Introduction to Fuzzy Sets Arithmetic and Logic Prof. Niladri Chatterjee Department of Mathematics Indian Institute of Technology minus Delhi

Lecture minus 22 Fuzzy Sets Arithmetic & Logic

Welcome students to the MOOCs lecture on fuzzy sets arithmetic and logic. This is lecture number

22.

(Refer Slide Time: 00:27)

FUZZY Selis Arithmetic 2 Logic -ecture - 22

And as you all know in the last class I started fuzzy logic.

(Refer Slide Time: 00:38)

We have seen that the barris of Propositional Logic i.e each statement can be given a Truth value either 1 (True) or 0 (False) in not practically justified.

We have seen that the basis of Propositional Logic that is each statement can be given a truth value either 1 that is True or 0 that is False is not practically justified.

(Refer Slide Time: 01:49)

To stress it further I give you another interesting example from logic, "Liar paradex" · Many versions · I consider a simple one

To stress the point further I give you another interesting example from logic known as Liar Paradox.

So there are many versions, I take a simple one.

(Refer Slide Time: 02:43)

Liar Paradox Suppose I make a statement S: What I am staking is False Querkan: What is the Truth value of S? As per propositional logic S True I.e. 1 False i.e. 0. ie. O.

Liar Paradox

Suppose I make a statement

S: What I am stating is False.

Question is what is the truth value of *S*?

So we know that is per propositional logic

S can have two truth values,

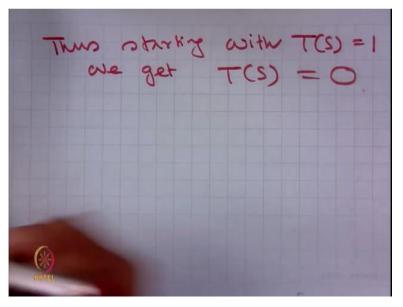
- True that is 1
- False that is 0.

(Refer Slide Time: 04:13)

Suppore 5 is Trime i.e T(s) = 1 => My statement S is True "What I am staking is Fabe" - True anat I am staking is False S in False T(S)

Suppose S is True that is T(S) = 1 \Rightarrow My statement S is True \Rightarrow 'What I am stating is False' ---this is True \Rightarrow What I am stating is False \Rightarrow S is False \Rightarrow T(S) = 0

(Refer Slide Time: 05:35)



Thus starting with T(S) = 1 we get T(S) = 0

(Refer Slide Time: 05:55)

Conversels: Suppose S is False. => T(s) = 0 => "What I am stating in Fabe" - Fabe => assat I am stating in True S in True T(S) = 1

Conversely,

Suppose S is False

 $\Rightarrow T(S) = 0$

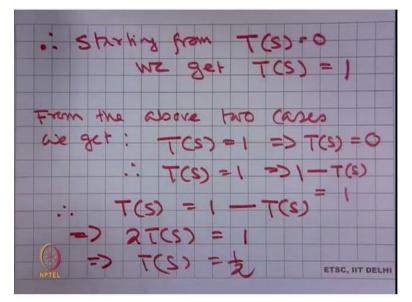
 \Rightarrow 'What I am stating is False', this statement is False

 \Rightarrow What I am stating is True

 \Rightarrow *S* is True

$$\Rightarrow T(S) = 1$$

(Refer Slide Time: 07:11)



Therefore, starting from T(S) = 0 we get T(S) = 1

From the above two cases we get:

 $T(S) = 1 \Rightarrow T(S) = 0$ $\therefore T(S) = 1 \Rightarrow 1 - T(S) = 1.$ $\therefore T(S) = 1 - T(S)$ $\Rightarrow 2T(S) = 1$ $\Rightarrow T(S) = \frac{1}{2}$ (Refer Slide Time: 08:31)

This says that we need to 80 beyond Propositional Logic to desire & new facts or to infor from given set of facto

This says that we need to go beyond propositional logic to derive new facts or to infer from given set of facts.

(Refer Slide Time: 09:20)

One possible extension is Multivalmed Logic Here Truther values not only 100 it can take other values also. The simplest form of Multivalued logic in 3-valued logic Here the Truth values are: えい 立, 0?

One possible extension is Multivalued Logic.

Here truth values are not only 1 or 0, it can take other values also.

The simplest form of multivalued logic is 3-valued logic. Here the truth values are $\{1, \frac{1}{2}, 0\}$.

Many researchers have given their interpretation with 3-valued logic.

(Refer Slide Time: 11:03)

Lukapiecoicz - 1930's. The main problem here in that how to ansign truth values for all the logic operations, in particular N, V, ->.

But the most popular one is by Lukasiewicz in the 1930s.

The main problem here is that how to assign truth values for all the logic operations. In particular, \land , \lor , \rightarrow

(Refer Slide Time: 12:08)

Since there are 3-values. given 2 logical variables G. b. we can have 9 possible pair of values: (1,1) (1,0) (0,1) (0,0) (した),(ちい),(ち、ち),(ち,0) Horse to arrigh Truth values for these.

Since there are 3 values given 2 logical variables *a*, *b*.

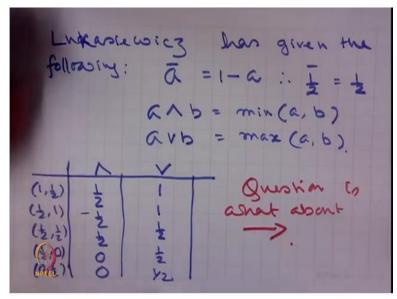
We can have 9 possible pair of values:

- (1,1)
- (1,0)
- (0,1)

- (0,0) $(1,\frac{1}{2})$ $(\frac{1}{2},1)$ $(\frac{1}{2},\frac{1}{2})$
- $-\left(\frac{1}{2},0\right)$
- $\left(0,\frac{1}{2}\right)$

How to assign truth values for these?

(Refer Slide Time: 13:56)



So Lukasiewicz has given the following:

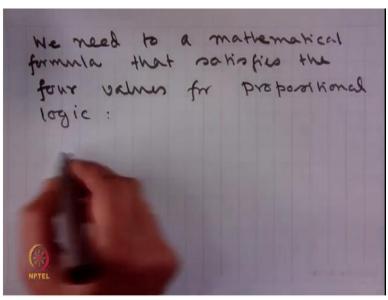
•
$$\bar{a} = 1 - a$$
 $\therefore \frac{\bar{1}}{2} = \frac{1}{2}$

- $a \wedge b = \min(a, b)$
- $a \lor b = \max(a, b)$

	Λ	V
$\left(1,\frac{1}{2}\right)$	$\frac{1}{2}$	1
$\left(\frac{1}{2},1\right)$	$\frac{1}{2}$	1
$\left(\frac{1}{2},\frac{1}{2}\right)$	$\frac{1}{2}$	$\frac{1}{2}$
$\left(\frac{1}{2},0\right)$	0	$\frac{1}{2}$
$\left(0,\frac{1}{2}\right)$	0	$\frac{1}{2}$

The question is what about implication?

(Refer Slide Time: 16:04)



We need to find a mathematical formula that satisfies the four values for Propositional Logic. (Refer Slide Time: 16:23)

Therefore, if we have two logical wariables a 26 them for the four pairs: (1,1), (1,0), (0,1) & (0,0) are should have the following: -> 0 a \cap

Therefore, if we have two logical variables a and b. Then for the four pairs (1, 1), (1, 0), (0, 1) and (0, 0).

We should have the following

а	b	$a \rightarrow b$
1	1	1
1	0	0
0	1	1
0	0	1

Because that is the truth value of $a \rightarrow b$ when we work with Propositional Logic.

(Refer Slide Time: 18:07)

Therefore ashers are are in the domain of multi-valued logic we need to get a mathematical for mula that should give the above q values for the appresaid bairs pairn. And at the same time the same formula should be applicable for other Truth values

Therefore, when we are in the domain of multi-valued logic, we need to get a mathematical formula that should give the above 4 values for the aforesaid pairs.

And at the same time the same formula should be applicable for other truth values.

(Refer Slide Time: 19:50)

Since one are atorking with 3-valued logic, with possible tenthe values $1, \pm 20$ there are pormula should such that for (1, 9), (1, 0) (0, 1) & (0, 0) it will give 1, 0, 1, 1 & we shalluse the same formula for $(1, \pm), (\pm, 1), (\pm, \pm), (\pm, 0)$ $(0, \pm), (\pm, 1), (\pm, \pm), (\pm, 0)$

Since we are working with 3-valued logic with possible truth values $1, \frac{1}{2}$ and 0.

Then $a \to b$ formula should be such that for (1,1), (1,0), (0,1) and (0,0) it will give 1,0,1 and 1 and we shall use the same formula for $(1,\frac{1}{2}), (\frac{1}{2},1), (\frac{1}{2},\frac{1}{2}), (\frac{1}{2},0)$ and $(0,\frac{1}{2})$

(Refer Slide Time: 21:39)

The problem is there may be more than one function for the same task. For illustration: G-DD = NRV b $3 a \rightarrow b = \min(1, 1-a+b)$ $3 a \rightarrow b = \max(1-a, \min(a,b))$ $a \rightarrow b = \max(1-a, \min(a,b))$ $a \rightarrow b = \max(1-a, \min(a,b))$ $a \rightarrow b = \max(1-a, \min(a,b))$ NPTEL

The problem is there may be more than one function for the same task.

For illustration:

- 1) $a \rightarrow b = \sim a \lor b$
- 2) $a \to b = \min(1, 1 a + b)$
- 3) $a \rightarrow b = \max(1 a, \min(a, b))$

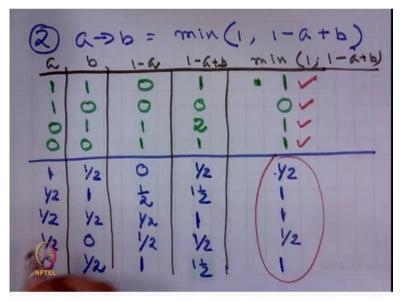
Let us examine these 3 formulae.

(Refer Slide Time: 23:15)



1) $a \rightarrow b = \sim a \lor b$					
	а	b	~a	$\sim a \lor b$	
	1	1	0	1	
	1	0	0	0	
	0	1	1	1	
	0	0	1	1	
	1	1/2	0	1/2	
	1/2	1	1/2	1	
	1/2	1/2	1/2	1/2	
	1/2	0	1/2	1/2	
	0	1/2	1	1	
		l	l	l	

(Refer Slide Time: 26:17)



Let us now consider the second formula

2)
$$a \to b = \min(1, 1 - a + b)$$

а	b	1-a	1 - a + b	$\min(1, 1-a+b)$
1	1	0	1	1
1	0	0	0	0
0	1	1	2	1

0	0	1	1	1
1	1/2	0	1/2	1/2
1/2	1	1/2	1/2	1
1/2	1/2	1/2	1	1
1/2	0	1/2	1/2	1/2
0	1/2	1	1/2	1
	1	l	l I	I

(Refer Slide Time: 30:15)

3	a-	əb =	max (1-	a, min (a, b)
_ a	15	1-6	min(a, b)	mane (1-a, min
1		0	1	17
- 1	0	0	0	oV
0	1	1	0	
0	0	1	0	• -
1	1/2	0	1/2	(Va
12	1	12	Y2.	12
Y2	42	1/2	V2	12
10	0	14	0	42
GS2	,	1/2	0	1/2
NPTEL	え			111

3) $a \rightarrow b = \max(1 - a, \min(a, b))$

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	а	b	1 – a	$\min(a, b)$	$\max(1-a,\min(a,b))$
	1	1	0	1	1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1	0	0	0	0
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	1	1	0	1
1/211/21/21/21/21/21/21/21/21/201/201/2	0	0	1	0	1
1/21/21/21/21/201/201/21/201/2	1	1/2	0	1/2	1/2
1/2 0 1/2 0 1/2	1/2	1	1/2	1/2	1/2
	1/2	1/2	1/2	1/2	1/2
0 1/2 1 0 1	1/2	0	1/2	0	1/2
	0	1/2	1	0	1

And we see that we get another set of values which are different from the earlier ones.

(Refer Slide Time: 33:40)

Thins are have several possible choices for a>b For multivalued logic Lukasiewicz proposed the formula Min(1, 1-a+b) to compute a=b

Thus we have several possible choices for $a \rightarrow b$.

For multivalued logic Lukasiewicz proposed the formula min(1, 1 - a + b) to compute $a \rightarrow b$ (**Refer Slide Time: 34:49**)

Note that the above formula can atork for any arbitrary values R 2 D E [0, 1] In our discussion on drawing informace from conditional statements are shall us Lukasiewicz formula to compute a->

Note that the above formula can work for any arbitrary values a and $b \in [0, 1]$.

In our discussion on drawing inference from conditional statements we shall use Lukasiewicz formula to compute $a \rightarrow b$.

(Refer Slide Time: 36:17)

A matural extension to 3-valued logic is n-valued logic $l_n = \sum_{n=1}^{\infty} \sum_{n=1}^{l} \sum_{n=1}^{\infty} \sum_{n=1}^{\infty}$ Two one can arriger on possible Forth values to a statement As n > 00 it covers nu retional autoburn in [0, 1]

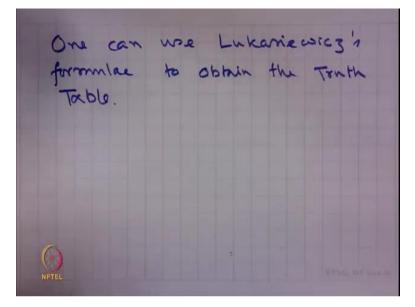
A natural extension to 3-valued logic is n-valued logic which is called

$$L_n = \left\{ 0 = \frac{0}{n-1}, \frac{1}{n-1}, \dots, \frac{n-1}{n-1} = 1 \right\}$$

Thus one can assign n possible truth values to a statement.

As $n \to \infty$ it covers all rational numbers in [0, 1]

(Refer Slide Time: 37:53)



One can use Lukasiewicz's formulae to obtain the truth table.

(Refer Slide Time: 38:24)

Propositional logic in helpful to validate an argument from a given set of premises to the conclusion. We cannot use propositional legic to establish the truth of a proposition ashore premises are not given.

The Propositional Logic is helpful to validate an argument from a given set of premises to the conclusion.

We cannot use propositional logic to establish the truth of a proposition whose premises are not given.

(Refer Slide Time: 40:00)

I All men are mortal Ram is a man. ." We can infor that Ram in also mortal But such an informa propositional logic because it does not analyze the internal "phrychire of "All men are mertal"

For example:

All men are mortal.

Ram is a man.

Therefore, we can infer that Ram is also mortal.

But such an inference cannot be drawn using propositional logic because it does not analyse the internal structure of 'All men are mortal'.

So, to propositional logic all men are mortal is one statement which can be True or False. It does not go inside to tell us that we can use this statement to justify that any particular person is also mortal.

And this is because all men covers all the people and therefore, to infer about each individual, we need to use some variable X. So that we can instantiate the value for X with some particular person and therefore from there we can infer that he is also mortal. This internal analysis of the structure is not present in propositional logic.

(Refer Slide Time: 42:57)

Hence comes the need to extend the gammat of Logic. in particular with the introduction variables next level of log ralle Predicale

Hence comes the need to extend the gamut of logic in particular with the introduction of variables. The next level of logic is called First Order Predicate Logic.

The next level of logic is called Thist Order I

(Refer Slide Time: 44:07)

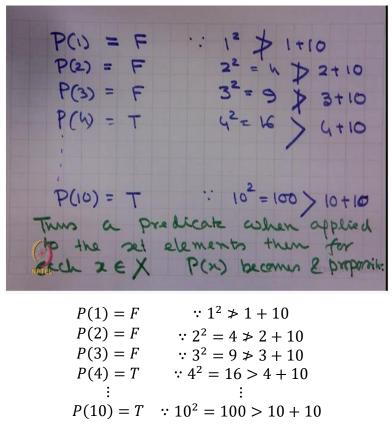
Predicate: A predicate in a Boolean function applied on come set X. 1.e P: X -> 20,13 EX: X = 7 1, 2, 3, --- 103 P(2) = "2" is greater them X+10 " ashern applied to the above set

So what is the predicate?

A predicate is a Boolean function applied on some set *X* that is $P: X \rightarrow \{0, 1\}$.

Say for example $X = \{1, 2, 3 ... 10\}$

And suppose the $P(x) = x^2$ is greater than $x + 10^2$. Therefore, when applied to the above set. (Refer Slide Time: 45:54)



Thus a predicate when applied to the set elements then for each $x \in X$, P(X) becomes a proposition. That is, it becomes a statement which says it is True or False.

(Refer Slide Time: 48:03)

In a similar any we can have pointry predicates: e.g P(x,y): x > y. If we apply on the set $\xi_{1,2,--}$ 103 P(10, 1) in T 3 Like that P(1,2) is F 3 can be obtained for all possible

In a similar way we can have binary predicates.

For example

P(x, y): x > y

If we apply on the same set $\{1, 2, 3 \dots 10\}$

Then we can see that P(10,1) is True, P(1,2) is False.

Like that can be obtained for all possible pairs.

(Refer Slide Time: 49:41)

Predicates allows us to use variables on its domain set. Ex. X: 2 set of all mon 3 .: Consider the predicate "man" man (John) : T man (mary) : F Noto suppose we agant to state "all men are mortal"

Predicates allows us to use variables on its domain set.

For example

Let *X* be the domain set, {set of all men}.

Therefore, consider the predicate man.

man(John): T
man(Mary): F

Now suppose we want to state 'all men are mortal'.

(Refer Slide Time: 51:25)

We define another predicate mortal (22) To state "all men are mostal" it is covering all the elements of the set X of men. Such a quantifier is called Universal quantifies? as it arrights Truth values for every element of the set X.

So we define another predicate is mortal(x), which is True if x is mortal, which is False if x is immortal.

Now to state 'all men are mortal', the focus now should go to this quantity all, it is covering all the elements of the set X of men.

Such a quantifier is called universal quantifier as it assigns truth values for every element of the set *X*.

(Refer Slide Time: 53:19)

Universal quantifier is denoted of Y (for all) .. To state "all men are mortal" we can have the fullowing logical expression (YX) (man(x) -> mortal(N))

The universal quantifier is denoted by \forall which means for all.

Therefore, to state all men are mortal we can have the following logical expression

 $\forall x \left(man(x) \rightarrow mortal(x) \right)$

(Refer Slide Time: 54:39)

Similary suppose we asont to state "All boys are tall" we can write (Ax) (boy (x) -> tall 60) Now support this is not tring. So we aran to see its negation

Similarly suppose we want to state

'All boys are tall'.

We can write

 $\forall x (boy(x) \rightarrow tall(x))$

Now suppose this is not True. So we want to see its negation.

What can be the negation?

(Refer Slide Time: 55:53)

The megation of "All boys are tall NOT All boys are Not thus The negation in : There must be at least 607 ashe in Not one tall (boy cn) ~ ~ tall (x))

The negation of 'All the boys are tall' is NOT 'All the boys are not tall'.

This is not the negation.

The negation is there must be at least one boy who is not tall.

That is $\exists x (boy(x) \land \neg tall(x))$

So this gives another quantifier.

(Refer Slide Time: 57:18)

This is called Existential quantifies. Ex PCAD is True if there is some element say & in X r.t prov is

This is called Existential quantifier. $\exists x, P(x)$ will be True if there is some element say $x \in X$ such that P(x) is True.

(Refer Slide Time: 58:10)

We can combine these quantifie in many asays . All men are taller than all women. HXEX HYEY taller (x, y) 3 Suppose we askart to say For any manx there is some womaky 3 y is taller

We can combine these quantifiers in many ways.

Suppose we want to say

1) All men are taller than all women.

So we can write it as $\forall x \in X \forall y \in Y \ taller(x, y)$

On the other hand

2) Suppose we want to say,

For any man x there is some woman $y \ni$ that y is taller than x.

(Refer Slide Time: 1:00:03)

Then we can express it as AXEX BREX taller (2, x) gression is hear to get Truth values for quantifiers. $\frac{\exists x P(x)}{\forall x P(x)} = \bigvee_{\substack{x \in X}} P(x)$ On the other bland $\bigotimes_{\substack{x \in X}} P(x) = \bigwedge_{\substack{x \in X}} P(x)$

Then we can express it as $f \forall x \in X \exists y \in Y \ taller(x, y)$

Question is how to get truth values for quantifiers?

$$\exists x P(x) = \bigvee_{x \in X} P(x)$$

That means if for any one of the elements x of X, P(x) is True. Then $\exists x P(x)$ is going to be True. On the other hand,

$$\forall x \ P(x) = \bigwedge_{x \in X} P(x)$$

(Refer Slide Time: 1:01:59)

Canonical representation · All the statements are sentences in a language So if we say man on =) 2 10 a man In a sinclar way tallon >> 2 is tall. Convical representation: 2 is P, anne Pins 2 is P, anne Pins

So before we stop I give you a canonical representation.

If we notice that all the statements are sentences in a language.

So if we say man(x)

 $\Rightarrow x$ is a man.

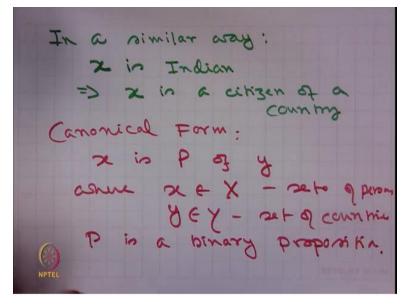
```
In a similar way tall(x)
```

```
\Rightarrow x is tall.
```

So canonical representation of such statements is:

x is P, where P is a predicate.

(Refer Slide Time: 1:03:41)



In a similar way, x is Indian is essentially we mean x is a citizen of a country. Therefore, the canonical form may be

```
x is P of y
```

, where $x \in X$ – set of persons, $y \in Y$ – set of countries and *P* is a binary proposition.

Okay students I stop here today. In the next class I shall start with fuzzy proposition. So in these 2 classes I have given you the basics of first order logic and propositional logic.

Of course logic has been extended to much more theoretical details. We have paraconsistent logic, description logic, default logic, defeasible logic there are many different forms of logic that has been developed theoretically for this course we shall look at fuzzy logic and we shall see how we can make inference from fuzzy logical propositions. Thank you.