Introduction to Fuzzy Sets Arithmetic and Logic
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Module - 4
Lecture - 11
Fuzzy Sets Arithmetic and Logic

Welcome students to lecture number 11 for the MOOCs course on Fuzzy Sets, Arithmetic and Logic. In the last class, we have seen multiplication of fuzzy numbers.

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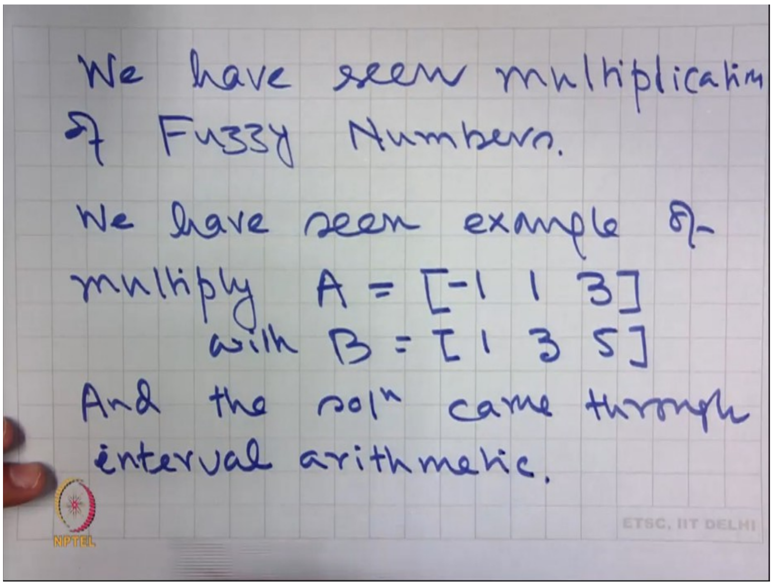

In particular, we have seen the example of multiplying is equal to the triangular fuzzy number with which is is equal to . And the solution came through interval arithmetic.

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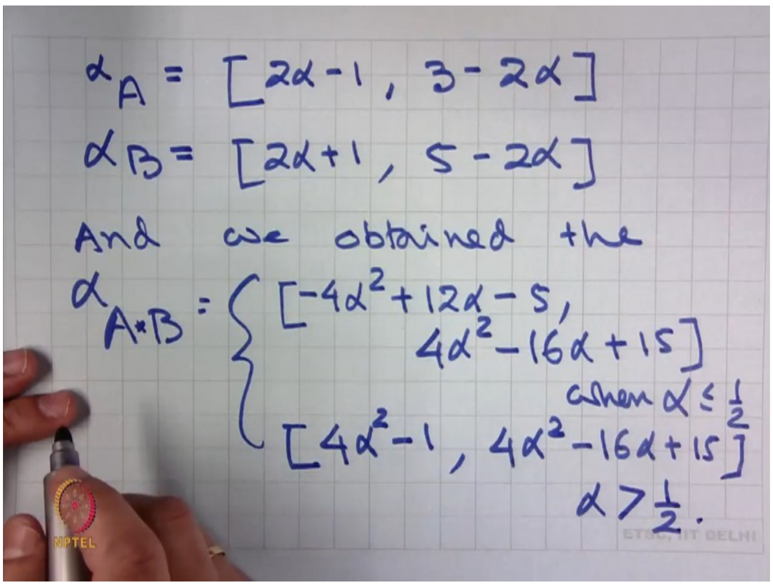

So, we had and .

And, we obtained the

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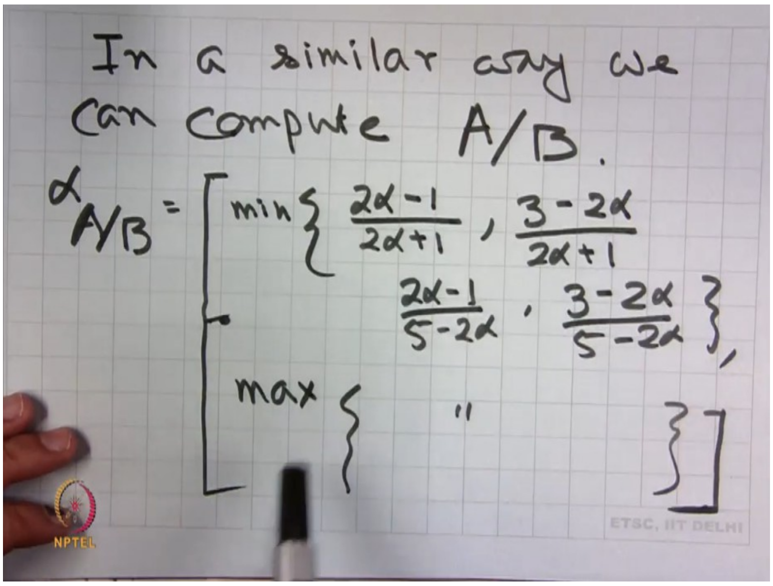

In a similar way, we can compute .

Since is a fuzzy number and is a fuzzy number, is also going to be a fuzzy number. And how do you get the -cut of this?

So,

So, we have to compute these 4 quantities. And then, depending upon the value of alpha, we have to see, in which range which of the expressions is minimum.

And the upper bound is going to be maximum of the same 4 quantities. And thus, we get a closed interval which is running for different values for ; the minimum of this to the maximum of the same 4 elements. The way we have drawn the curve for multiplication; in a similar way, you can draw the curves for the 4 functions for is equal to to. And from there, you can calculate exactly the -cut for different values of . I leave that as an exercise. But, I am giving you the answer.

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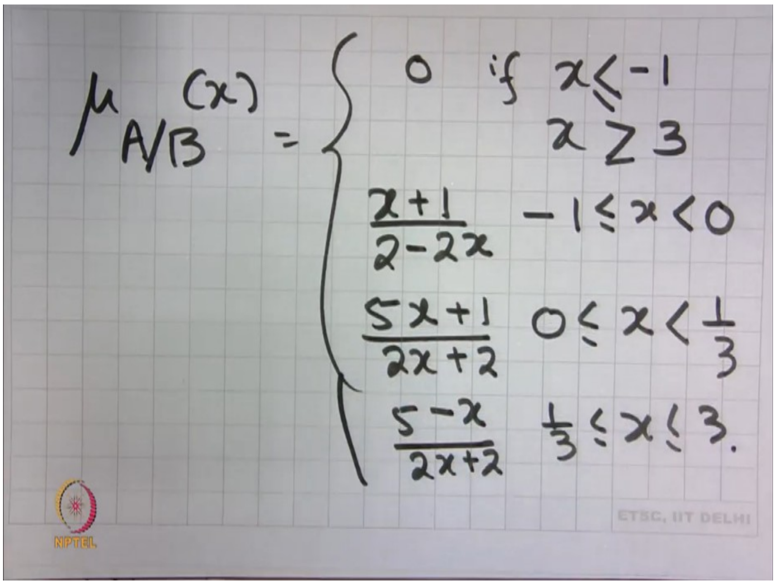

The membership function

Thus, the membership function changes in different intervals.

And we can see that, since we had and , the minimum value that is coming out to be . And the maximum value will come when I am dividing by , that is . Therefore, the support of is going to be from to. I leave this as an exercise for you, to complete this and see whether you can actually arrive at this answer, when you are using the analytical aspect of the 4 curves that I have just mentioned; and see how they are behaving for different values of between to

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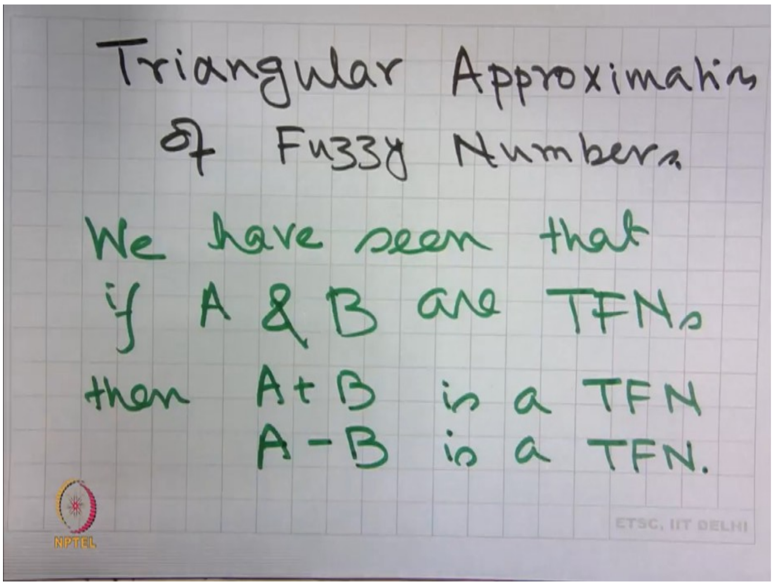

Now, let me proceed with triangular approximation of fuzzy numbers.

We have seen that, if and are triangular fuzzy numbers, then is a triangular fuzzy number; and similarly is a triangular fuzzy number.

The advantage of triangular fuzzy number, I have already explained. That because of the linearity of the left-hand side of the membership function and the right-hand side of the membership function, it is very easy to calculate the -cuts.

And also to compute the membership functions very easily from the linear aspect of the and . Hence, we have a constant tendency to convert a fuzzy number into a TFN, because that makes our life very easy.

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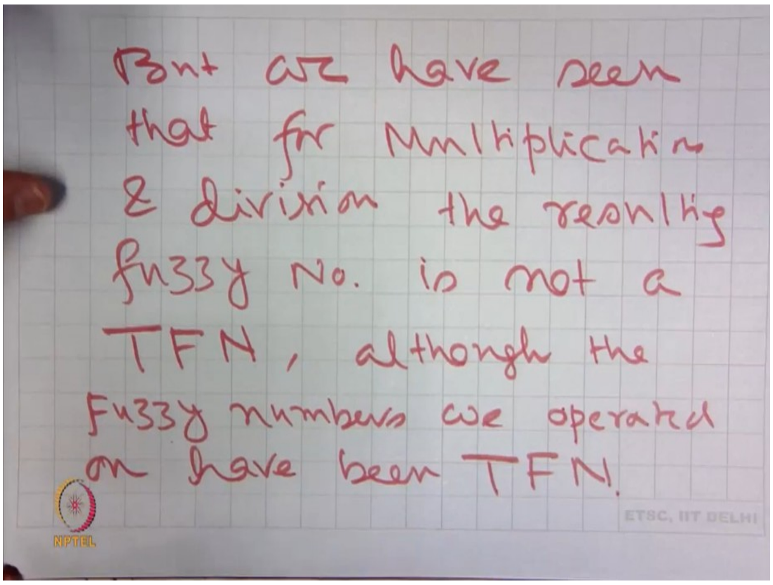

But we have seen that, for multiplication and division, the resulting fuzzy number is not a triangular fuzzy number, although the fuzzy numbers we operated on have been triangular fuzzy number.

Hence comes the question of linear approximation of a fuzzy number. So, let us first start with multiplication.

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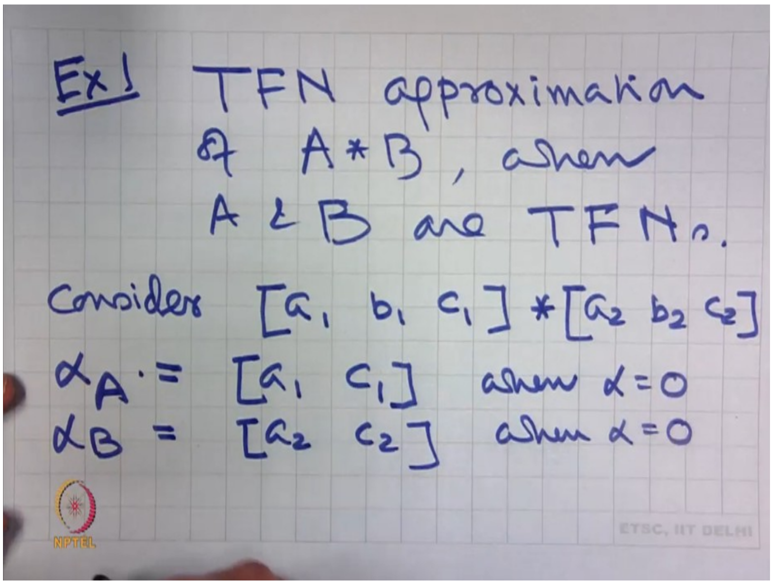

Example 1: TFN approximation of when and are triangular fuzzy numbers.

How to do that?

So, consider .

We can get the following when and when

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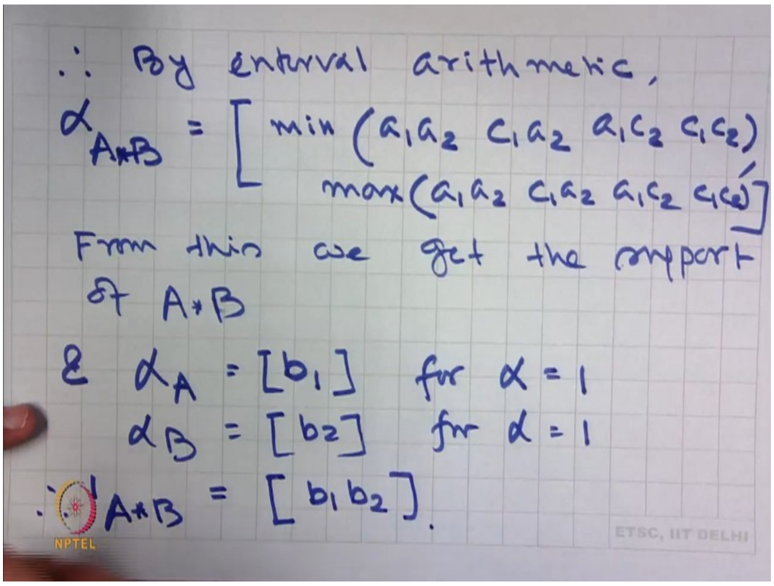

Therefore, by interval arithmetic,

So, from this, we get the support of .

And for and for .

Therefore, by interval arithmetic

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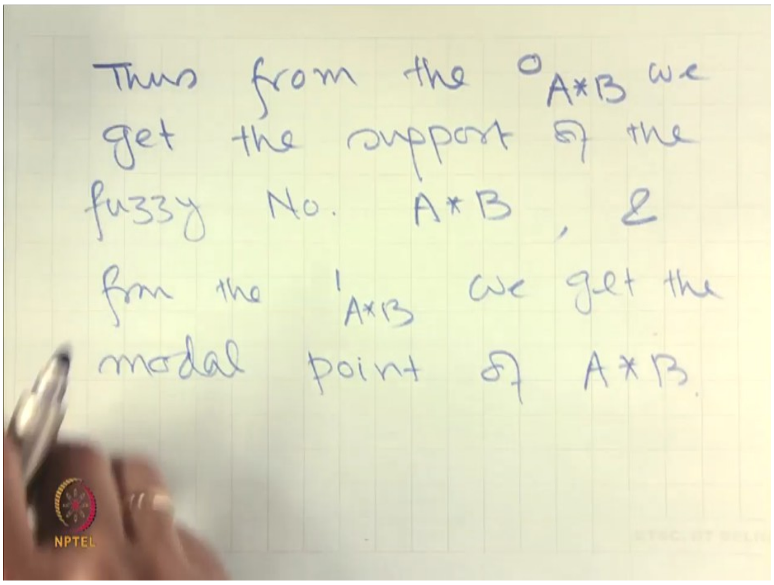

Thus, from the , we get the support of the fuzzy number . And from the we get the modal point of .

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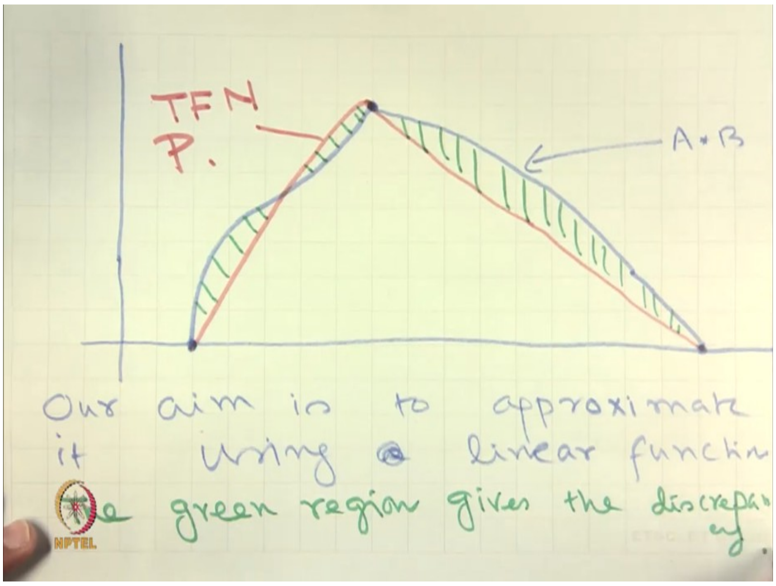

So, if we plot it, we got the 3 important points for . But the problem is, the and , instead of being linear, they have a quadratic form or combination of 2 different quadratic functions. As we have seen in our last class that we have got something like this as the membership function for . Our aim is to approximate it using a linear function or using linear functions.

That is, we are trying to have lines like this, so that the fuzzy number is approximated by the TFN. Let us call it .

How to do that?

We shall look at this region. This is the discrepancy between the approximated straight line and the actual curve. This is on the left-hand side and this is on the right-hand side. So, the green region gives the discrepancy.

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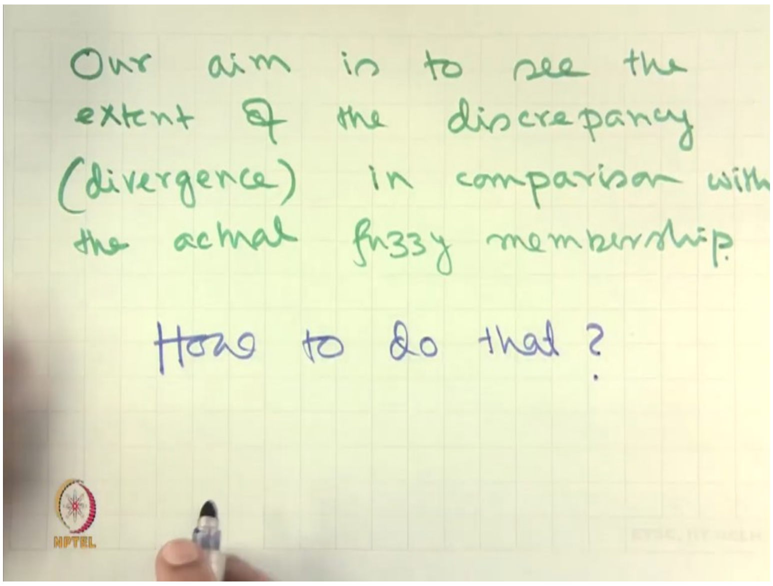

Our aim is to see the extent of the discrepancy, which we often call divergence, in comparison with the actual fuzzy membership.

Question is: How to do that?

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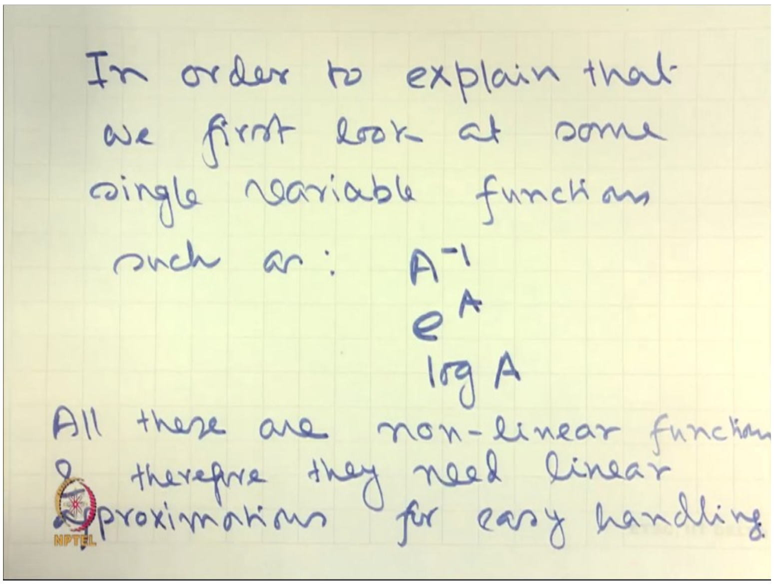

In order to explain that, we first look at some single variable functions such as . All these are non-linear functions. And therefore, they need linear approximations for easy handling.

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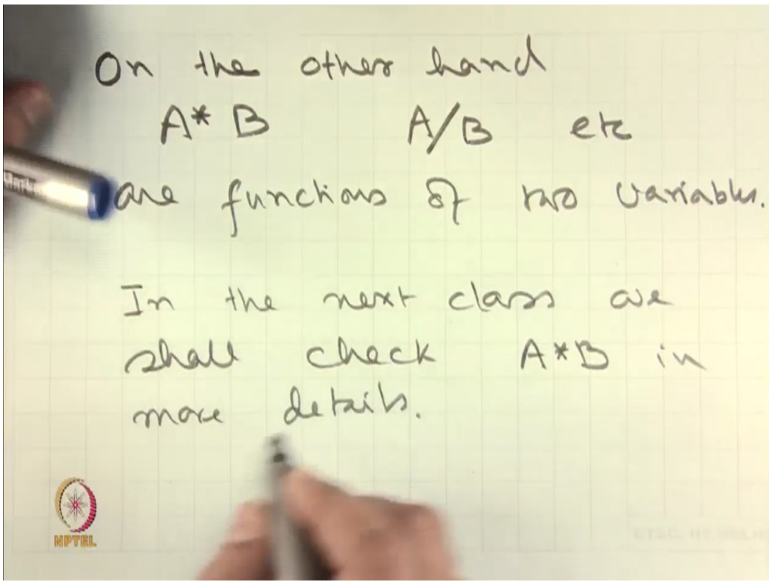

On the other hand, are functions of 2 variables. So, in the next class, we shall check in more details.

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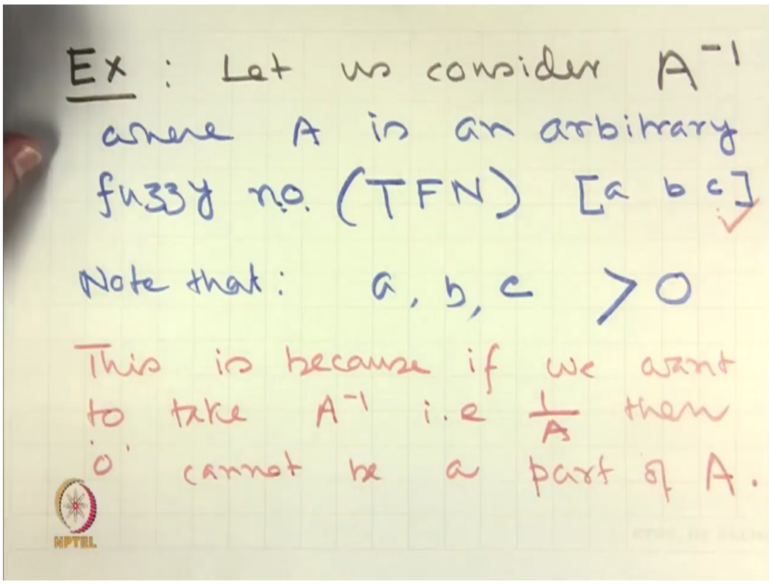

So, example: Let us consider , where is an arbitrary fuzzy number.

That is, TFN, triangular fuzzy number .

Note that . This is because, if we want to take , that is ; then, cannot be a part of

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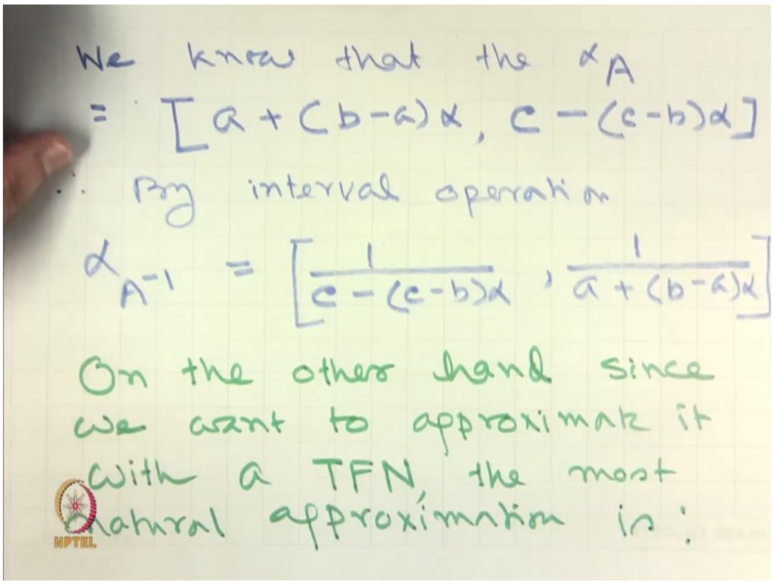

We know that the

Therefore, by interval operation,

On the other hand, since we want to approximate it with a TFN, the most natural approximation is

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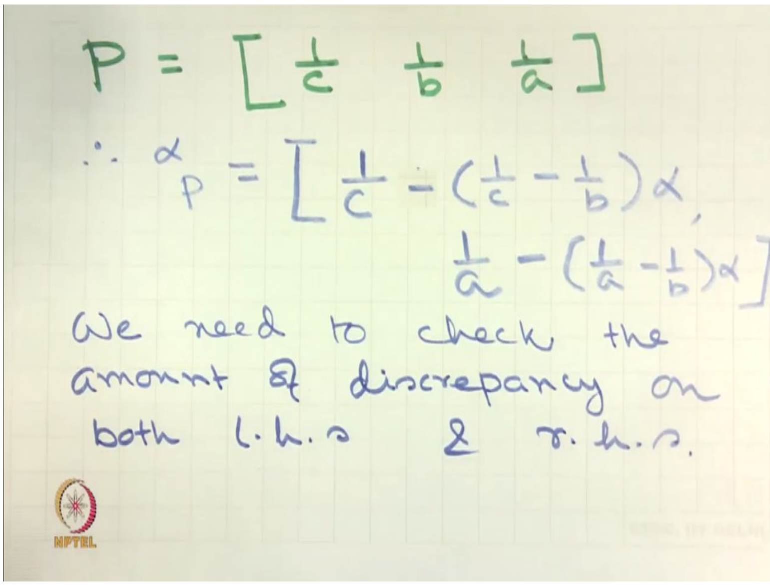

This should be very clear.

We need to check the amount of discrepancy on both left-hand side and right-hand side.

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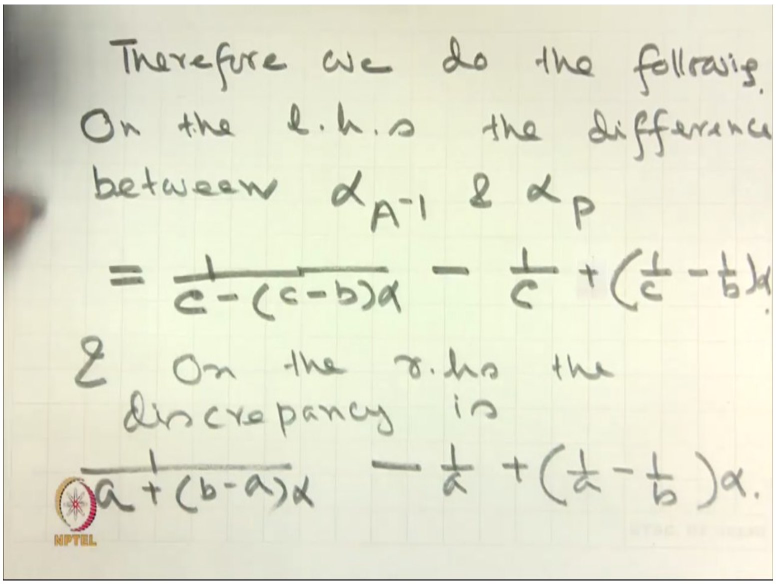

Therefore, we do the following.

On the left-hand side, the difference between and

And on the right-hand side, the discrepancy is

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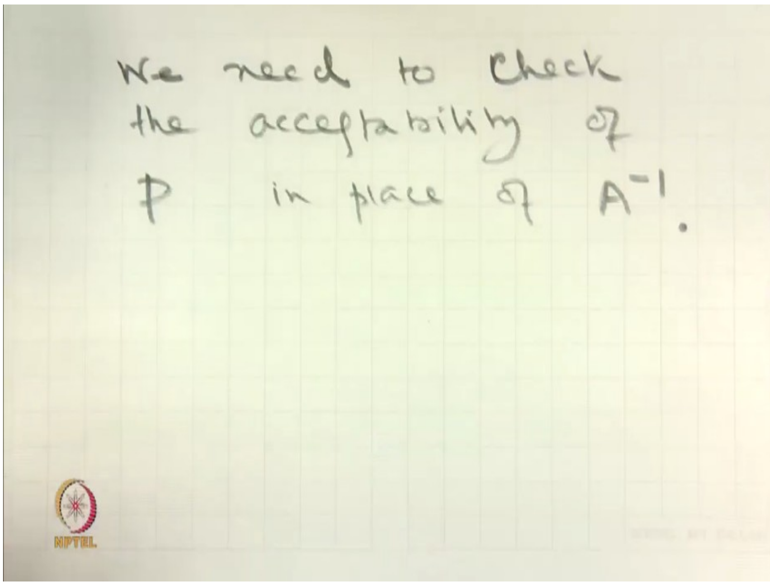

We need to check the acceptability of in place of

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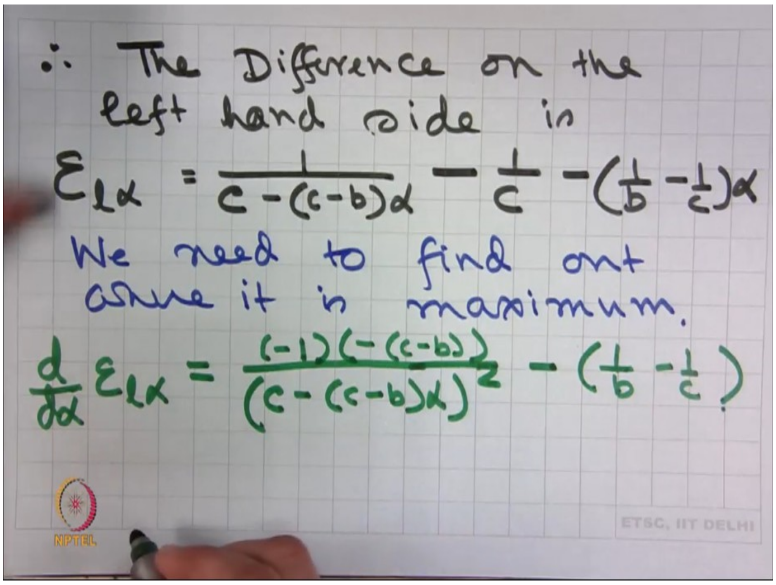

The difference on the left-hand side is, if we call it ,

So, this is a function of . We need to find out where it is maximum. So, we differentiate it with respect to . So, I am differentiating this.

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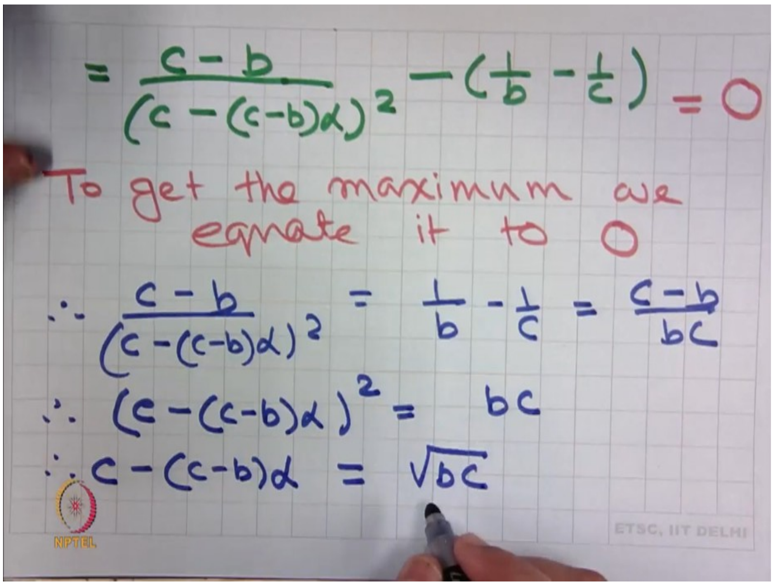

S0, to get the maximum, we equate it to .

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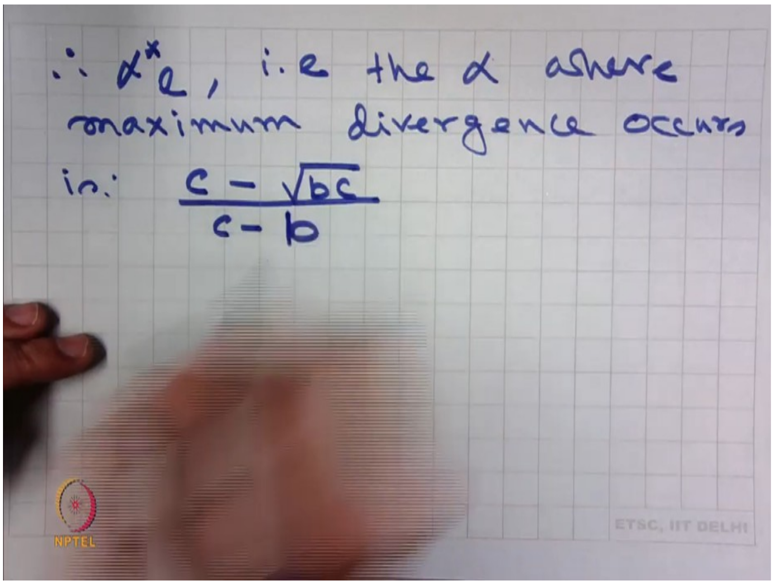

Therefore, , that is the where maximum divergence occurs is

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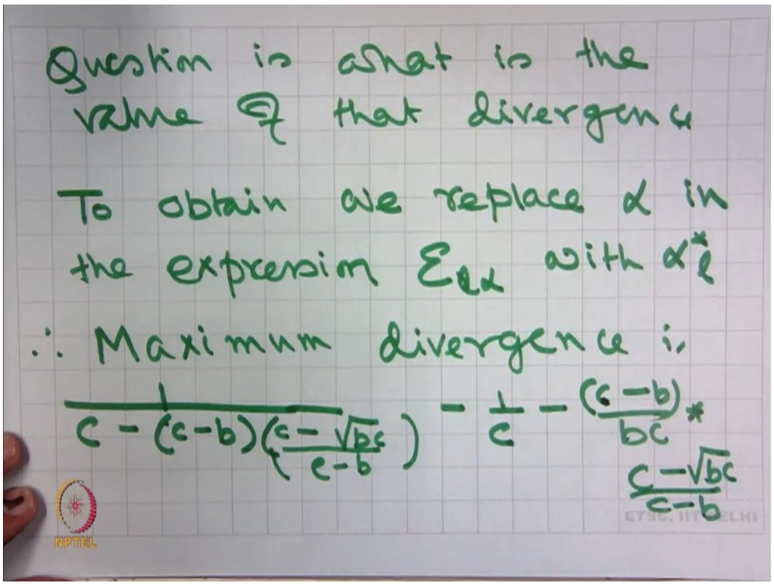

Question is: What is the value of the divergence?

In order to obtain that, we replace in the expression with .

And we get the following.

Therefore, maximum divergence is

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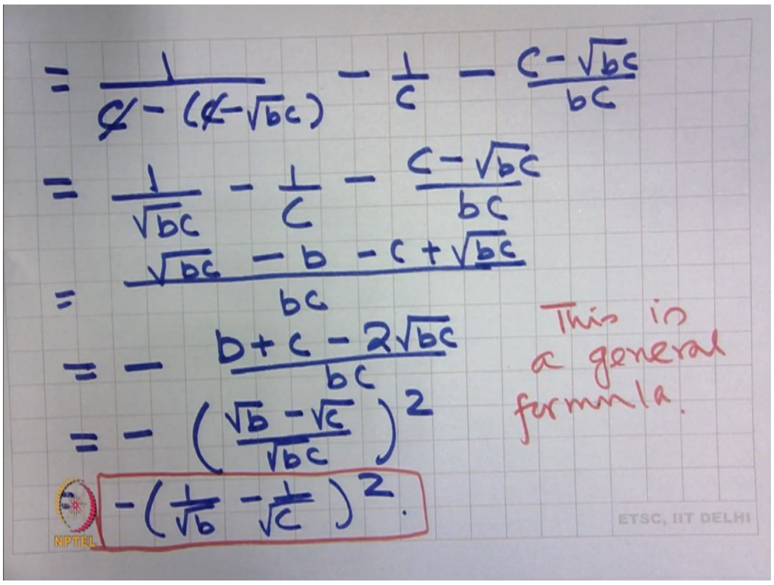

Which is equal to; if you simplify,

So, this is a general formula. For a particular problem, we can substitute the value for ; and we can get the maximum divergence.

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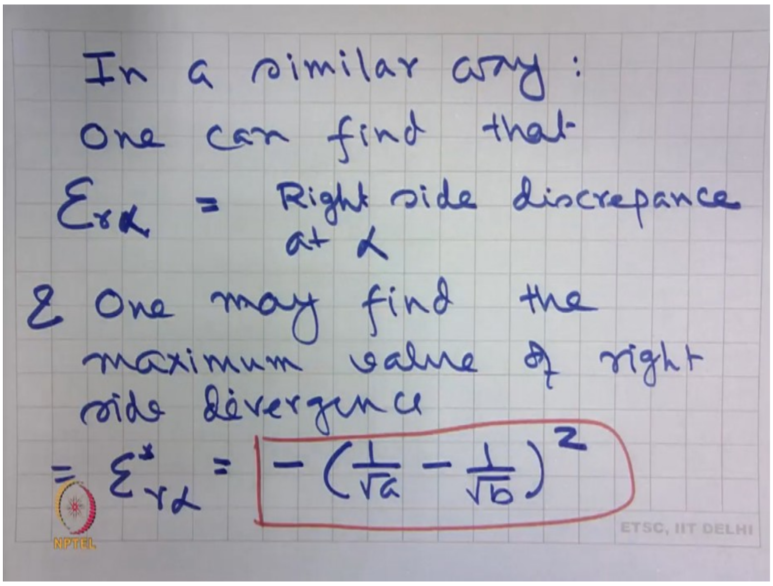

In a similar way, one can find that ; that is, right-side discrepancy at . And one can find the maximum value of right-side divergence is equal to

So, these 2 values are very important for us to solve the problem. So, let me give you an example.

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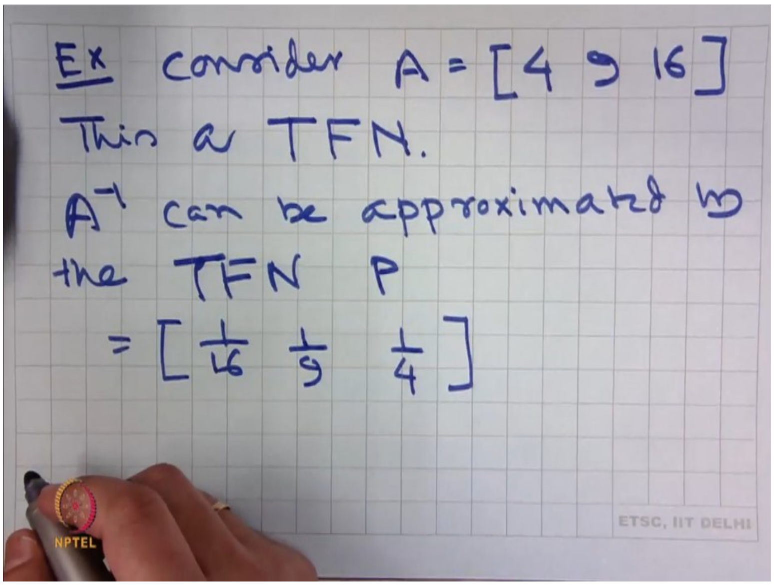

Consider . This is a triangular fuzzy number.

Therefore, can be approximated by the fuzzy, TFN

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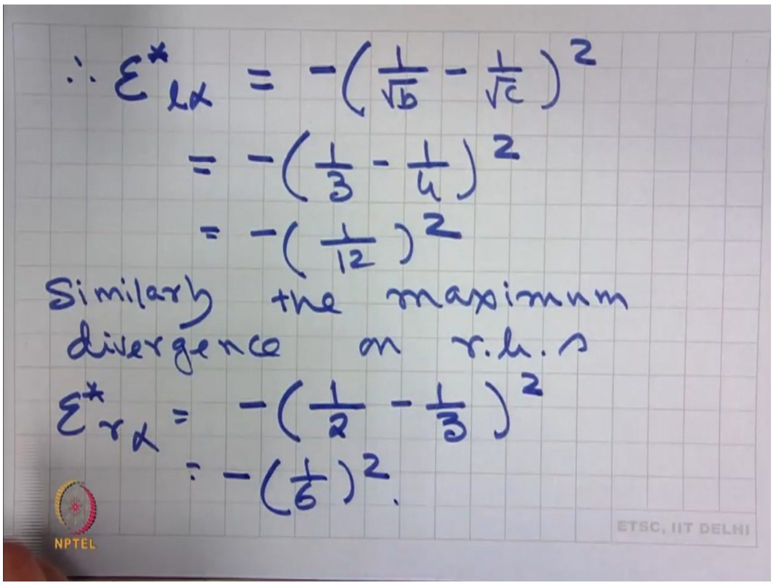

And we have the maximum divergence on left-hand side is

Similarly, the maximum divergence on right-hand side,

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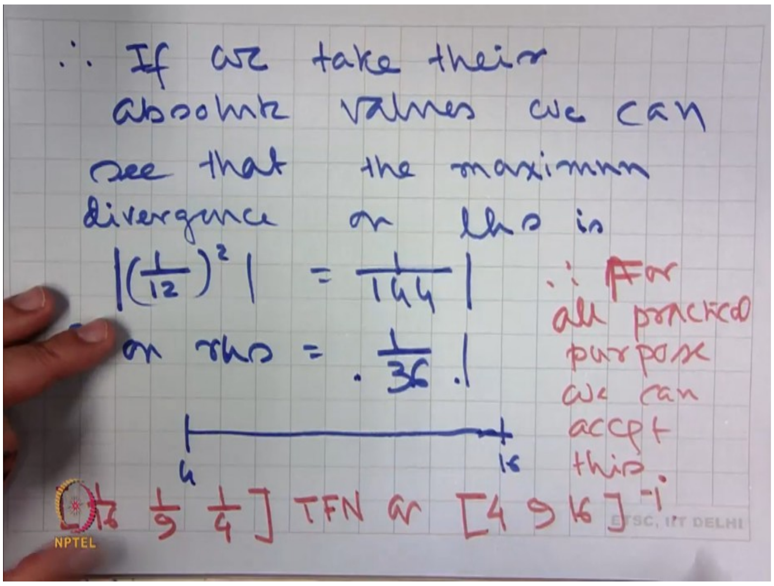

Therefore, if we take their absolute values, we can see that the maximum divergence on left-hand side is . And on right-hand side is equal to

Question is, whether it is acceptable or not?

As we can see, these are pretty small numbers in comparison with the overall spread of the fuzzy number, which is from to . That was the support for original number with the peak value at .

In comparison, these are very small. So, for all practical purpose, we can accept this.

That is, ; this TFN as .

Okay students, let me consider another problem for approximation of exponentiation of fuzzy numbers.

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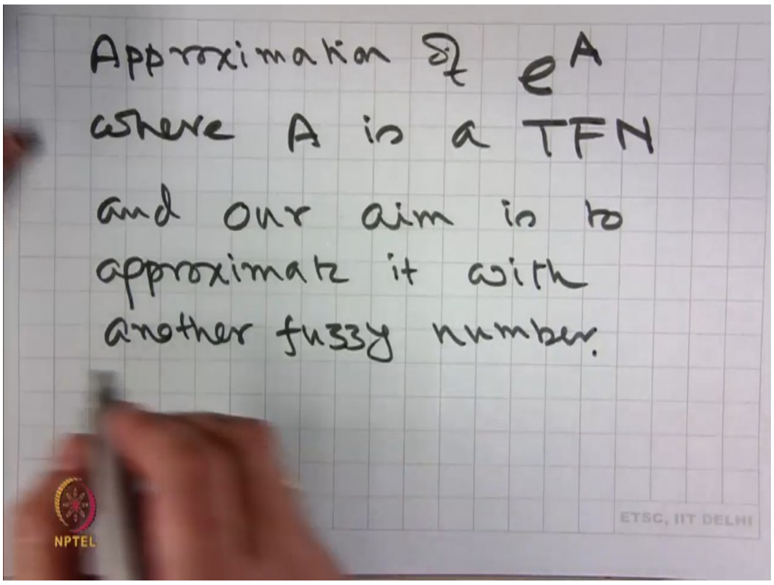

, where is a TFN. That is a triangular fuzzy number. And our aim is to approximate it with another fuzzy number.

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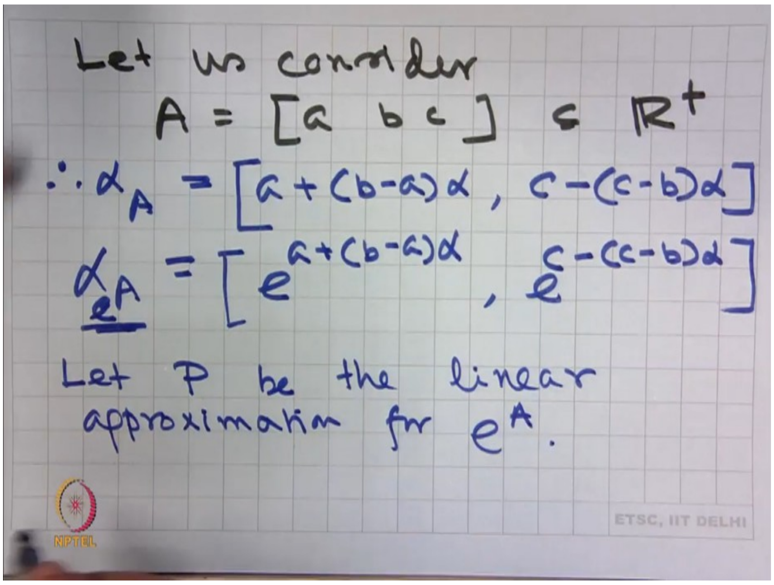

So, let us consider , where That is, all and are positive real numbers.

So, as before, we are applying the arithmetic operations on corresponding -cuts to get -cut of the result, which is

Let be the linear approximation for

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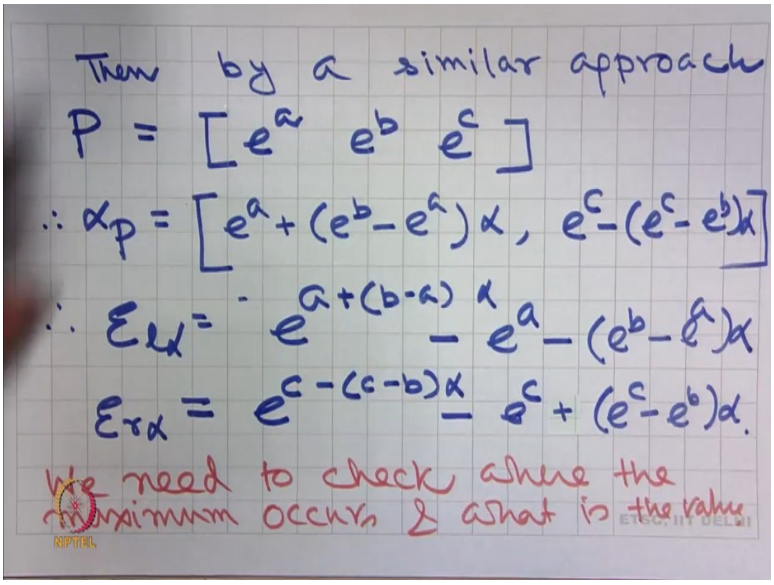

Then, by a similar approach, can be the triangular fuzzy number .

Since all and are positive, therefore exponentiation is an increasing function. So, we need not worry about their relative ordering. And we are going to get this.

We need to check where the maximum occurs and what is the value.

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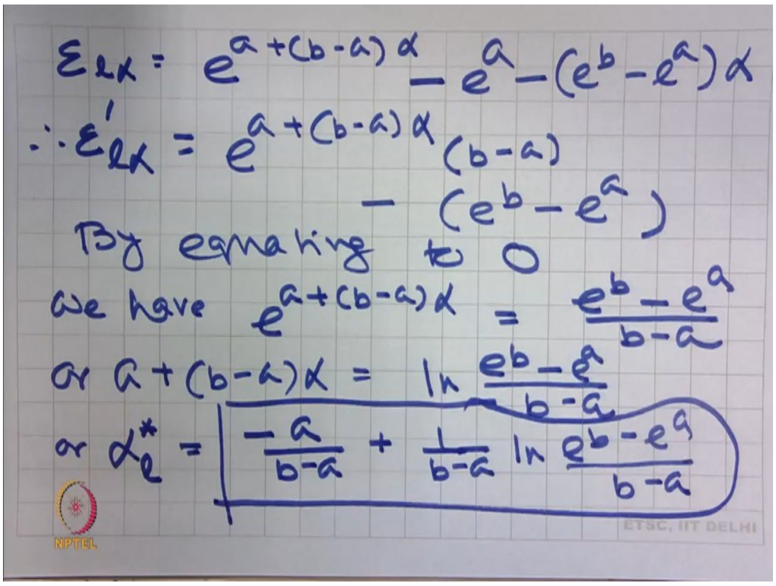

So,

By equating to 0, we have

So, this is the value of where the maximum divergence will occur on the left-hand side.

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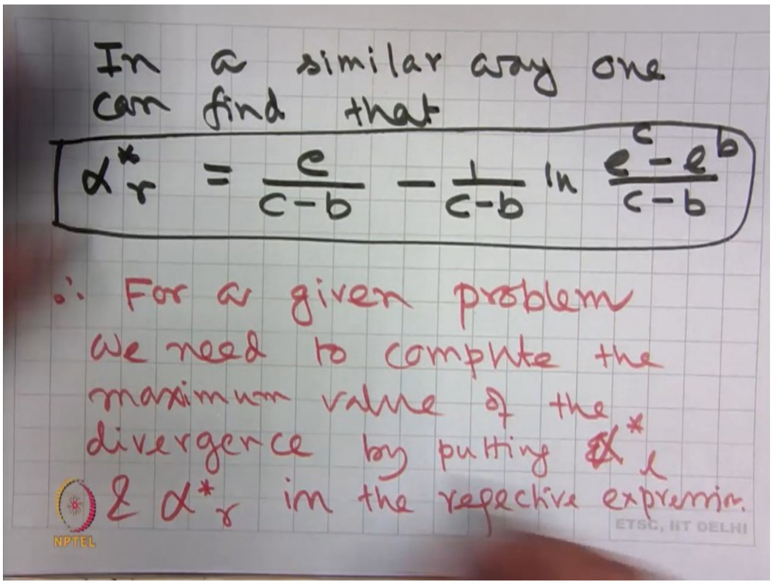

In a similar way, one can find that

Therefore, for a given problem, we need to compute the maximum value of the divergence by putting in these values in the respective expression. So, let me give you an example.

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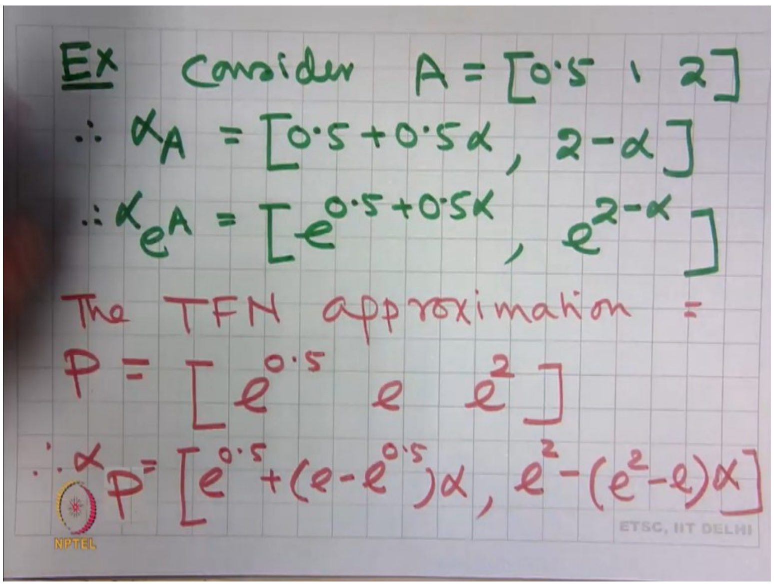

Consider

On the other hand, the TFN approximation is equal to

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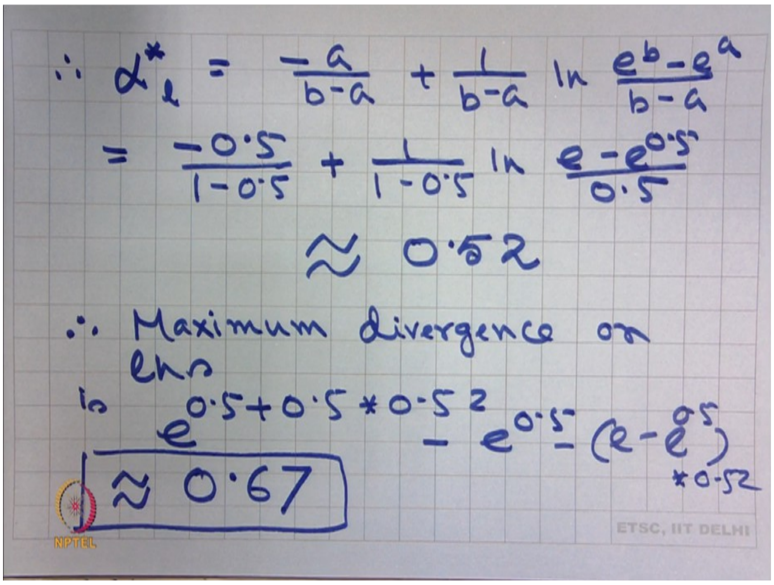

And therefore, maximum divergence on LHS is; by putting this value, is

So, this is the maximum divergence on the left-hand side.

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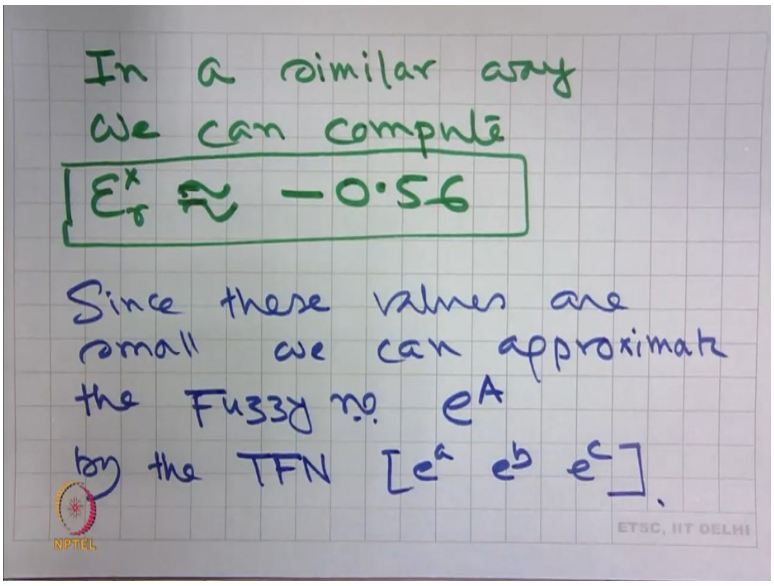

In a similar way, we can compute maximum divergence on the right-hand side

Therefore, if we compare this divergence in comparison with the real number and , we find that these values are quite small in comparison with the overall triangular fuzzy number.

Hence, for a practical application, we may use; since these values are small, we can approximate the fuzzy number by the TFN

Okay friends, I stop here today. In the next class, I shall show how we can use triangular approximation for a function between 2 numbers. In particular, I shall work on fuzzy multiplication. And then, I will discuss how to order a set of fuzzy numbers linearly. Or in other words, how to sort a set of fuzzy numbers. Okay friends, thank you so much.