per the problem.

Suppose we assign the membership to the set *Tall* as follows:

At 5' it is 0, at 5'10'' it is 1 and from there it remains 1 and between 5' to 5'10'' it grows linearly. Then  $\mu_{Tall}(5'8'') = 0.8$ 

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Therefore, If p = John is Tall then

T(p) = T(John is Tall)

= T(height of John is Tall)

= T(height(John) is Tall)

$$= T(5'8'' \text{ is Tall})$$
$$= T(\mu_{Tall}(5'8''))$$
$$= T(0.8)$$

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Here if we interpret T as a fussy set on [0, 1] &the prempership to T is T(p) = p. ·· T(0.8) = 0.8 or The Truth value of the above proposika is 0.8.

Now if we interpret T as a fuzzy set on [0, 1] and the membership to T is

T(p) = p $\therefore T(0.8) = 0.8$ 

Or the truth value of the above proposition is 0.8.

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Therefore for unconditional 2 unqualified FU333 propon propositions of the form: p: V in F. then for a given value up V,  $T(P) = f_F(D)$ 

Therefore, for unconditional and unqualified fuzzy propositions of the form:

p = V is F

then, for a given value v of V,  $T(p) = \mu_F(v)$ 

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In a similar way if we usant the truth value for the proportion John is Very Tall Then  $T(P) = \int_{Very Tall} (5'8'')$  $= (\mu_{(s's')})^2$ = (0.8)2 = 0.64

In a similar way, if we want the truth value for the proposition

John is Very Tall Then  $T(p) = \mu_{Very Tall}(5'8'') = (\mu_{Tall}(5'8''))^2 = (0.8)^2 = 0.64$ 

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Uncondional 2 qualified. Typical form of such propositions in: p: V is F is S. e.g Tobay is that is very True. 2 We need membership functions for both the sels.

Let us now look at statements of the form

## 2. Unconditional and qualified.

Typical form of such propositions is:

p = V is F is S.

For example: Today is hot is Very True.

Thus, we have two fuzzy terms one is *hot*, one is *Very True* and we need membership functions for both the sets.

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Suppose we assign membership values to *hot* as follows:

At 0°C it is 0, up to 20°C it is 0 between 20 - 35°C it grows linearly. I am keeping the functions linear to keep the computation simple, if the membership is not linear you have to look at the membership value accordingly.

And suppose today's temperature is 30°C then

$$\mu_{Hot}(30) = \frac{2}{3} = 0.67$$

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Now for *True*, we typically use the following membership function.

This goes linearly from 0 to 1 in the set 0 to 1.

Therefore *Very True* is going to be square of *True*. And in a similar way by taking square root we get the membership for a *Dilation* say *Fairly True*.

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In a similar way, the fuzzy set membership for *False* is given as:

At 0 it is 1 and at 1 it is 0 and we get a straight line.

Therefore, by taking square we get the membership to the concentrator *Very False* and by considering square root we get the membership for *Fairly False*.

As I have discussed there can be other concentrators and there can be other dilators also but for this class we are using  $\alpha^2$  for concentration and  $\sqrt{\alpha}$  for dilation.

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Another form of qualifier is *likely* which may have the following type of membership.

At 0.5 it is 0. So as we go from 0.5 to 1 if we keep it linearly increasing. This is called *Likely*. From 0.5 to 0 it is linearly decreasing and from 0.5 to 1 it is 0. This is *Unlikely* and we can also use concentration here *Very Likely* and *Very Unlikely*. So this allows us to give membership to

different qualifiers. With that background now let us look at the original problem.

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Our statement was: Today is hot is very True

So if we look at the original problem.

We find that if today's temperature is 30°C its membership is  $\frac{2}{3} = 0.67$ .

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Therefore to obtain the truth value for p: V is F is S. we first compute = (V).)

Therefore, to obtain the truth value for

$$p = V$$
 is F is S,

We first compute  $\mu_F(V)$  and then we look at it is membership to S

$$\therefore T(p) = \mu_S(\mu_F(V))$$

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T(b): T (Today is Let is Very True) = Miery True (Today is hot) = Miery True (In that (Temperature of today)) = Miery True (In that (30°)) = Miery True = Miery True

$$\therefore$$
  $T(p) = T(Today is hot is very True)$ 

 $= \mu_{Very True} (Today is hot)$   $= \mu_{Very True} (\mu_{Hot} (temperature of Today))$   $= \mu_{Very True} (\mu_{Hot} (30^{\circ}\text{C}))$   $= \mu_{Very True} \left(\frac{2}{3}\right)$   $= \left(\mu_{True} \left(\frac{2}{3}\right)\right)^{2}$   $= \left(\frac{2}{3}\right)^{2} = \frac{4}{9} \approx 0.44$ 

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= (A Free (3/2))<sup>2</sup> = (3/3)<sup>2</sup> = 4 × 0.44 In this way we can the Truth value of. get unconditional & qualified statements (\*

In this way we can get the truth value of unconditional and qualified statements. (**Refer Slide Time: 43:05**)

t us consider "onditional unanalified statement Eq: (2D 000

Let us now consider

3. Conditional and unqualified statements.

For illustration, consider:

If the effort is hard then result is good.

Thus, it is a conditional statement. Because the *result is good*, this phrase, its truth value depends upon the conditional statement, *the effort is hard*.

Question is how to assign truth values to such propositions?

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expressed art value the 8 & hours m. baris 2000

So suppose the fuzzy set *hard* with respect to *effort* is expressed with the value of number of hours spent.

Say for example:

$$hard = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \dots + \frac{1}{10} \right\}$$

And suppose goodness of result is computed on the basis of marks.

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And we use the following:

$$\mu_{good}(x) = \begin{cases} 0 & x \le 50 \text{ or } x > 100\\ \frac{x - 50}{50} & 50 \le x \le 100 \end{cases}$$

That is if someone gets 85 marks, then  $\mu_{good}(85) = \frac{35}{50} = 0.7$ 

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Thus we need to associate two fuzzy sels. Canonical form for conditional & unqualified toroporations in P: If X in A then arres X taken values X in B form a set V. & A in a fussy set defined over V.

Thus we need to associate two fuzzy sets because the canonical form for conditional and unqualified propositions is

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p: If X is A then Y is B,
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where *X* takes values from a set *V* and *A* is a fuzzy set defined over *V* and *Y* takes values from a set *W* and *B* is the fuzzy set defined over *W* 



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So with respect to our example

X = effort which takes values number of hours  $[0 \dots 10] = V$  and A is the fuzzy set defined on V such that  $\mu_A(a) = \frac{a}{10}$ 

And similarly we had Y = result which takes values in [50, 100] which is W and

$$\mu_B(b) = \frac{b - 50}{50} \quad 50 \le b \le 100$$

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In this set up to option the truth value for p we proceed as follows. Given a & b we associate a with b wing Implication If a then b a implies b Now for emplication we may

Therefore, in this setup to obtain the truth value for p we proceed as follows.

Given *a* and *b* we associate *a* with *b* using implication.

Because if *a* then  $b \rightarrow a$  implies *b*.

Now for implication we may use Lukasiewicz scheme which says

$$a \rightarrow b = \min(1, 1 - a + b)$$

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a=>b= min (1, 1-a+b) Suppose # of hours spin is 8. ... Mard fort) = 0.8 2 suppose marks obtained 9 85. ... figood (85) = 0.7 We need to associate b=0.7 through Unkarrie wicz

So suppose number of hours spent is 8,

$$\therefore \mu_{hard}(effort) = 0.8$$

and suppose marks obtained is 85

$$\therefore \mu_{good}(85) = 0.7$$

We need to associate these two, a = 0.8 and b = 0.7 through Lukasiewicz.

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And we get  $\mathfrak{T}(a, b) = \mathfrak{T}(0.8, 0.7) = \min(1, 1 - 0.8 + 0.7) = 0.9$ , where  $\mathfrak{T}$  means implication through Lukasiewicz.

Thus, T(p) when effort is equal to 8 hours and marks is equal to 85 is 0.9.

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e fourth type of propositions Conditional & qualified. Soy for example effort in hard then remit in good in Very True m proceeding as before shall get 0.9 fm

4. Conditional and Qualified.

Say for example,

If effort is hard then result is good is very true.

Then proceeding as before we shall get 0.9 for *If effort is hard then marks is good or result is good*. Now we have to look at this qualifier and for that what we will do.



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 $\therefore T(p) = \mu_{Very \, True}(0.9) = \left(\mu_{True}(0.9)\right)^2 = 0.81$ 

Ok friends I stop here today. In the next class I shall look at fuzzy implication and I look at how to handle them for reasoning with fuzzy statements, thank you.