Introduction to Fuzzy Set Theory, Arithmetic and Logic
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Lecture 01
Fuzzy Sets Arithmetic and Logic

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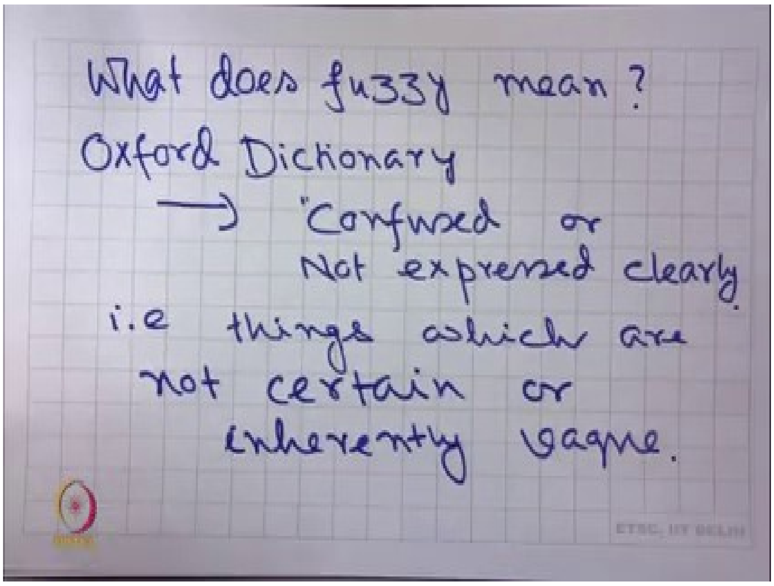

Welcome students to the first lecture on fuzzy sets arithmetic and logic what is fuzzy or what does fuzzy mean? If you look at the oxford dictionary it says that, Fuzzy is something like confused or not expressed clearly. That is fuzzy talks about things which are not certain or inherently vague one may wonder why should, we deal with things that are inherently vague. But it is our day-to-day use we see that we use terms which are actually not certain or not precise.

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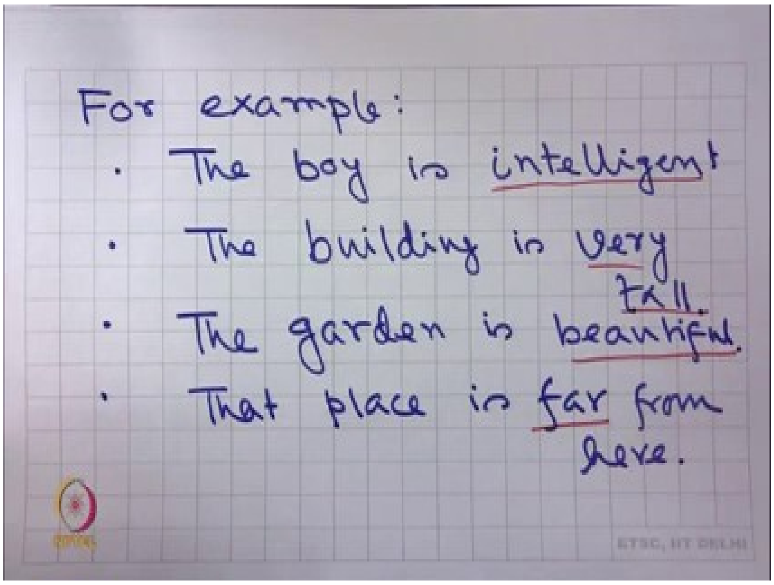

For example: The boy is intelligent, The building is very tall, The garden is beautiful, That place is far. If we focus on the terms intelligent, very tall, beautiful, far we all have some intuitive feeling of what actually they convey but, if I ask you the preciseness of each of this treatment. What do you exactly mean by intelligent? What do you exactly mean by very tall? Similarly beautiful or far, it is very clear that we do not have very precise meaning.

And that meaning may differ from person to person say for someone of height 5 feet 4, somebody of height 5 feet 10 inches maybe very tall. But for someone of height 5 feet 8 he may think a person of height 5 feet 10 is just tall or in other words, basically I am looking at statements although some intuitive feeling is there the preciseness is not there that we typically have in mathematics.

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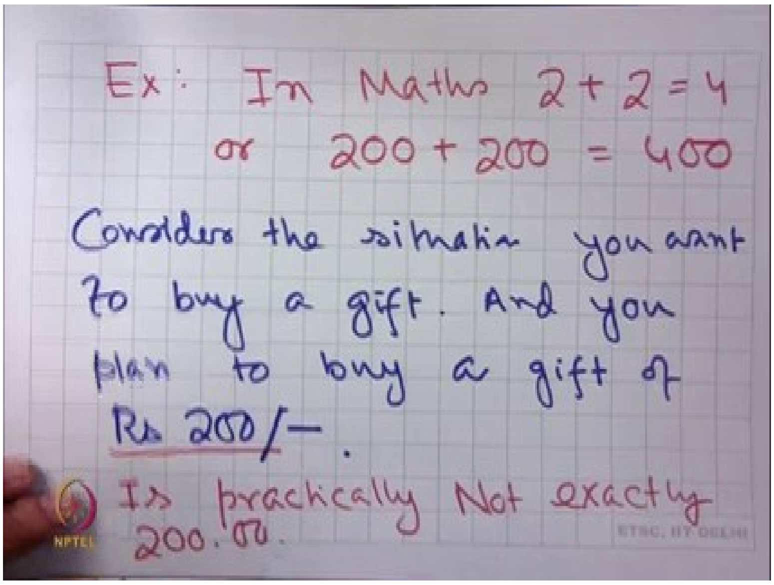

For example: In maths or say I call say . Now consider the situation you want to buy a gift and you plan to buy a gift of rupees 200. What does it mean? Does it mean that if you like a gift of say price 190 rupees, you are not going to buy it, does it mean that if you get something which you actually like and present to your friend but it costs 205 you are not going to buy it? We all know that that is not correct so this 200 is practically not exactly 200.

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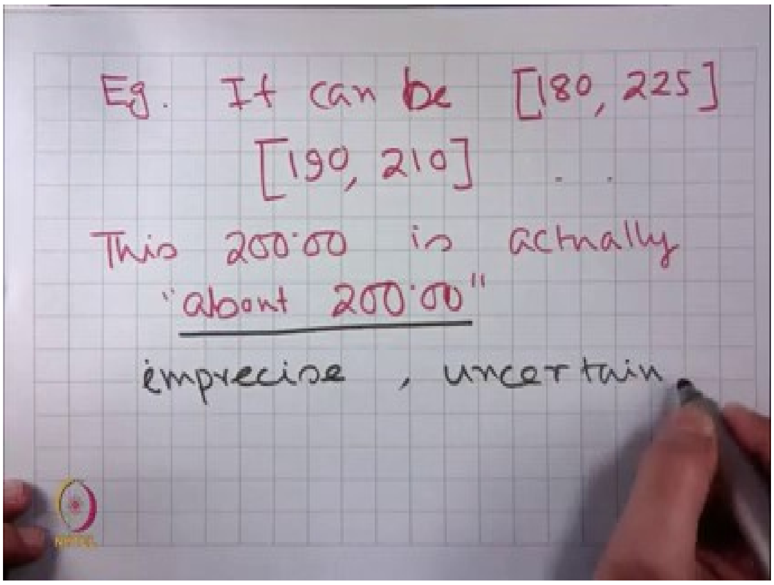

In fact, it can be an interval of values around 200 say, for example it can be say 180 to 225 for someone else it can be say 190 to 210 something like that. Therefore, these 200 is actually something like say 'about 200' and therefore, this term introduces impreciseness and uncertain.

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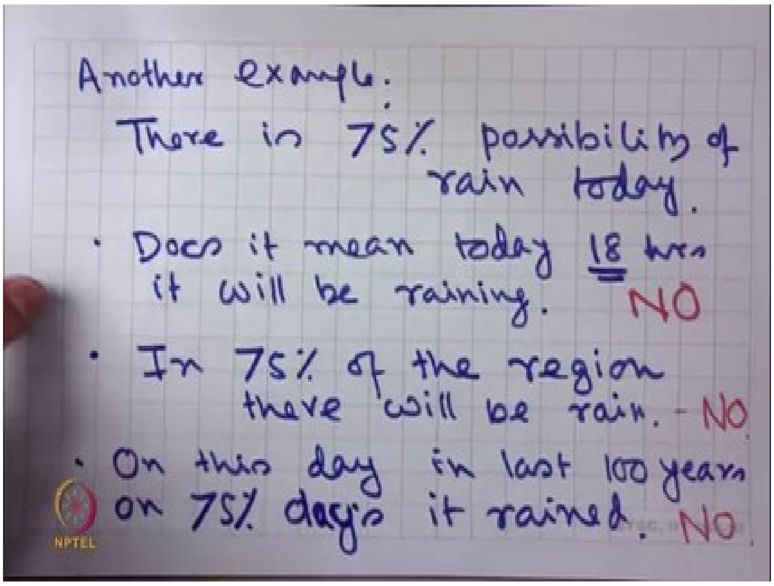

Let me give you another example, There is 75% possibility of rain today. What does it mean? Does it mean that today 18 hours it will be raining, why because 18 hours is 75% of total duration of a day. We know that that is not correct, no. Does it mean that in 75% of the region there will be some rain, no, that is also not correct. Also if we say on this day in last 100 years on 75% occasions or 75% days it rained that is also not correct.

What I mean to say this is a statement that we come across on regular basis as a prediction of whether but we do not have a precise meaning of what actually it is.

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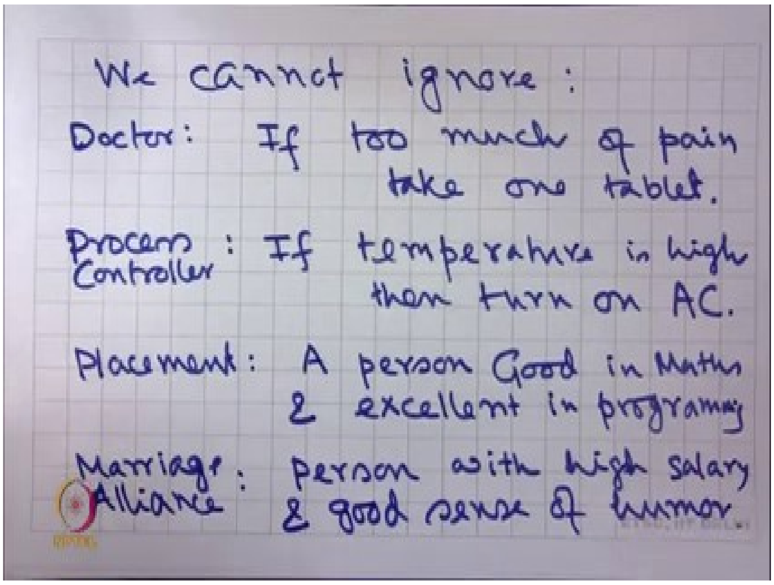

But can we ignore? We cannot ignore. Why? Say for example, a doctor may say if too much of temperature or say if too much of pain take one tablet. A process controller may say if temperature is high then turn on the AC. A placement may look for a person good in mathematics and excellent in programming. And say marriage alliance person with high salary and good sense of humour.

So what I have shown that in different aspects of our life we can see that there is a scope of fuzzy statements and we need to reason with this uncertain or imprecise statements to come to a conclusion.

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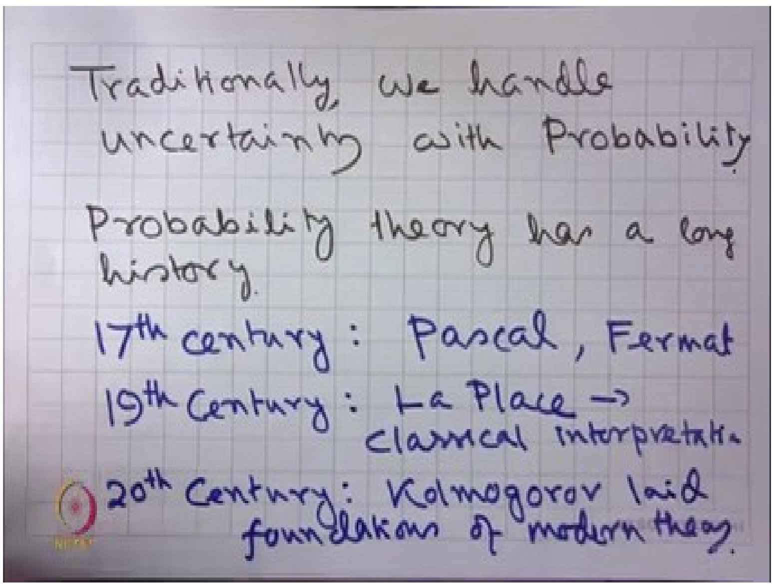

Now traditionally, we handle uncertainty with probability. Say for example, if I toss a coin 100 times. How many times we will get a head that is very uncertain. But we can make mathematical models for different outcomes that one may get in repeated trials of the experiment. So, probability theory has a long history. 17th century people like Pascal, Fermat they have started developing models for probabilistic happening of different outcomes.

19th century Laplace has given classical interpretation, 20th century in fact, early 20th century called Kolmogorov laid foundations of modern theory of probability. Or in other words, probability is being dealt with for several centuries and which has huge applications in different aspects of modelling physical phenomenon and other things.

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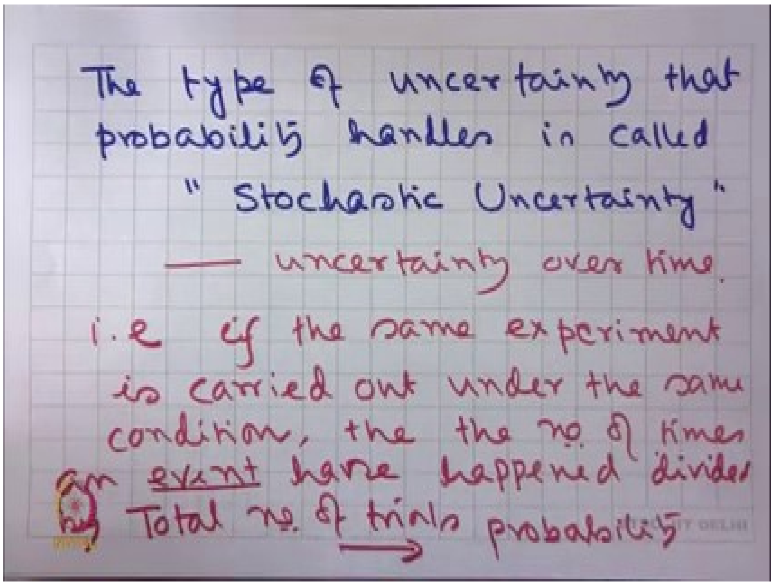

But the type of uncertainty that probability handles is called 'stochastic uncertainty' that is uncertainty over time. That is if the same experiment is carried out under the same conditions then, the number of times an event has happened divided by the total number of experiments a total number of trials gives the probability of that event. So, like that we calculate the probability of an event.

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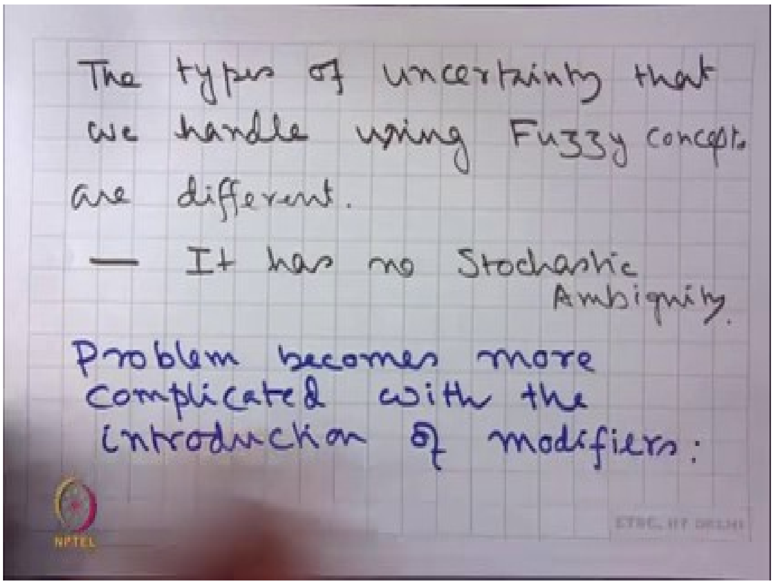

But the types of uncertainty that we handle using fuzzy concepts are different. It has no stochastic ambiguity. It is not that, the same boy out of 100 times, I am going to say tall and another 10 times out of 100 I am going to say short or another 10 times of medium, no. It is not that sort of ambiguity that I am talking about what we are talking about is an inherent vagueness in the definition of the term tall.

What is tallness and not only that not only it is with respect to one person for different persons it may have different interpretations about what is tall, this the problem becomes more complicated with the introduction of modifiers

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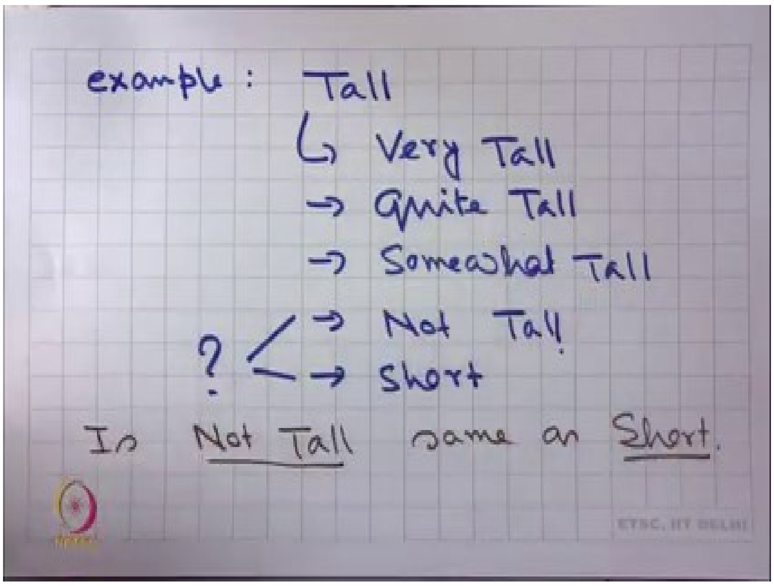

For example, from tall we may think of very tall we may think of quite tall, we may think of somewhat tall, we may think of not tall and with respect to height we can also think of short. The interesting thing is, are they same? Is 'not tall' same as 'short'. Somewhere there is a subtle difference in meaning with respect to the terms not tall and short. Similarly, a person may be tall but need not be very tall.

And the person who is somewhat tall is perhaps different from a person who is tall. So, this I have shown with respect to one term tall but similar expressions actually exists with respect to different adjectives and their modifiers and in different sentences where we will have lot of ambiguity.

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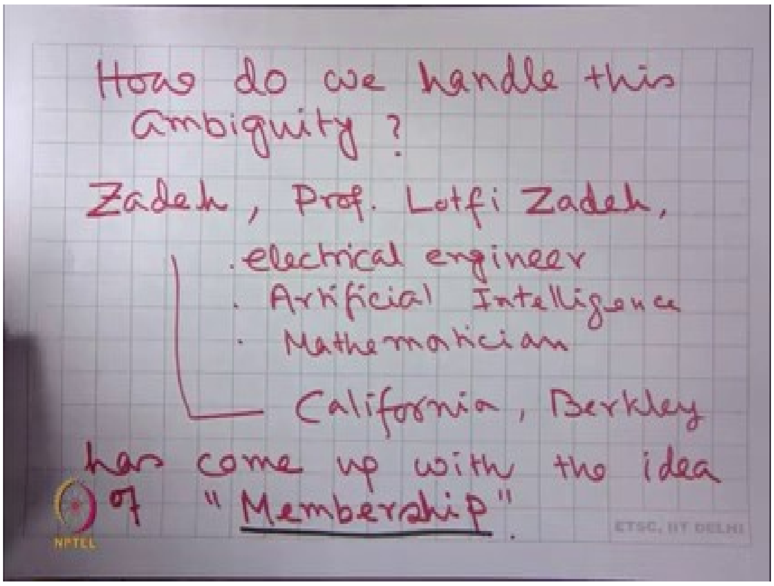

Question is how do we handle this ambiguity? Zadeh, that is Professor Lotfi Zadeh. He is an electrical engineer, he is an AI person, he is a mathematician coming from university of California, Berkeley. He passed away recently has come up with the following idea. What is that idea? This idea is called the concept of membership and perhaps despite being a genius who has worked in different fields of science.

He will be mostly remembered for his contribution or development of the concept of fuzzy and developing the basic mathematical theory for dealing with such uncertainties and that came in the year 1965. So, in comparison with probability, fuzzy is much, much younger. It is comparatively a much newer subject but it has a lot of importance a lot of interesting aspects which you will study in this course.

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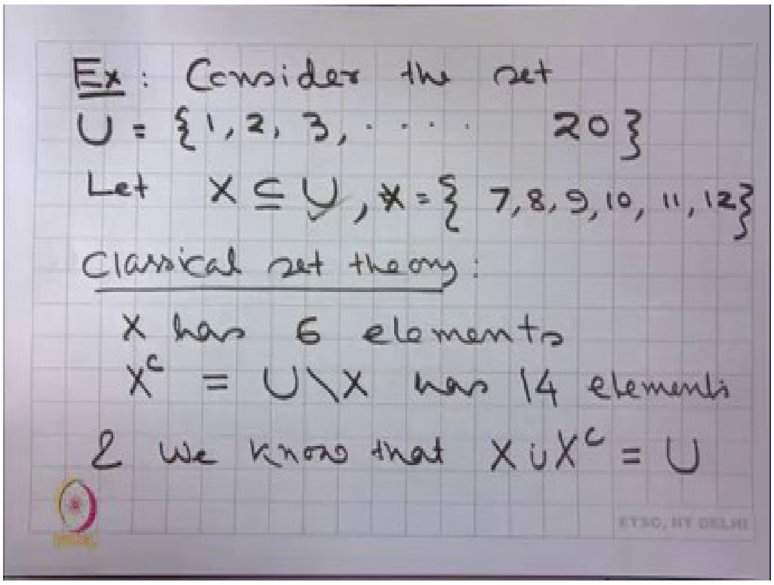

So, let me give you an example, consider the set

call it my universe. Let and . Now when we look at from in classical set theory.

Therefore, has 6 elements. with respect to is equal to has 14 elements and if an element belongs to it does not belong to and if one element belongs to the it does not belong to the set .

And we know that . So, this is the fundamental of classical set theory.

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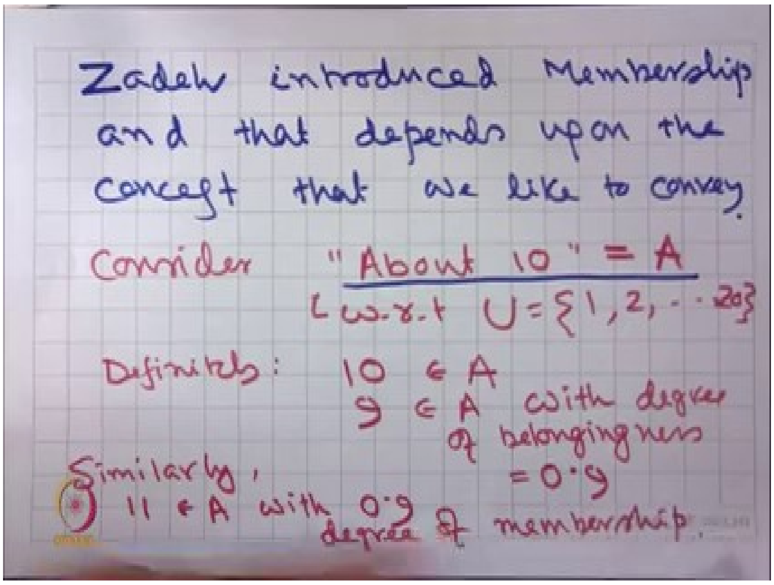

Zadeh introduced membership as I have already said, and that depends upon the concept that we like to convey. Consider for example with respect to

.

Therefore, definitely . Let me call the set , definitely belongs to , it is close to . So, we can say it is but perhaps its membership with degree of belongingness maybe say 0.9. Similarly say with degree of membership.

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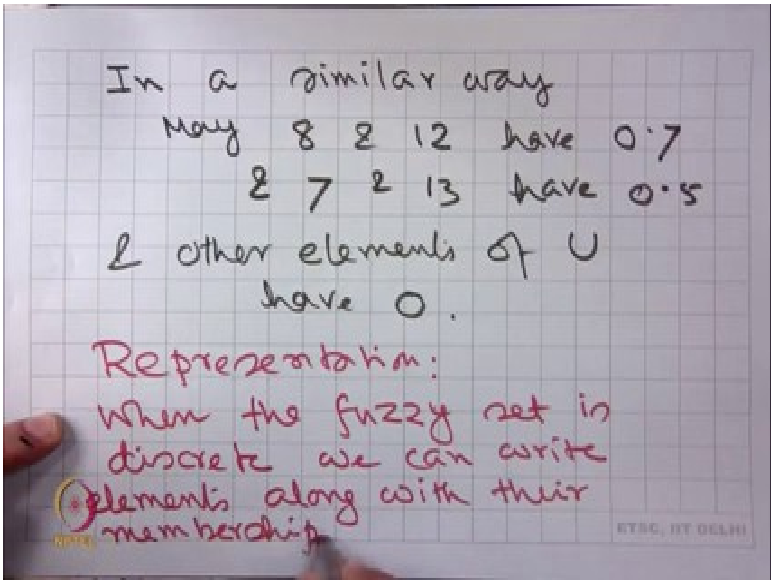

In a similar way, maybe and have , and and have say and other elements of have . That means we do not consider say to and to to be numbers or . How do you represent such a set? When the fuzzy set is discrete we can write elements along with their memberships.

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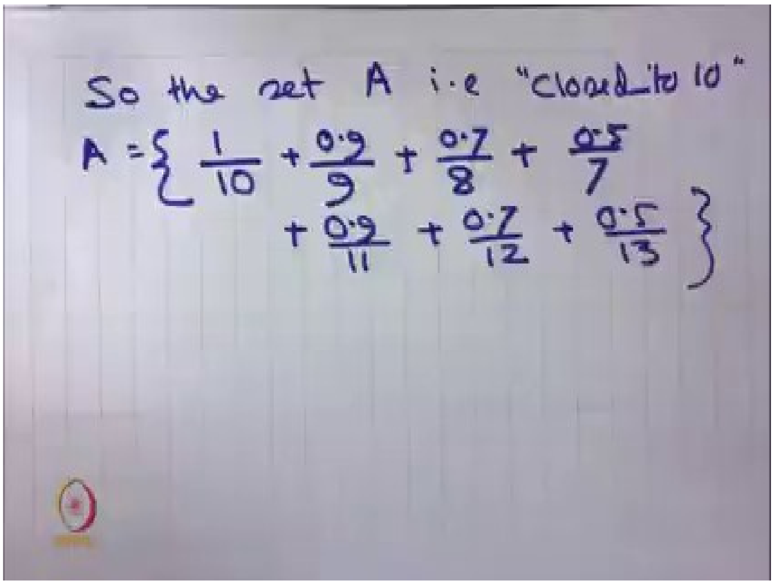

So, the set , let me call it instead of .

So, this new set , which actually interprets the term has these element with different membership values. So, when things are discrete this is one way of representing that.

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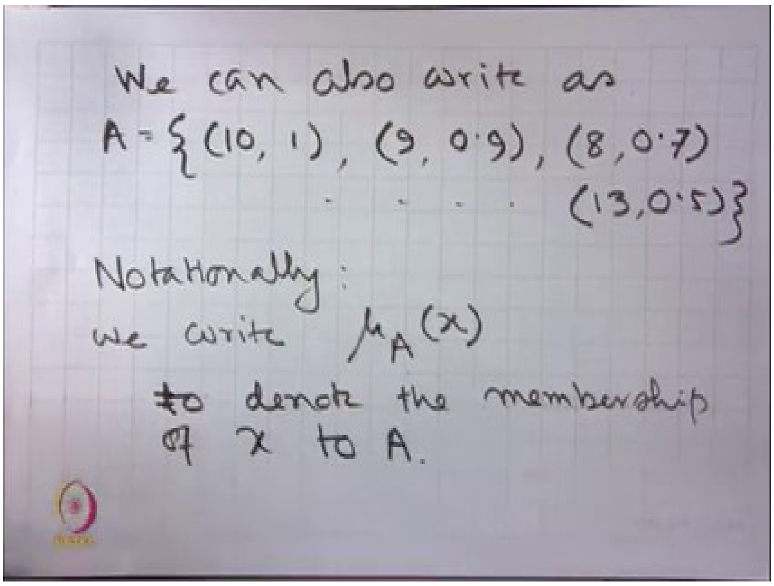

Another way is, we can also write as

So, this is another way of representing a fuzzy set when the number of elements are small we can associate with each number its degree of membership. Notationally we write . So, is the membership of to .

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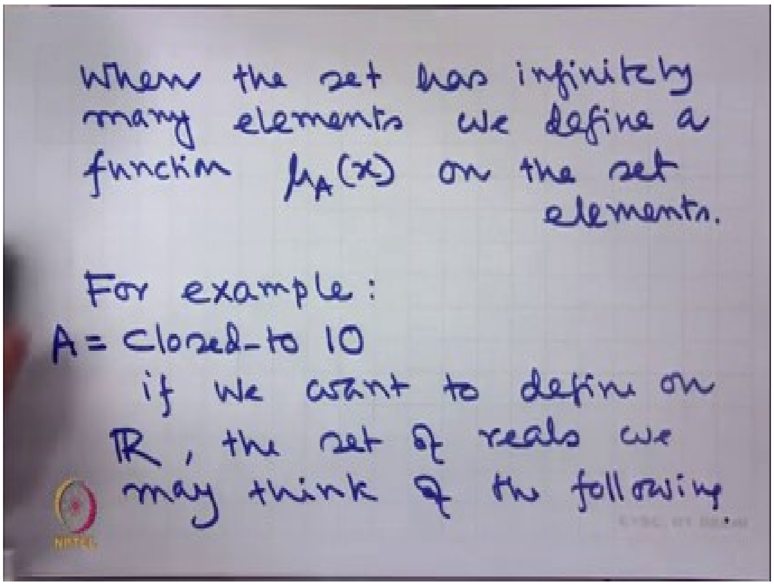

When the set has infinitely many elements then, of course, we cannot write it as pairs of value and its membership or in other words by mentioning each element along with its membership values. So, what we do we define a function on the set the elements okay. For example say which you have called , if you want to define on the set of reals, we may think of the following.

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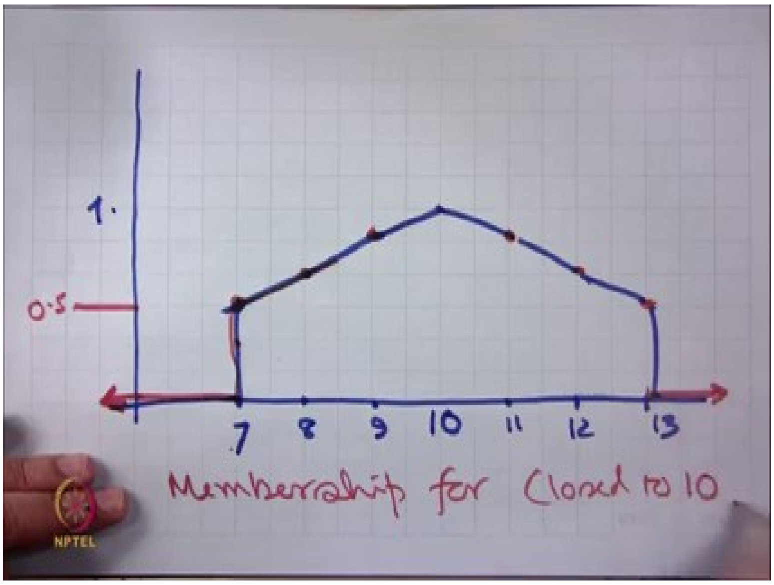

Suppose this is 10 and at 10 which membership value is 1, suppose it is 9 it is 8 this is 7, 11 it is 12, it is 13. So, as I said below 7 we do not consider it to be close to 10, above 13 we do not consider it to be close to 10. So, in this region this goes to infinity and this goes to infinity this goes to minus infinity and this goes to infinity it is all 0. At 7 we give a value say 0.5, so the curve starts from this point similarly at 13 we may give 0.5.

At 8 we have given something like 0.7 and at 12 we have given something like 0.7 at 9 we have given something like 0.9 and 11 give something like 0.9 and what about the rest of the real numbers suppose we define linear function between 7 to 8 connecting these 2 points between 8 to 9 connecting these 2 points and between 9 to 10 connecting these 2 points and in a similar way. So this is the function that we get to describe the membership of different real numbers between 7 to 13 as member of the set .

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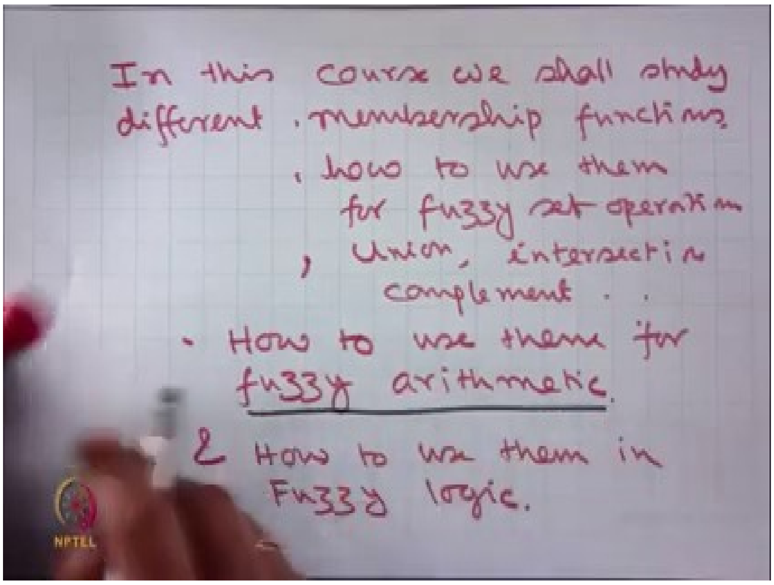

So, in this course we shall study different membership functions. How to use them for fuzzy set operations such as Union, Intersection, Compliment etcetera. How to use them for fuzzy arithmetic and how to use them in Fuzzy the logic. Let me now give an example of a fuzzy arithmetic.

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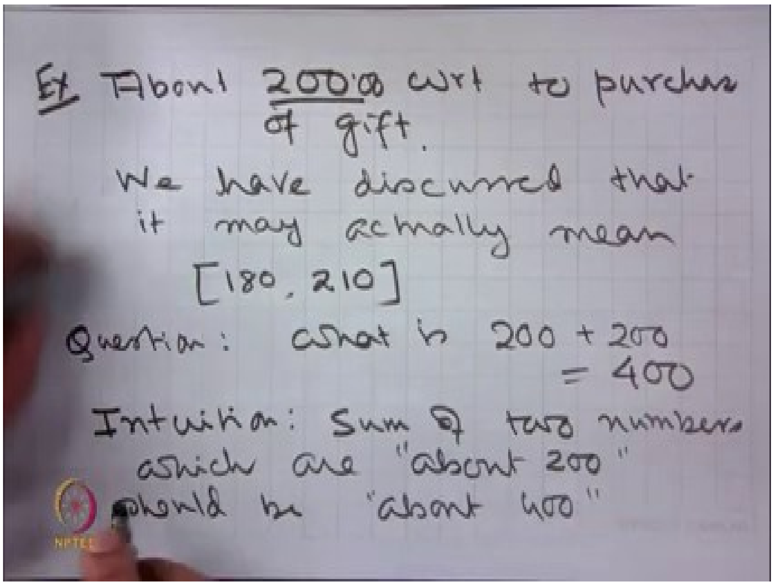

So, I was talking about ‘’ with respect to purchase of gift. And we have discussed that it may actually mean say with membership values given for different numbers the way I have just shown with respect to say ‘’. Here instead of 10 we are looking at say something like 200. Question is what is we all know it is 400 therefore intuitive believe is that, sum of 2 numbers which are should be .

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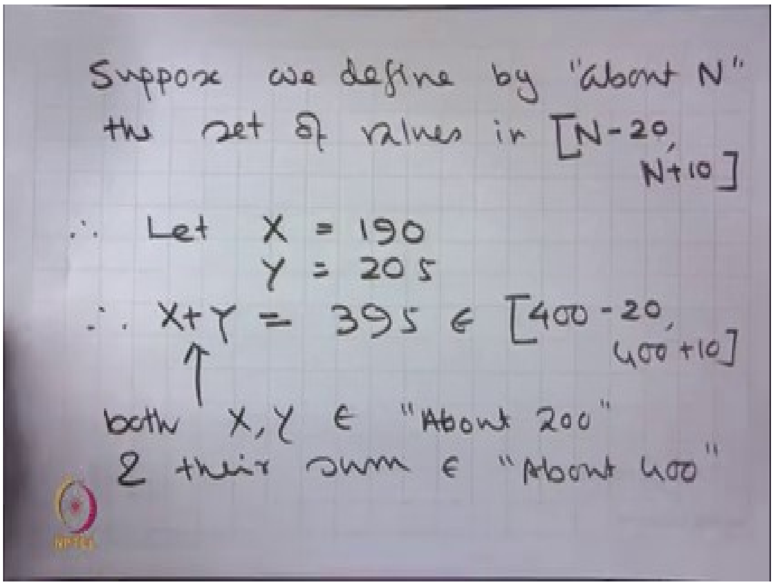

Now what is , we have not defined clearly but from the example suppose we define by ‘’ the set of values in , just now we have shown , is . Therefore, let , therefore . So, we can say these numbers and which belonged to and their sum belongs to .

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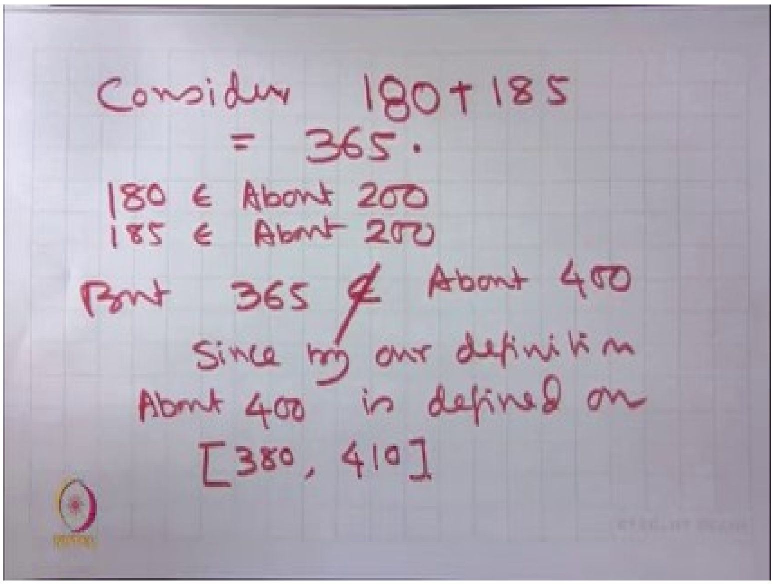

But now consider , , .

But, . Since, by our definition is defined on [380, 410]. Therefore, it is clear that doing arithmetic with fuzzy sometimes do not match our natural integer.

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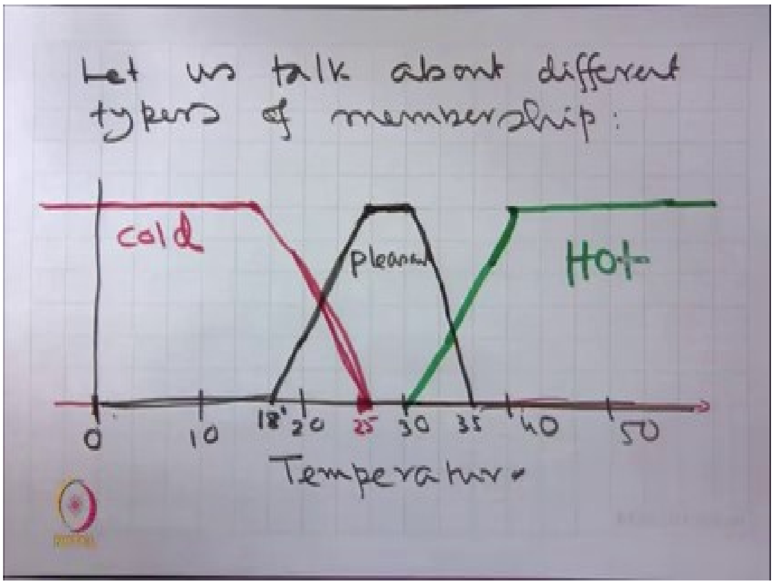

Now let me talk about different types of membership. Consider temperature say this is 0 degree, 10 degrees, 20 degrees, 30 degrees, 40 degrees and 50 degrees. What is cold? Being in India we can assume that 0 to 10 or so 0 to 15 is cold for everywhere. Therefore, when I am defining that set cold I give full value 1 membership value 1 to all temperature between 0 to 15. But as the temperature goes above 13 the degree of coldness decreases and suppose we decrease it linearly.

And at 25 we say that it is not cold anymore so you remember see its membership goes 0 to the infinity. And below 0 we can consider that it is always 1. So, this is the type of graph we may get for defining the membership function for cold. What about pleasant? Suppose one thinks that 25 degrees is a very pleasant temperature, 25 to 30 degree one finds pleasant but as it goes below 25 it starts getting colder and perhaps say up to 18 degrees one may feel pleasant.

On this side on this side say maybe up to 35 degrees so the membership for pleasant may have a shape like this in outer outside this it is 0 and this is what it looks like. What about hot? One may think that actually anything above 30 is hot but definitely anything above 40 is surely hot. So, it goes like this and between 30 to 40 it may grow linearly like this. So, we can see different shapes of membership functions. Similar things can be shown with respect to another example.

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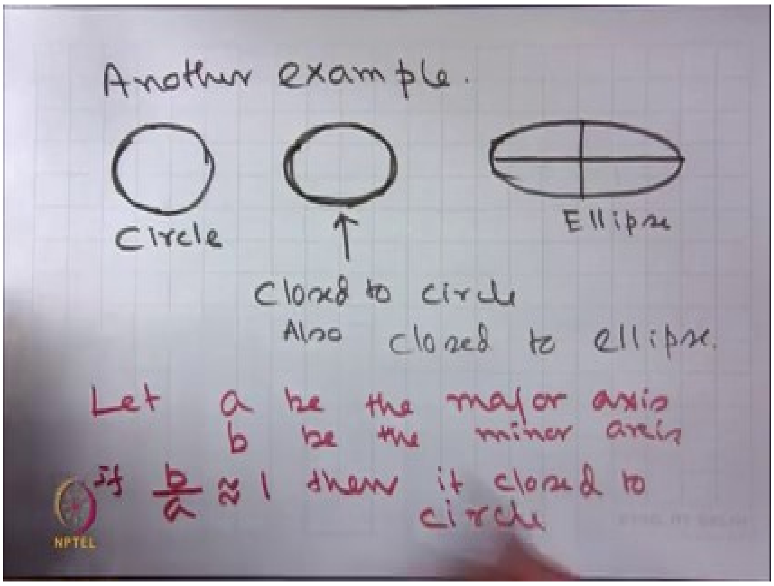

If I draw a shape like this, we can identify this to be a circle. On the other hand, if I draw a shape like this, we may consider this to be an ellipse. What about a shape something like this? One may think it is close to circle, one may think closes to ellipse. Therefore, what I wanted to emphasize on is that, our concept of the shape circle and ellipse depends upon the major axis and minor axis. So, let , be the major axis and be the minor axis. So, if is approximately 1 then it is close to a circle.

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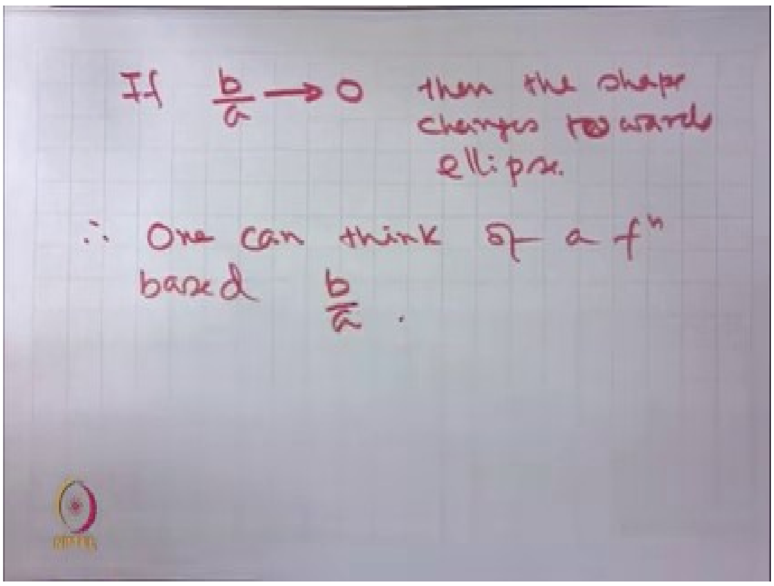

If goes to 0 then the shape changes too towards ellipse. Therefore, one can think of a function based on such that we can define the membership to a circle or ellipse using a function of . In a similar way we can take up many physical phenomena or situations and we can try to develop membership function for describing the membership to different fuzzy sets.

So, in this talk I tried to give you a brief introduction towards fuzzy set, the membership and some intuitive idea of what it says and how to deal with the vagueness in the different fuzzy terms that we use in our daily life. In the next class I will start with fuzzy set and first I will distinguish the difference between crisp set and fuzzy set and their operations using fuzzy membership functions, thank you.