Introduction to Fuzzy Sets, Arithmetic and Logic
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Module - 4
Lecture - 10
Fuzzy Sets, Arithmetic and Logic

Welcome students to lecture number 10 on the MOOCs course on fuzzy sets, arithmetic and logic. If you remember, we were working on fuzzy arithmetic.

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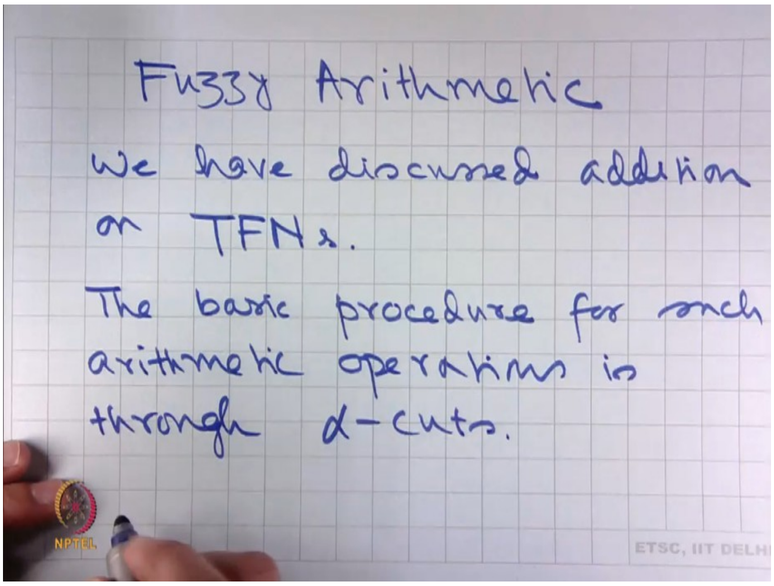

In particular, we have discussed addition on triangular fuzzy numbers.

We have shown that can be done in two ways. The basic procedure for such arithmetic operations is through alpha-cuts.

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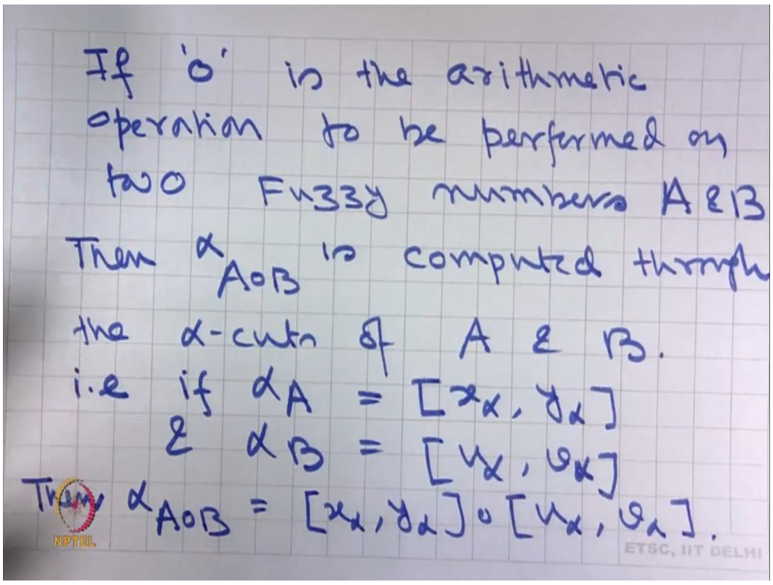

If is the arithmetic operation to be performed on two fuzzy numbers and . Then alpha-cut of is computed through the -cuts of and .

That is, if -cut of and -cut of

You have to understand that these are closed intervals on the real line.

Then, -cut of is computed through the interval operation on and the interval .

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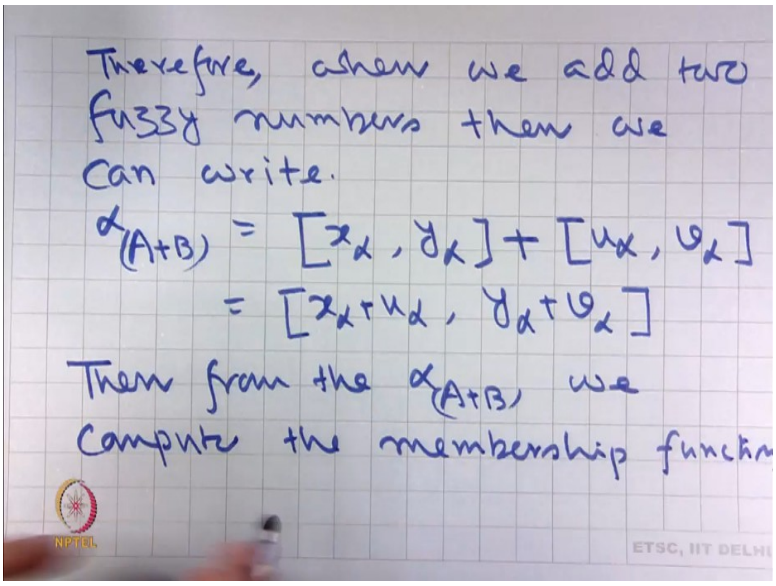

Therefore, when we add two fuzzy numbers, then we can write

This, we have already seen in the last class. And then, what we do. Then, from , we compute the membership functions.

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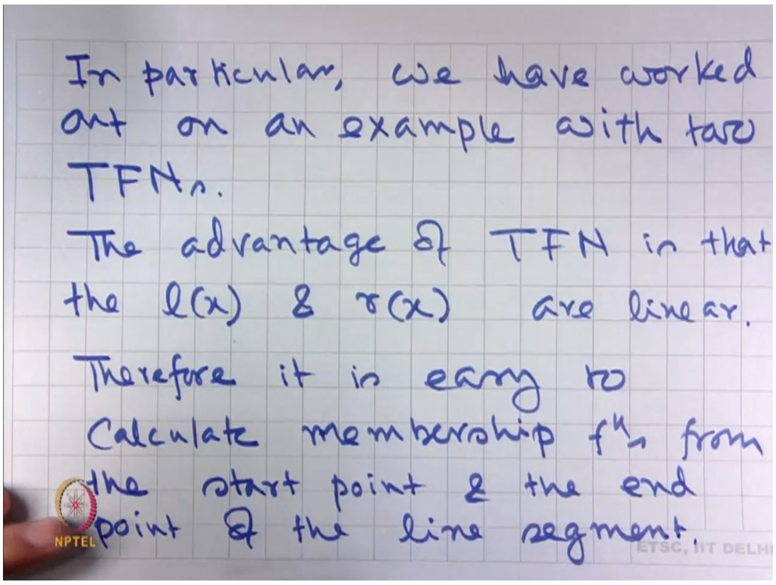

In particular, we have worked out on an example with two triangular fuzzy numbers.

Why generally we use TFNs?

The advantage of TFN is that, the and ; that is the membership functions on the left side and membership function on the right side of the fuzzy number are linear.

And therefore, it is easy to calculate membership functions from the start point and the end point of the line segment.

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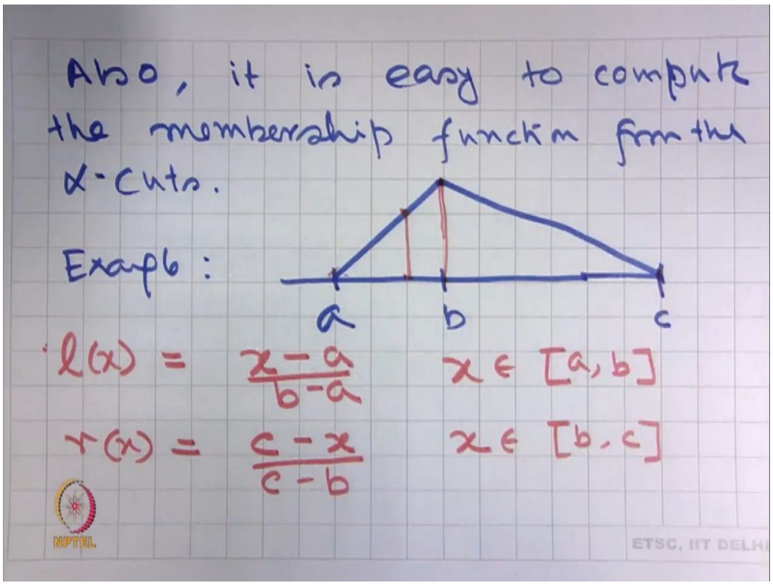

Also, it is easy to compute the membership function from the alpha-cuts.

Example: Suppose, we have a fuzzy number like this.

Between to , it is linear; and between to , it is linear.

Then, for

That you get from simple geometry and the similarity of triangles of, similarity of triangles between this and this.

In a similar way, for

As you can see that putting the value of , we get . But putting the value of we get 0.

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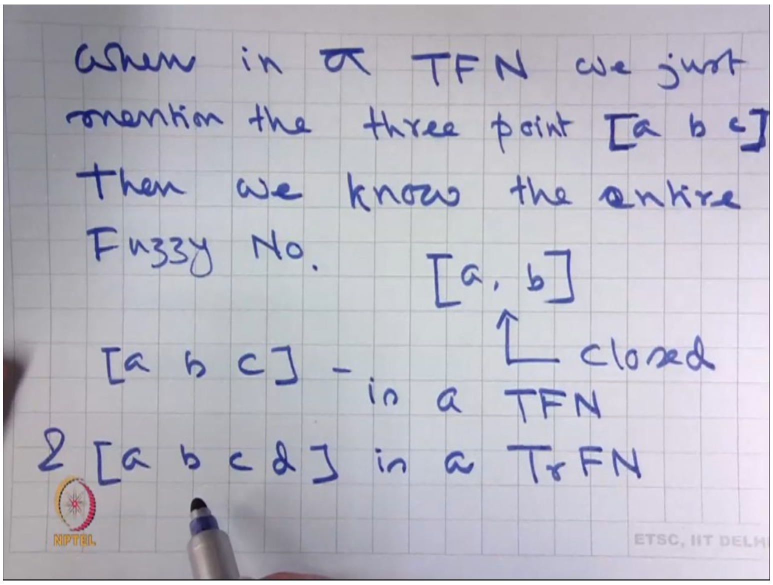

Therefore, when in a TFN, we just mention the three points ; then, we know the entire fuzzy number. Please note that, when I am using a TFN, I am not putting any comma. And there are numbers which is different from , which is basically a closed interval.

So, you have to remember that is a TFN and is a trapezoidal fuzzy number which will have a shape very similar to a triangular fuzzy number. Only thing is that, between to , it will have a membership value 1.

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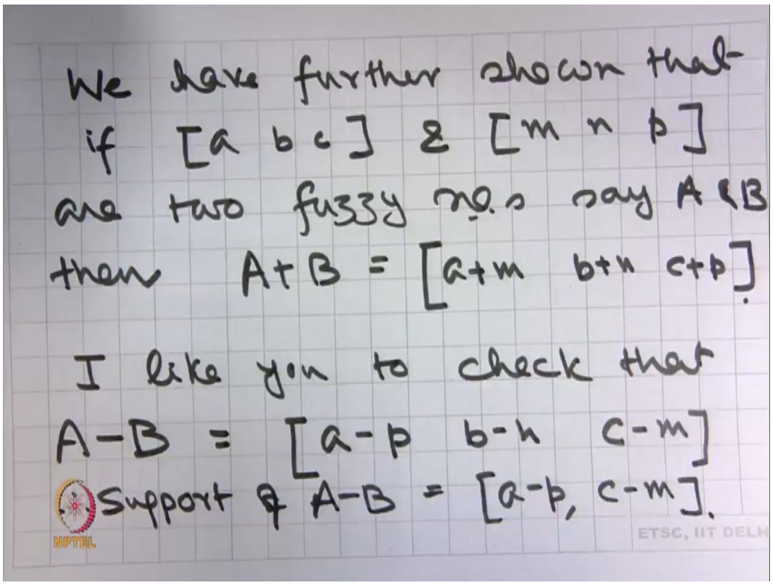

We have further shown that, if and are two fuzzy numbers, say and ; then is actually the TFN .

You can prove it in general. But in the last class, I have given you an example. And I have shown; by using the interval operations on the -cuts of and , we have actually got this to be the answer. I like you to check that

How do I get it?

Because the minimum value; when I am subtracting from , the minimum value will be; when I take the smallest value here and subtract from it the largest value here. That is how I get to be the smallest point in .

In a similar way, it is going to be for the maximum point.

Therefore, the support of is going to be the interval . And since we know the 3 points of the TFN, we know how to find the membership functions for all belonging to this interval. And just now I have shown how to calculate that.

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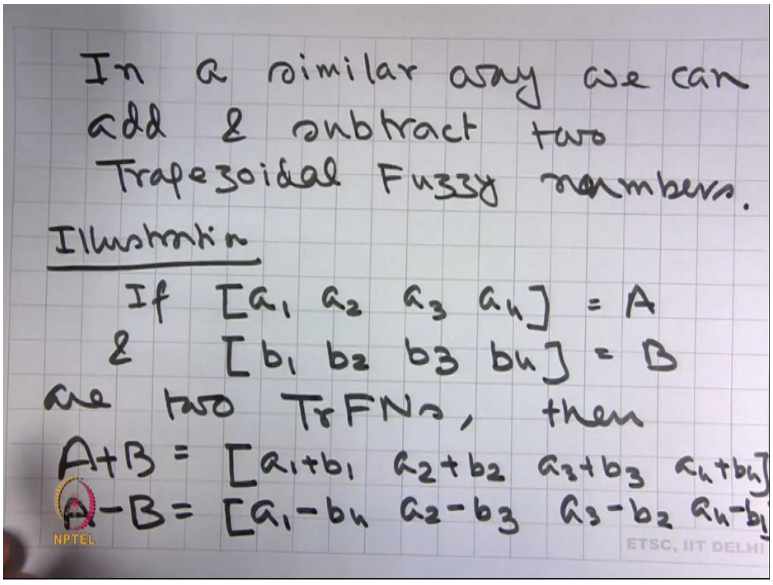

In a similar way, we can add and subtract two trapezoidal fuzzy numbers.

Illustration:

If and are 2 trapezoidal fuzzy numbers.

Then, is also a trapezoidal fuzzy number which is And .

I am not working out this, because this is simple, because the membership functions are linear.

I want you to verify this by taking two arbitrary trapezoidal fuzzy numbers and actually working out on them using the interval operations.

Let me now change my focus to fuzzy multiplication.

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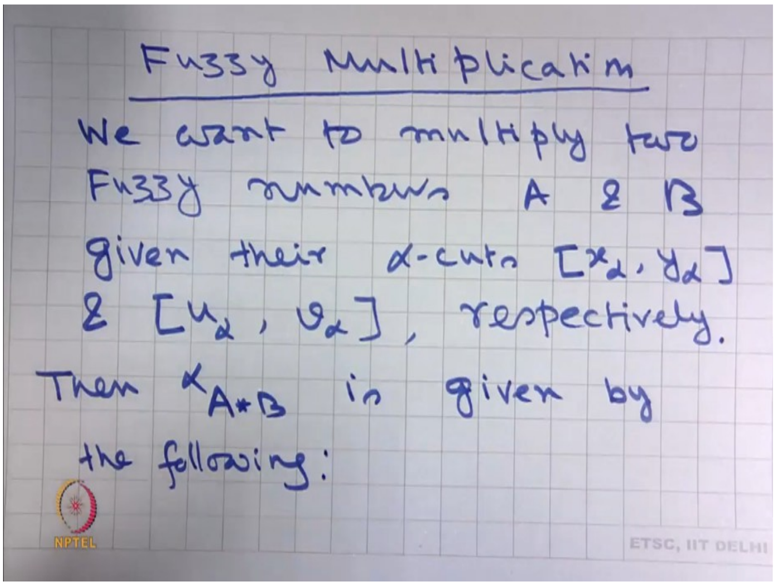

We want to multiply two fuzzy numbers and Given their -cuts and respectively. Then, is given by the following:

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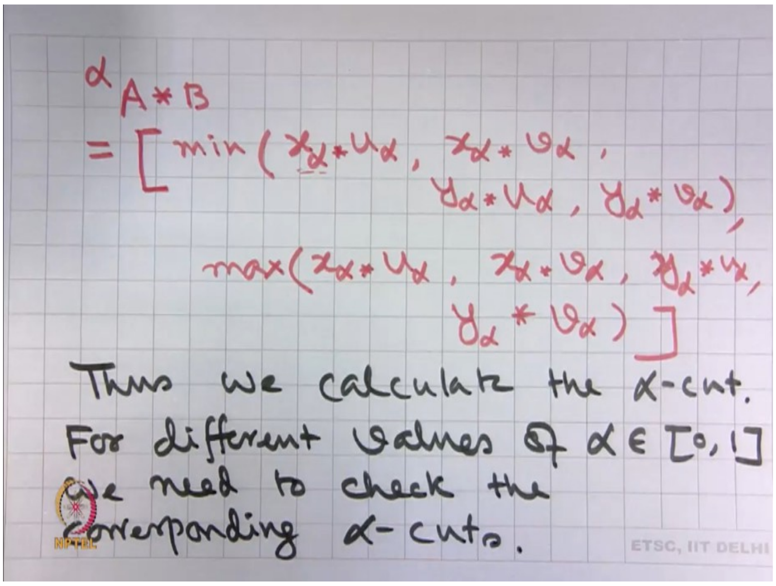

So, depending upon the sign of and , we may get different expressions to be the minimum. So, in each individual case, we will have to compute these and we have to see which one of them is minimum. So, that is going to give you the lower bound of the -cut. And the upper bound is going to be very similar expression. But, in this case, we are going to consider the . So, this interval that we get is going to give us the alpha-cut.

But, the difference with addition or subtraction is that, it is not that one function will remain the minimum for all alpha between to . So, for different values of , we need to check the corresponding alpha-cuts. Now, I illustrate this using an example.

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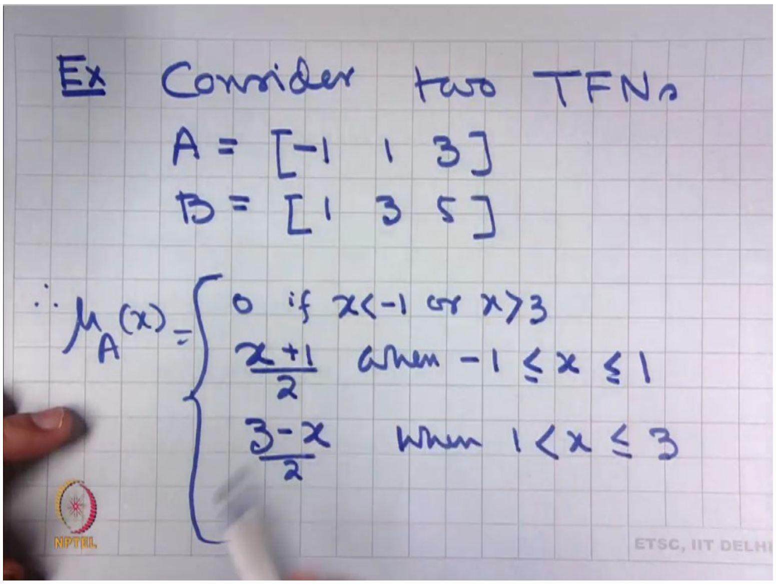

Consider two TFNs;

 and

This we can obtain very easily by considering these 3 values a b c; and then applying the formulae I have given just some time back.

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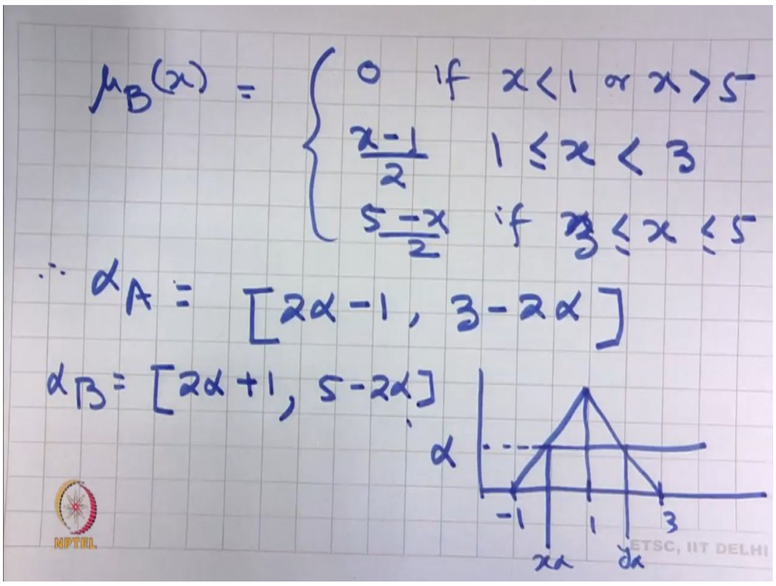

Similarly,

Therefore,

How do I get it? So, our . So, this is the shape of . So, for any alpha, I am looking at and .

So, we can use the geometry again or the similarity of these triangles again, to compute . In a similar way,

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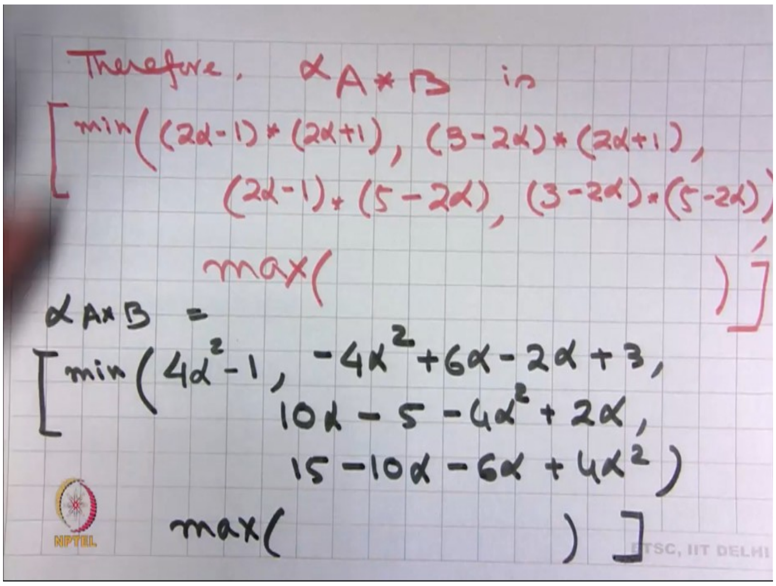

Therefore, is

So, let us first calculate the values.

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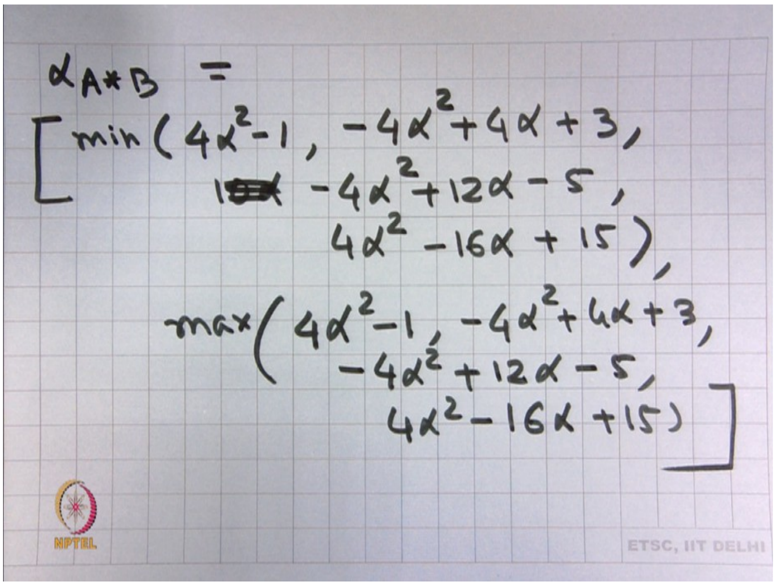

Which, when we simplify we get,

Fairly complicated terms. And therefore, we need to check for different values of alpha; which one of them is going to be minimum and which one of these is going to be maximum.

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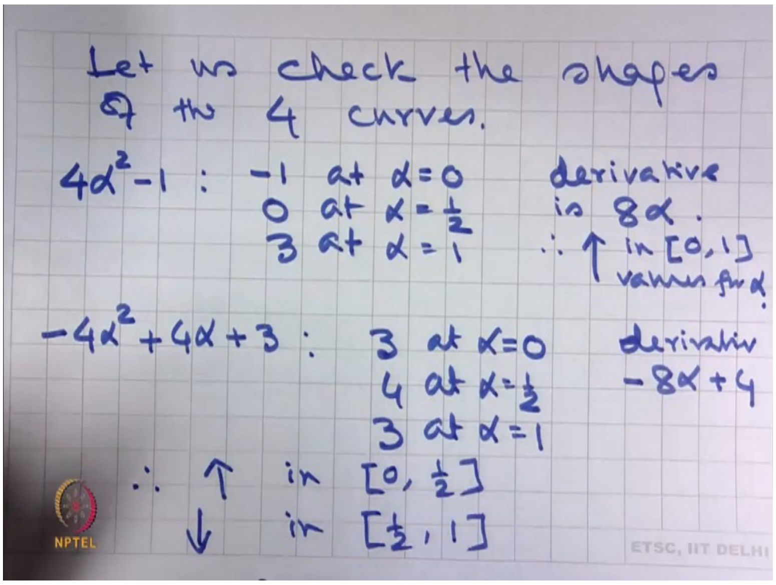

So, let us try to understand the shapes of these four curves.

at

at

at

And its derivative is . Therefore, it is increasing in values for .

The next equation is:

: at

at

at

And its derivative is .

Therefore, increasing in and decreasing in . From the sign of the derivative, we can calculate. In , it is going to be positive; therefore, it is increasing. From , this value is negative; therefore, it is decreasing.

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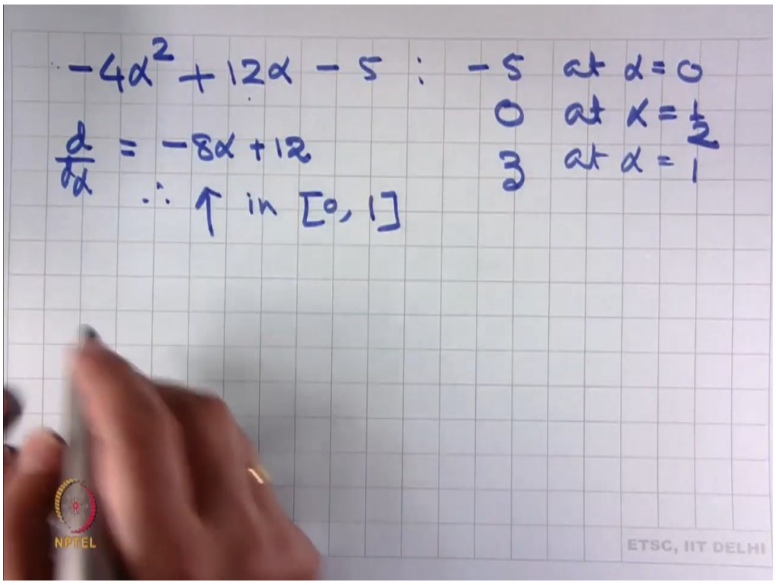

The third equation is:

: at

at

at

Derivative is .

Therefore, it is increasing in values for , because throughout it is remaining positive.

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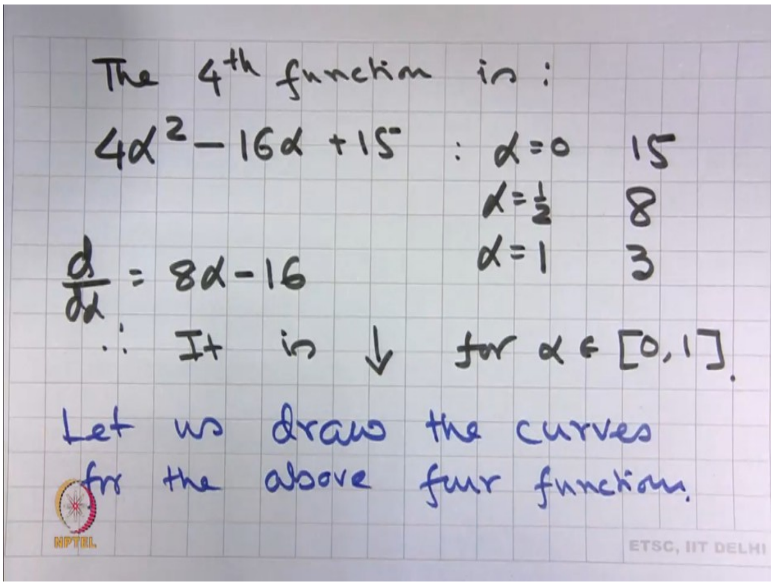

And the fourth function is:

: at

at

at

And derivative is equal to . Therefore, it is decreasing for. Now that we have seen the nature of the 4 curves, let us draw the curves for the above four functions. Note that and

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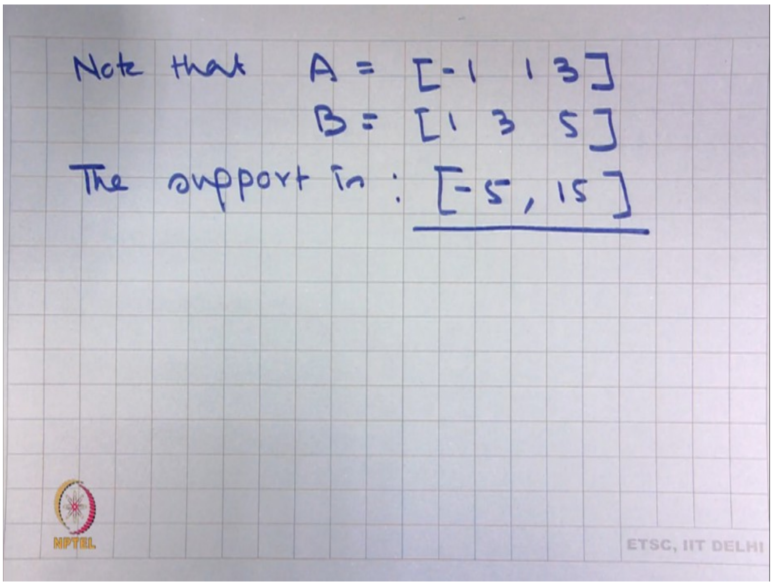

Therefore, the minimum value that can happen is and the maximum value that can happen is . Therefore, we have to plot the curves for .

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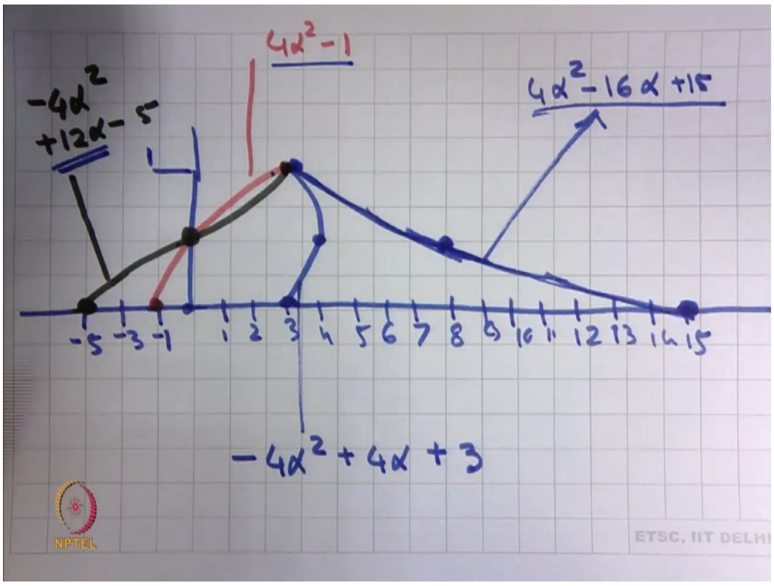

Therefore, what is going to be the minimum?

We can see that between to , the minimum is given by this function:. Between to , the minimum is given by . But the maximum for the entire range of is given by

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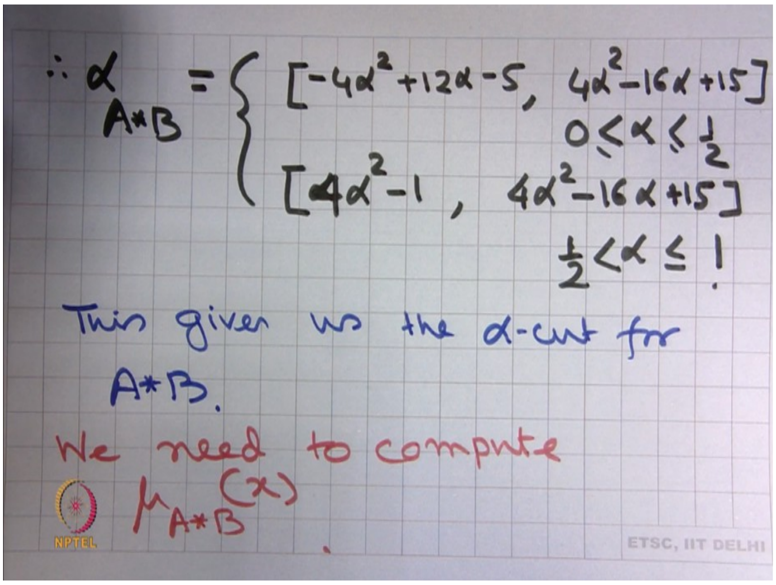

Therefore,

So, this gives us the -cut for . Now, we need to compute the membership functions. How to do that? We do in the following way.

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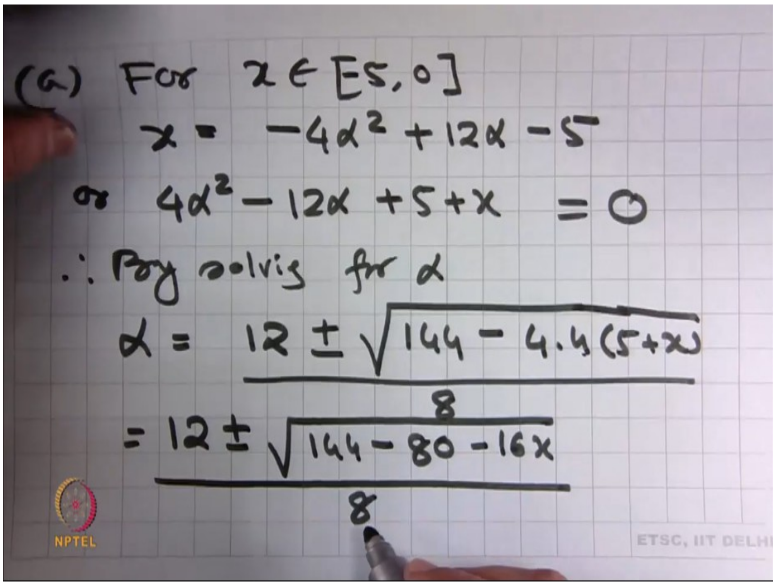

For

The equation is

Or

Therefore, by solving for ,

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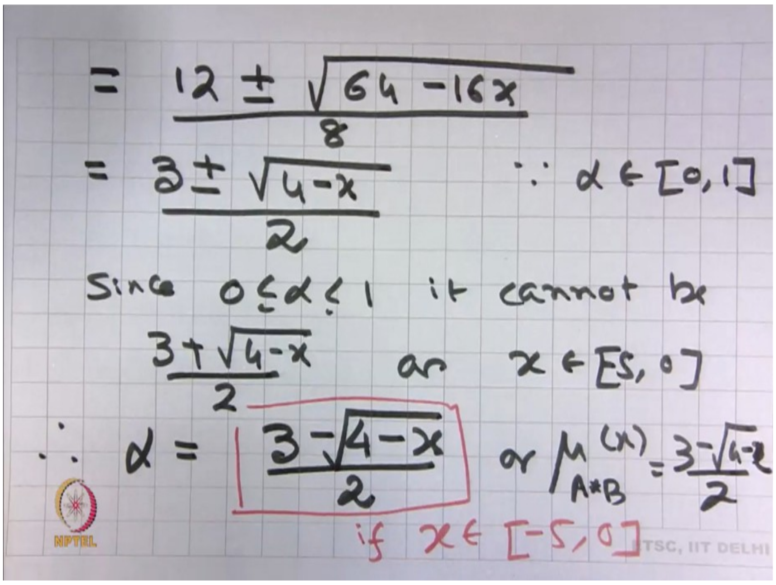

Since, , it is, since if this is going to be positive, then is going to be above 1.

Therefore, since , it cannot be , as .

Because, in that case, it is going to be above .

Therefore,

Or . So, this gives us the membership value, if

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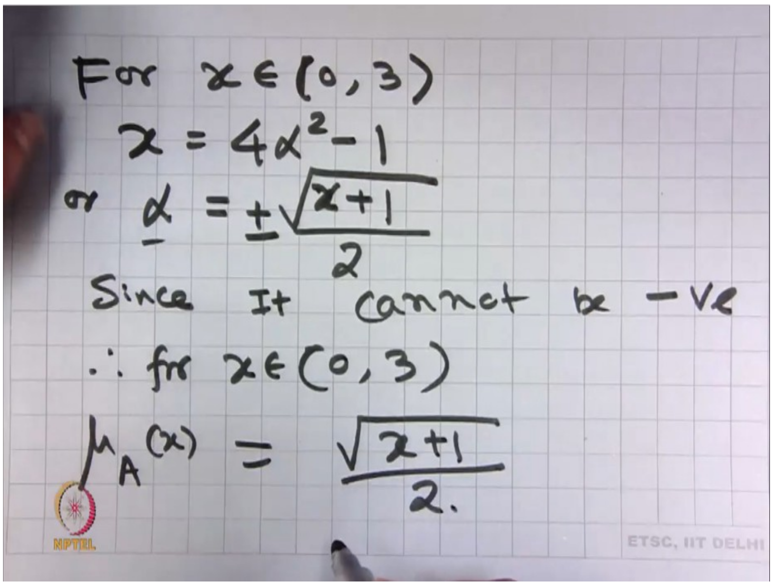

For , we have the equation

Or

Since it cannot be , therefore for , which we denote by is equal to

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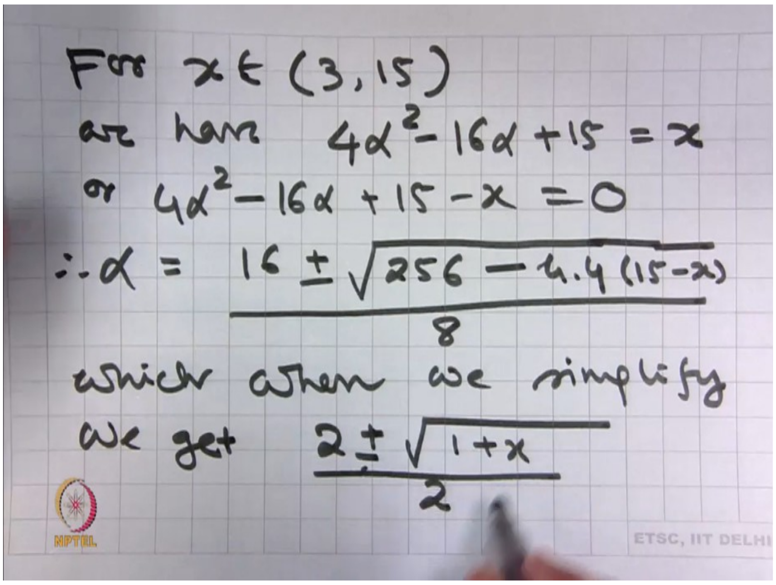

And finally, for

We have or .

Therefore, which when we simplify, we get .

We cannot take , because in that case it is going to be above . Therefore, we will look at

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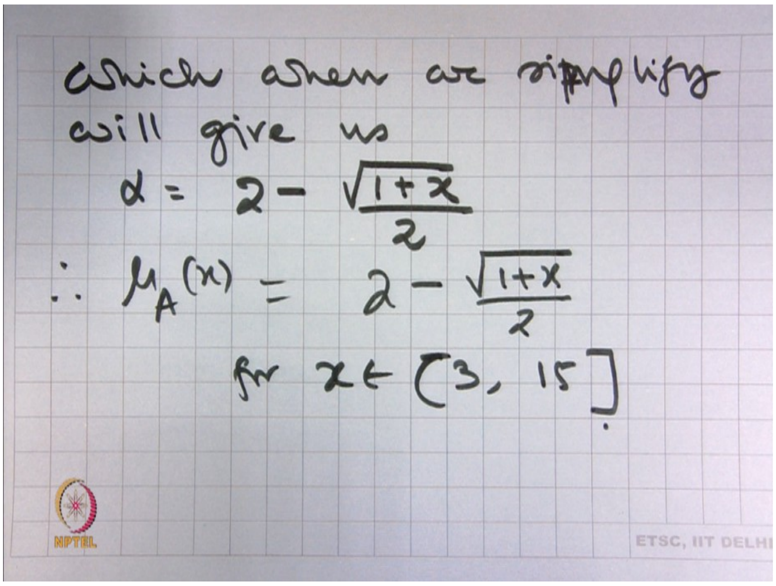

Which when we simplify will give us

Therefore,   for . In this way, we get the membership functions for for the entire support of

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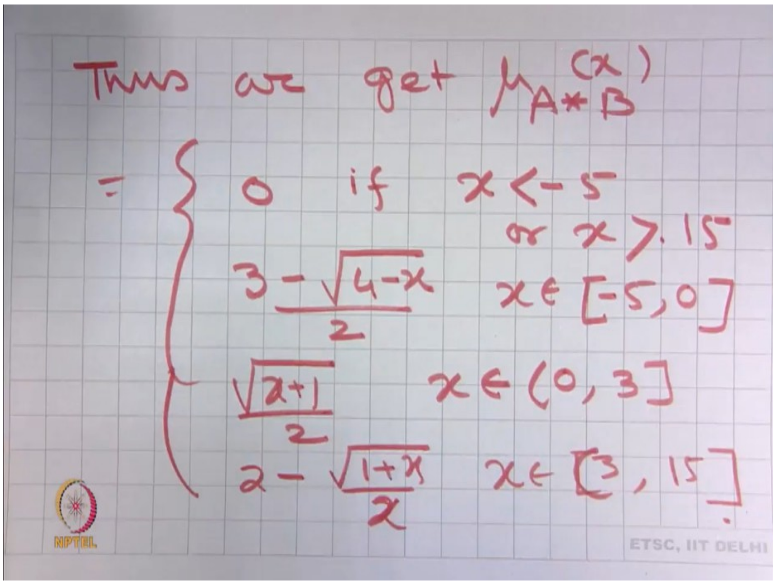

Thus, we get

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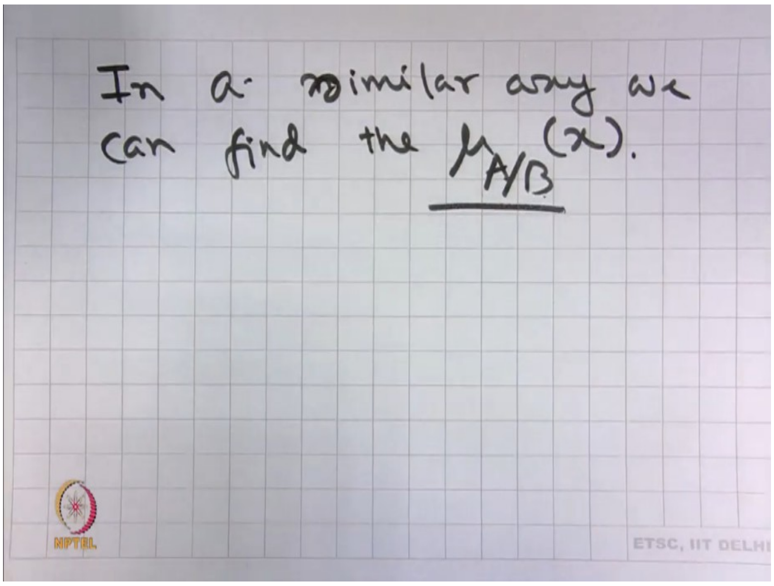

In a similar way, we can find the , which is a fuzzy number when does not contain in it. And compute the membership functions for different belonging to the fuzzy number .

Okay students, I stop here today. Thank You.