Introduction to Fuzzy Sets Arithmetic and Logic
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Lecture-23
Fuzzy Sets Arithmetic and Logic

Welcome students to the MOOCs course on fuzzy sets arithmetic and logic. This is lecture number 23 and we know that we are working on fuzzy logic.

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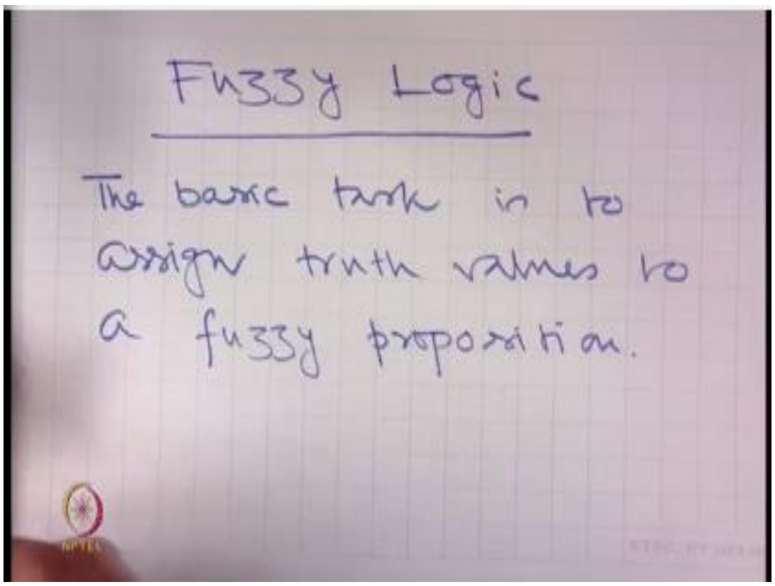

The basic task is to assign truth values to a fuzzy proposition.

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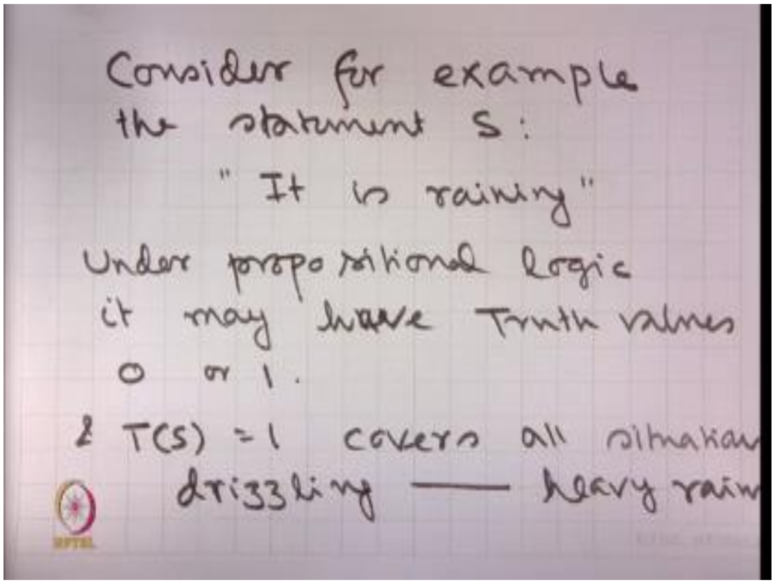

Consider for example the statement S:

It is raining

Under propositional logic it may have truth values, 0 or 1.

And $T(S) = 1$ covers all situations from drizzling to heavy downpour say heavy rains.

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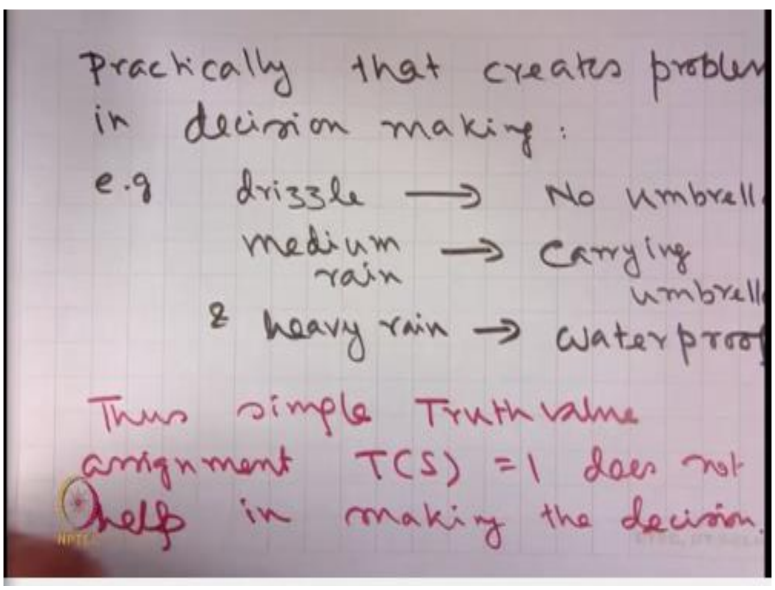

But practically, that creates problem in decision making

For example:

drizzle → no umbrella to be taken

medium rain → one can think of carrying umbrella heavy rain → that one will carry waterproof.

Thus simple truth value assignment $T(S) = 1$ does not help in making the decision.

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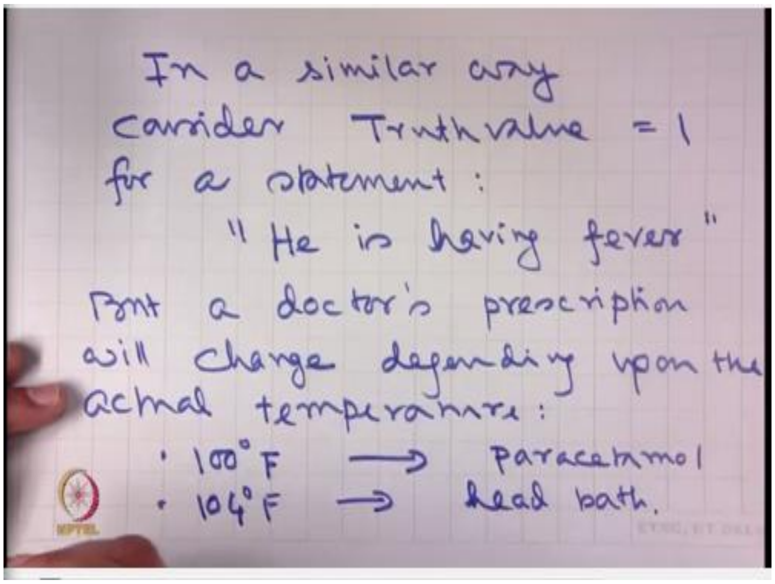

In a similar way, consider Truth Value = 1, for a statement:

He is having fever.

But a doctor's prescription will change depending upon the actual temperatures.

100°F → paracetamol

104°F → head bath

Thus what I wanted to emphasize on is that simple $0, 1$ truth values do not help us in making a decision in real life situation.

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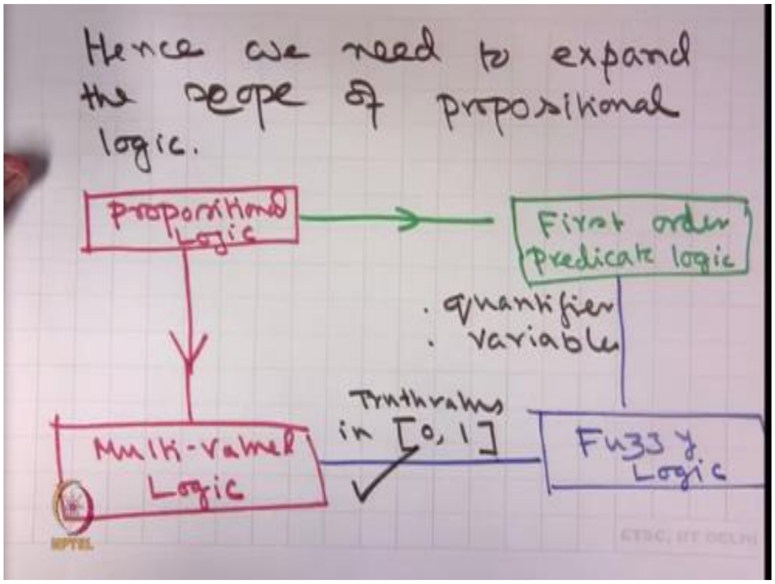

Hence, we need to expand the scope of propositional logic.

And we have seen the following propositional logic, we have expanded it to take care of multi-valued logic. And another way of expansion was through first order predicate logic. And together we got that we will use in fuzzy logic.

First order predicate logic gives us the concept of quantifier and also the concept of variables and multi-valued logic allows us to use truth values in the entire interval $[0, 1]$.

In today's lecture I shall focus on this aspect.

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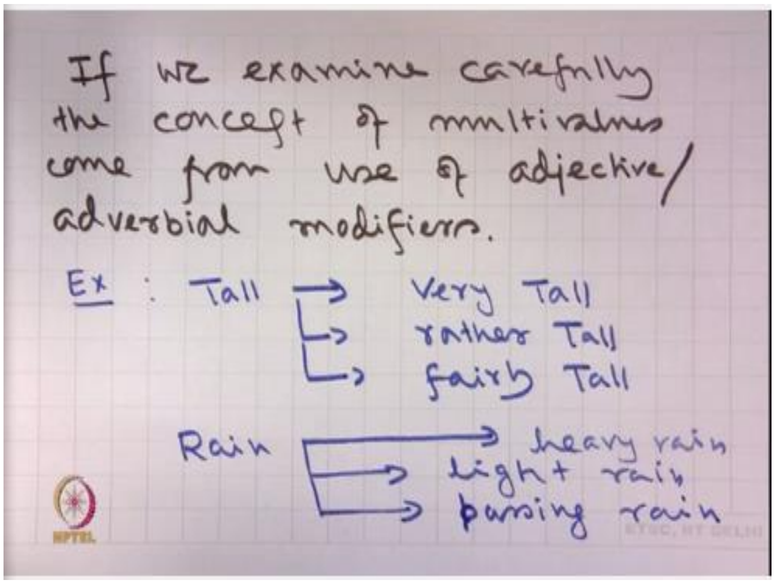

Now if we examine carefully the concept of multi-values come from use of adjective or adverbial modifiers.

For example:

Tall, we can get *Very Tall*, we may get *Rather Tall*, we may get *Fairly Tall*.

Similarly, from *Rain* we can get *Heavy Rain*, *Light Rain*, *Passing Rain*.

So these are the terms *Very*, *Rather*, *Fairly*, *Heavy*, *Passing*, *Light*, that actually affect the truth values of a fuzzy proposition.

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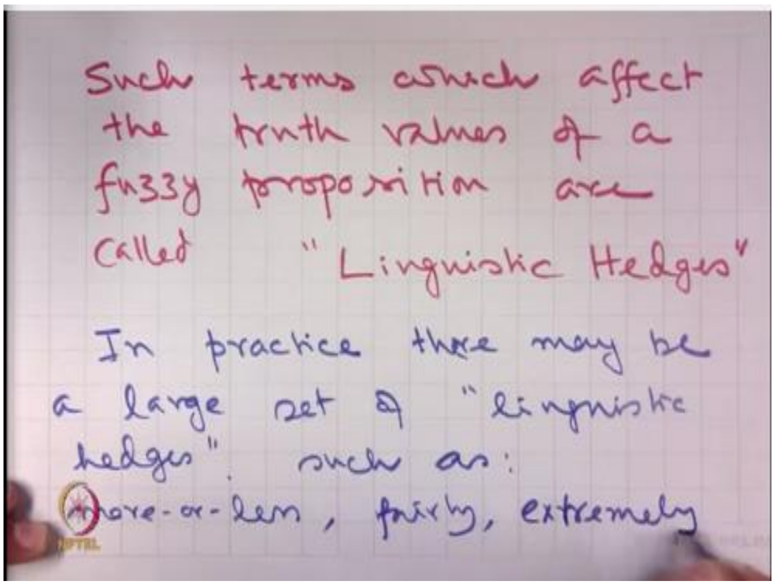

Such terms which affect the truth values of a fuzzy proposition are called *Linguistic Hedges*.

In practice there may be a large set of *Linguistic Hedges*, such as:

more or less, *fairly*, *extremely*.

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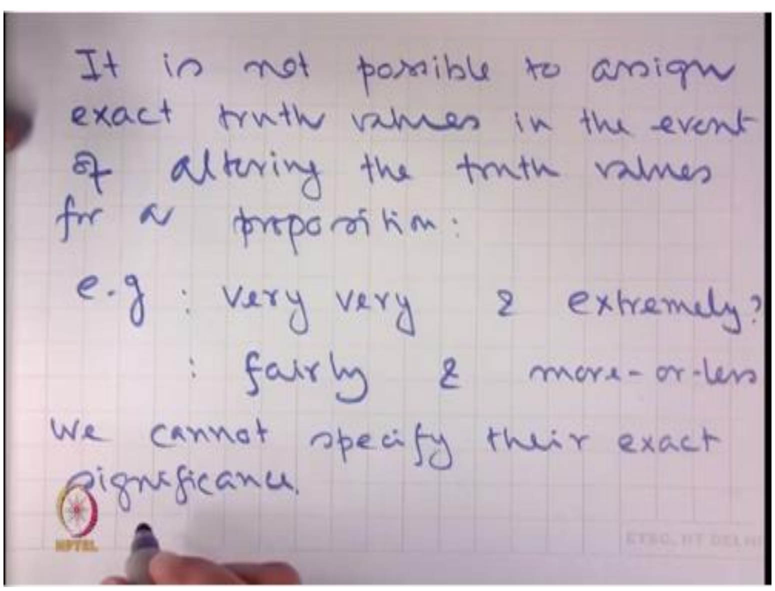

It is not possible to assign exact truth values in the event of altering the truth values for a proposition.

For example:

What is the difference between *Very Very* and *Extremely*?

What is the difference between *Fairly* and *More or Less*?

We cannot specify their exact significance. Hence what we try, we try to classify them into different groups.

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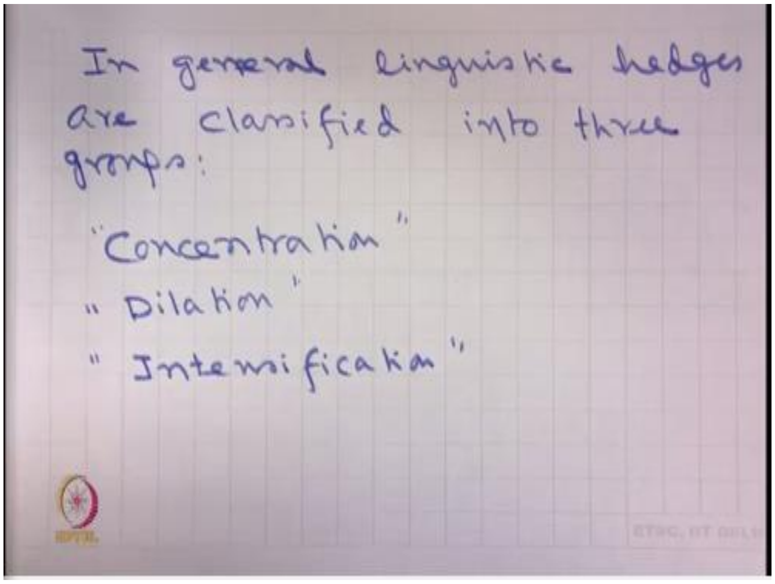

In general, linguistic hedges are classified into three groups:

What are they?

- *Concentration,*
- *Dilation*
- *Intensification.*

So let us discuss this one-by-one

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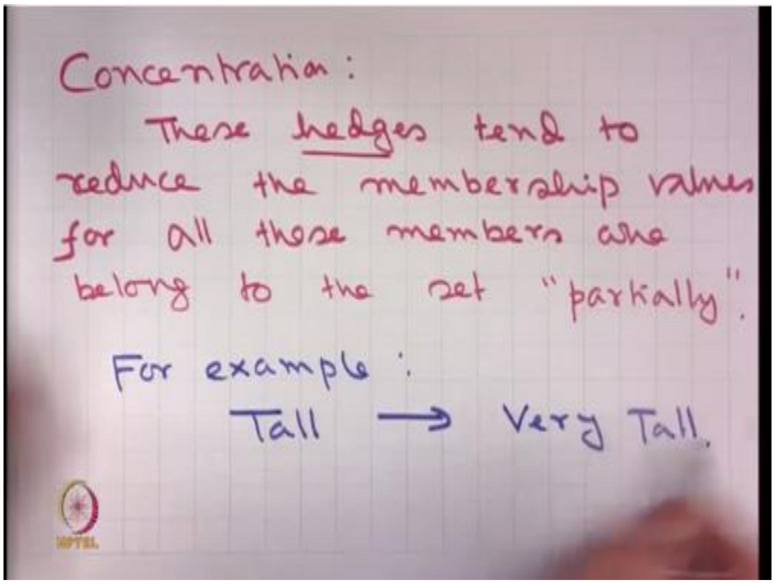

Concentration:

These hedges tend to reduce the membership values, for all those members who belong to the set ‘partially’.

That means if the membership is 0 or the membership is 1, then their membership values are not altered by these hedges but for all other their values are reduced to the new class.

Say for example:

$$Tall \rightarrow Very\ Tall.$$

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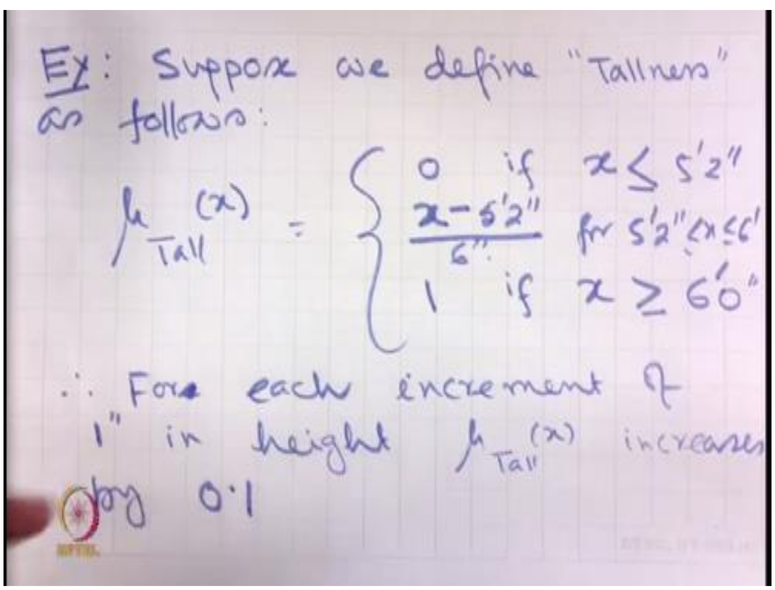

For illustration let us take this example,

Suppose we define *Tallness* as follows:

$$\mu_{Tall}(x) = \begin{cases} 0 & \text{if } x \le 5'2'' \\ \dfrac{x - 5'2''}{10''} & \text{if } 5'2'' \le x \le 6' \\ 1 & \text{if } x \ge 6' \end{cases}$$

Therefore, for each increment of $1''$in height $\mu_{Tall}(x)$ increases by 0.1.

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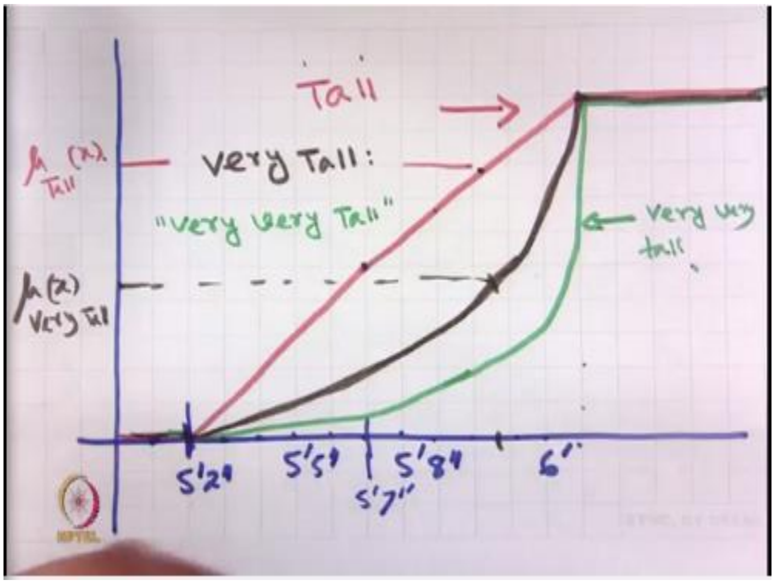

So let us plot it. We get following straight line as the membership to the fuzzy set *Tall*.

Question is what is a *Very Tall*?

Now it is clear that anybody who is below $5'2''$ will have membership 0 and by the definition of concentrator anybody above $6'$ will also have membership 1. In between obviously if somebody's membership is given here, we expect that membership to the set *Very Tall* will be slightly less. So suppose we draw the membership to *Very Tall* by using a curve and it is clear from the graph that for any height x, $\mu_{Tall}(x) \ge \mu_{Very\ Tall}(x)$. We can go even further suppose we want to give the membership to the set *Very Very Tall*, so how it will look like.

So we will concentrate *Very Tall* further and we can expect a shape like this which will give us the membership for *Very Very Tall*.

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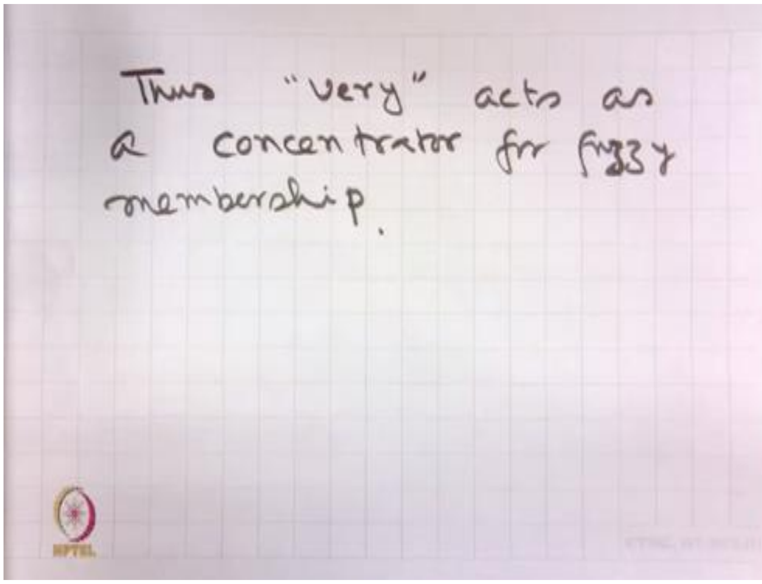

Thus, *Very* acts as a concentrator for fuzzy membership.

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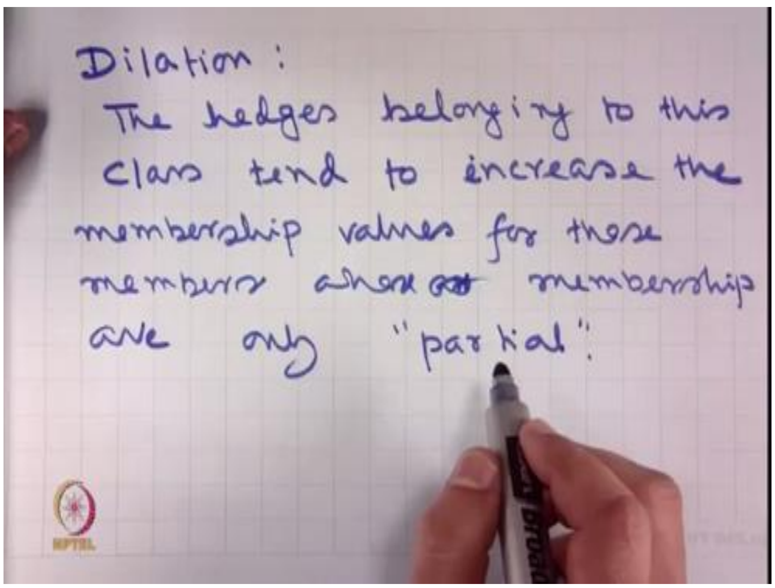

Dilation:

The hedges belonging to this class tend to increase the membership values for those members whose membership are only *partial.*

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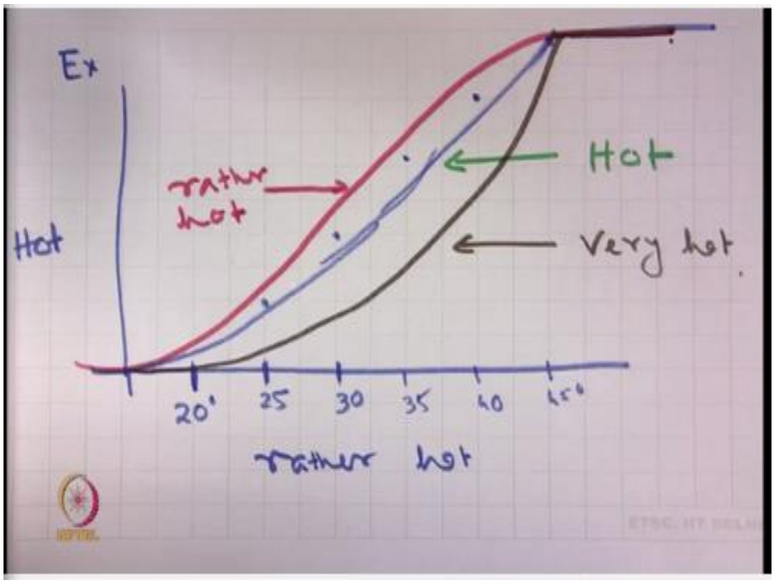

Example:

Consider temperature and suppose we are looking at *Hot*

Thus the straight line between 20℃ and 45℃ is the membership value for Hot, then what happens to *Rather Hot*. We expect that membership of these temperatures to the set *Rather Hot* is going to be a little bit more than *Hot* therefore we may expect that the membership is represented by a curve above the straight line for *Hot*.

And as we have discussed sometime back the membership to *Very Hot* is represented by a curve below the straight line for *Hot*. I hope that distinction is clear.

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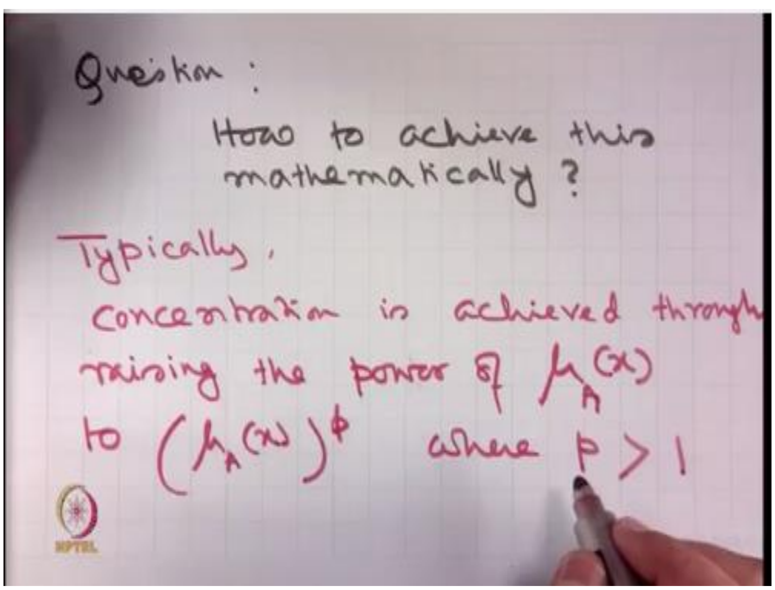

Question is how to achieve this mathematically?

Typically, concentration is achieved through raising the power of $\mu_A(x)$ to $\left(\mu_A(x)\right)^p$ where $p > 1$

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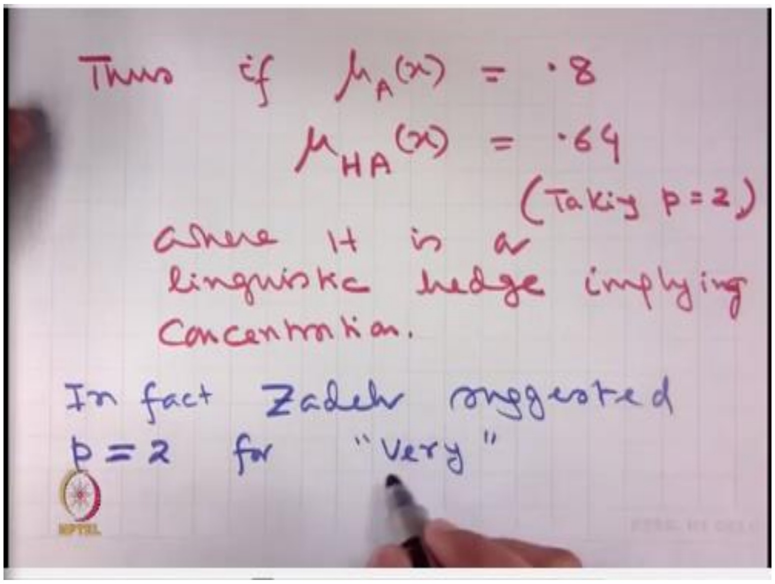

Thus if $\mu_A(x) = 0.8$

$\mu_{H(A)}(x) = 0.64$ (Taking $p = 2$), where H is a *Linguistic Hedge* implying concentration.

In fact, Professor Zadeh suggested $p = 2$ for *Very*.

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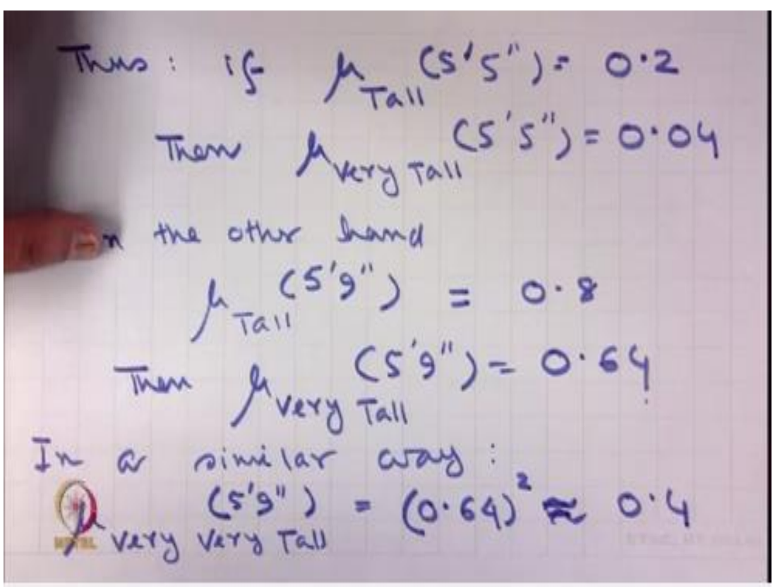

Thus, if $\mu_{Tall}(5'5'') = 0.2$ then, $\mu_{Very\,Tall}(5'5'') = 0.04$

On the other hand, $\mu_{Tall}(5'9'') = 0.8$ then, $\mu_{Very\,Tall}(5'9'') = 0.64$

In a similar way, $\mu_{Very\,Very\,Tall}(5'9'') = (0.64)^2 \approx 0.4$

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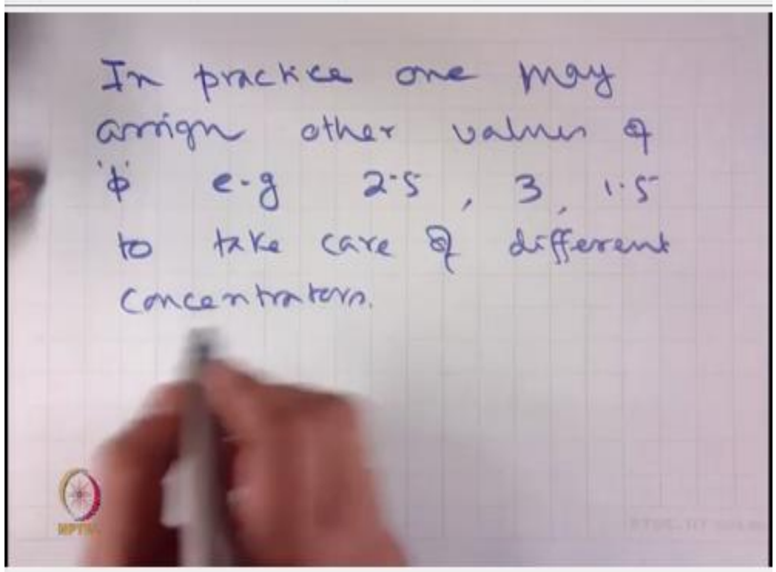

In practice one may assign other values of p

e.g: say $2.5, 3, 1.5$ to take care of different concentrators.

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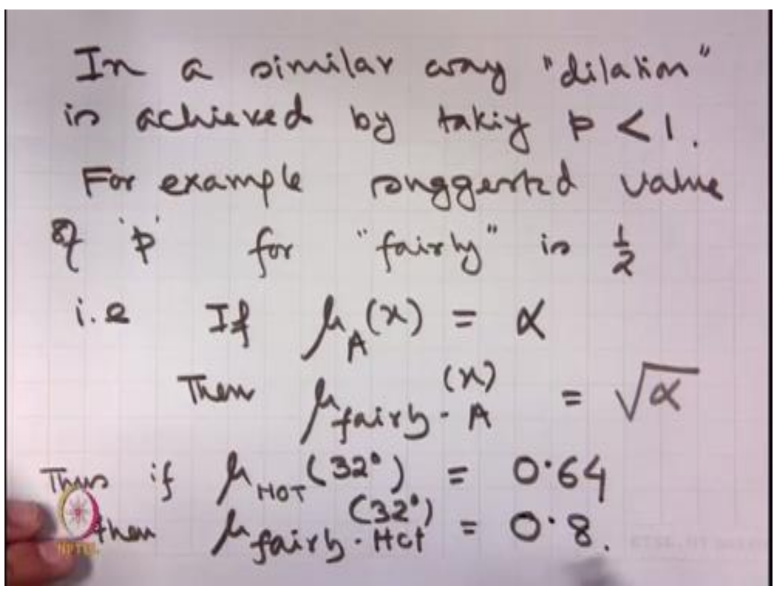

In a similar way *Dilation* is achieved by taking $p < 1$

For example, suggested value of p for *fairly* is $\frac{1}{2}$.

That is if $\mu_A(x) = \alpha$ then, $\mu_{fairly\ A}(x) = \sqrt{\alpha}$

Thus, if $\mu_{Hot}(32\ ℃) = 0.64$ then, $\mu_{fairly\ Hot}(32\ ℃) = 0.8$

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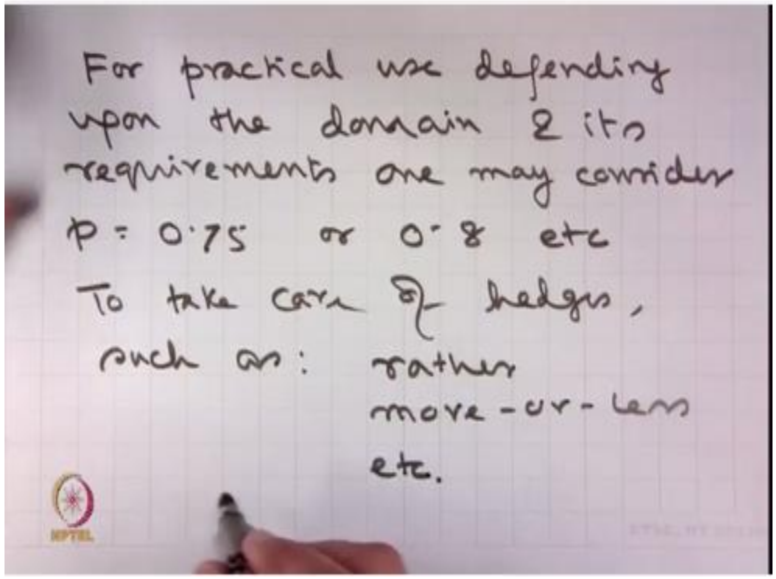

For practical use depending upon the domain and its requirements one may consider $p = 0.75$ or 0.8 etc. To take care of hedges such as: *Rather, More or Less* etc.

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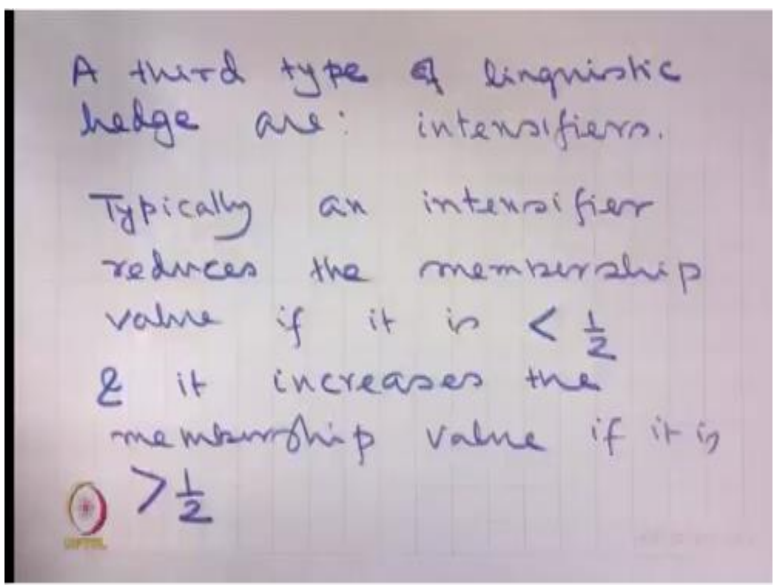

A third type of linguistic hedge are intensifiers.

Typically, an intensifier reduces the membership value if it is $< \frac{1}{2}$ and it increases the membership value if it is $> \frac{1}{2}$

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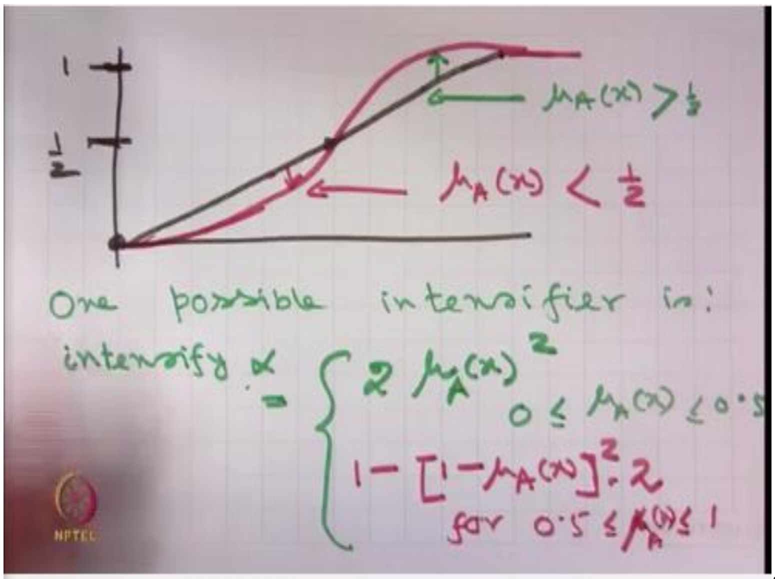

One possible intensifier is:

$$\mu_{intensify(A)}(x) = \begin{cases} 2\,(\mu_A(x))^2 & \text{for } 0 \le \mu_A(x) \le 0.5 \\ 1 - 2[1 - \mu_A(x)]^2 & \text{for } 0.5 \le \mu_A(x) \le 1 \end{cases}$$

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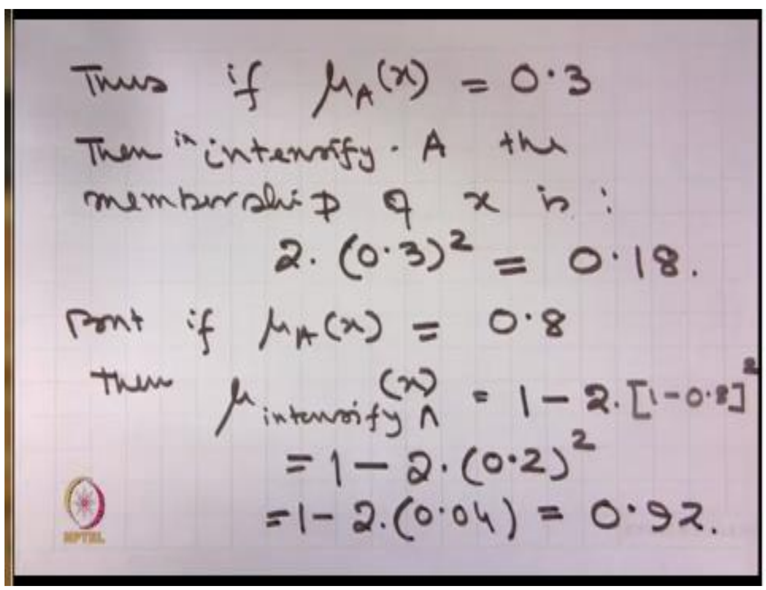

Thus if $\mu_A(x) = 0.3$ then, in $intensify\ A$ the membership of x:

$$2(0.3)^2 = 0.18$$

But if $\mu_A(x) = 0.8$ then

$$\mu_{intensify(A)}(x) = 1 - 2[1 - 0.8]^2 = 1 - 2(0.2)^2 = 1 - 2(0.04) = 0.92$$

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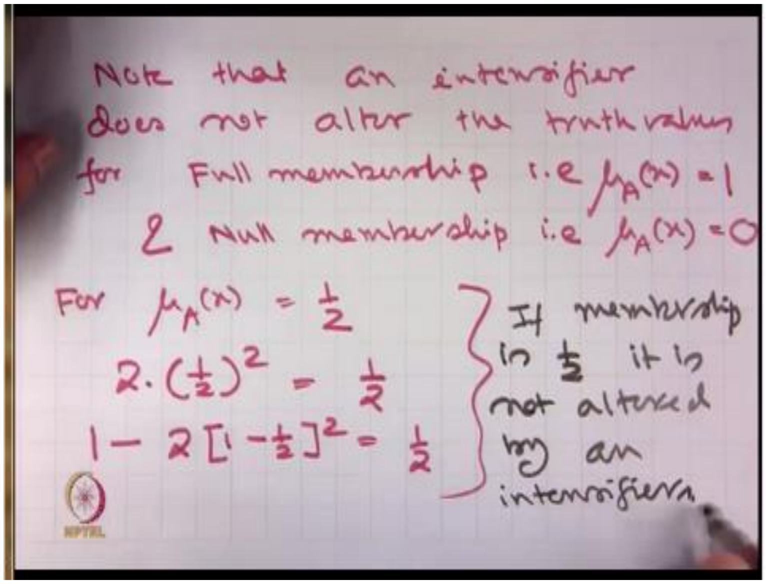

Note that an intensifier does not alter the truth values for full membership that is $\mu_A(x) = 1$ and null membership that is $\mu_A(x) = 0$

For $\mu_A(x) = \frac{1}{2}$

$$2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$1 - 2\left[1 - \frac{1}{2}\right]^2 = \frac{1}{2}$$

Thus, if membership is $\frac{1}{2}$ it is not altered by an intensifier.

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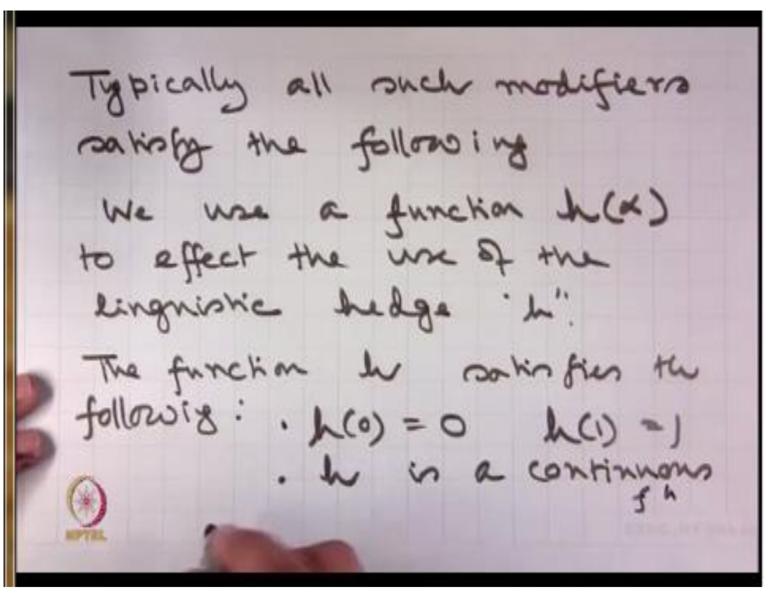

Typically, all such modifiers satisfy the following:

We use a function $h(\alpha)$ to effect the use of the *Linguistic Hedge* say h.

The function h satisfies the following:

- $h(0) = 0, h(1) = 1$
- h is a continuous function.

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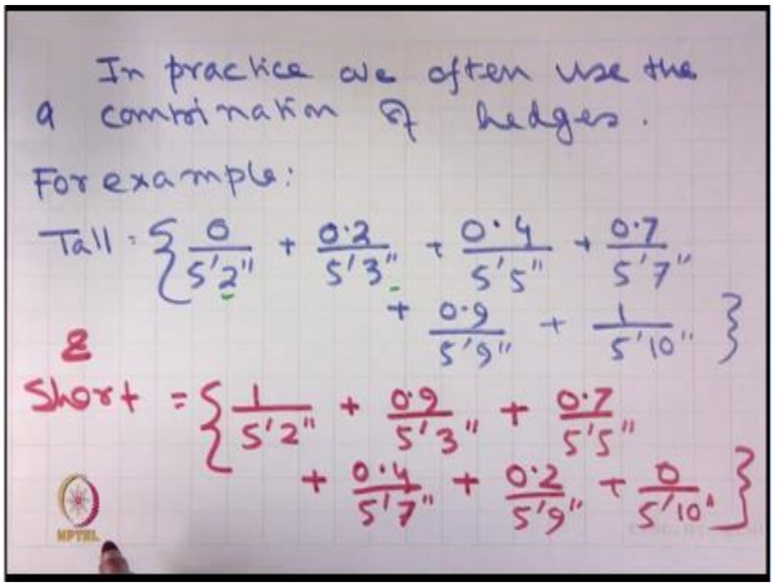

In practice we often use the combination of hedges.

For example:

Suppose for $Tall$ we use the following:

$$Tall = \left\{\frac{0}{5'2''} + \frac{0.2}{5'3''} + \frac{0.4}{5'5''} + \frac{0.7}{5'7''} + \frac{0.9}{5'9''} + \frac{1}{5'10''}\right\}$$

So suppose this is the membership values we assign to class $Tall$ for different heights.

Suppose $Short$ is defined as

$$Short = \left\{\frac{1}{5'2''} + \frac{0.9}{5'3''} + \frac{0.7}{5'5''} + \frac{0.4}{5'7''} + \frac{0.2}{5'9''} + \frac{0}{5'10''}\right\}$$

Suppose these are the two fuzzy sets defined on the subset of heights.

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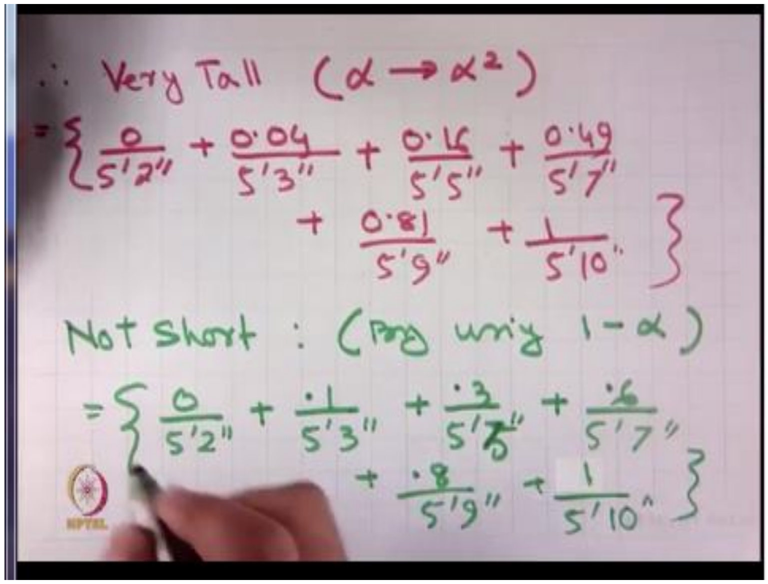

Therefore, $Very\ Tall$ is going to be by making α going to α^2. $(\alpha \rightarrow \alpha^2)$

$$Very\ Tall = \left\{\frac{0}{5'2''} + \frac{0.04}{5'3''} + \frac{0.16}{5'5''} + \frac{0.49}{5'7''} + \frac{0.81}{5'9''} + \frac{1}{5'10''}\right\}$$

And $Not\ Short$ by using $1 - \alpha$

$$Not\ Short = \left\{\frac{0}{5'2''} + \frac{0.1}{5'3''} + \frac{0.3}{5'5''} + \frac{0.6}{5'7''} + \frac{0.8}{5'9''} + \frac{1}{5'10''}\right\}$$

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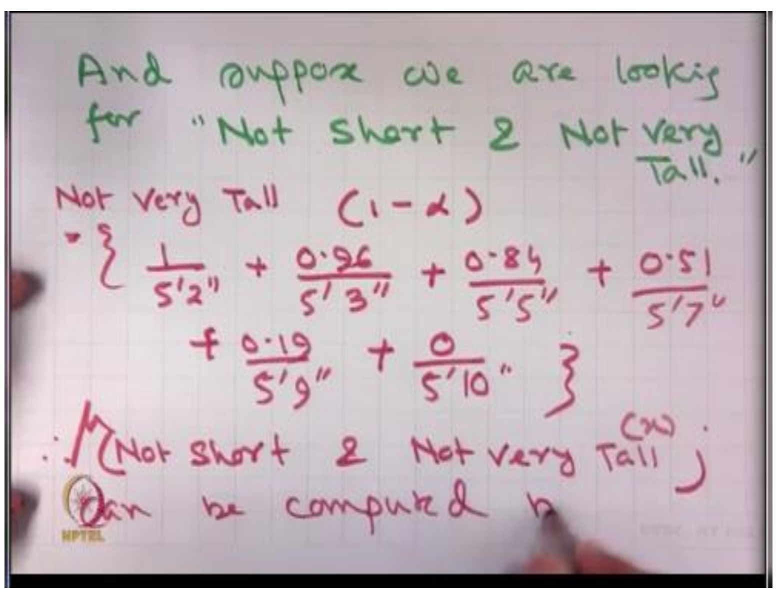

And suppose we are looking for $Not\ Short$ and $Not\ Very\ Tall$.

We want to find membership of say boys to this set. So, in order to do that we first need to compute $Not\ Very\ Tall$ and that set is going to be by again taking $1 - \alpha$ and applying it on $Very\ Tall$

$$Not\ Very\ Tall = \left\{\frac{1}{5'2''} + \frac{0.96}{5'3''} + \frac{0.84}{5'5''} + \frac{0.51}{5'7''} + \frac{0.19}{5'9''} + \frac{0}{5'10''}\right\}$$

Therefore, $\mu_{(Not\ Short\ and\ Not\ Very\ Tall)}(x)$ can be computed by taking minimum of $\mu_{Not\ Short}(x)$ and $\mu_{Not\ Very\ Tall}(x)$

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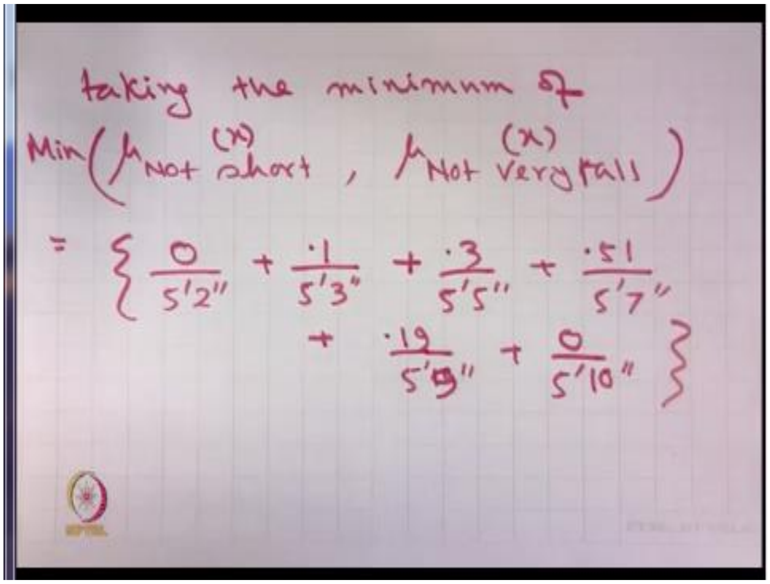

$$\min\left(\mu_{Not\ Short}(x), \mu_{Not\ Very\ Tall}(x)\right) = \left\{\frac{0}{5'2''} + \frac{0.1}{5'3''} + \frac{0.3}{5'5''} + \frac{0.51}{5'7''} + \frac{0.19}{5'9''} + \frac{0}{5'10''}\right\}$$

So in this way we can use a combination of *Linguistic Hedges* and from there by using the formulae that we learnt already, we can compute the membership values for different x belonging to the set by combining them suitably.

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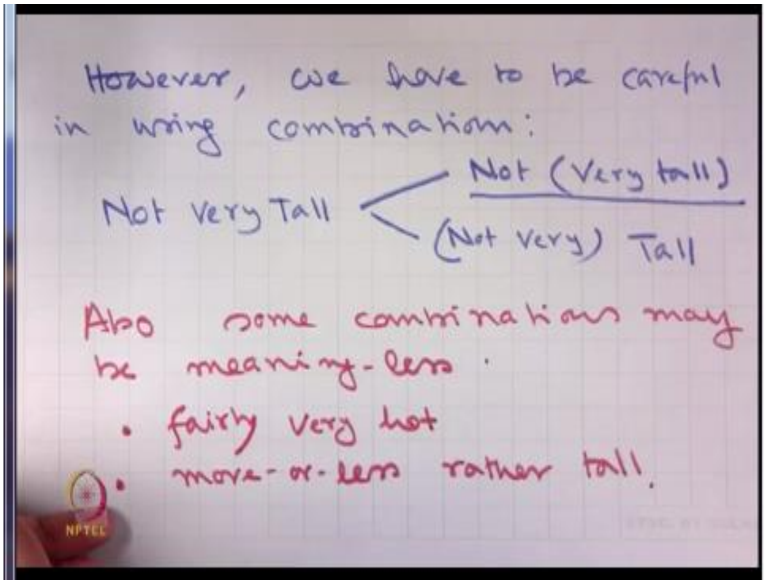

However, we have to be careful in using combinations.

For example: What is $Not\ Very\ Tall$?

Is it $Not(Very\ Tall)$ or is it $Not\ Very\ (Tall)$

This is a linguistic question and perhaps an answer is not very clear.

In the example earlier I have used this definition.

Also some combinations maybe meaningless, say for example:

Fairly very hot,

More or less rather tall,

Such combinations may not really have any meaning but by using formulae we may be able to compute their values the membership values for different entities but its practical significance is very doubtful.

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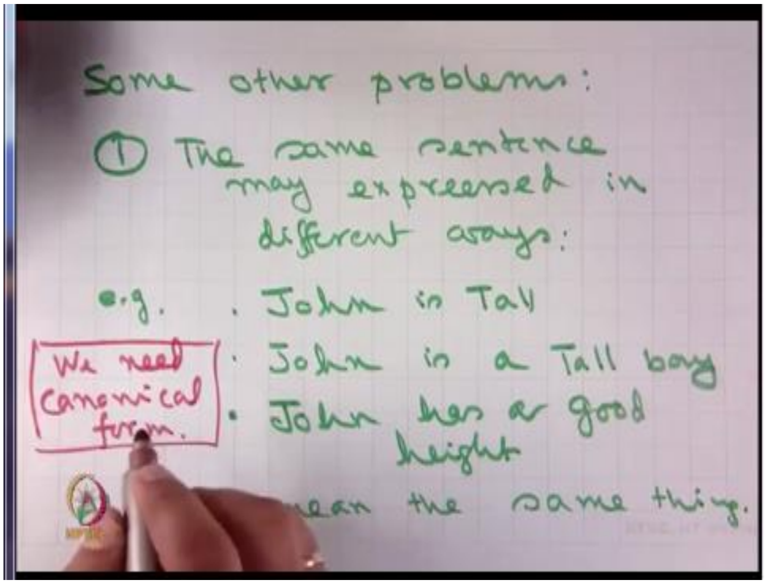

Some other problems

1. The same sentence may be expressed in different ways

   e.g.

   - *John is tall,*
   - *John is a tall boy,*
   - *John has a good height*

   All may mean the same thing; therefore, we need canonical form.

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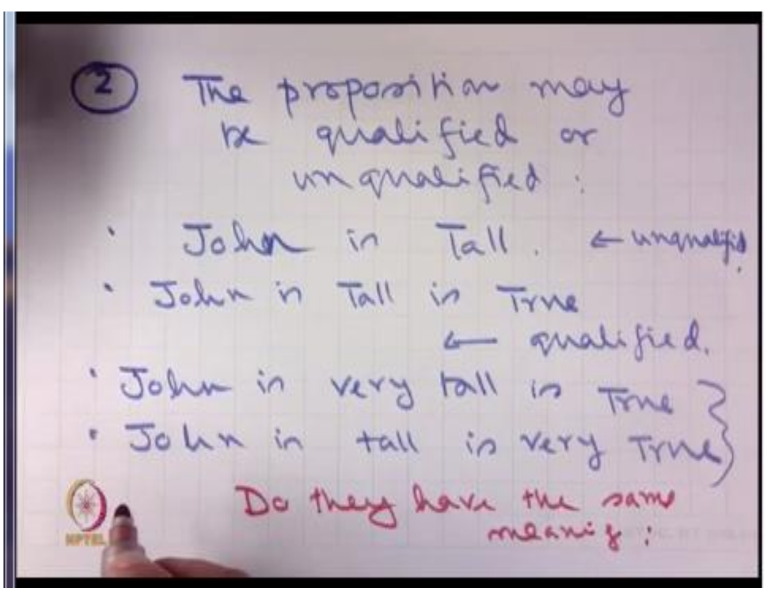

2. The propositions may be qualified or unqualified.

- *John is Tall* is unqualified.
- *John is Tall is True* is qualified.
- *John is Very Tall is True*
- *John is Tall is Very True*.

John is Very Tall is True and *John is Tall is Very True* both are qualified but do they have the same meaning?

We need to understand how to deal with such propositions. Before I stop I give you one more point.

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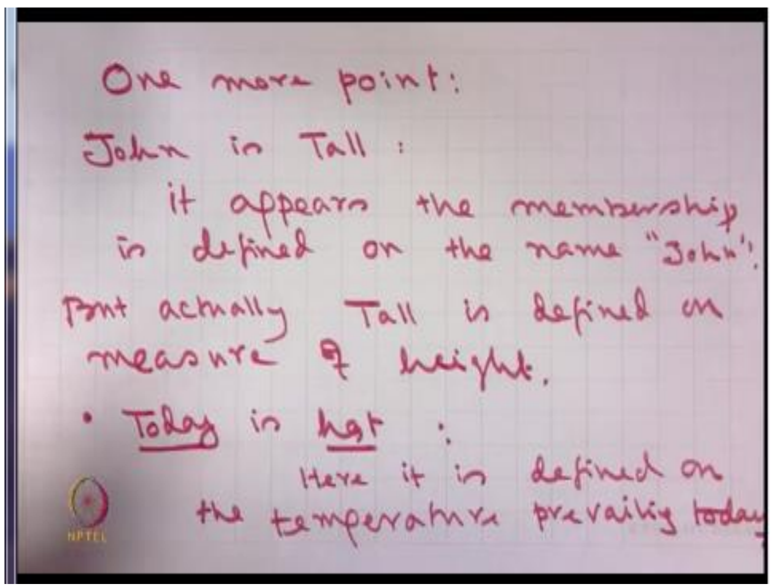

When we say *John is Tall*:

it appears the membership defined on the name *John*.

But actually *Tall* is defined on measure of height.

Similarly, *Today is Hot,*

Hot is not defined on that date of *Today*, here it is defined on the temperature prevailing today.

So when we talk about fuzzy propositions we need to take care of these as well.

Ok friends I stop here today. In the next class I shall discuss different fuzzy propositions and means of handling them to obtain their truth values, thank you.