Introduction to Fuzzy Sets Arithmetic and Logic

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Lecture-24
Fuzzy Sets Arithmetic and logic

Welcome students to the MOOCs lecture series on fuzzy sets arithmetic and logic this is lecture number 24.

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And in this lecture we shall look at truth values of a Fuzzy Proposition.

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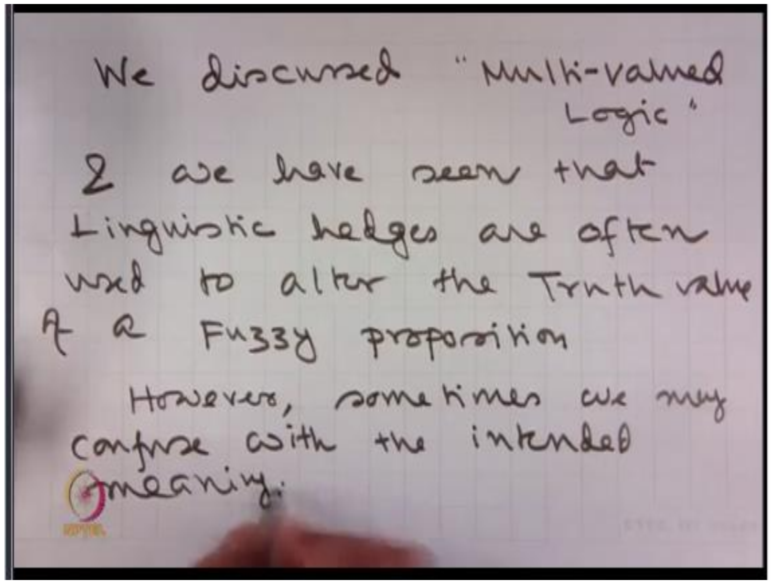

Recall that, in the last class we discussed *Multi-Valued Logic* and we have seen that *Linguistic Hedge*s are often used to alter the truth value of a Fuzzy Proposition.

However, sometimes we may confuse with the intended meaning.

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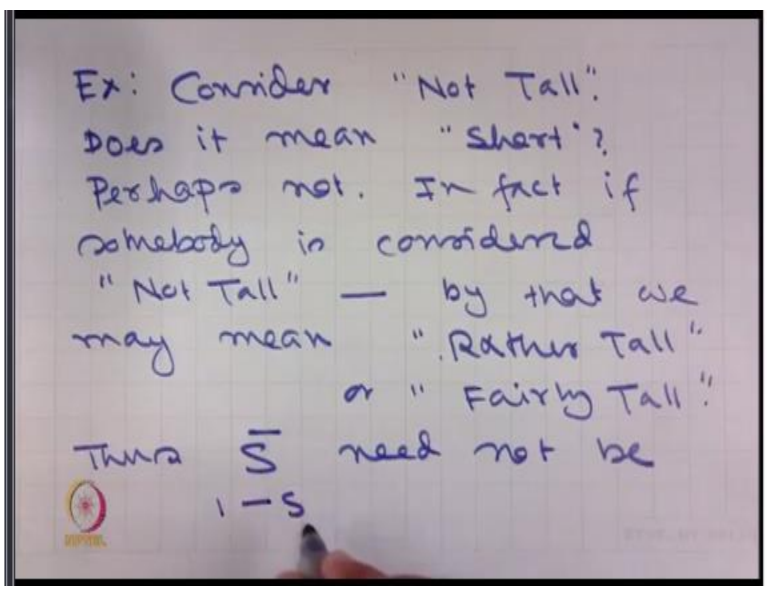

For example:

Consider $Not\ Tall$.

Does it mean $Short$?

Perhaps not, in fact if somebody is considered $Not\ Tall$ by that we may mean he is $Rather\ Tall$ or $Fairly\ Tall$.

Thus $\bar{S}$ need not be $1 - S$

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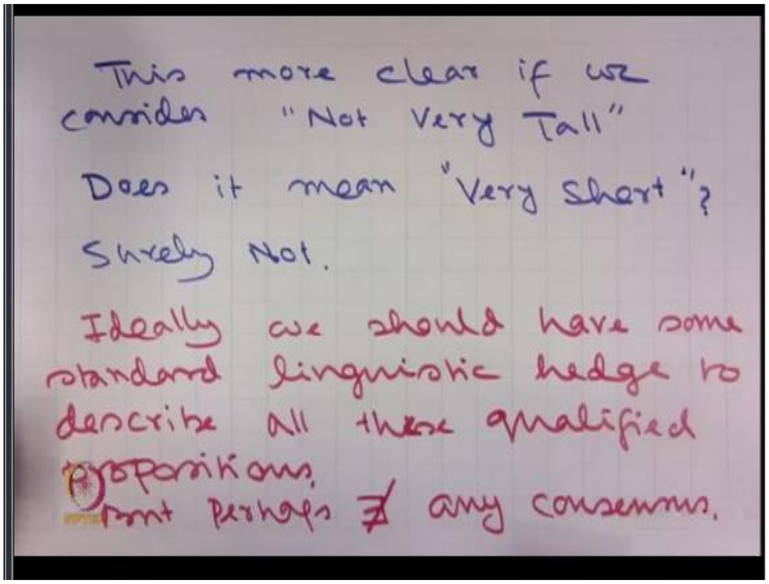

This is clearer if we consider $Not\ Very\ Tall$.

Does it mean $Very\ Short$?

Surely not.

Ideally we should have some standard linguistic hedge to describe all these qualified propositions.

But perhaps there does not exist any consensus.

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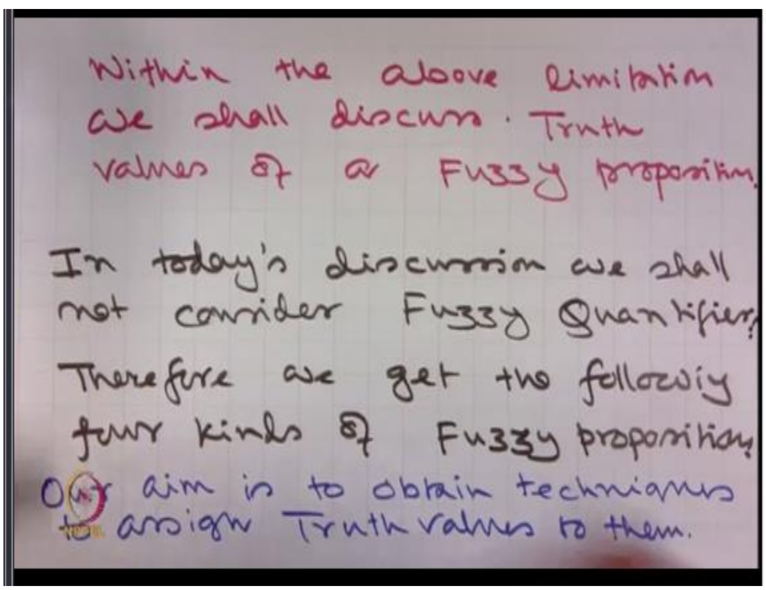

Within the above limitation we shall discuss truth values of a Fuzzy Proposition.

In today's discussion we shall not consider Fuzzy Quantifiers.

Therefore, we get the following 4 kinds of Fuzzy Propositions. Our aim is to obtain techniques to assign truth values to them.

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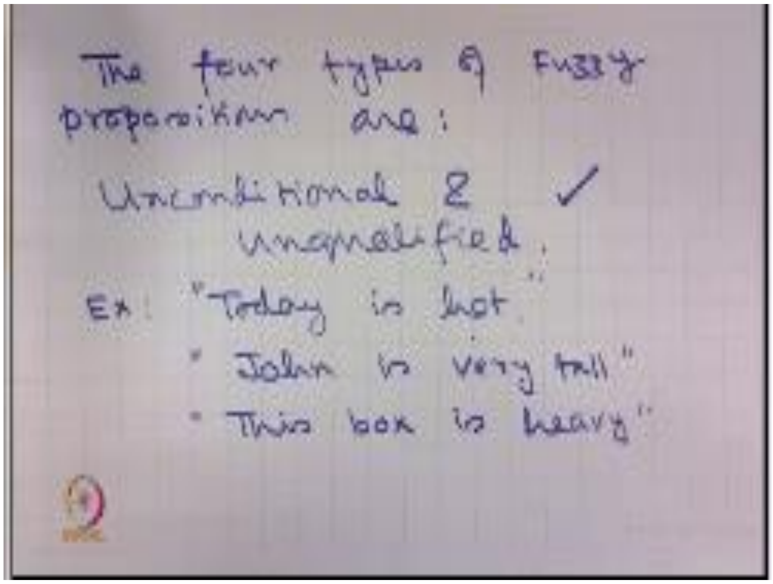

Now the 4 types of fuzzy propositions are:

1. Unconditional and Unqualified.

   Example:

   Today is hot,

   hot is a fuzzy term. Therefore, this is a fuzzy proposition. It is unconditional because it is not constrained by any condition like if something happens then today is hot. Similarly, the statement is unqualified that means that we are not assigning any fuzzy truth value to the statement itself.

   Similarly, we can have say something like

   John is very tall,

   This box is heavy.

   These are all propositions of the type unconditional and unqualified.

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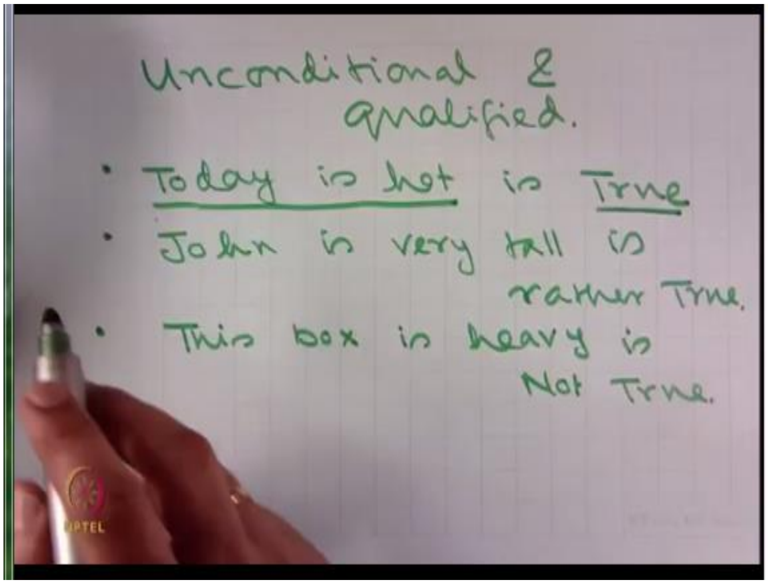

2. Unconditional and qualified.

   Say for example:

   Today is hot is True

   John is very tall is rather True or

   This box is heavy is not True.

   Thus we can see that on one hand we have a fuzzy statement and then we have a qualifier which qualifies the truth value of the entire statement.

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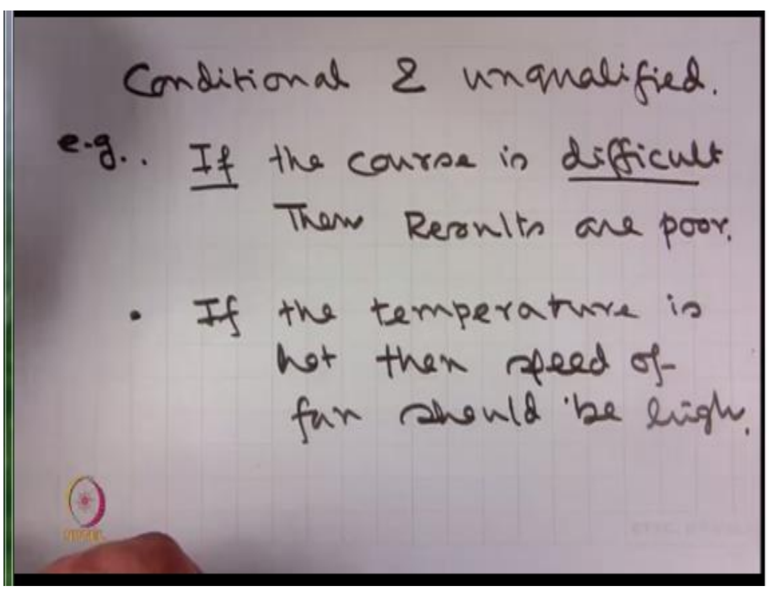

3. Conditional and Unqualified.

   Examples:

If the course is difficult then results are poor.

If the temperature is hot, then speed of fan should be high.

As you can understand that these are conditional because the overall truth value of the statement depends upon the condition given in the form of *if-then.*

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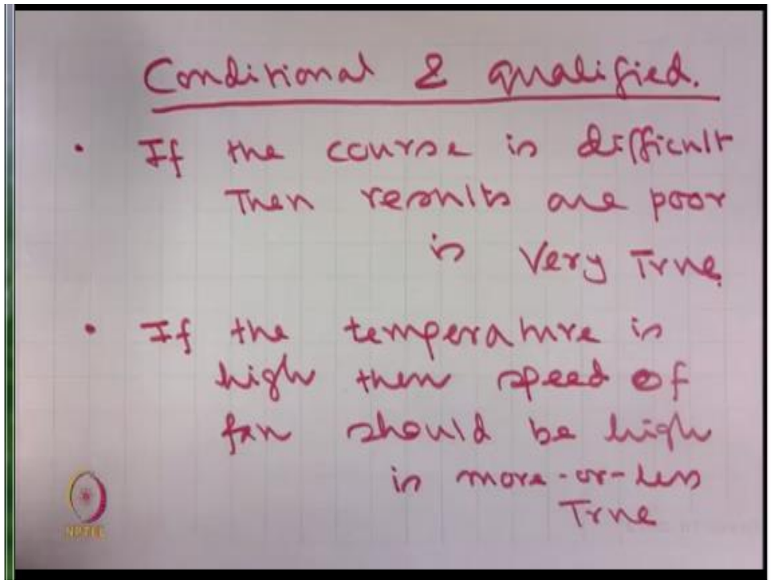

4. Conditional and Qualified.

   For example:

   If the course is difficult then results are poor is very true.

   If the temperature is high, then speed of fan should be high is more or less true.

   So these are the statements of the type conditional and qualified.

   The question is how do we assign truth values to different types of fuzzy propositions.

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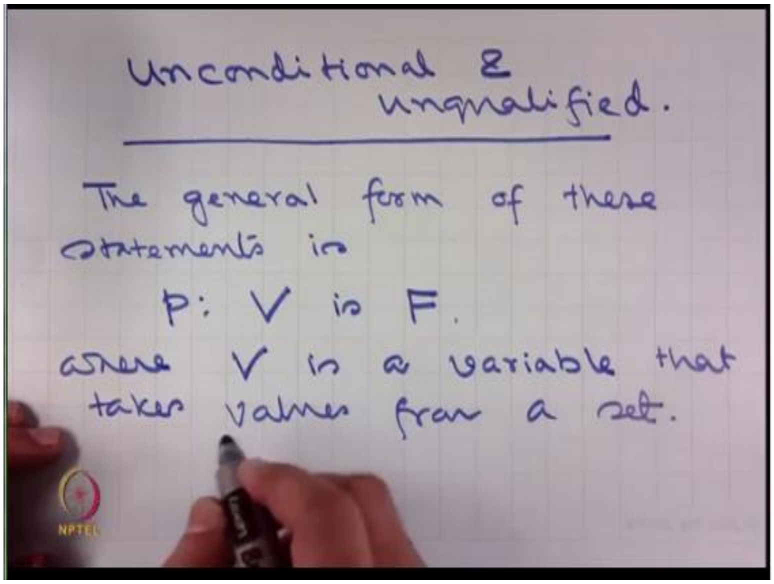

1. Unconditional and Unqualified.

The general structure or the general form of these statements is

$$p\colon\ V \text{ is } F,$$

where V is a variable that takes values from a set.

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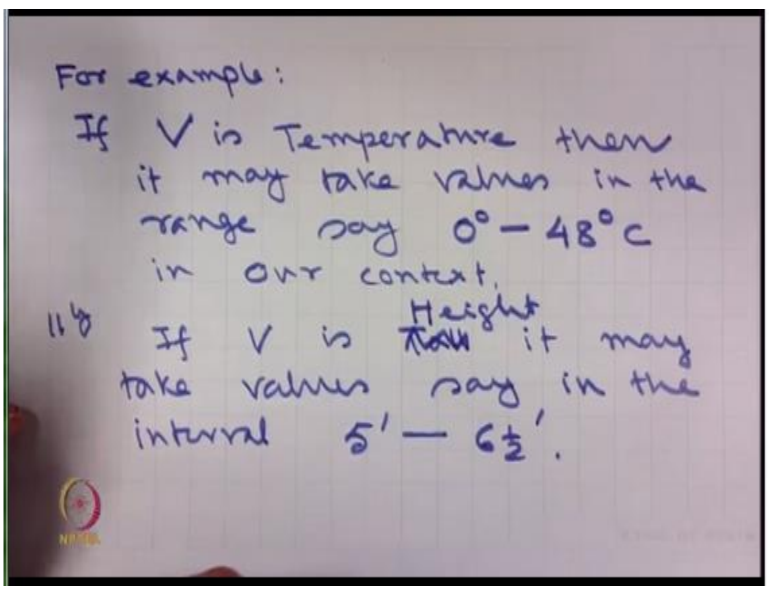

For example:

If V is temperature then it may take values in the range $0 - 48$°C in our context.

Similarly, if V is height it may take values, say in the interval $5' - 6'6''$

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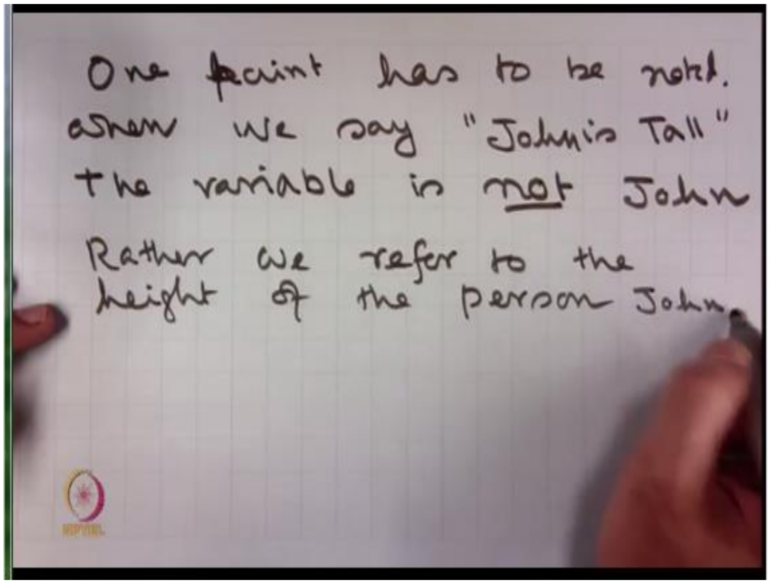

Now, one point has to be noted.

When we say *John Is Tall,* the variable is not *John* rather we refer to the height of the person *John*.

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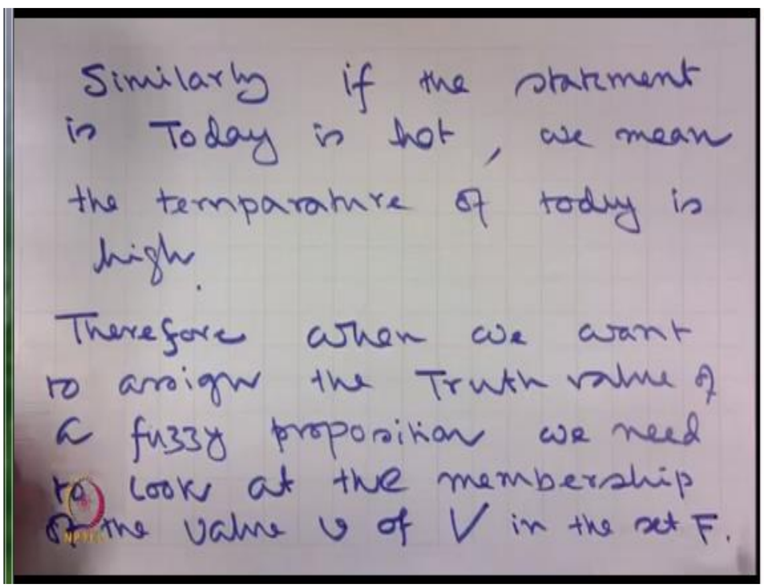

Similarly, if the statement is *Today is Hot* we mean the temperature of today is high.

Therefore, when we want to assign the truth value of a fuzzy proposition we need to look at the membership of the value v of V in the set F.

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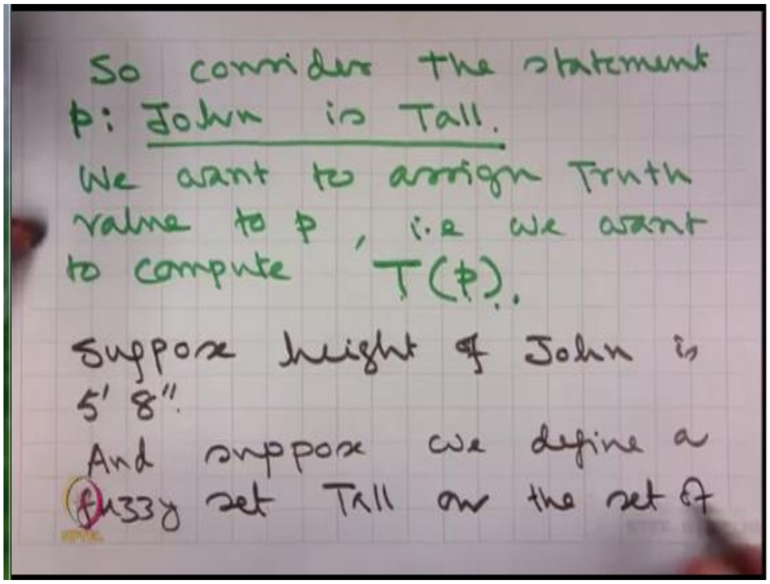

So consider the statement

$p{:}\,John\ is\ Tall$

We want to assign truth value to p, that is we want to compute $T(p)$ that is truth value of the statement or of the proposition p.

Now suppose height of $John$ is $5'8''$

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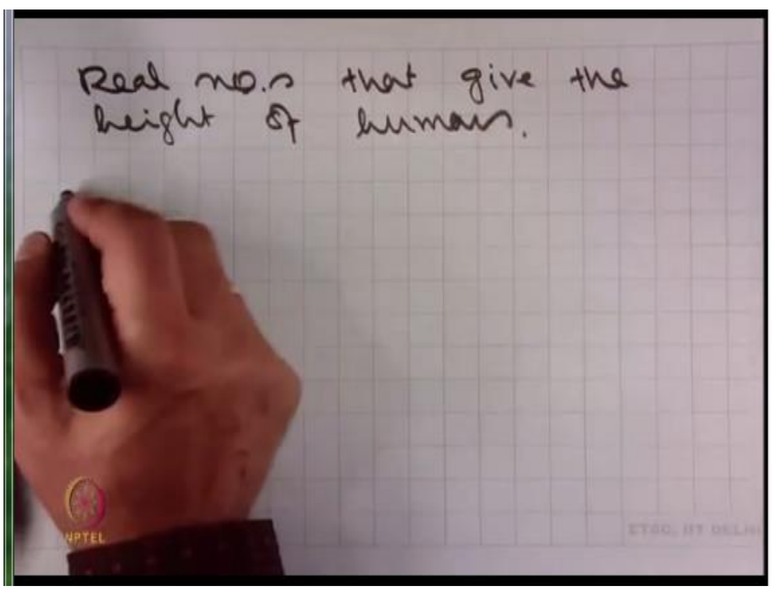

And suppose we define a fuzzy set $Tall$ on the set of real numbers that give the height of human beings.

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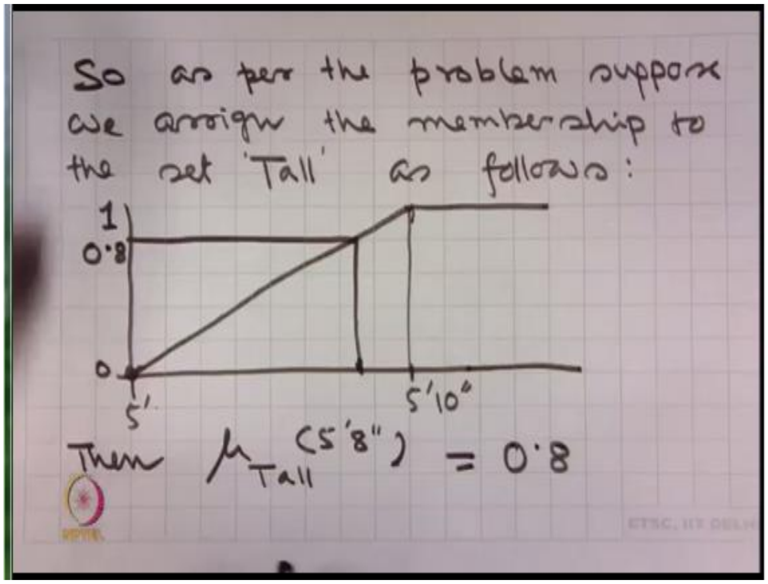

So as per the problem.

Suppose we assign the membership to the set $Tall$ as follows:

At $5'$ it is 0, at $5'10''$ it is 1 and from there it remains 1 and between $5'$ to $5'10''$ it grows linearly.

Then $\mu_{Tall}(5'8'') = 0.8$

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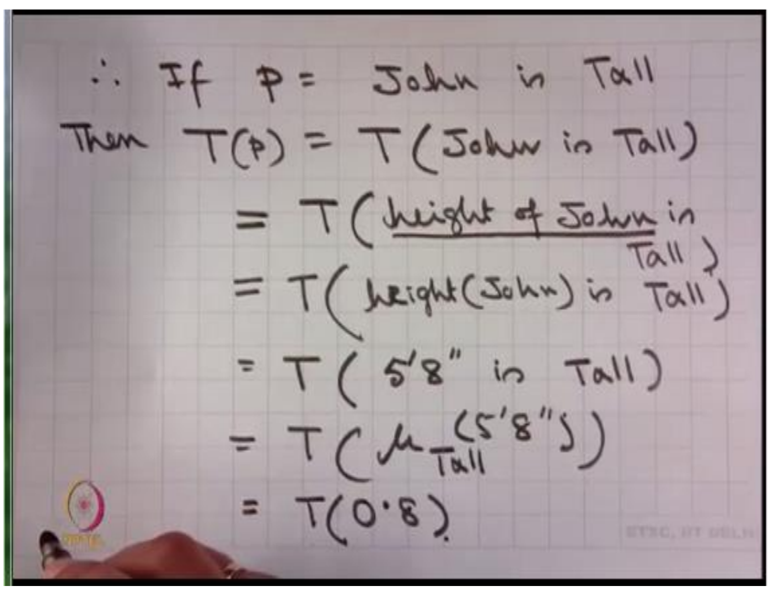

Therefore, If $p =$ $John\ is\ Tall$ then

$T(p) = T(John\ is\ Tall)$

$\quad = T(height\ of\ John\ is\ Tall)$

$\quad = T(height(John)\ is\ Tall)$

$= T(5'8'' \ is \ Tall)$

$= T\left(\mu_{Tall}(5'8'')\right)$

$= T(0.8)$

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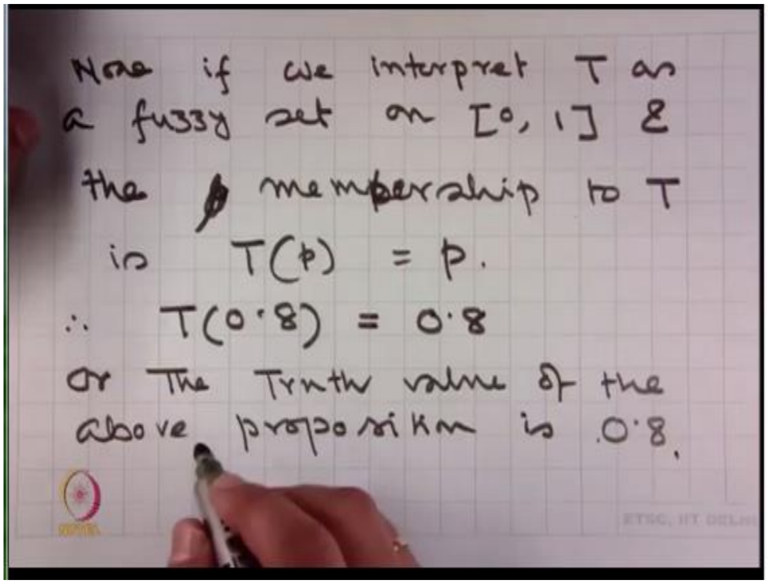

Now if we interpret T as a fuzzy set on $[0, 1]$ and the membership to T is

$$T(p) = p$$

$$\therefore T(0.8) = 0.8$$

Or the truth value of the above proposition is 0.8.

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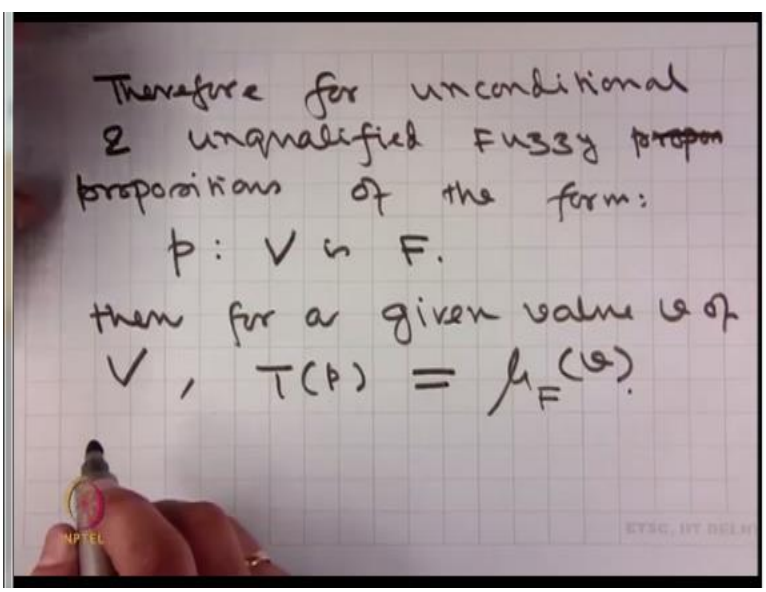

Therefore, for unconditional and unqualified fuzzy propositions of the form:

$$p = V \text{ is } F$$

then, for a given value v of V, $T(p) = \mu_F(v)$

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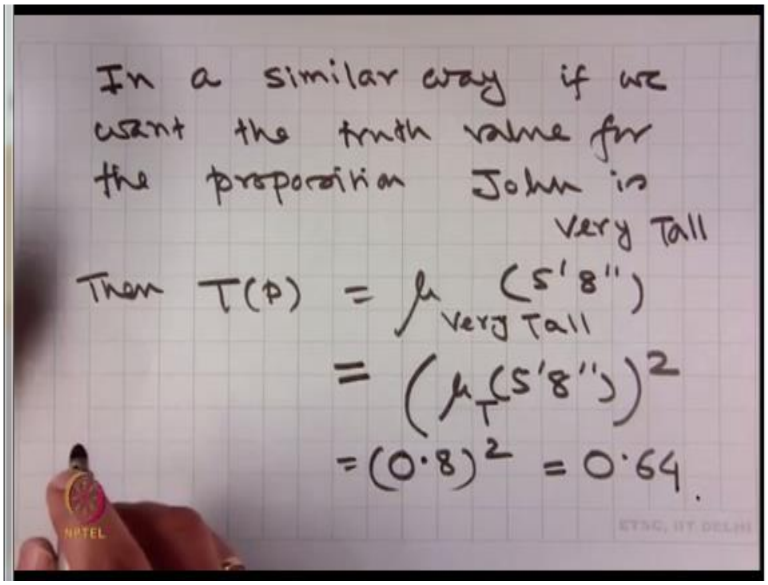

In a similar way, if we want the truth value for the proposition

$$John\ is\ Very\ Tall$$

Then $T(p) = \mu_{Very\ Tall}(5'8'') = \left(\mu_{Tall}(5'8'')\right)^2 = (0.8)^2 = 0.64$

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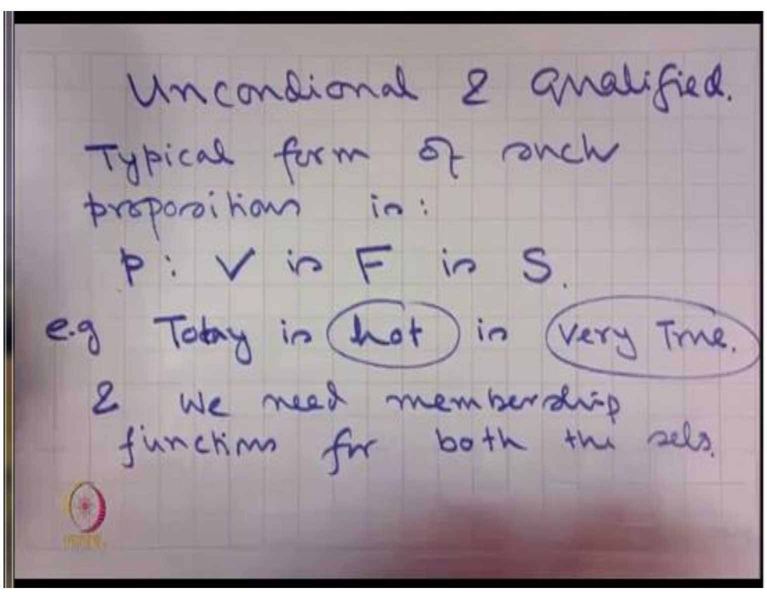

Let us now look at statements of the form

2. Unconditional and qualified.

Typical form of such propositions is:

$$p = V \text{ is } F \text{ is } S.$$

For example: $Today\ is\ hot\ is\ Very\ True$.

Thus, we have two fuzzy terms one is hot, one is $Very\ True$ and we need membership functions for both the sets.

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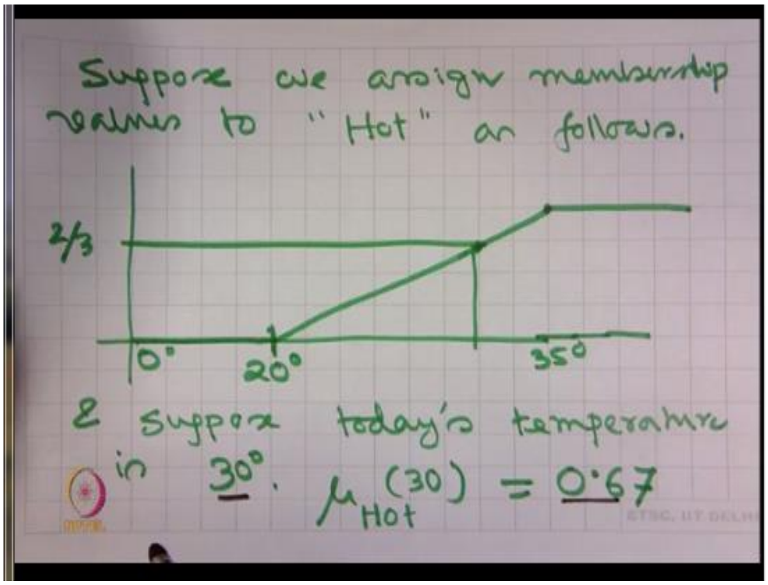

Suppose we assign membership values to hot as follows:

At 0°C it is 0, up to 20°C it is 0 between $20 - 35$°C it grows linearly. I am keeping the functions linear to keep the computation simple, if the membership is not linear you have to look at the membership value accordingly.

And suppose today's temperature is 30°C then

$$\mu_{Hot}(30) = \frac{2}{3} = 0.67$$

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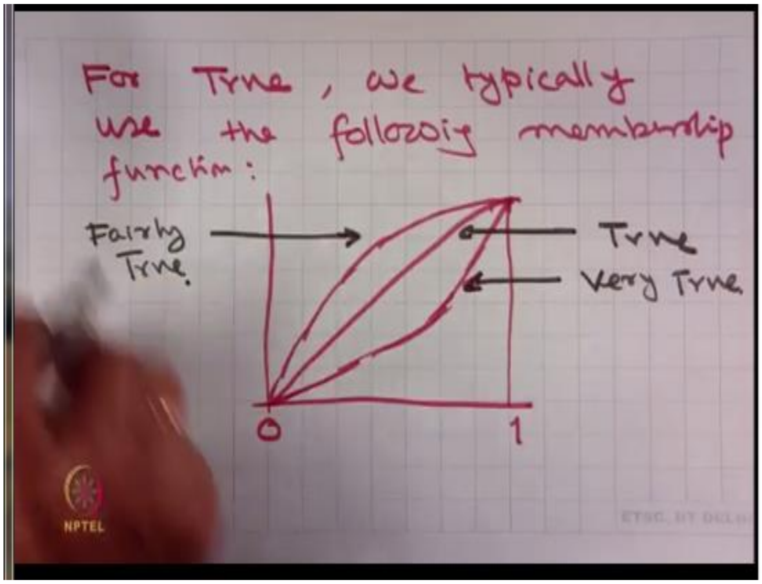

Now for $True$, we typically use the following membership function.

This goes linearly from 0 to 1 in the set 0 to 1.

Therefore $Very\ True$ is going to be square of $True$. And in a similar way by taking square root we get the membership for a *Dilation* say $Fairly\ True$.

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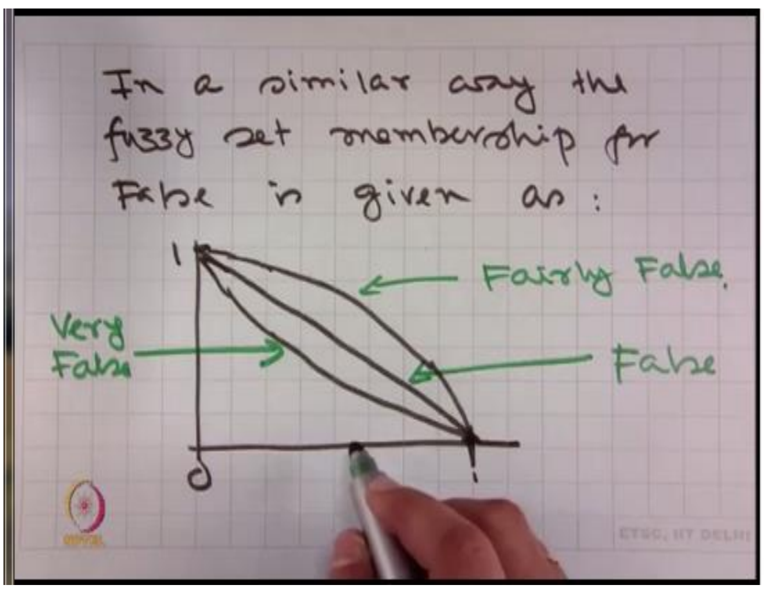

In a similar way, the fuzzy set membership for $False$ is given as:

At 0 it is 1 and at 1 it is 0 and we get a straight line.

Therefore, by taking square we get the membership to the concentrator $Very\ False$ and by considering square root we get the membership for $Fairly\ False$.

As I have discussed there can be other concentrators and there can be other dilators also but for this class we are using α^2 for concentration and $\sqrt{\alpha}$ for dilation.

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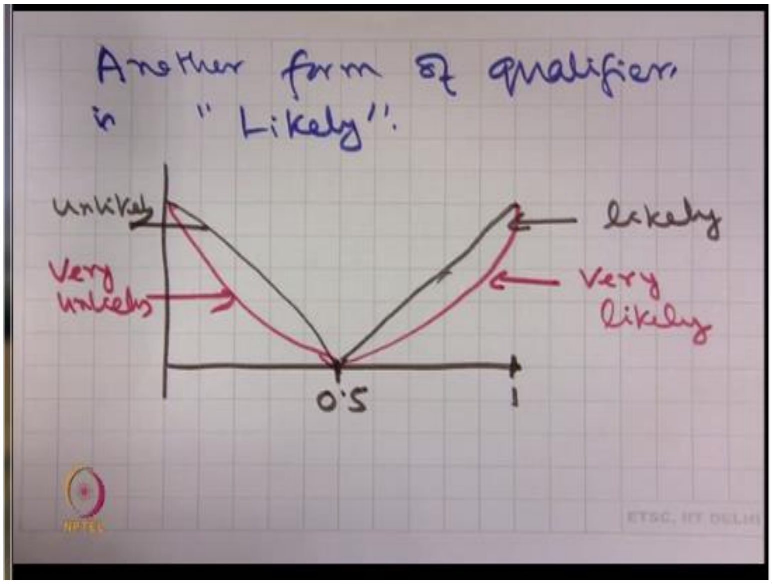

Another form of qualifier is $likely$ which may have the following type of membership.

At 0.5 it is 0. So as we go from 0.5 to 1 if we keep it linearly increasing. This is called $Likely$. From 0.5 to 0 it is linearly decreasing and from 0.5 to 1 it is 0. This is $Unlikely$ and we can also use concentration here $Very\ Likely$ and $Very\ Unlikely$. So this allows us to give membership to different qualifiers. With that background now let us look at the original problem.

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Our statement was: $Today\ is\ hot\ is\ very\ True$

So if we look at the original problem.

We find that if today's temperature is $30°C$ its membership is $\frac{2}{3} = 0.67$.

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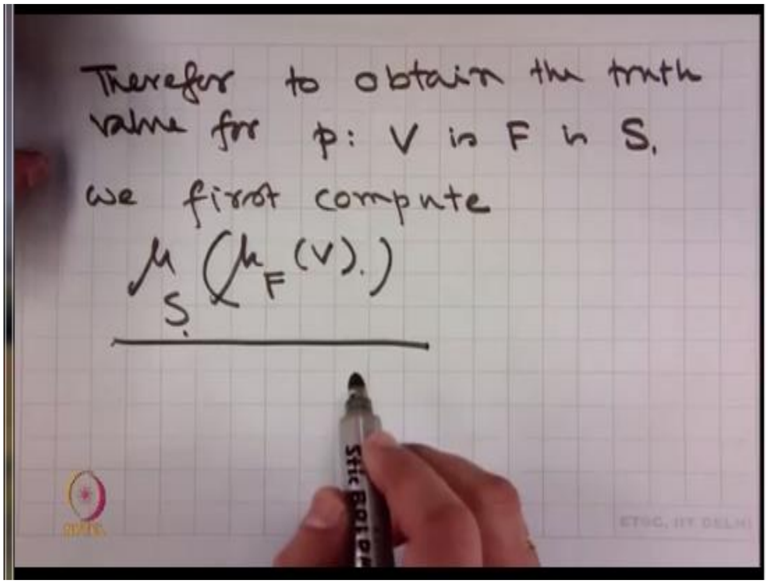

Therefore, to obtain the truth value for

$$p = V \text{ is } F \text{ is } S,$$

We first compute $\mu_F(V)$ and then we look at it is membership to S

$$\therefore T(p) = \mu_S(\mu_F(V))$$

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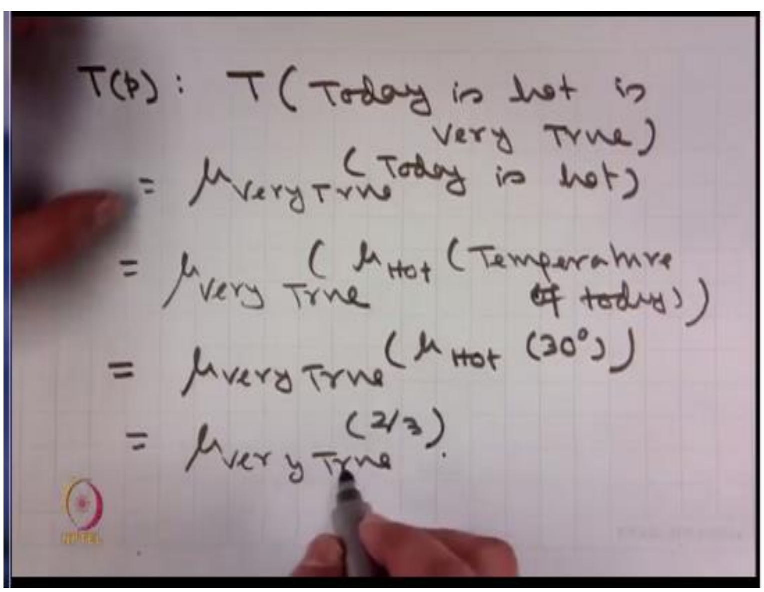

$$\therefore T(p) = T(Today\ is\ hot\ is\ very\ True)$$
$$= \mu_{Very\ True}(Today\ is\ hot)$$
$$= \mu_{Very\ True}\big(\mu_{Hot}(temperature\ of\ Today)\big)$$
$$= \mu_{Very\ True}\big(\mu_{Hot}(30℃)\big)$$
$$= \mu_{Very\ True}\left(\frac{2}{3}\right)$$
$$= \left(\mu_{True}\left(\frac{2}{3}\right)\right)^2$$
$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9} \approx 0.44$$

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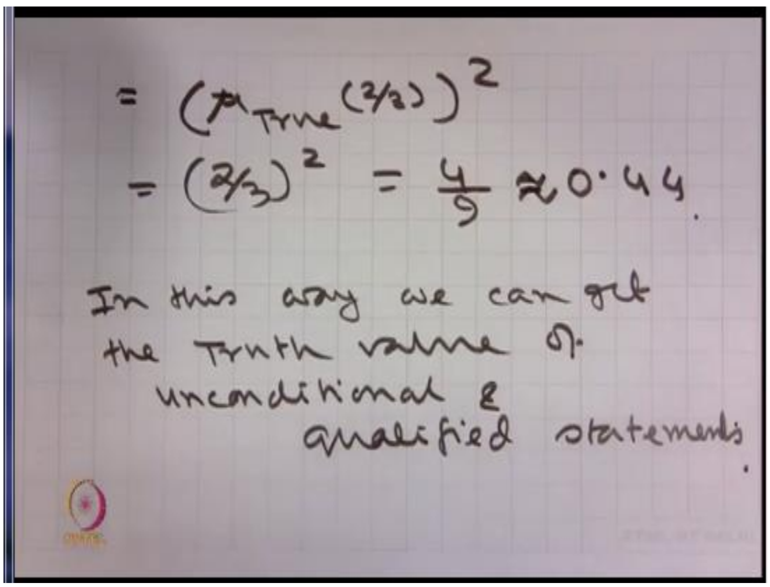

In this way we can get the truth value of unconditional and qualified statements.

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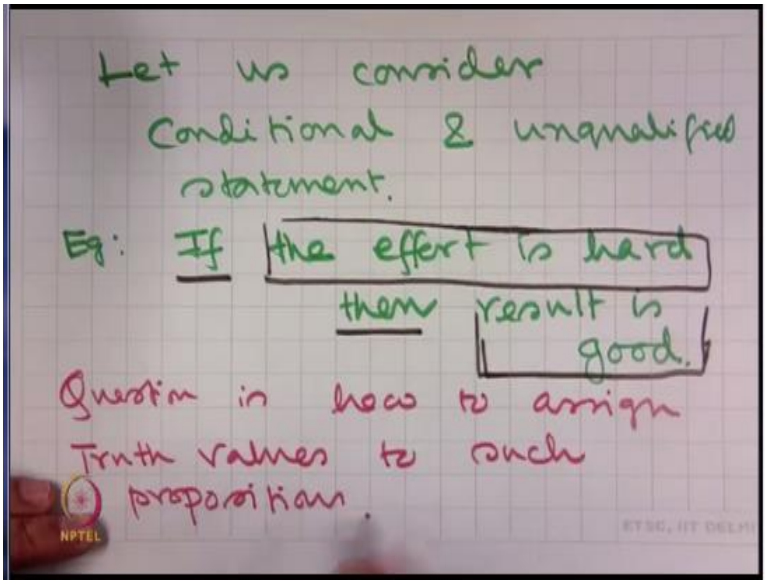

Let us now consider

3. Conditional and unqualified statements.

   For illustration, consider:

   If the effort is hard then result is good.

   Thus, it is a conditional statement. Because the *result is good*, this phrase, its truth value depends upon the conditional statement, *the effort is hard*.

   Question is how to assign truth values to such propositions?

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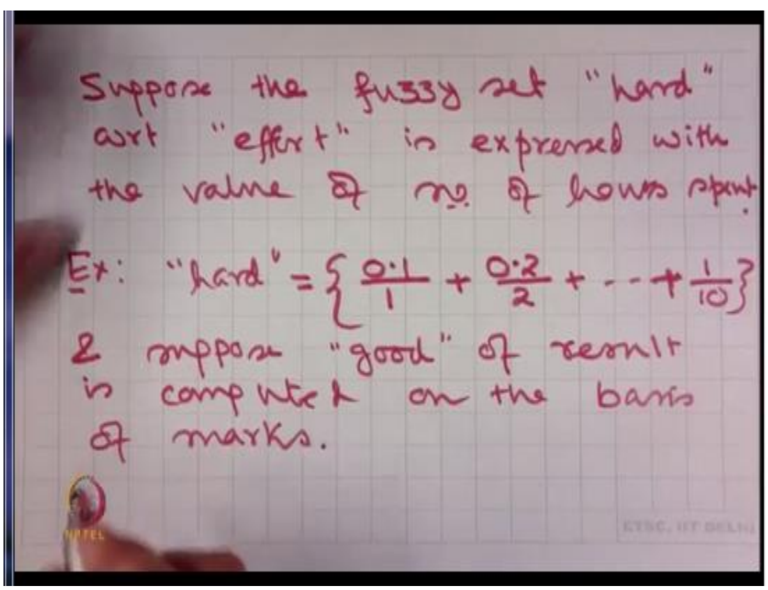

So suppose the fuzzy set $hard$ with respect to $effort$ is expressed with the value of number of hours spent.

Say for example:

$$hard = \left\{\frac{0.1}{1} + \frac{0.2}{2} + \cdots + \frac{1}{10}\right\}$$

And suppose *goodness* of result is computed on the basis of marks.

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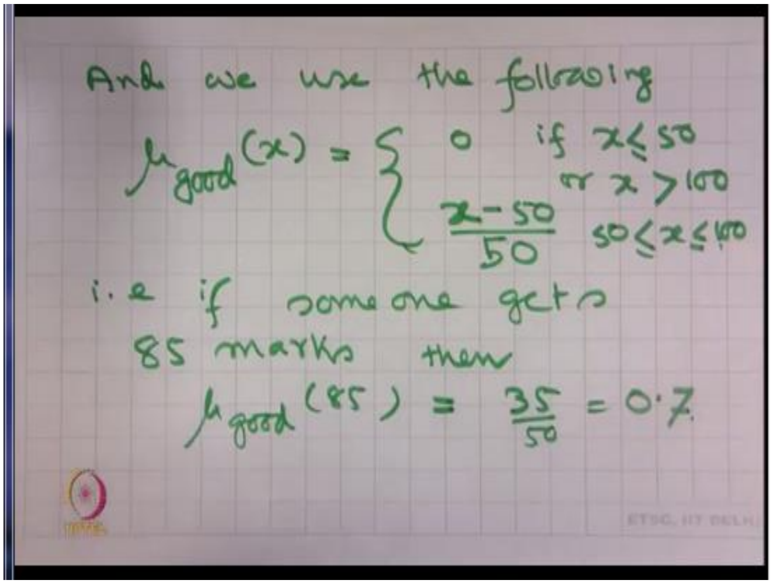

And we use the following:

$$\mu_{good}(x) = \begin{cases} 0 & x \leq 50 \; or \; x > 100 \\ \dfrac{x-50}{50} & 50 \leq x \leq 100 \end{cases}$$

That is if someone gets 85 marks, then $\mu_{good}(85) = \frac{35}{50} = 0.7$

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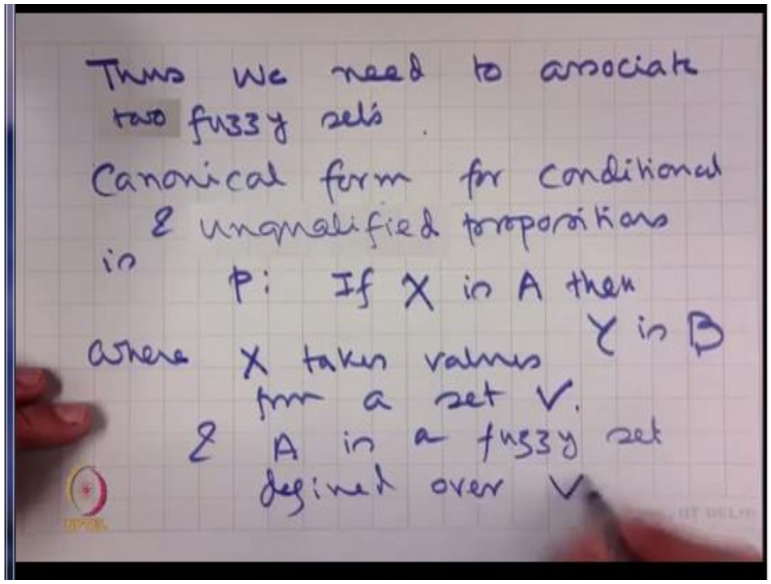

Thus we need to associate two fuzzy sets because the canonical form for conditional and unqualified propositions is

$$p: If\ X\ is\ A\ then\ Y\ is\ B,$$

where X takes values from a set V and A is a fuzzy set defined over V and Y takes values from a set W and B is the fuzzy set defined over W

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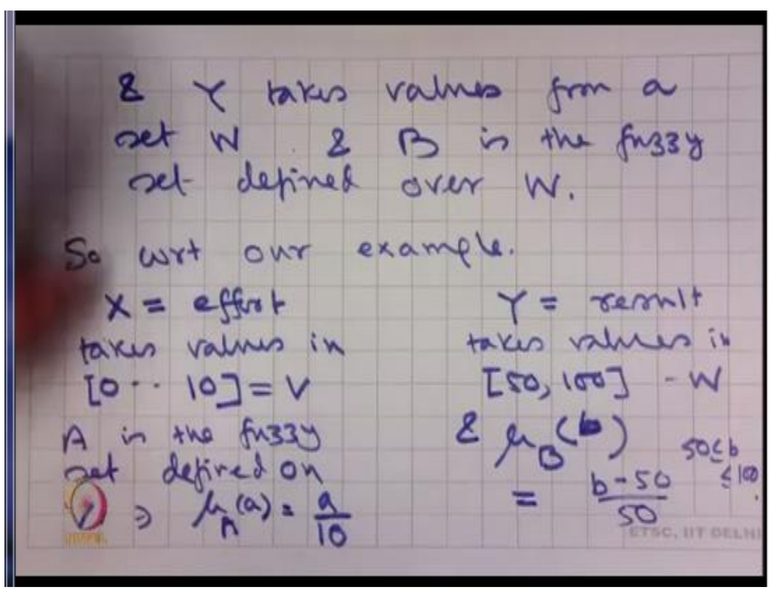

So with respect to our example

$X = effort$ which takes values number of hours $[0 \ldots 10] = V$ and A is the fuzzy set defined on V such that $\mu_A(a) = \frac{a}{10}$

And similarly we had $Y = result$ which takes values in $[50, 100]$ which is W and

$$\mu_B(b) = \frac{b - 50}{50} \quad 50 \le b \le 100$$

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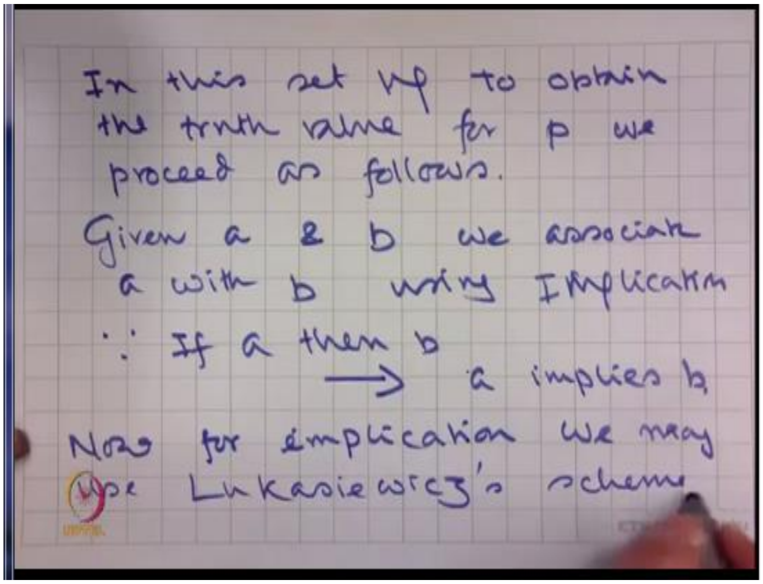

Therefore, in this setup to obtain the truth value for p we proceed as follows.

Given a and b we associate a with b using implication.

Because if a then $b \to a$ implies b.

Now for implication we may use Lukasiewicz scheme which says

$$a \to b = \min(1, 1 - a + b)$$

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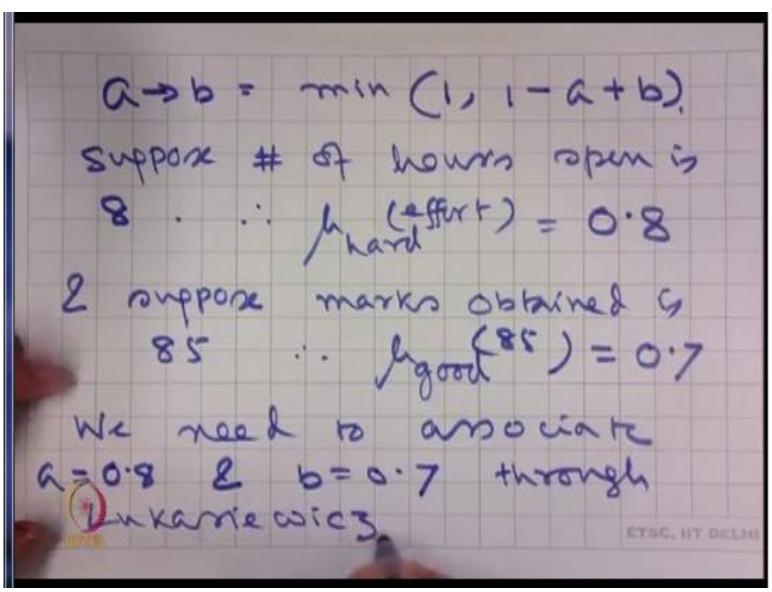

So suppose number of hours spent is 8,

$$\therefore \mu_{hard}(effort) = 0.8$$

and suppose marks obtained is 85

$$\therefore \mu_{good}(85) = 0.7$$

We need to associate these two, $a = 0.8$ and $b = 0.7$ through Lukasiewicz.

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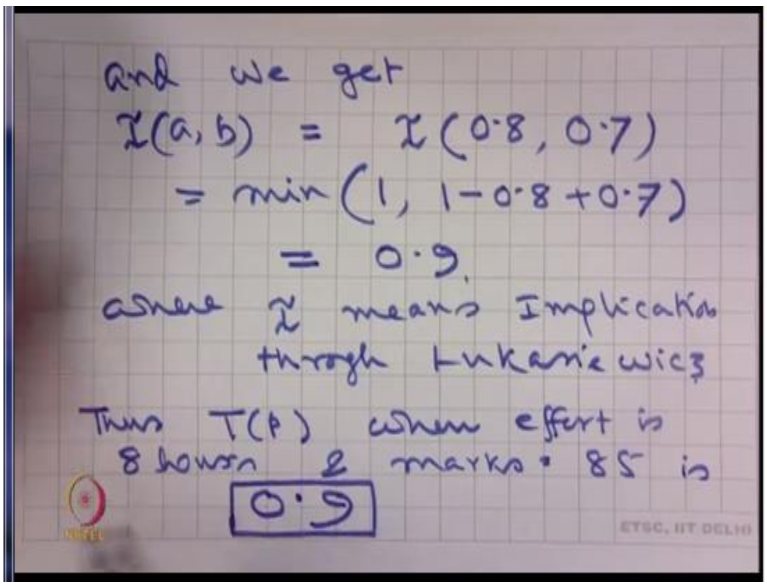

And we get $\mathfrak{T}(a, b) = \mathfrak{T}(0.8, 0.7) = \min(1, 1 - 0.8 + 0.7) = 0.9$, where $\mathfrak{T}$ means implication through Lukasiewicz.

Thus, $T(p)$ when effort is equal to 8 hours and marks is equal to 85 is 0.9.

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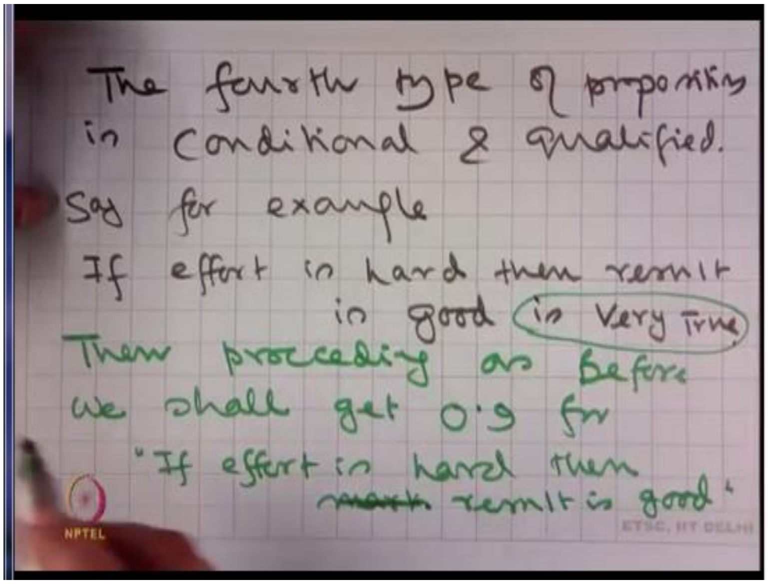

4. Conditional and Qualified.

   Say for example,

   If effort is hard then result is good is very true.

   Then proceeding as before we shall get 0.9 for *If effort is hard then marks is good or result is good*. Now we have to look at this qualifier and for that what we will do.

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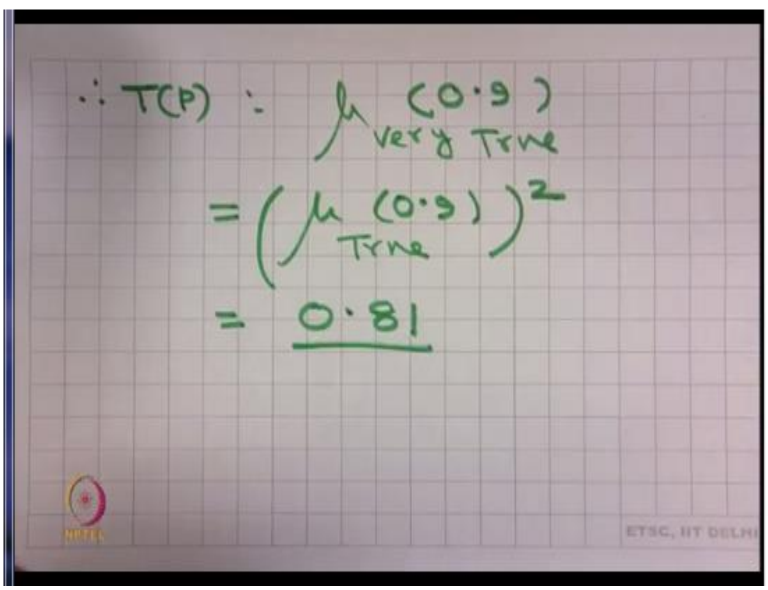

$$\therefore T(p) = \mu_{Very\ True}(0.9) = \left(\mu_{True}(0.9)\right)^2 = 0.81$$

Ok friends I stop here today. In the next class I shall look at fuzzy implication and I look at how to handle them for reasoning with fuzzy statements, thank you.