Partial Differential Equations
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Lecture – 2.10
General Nonlinear Equations 1
Search for a Characteristic Direction

You are welcome to this lecture. We continue to study the first order partial differential equations, but now we move on to general nonlinear equations from Quasilinear equations that we have been considering so far. So, in general nonlinear equations, we are going to have 4 lectures in which we will establish the existence and uniqueness of solutions to Cauchy problems for general nonlinear equations. In this lecture, we are going to discuss about search for a characteristic direction.

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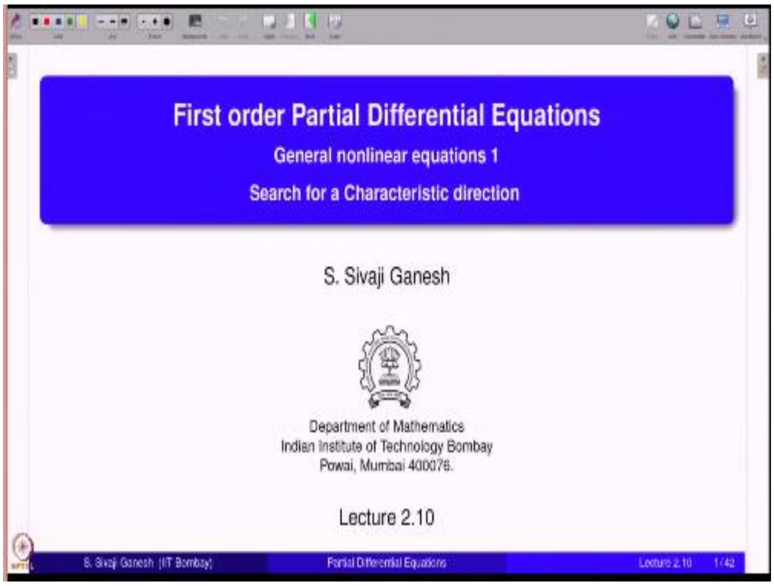

So, we will be making regular comparisons with the Quasilinear case from time to time.

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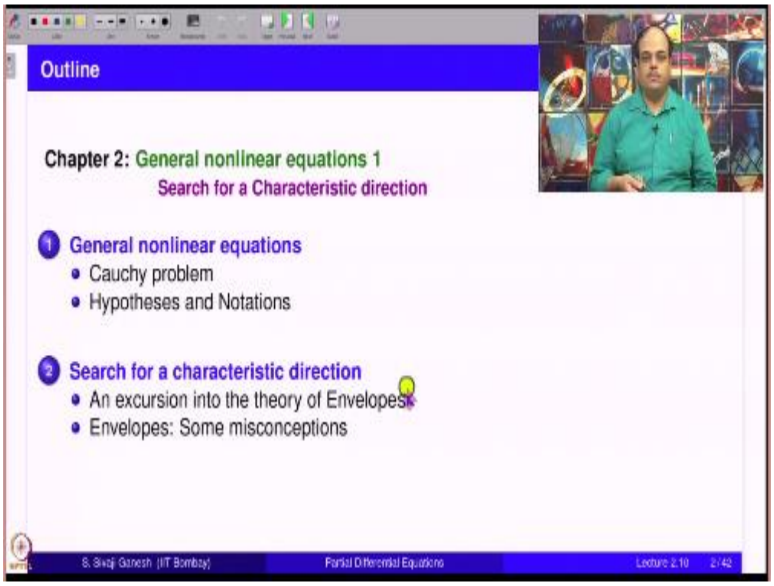

So, first we set up the notation for the Cauchy problem for general nonlinear equations and the hypothesis. Then in the search of characteristic direction, we will be led to the study of theory of envelopes. And since we are anywhere doing the theory of envelopes, I would like to point out a few misconceptions which are there in the form of language sentences.

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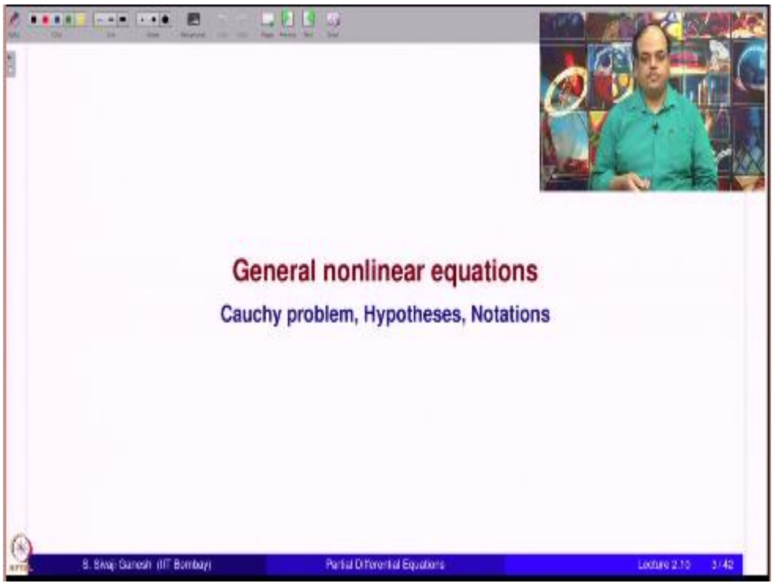

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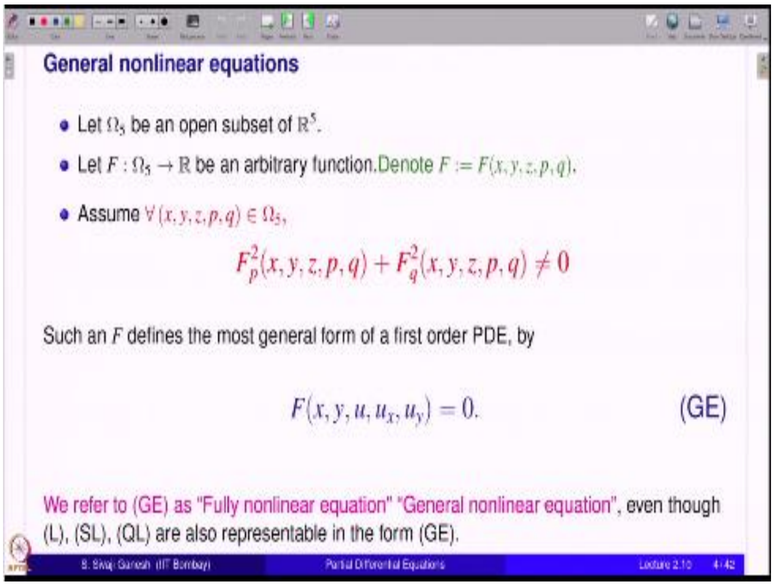

So, to define a general nonlinear equation, first order, we need to set up some notations. The equation is going to feature the independent variables x and y. So, we are going to deal still with the equations with the 2 independent variables. So, x and y are 2 in number, then the u, ux and ui. There are 5 quantities which are appearing in the equation. Therefore, we consider an open set in our file called omega 5.

Recall omega d, you would like to use it for R d. Then let F be any arbitrary function. Denote F by F of x, y, z, p, q. That means, whenever we are going to differentiate F of something of a function of x y, then we need to do chain rule. Then we should know what are the variables we are using for the function F. They are namely x, y, z, p, q. And we need to assume that F p and F q both cannot be 0 at the same point.

That means, at every point in the domain, at least one of their F p or F q is not zero. That is expressed by writing this equation, for every x, y, z, p, q in omega 5 F p square plus F q square is not equal to 0 which means that F has to be differentiable. I said arbitrary function here, but this already suggests it should be a differentiable function. A precise hypothesis we will also see when we are going to prove theorems till then this is fine.

So, such an F defines a more general form of a first order PDE. Of course, first order PDE is defined for any arbitrary function which is going to come now. Okay, this always makes sense, even if F is not differentiable. But we want to say that at least one other first order derivative appears in F. That is made sure we are asking F p squared plus F q square is nonzero.

This is a differential way of expressing that u x and u y, one of them features in the equation all the time. So, we refer to this GE. So, we used L for linear, SL for semilinear, QL for quasilinear, GE, we use it to denote the most general form of a first order PDE. So, sometimes we call it fully nonlinear equation. Sometimes we call the general nonlinear equation.

So, you should not be confused because a fully nonlinear is a very standard usage for these kind of equation. And I am calling it general nonlinear equation, a mild, step down from the word fully. Even though we know that L, SL, QL are all representable in the form of GE, we know that. But still in this form, whenever we are dealing, we are doing the theory for this; we do not know whether it is going to be linear or semi linear. So, that is why we call it general nonlinear equation.

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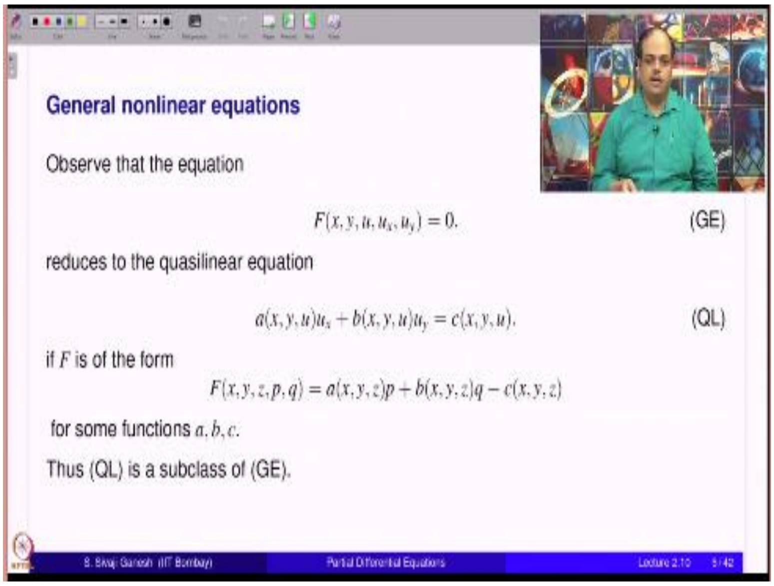

Now, this general nonlinear equation reduces to Quasilinear equation which is here, when the function F is of what type? F of x, y, z, p, q should be equal to? So note x, y, z, p, q stands for x, y, z, p, q in the place of z, u comes, in the place of p, u x comes, in the place of q, u y comes. Therefore F of x, y, z, p, q in the case of Quasilinear equation is a of x, y, z into p + b of x, y, z into q - c of x, y, z, for some functions a, b, c.

So, therefore QL is a subclass of GE. So Quasilinear equations are contained in general nonlinear equations.

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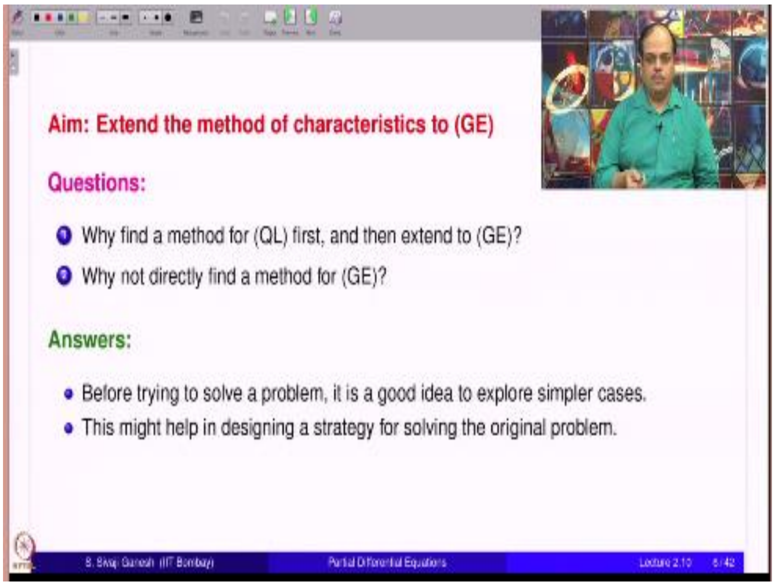

Now, aim is to extend the method of characteristics to the general equation, general nonlinear equation. Of course, questions come first that why find a method for Quasilinear first and then try to extend to GE. Why not directly find a method for GE? Of course, there are answers. Before trying to solve a problem, it is a good idea to explore simpler cases. And this might help in designing a strategy for solving the more general form that is the original problem GE.

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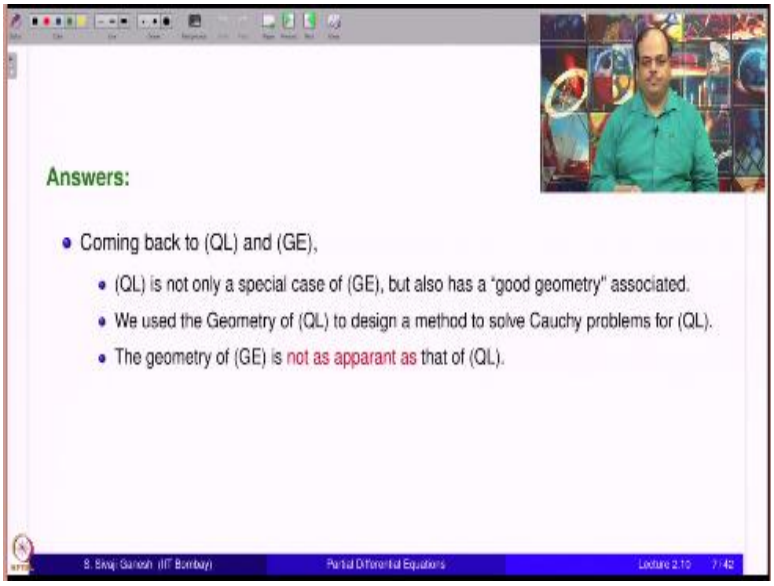

Now, coming back to Quasilinear and general nonlinear equation QL is not only a special case. Yes, it is a special case, not only a special case, but also has a good geometry associated to it. We devoted a lecture for geometry of Quasilinear equations. So, we use a geometry of QL to design a method to solve Cauchy problems. The geometry of GE is not as apparent as that of QL.

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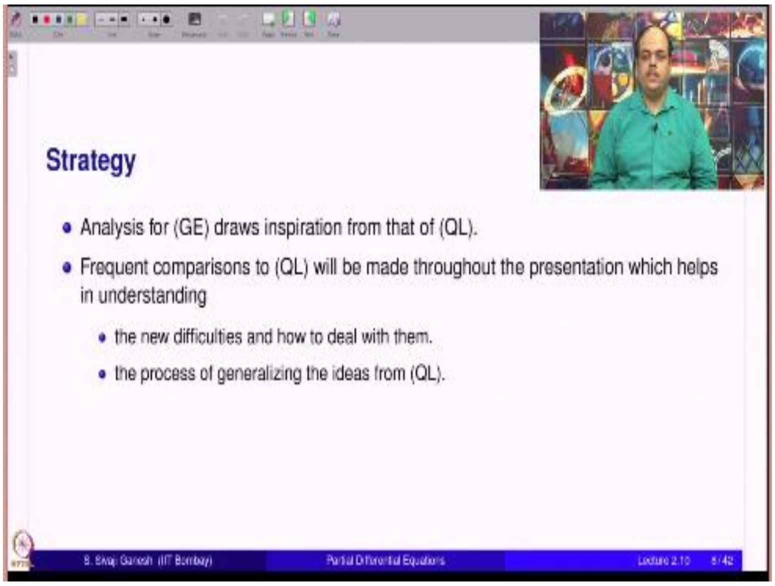

Strategy is analysis for GE draws inspiration from that of QL Quasilinear case. So, frequent comparisons to Quasilinear case will be made throughout the presentation, which helps in understanding the new difficulties and how to deal with them. The process of generalising the ideas from the Quasilinear case.

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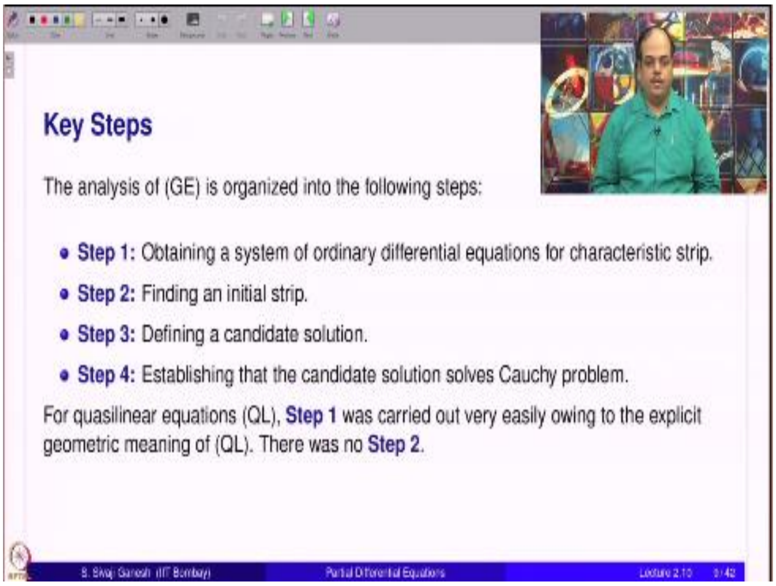

The key steps in the analysis of general nonlinear equation is organised into the following steps. The key steps are this step 1, obtaining a system of ODEs for what is called a characteristic strip. They will be defined soon. But I am just writing down the main steps involved so that we can keep our progress. We can keep tabs on the progress. Where are we, at each step. Step 2 is finding an initial strip. Step 3 is defining a candidate solution.

This was also there in QL case. This was I think, step 2 there, in QL. Defining a candidate solution and checking that can candidate solution is indeed a solution to the Cauchy problem. That is the last step. So, for Quasilinear equations, step 1 was carried out very easily. In fact, there was no characteristic strip there, characteristic curve. It was owing to the explicit geometric meaning of QL. There was no step 2.

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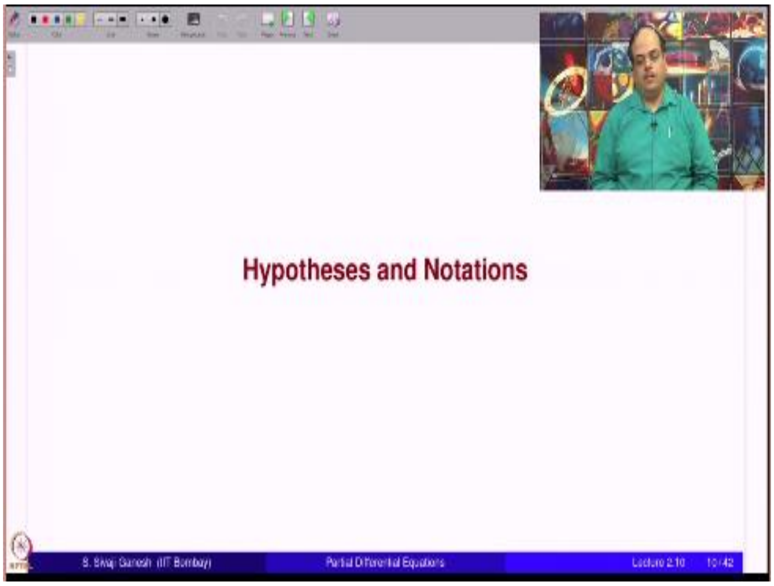

And then step 3 and step 4, were called step 2 and step 3 in QL case.

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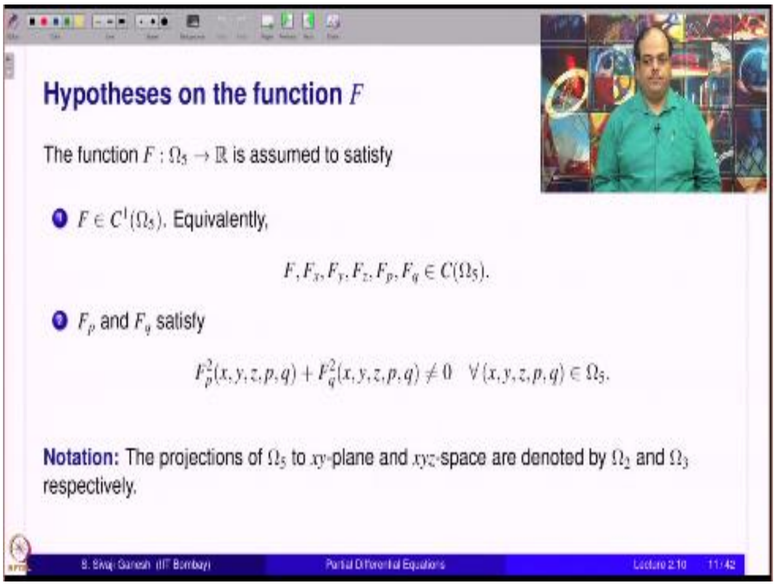

Now hypothesis on the function F. F is assumed to satisfy C 1 function on the domain omega 5, where it is defined. And assuming it is C 1 omega 5 is same as saying all the partial derivatives on the function are continuous functions on omega 5. This is equalent to that. And

F p and F q satisfy this condition. We already discussed about this. This makes sure that F of x, y, u, u x, u y = 0 is a first order PDE at every point.

The projection of omega 5 to xy plane is called omega 2 and to xyz space is denoted by omega 3.

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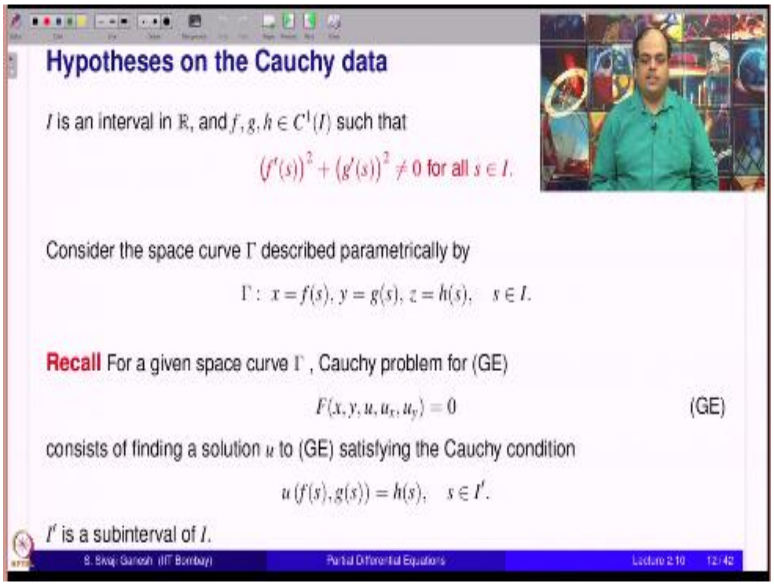

Now Cauchy data. We take an interval I in R and we take 3 functions which are C 1 functions such that f prime square plus g prime square is not equal to 0 for all s in I. Consider a space curve gamma described parametrically by x = f s, y = g s, z = h s. We call this condition meant that projection of gamma to omega 2 which was called gamma 2 is a regular curve.

For a given space curve, we already define what is the meaning of Cauchy problem. Cauchy problem for GE consists of finding a solution that is a function which satisfies the differential equation and also such that it satisfies this condition: u of f s, g s equal to h s, s in I dash. We do not require s to be in I, as I already reminded you many times this is a notion of a kind of local solution. So, I prime is a sub interval of I.

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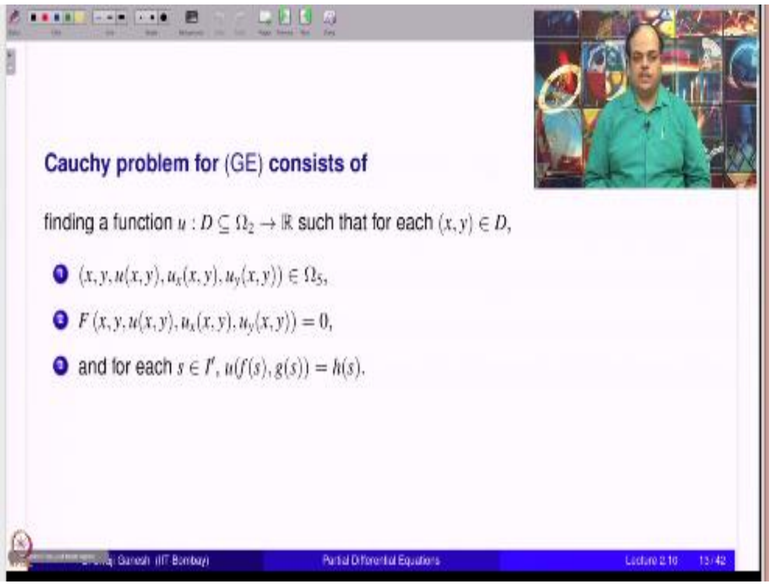

So Cauchy problem for GE consists of finding a function u with a domain D which is contained in omega 2 such that for every x, y in D. This 5 tuple belongs to omega 5. And therefore I can apply F to it and F of that should be 0. And for each s in I dash, you have f s, g s = h s for some I dash sub interval of I.

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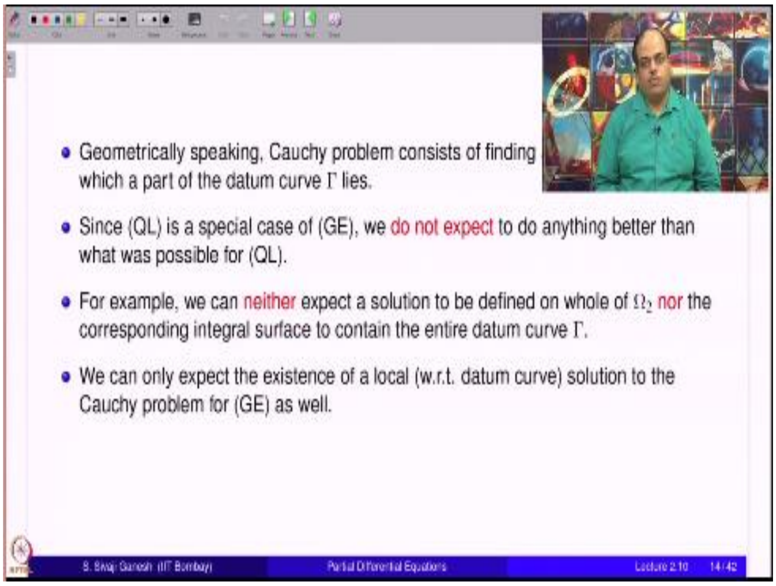

So, geometrically speaking Cauchy problem consists of finding an integral surface on which a part of the datum curve lies. Since Quasilinear equation a special case of GE, you do not expect anything better than what was possible for Quasilinear equations. Because any result that you show for a general GE equation continues to hold for QL. Therefore, what you do not have in QL, you do not expect for GE.

So, on the other hand, you can ask the questions what is true for QL, is it true for GE? That is a question one can ask. So, for example, we can neither expect a solution to be defined on whole for omega 2 nor the corresponding integral surface contains entire datum curve gamma. That is, we cannot expect solutions which are global with respect to domain.

Similarly, we cannot even expect solution which are global with respect to the datum curve because that is not true for QL. So, same thing holds here. So, we can only expect the existence of a local solution with respect to datum curve. This is what we have proved for the Quasilinear equation and the corresponding Cauchy problem. Therefore, we can expect this for Cauchy problem for GE as well.

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Now, search for a characteristic direction.

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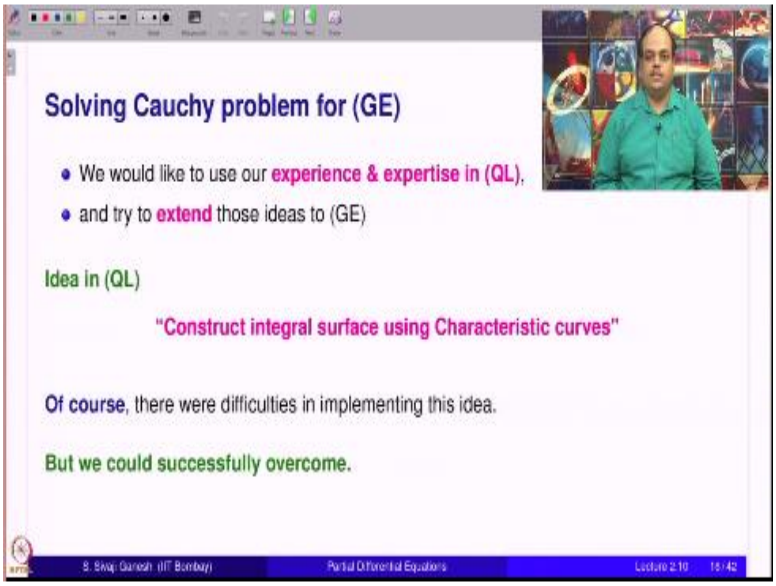

Because that is what we had, the starting point for QL. So, we would like to use our experience and expertise in Quasilinear equations and try to extend these ideas to general equation. What is the idea in Quasilinear case? Construct integral surface using characteristic curves. This was the idea. Of course, there were difficulties in implementing this idea. But, we could successfully overcome all these difficulties and actually implement this idea.

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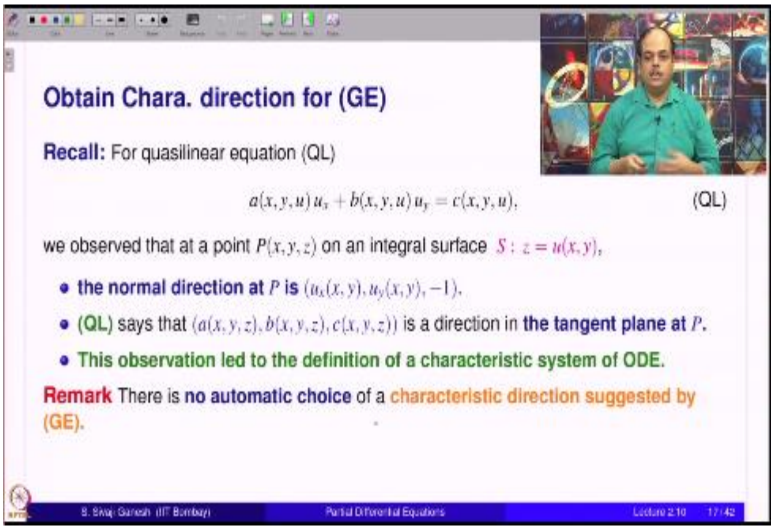

So, therefore, we would like to have something similar for GE. So, here we had characteristic curves which are defined by characteristic direction. Now, we ask the question, what is the characteristic direction for GE? How do we get it? Recall that for Quasilinear equations, the equation is au x + bu y = c. We observed that at a point on an integral surface, the normal direction will be u x, u y, -1.

And the equation tells us that abc dot product with u x, u y, -1 = 0. It means the direction abc is in the tangent plane. It is a direction in the tangent plane at P. This observation led to the definition of a characteristic system of ODE. Once we understood that these are direction in the tangent plane, then we thought of a curve which lives on the surface and such that the tangential direction is abc.

That led us to the definition of characteristic ODE and solutions trace was characteristic curves and their union gave us integral surface for the case of Quasilinear equation. Now, remark that there is no automatic choice of characteristic direction suggested by the general equation. The QL suggested very easily abc is perpendicular to u x, u y, -1.Therefore, abc is a direction in the tangent plane, but GE is simply says F of x, y, u, u x, u y = 0. So, there is no directly suggested characteristic direction by the equation GE.

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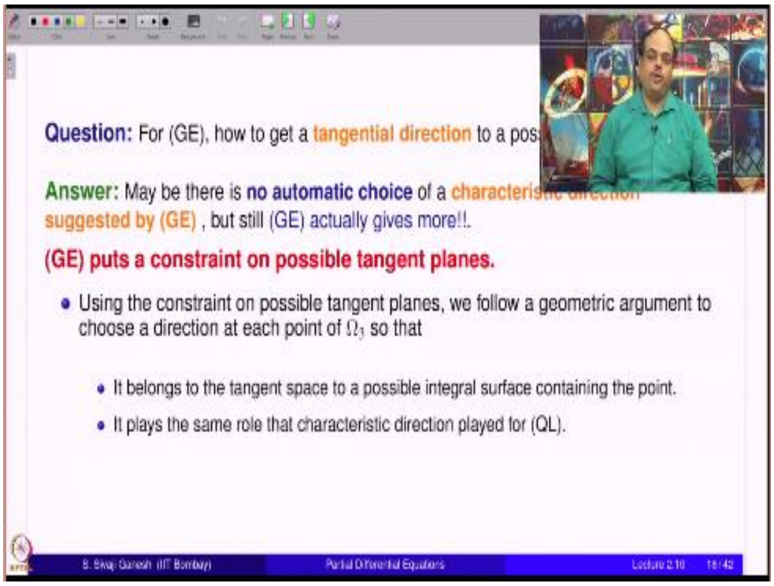

Now, how to get a tangential direction to a possible integral surface? Answer to this question is yes, maybe there is no automatic choice, very visible choice from the equation for a characteristic direction. But still it gives something more actually, it does not give one direction in the tangent plane, maybe it gives entire tangent plane. That is a possible tangent plane. We will see that. So GE puts a constraint on possible tangent planes. That is all it says.

Now we have to figure out one direction, which is going to be in the tangent plane. And in a consistent way as well. Consistent as we change the point. We will see that. Using the constraint on possible tangent planes, we are going to see what is this constraint. We follow a geometric argument to choose a direction at each point of omega 3, so that it belongs to the tangent space to a possible integral surface containing the point.

It plays the same role that characteristic direction played for QL. So, how are we going to get somebody who looks like a characteristic direction for GE. We look at what is a constraint on the possible tangent planes and follow a geometric argument and then we will be able to choose one direction at each point which serves a similar purpose as characteristic direction did for QL. That is the summary here.

Now, let us ask how do you find? First assume that such a thing is there. That is there is an integral surface. Somebody gave you integral surface. Understand that and then pretend that you do not know integral surface and try to see how much of this information will be useful in getting what you want.

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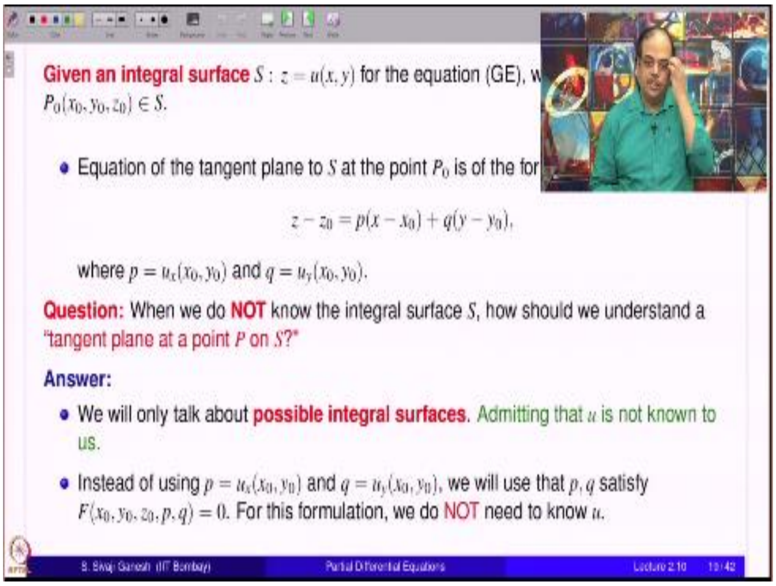

So, given an integral surface for the GE where u is a C 1 function and P 0 is a point on integral surface. In other words, z 0 = u of x 0, y 0. That is the meaning. S is defined like that. The third coordinate is u of first 2 coordinates. So, equation of the tangent plane to S at the point P 0 is of this form. It looks like this, It is a equation of a plane passing through the point x 0, y 0, z 0 having normal p, q – 1.

What are p and q? p is u x x 0, y 0, q is u y at the point x 0, y 0. p, q – 1 that is u x, u y - 1 is a normal to the surface and then it will be normal to the tangent plane. So and the tangent plane passes through the point x 0, y 0, z 0. So, this is an equation. When we do not know the integral surface, how should we understand a tangent plane at a point P on S. S itself is not known.

So, we will only talk about possible integral surfaces and possible tangent planes to that, at points of that. So admitting that u is not known to us, what is left of this equation? It is still the equation of a plane where I cannot write what p and q are like this, because u is not known. But if at all p and q are going to be tangent planes to the integral surface, p and q are required to satisfy the condition F of x 0, y 0, z 0, p, q = 0.

So, we will not use this. This is explicit information which is known only if you know u. Now, we are saying no I do not know u, admit admitting that u is not known we won't find u. So, u is not known. So, this is the equation of the plane anytime. Only thing is p, q - 1 is supposed to be the normal to a possible integral surface and that is required to satisfy this constraint, F of x 0, y 0, z 0, p, q = 0.

Because if at all, you knew your integral surface, this equation will be satisfied: x 0, y 0, z 0, u x at x 0, y 0, u y at x 0, y 0. Now, I do not know this u. Therefore, I simply put p and q and this constraint. So, this is the equation of a tangent plane passing through a point x 0, y 0, z 0. And this is a constraint on the normals. For this formulation, we need not we do not need to know u.

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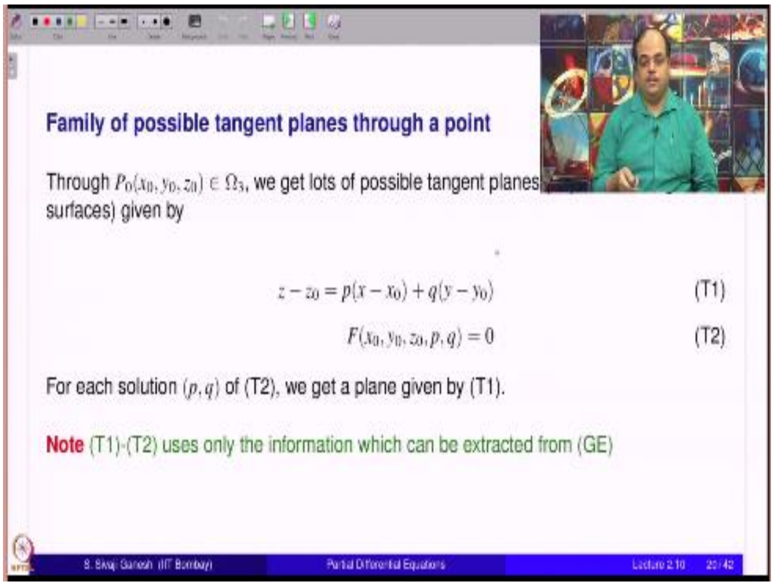

So, that is why the word possible keeps on coming here. Family of possible tangent planes through a point. Take a point in omega 3. We get lots of possible tangent planes, 2 possible integral surfaces, given by this relation. As many as solutions of T2 R. For every solution of T2, you can associate a plane given by T1. We do not know how many solutions will be there for T2. It is a nonlinear equation.

So, for each solution p, q of T2, in fact, what is to be expected is T2 is an equation involving 2 parameters p and q and 1 constraint. Therefore, what we expect here is 1 family of solutions will be there, corresponding to that you will have 1 family of planes. So, for each solution of T2, p, q of T2 we get a plane. That is true. T1-T 2 uses only the information which can be extracted from the equation.

For example, T1 it uses nothing. It just the equation of plane we always write. Only unknown quantities are p and q. They are supposed to satisfy this T2 which is coming from GE. So, I am not making use of any explicit knowledge of unknown integral surface here. I do not know integral surfaces. I am proposing this T1 f planes, in fact going to be a family of planes, which are one parameter family because 2 parameters are tied by one equation essentially means only one of them is free. Therefore, this one parameter family of possible tangent planes.

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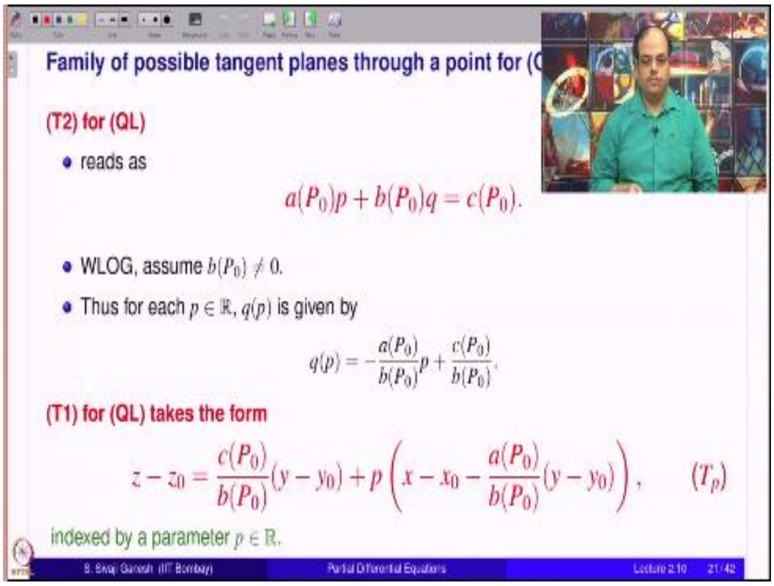

Now T2 let us see what it means for the Quasilinear equation case. So, we can explicitly write down T2 for Quasilinear equations. This is 1 where P 0 is the point x 0, y 0, z 0. And this equation F of x, y, z, p, q = 0 is this. It reads as this. From here, one of them is going to be nonzero either a or b. Let us assume b is nonzero. Then I can divide by that. Therefore, I can solve for q in terms of p. So, you see q is a function of p.

Now, I take this and substitute in T1. Therefore, T1 looks like this. So, this is the equation of a possible tangent plane where p is free. So, It is a 1 parameter family of possible tangent planes. The only property is that they pass through the point x 0, y 0, z 0. So, you have a point x 0, y 0, z 0. Now, you are looking at a plane like that. Another plane, a plane is infinite. These are planes in 3d and another plane like that and so on.

The only common thing is this point P 0. They all pass through this one So, It is a 1 parameter family of possible tangent class, index by this parameter p in R.

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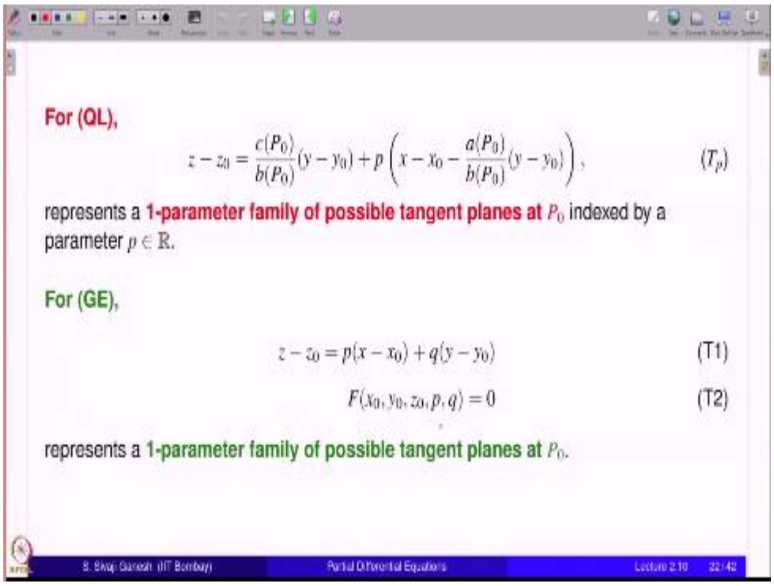

It is because, why how did we get this explicitly this p here? Because we could solve from the equation we express q in terms of p explicit expression and then we substitute it and got. Now for GE also we would like to do the same thing. Let us continue for QL. This is the equation. It represents a one parameter family of possible tangent planes indexed by parameter p. So, for GE exactly this. This is T1, T2 as written earlier.

So, this represents a one parameter family of possible tangent planes at P 0. So, maybe we may have to analytically impose some conditions, which say that q can be solved in terms of p. Atleast locally for around a fixed value of p let us say p = p 0, small p 0, you can express q as a function of p or vice versa for a fixed value of q = q 0. You can express p as a function of q, then go and substitute here we will get a 1 parameter family of possible tangent planes.

In the case of QL, these 2 equations reduced to just this. That is because we could express from here q as a function of p, went back and substituted in T1 and we got this. It is a family of tangent planes indexed by the parameter p in R. In the case of nonlinear equations, we do not expect that we can have solutions, global solutions. Therefore, what typically happens is q is a function of p in a small neighbourhood of some fixed value of p.

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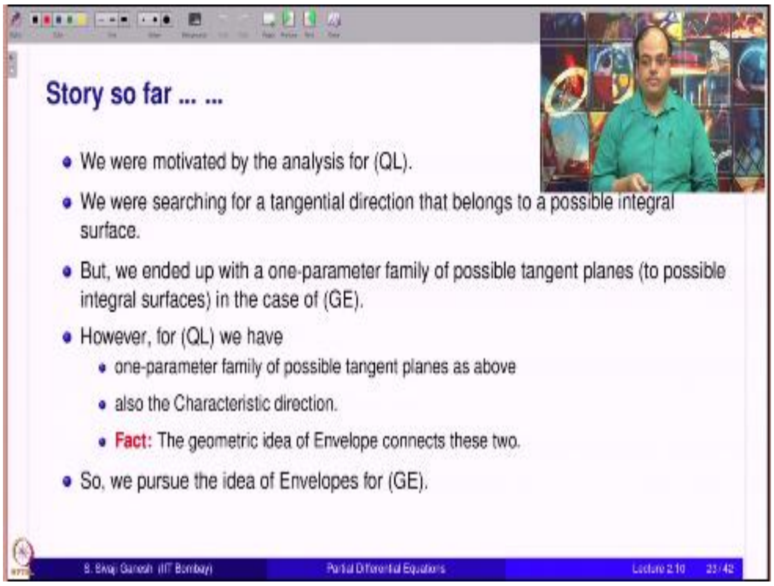

We will see the rigours of the details later, rigorous details. Story so far, we were motivated by the analysis of Quasilinear equations, we were searching for a tangential direction that belongs to a possible integral surface. But we ended up with a 1 parameter family of possible tangent planes. Okay, now we need to pick up 1 direction from this. How do you pick up that? That is a problem. Let us see how that is work.

However, for Quasilinear equations, we have one parameter family of possible tangent planes as above in the previous slide. And also the characteristic direction, we also had a characteristic direction. And the fact the geometric idea of envelope connects these two. The envelope of the one parameter family of tangent planes, possible tangent planes that we described earlier, envelope of that turns out to be a cone.

And the characteristic direction turns out to be a generator of the cone. So, therefore, there is a connection. So, we pursue this idea of envelopes for general equations.

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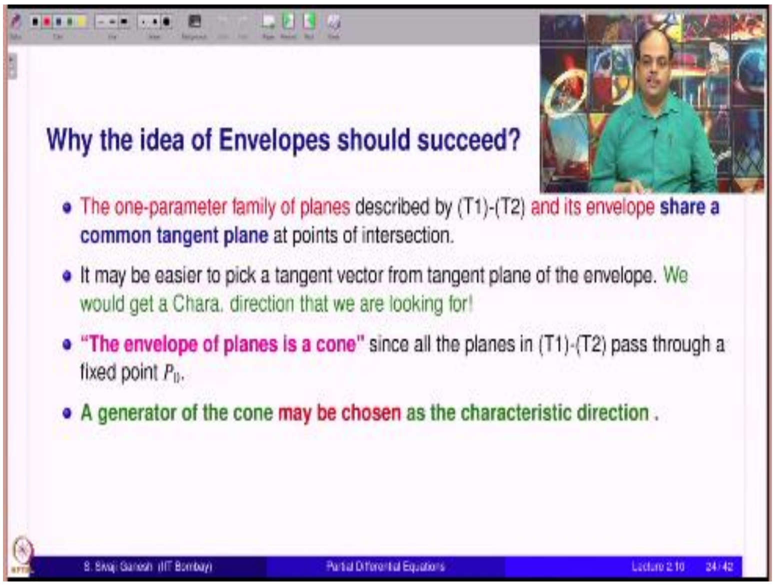

Now, why the idea of envelopes should succeed? The one parameter family of planes described by T1 T2 and its envelope share a common tangent plane. That is a property of envelope. If you have one parameter family planes and you take each envelope, whenever this envelope intersects any member of the family it intersects, it touches. That means, they share a common tangent plane.

Therefore, if you can choose a tangential direction from the envelope that will also be tangential direction in the further surface. It will also be a direction in the family of planes that we are considering, which is what we want. So, we hope that envelope is a good thing, where it is easier to pick up that particular direction which was a case for quasilinear. So, it may be easier to pick a tangent vector from the tangent plane of the envelope.

We would get a characteristic direction that we are looking for. Envelope of planes is a cone. I have put it in quotes because I am going to explain later. Since all the planes pass through a point, a fixed point 0 so a generator of the cone may be chosen as the characteristic direction. That was the idea.

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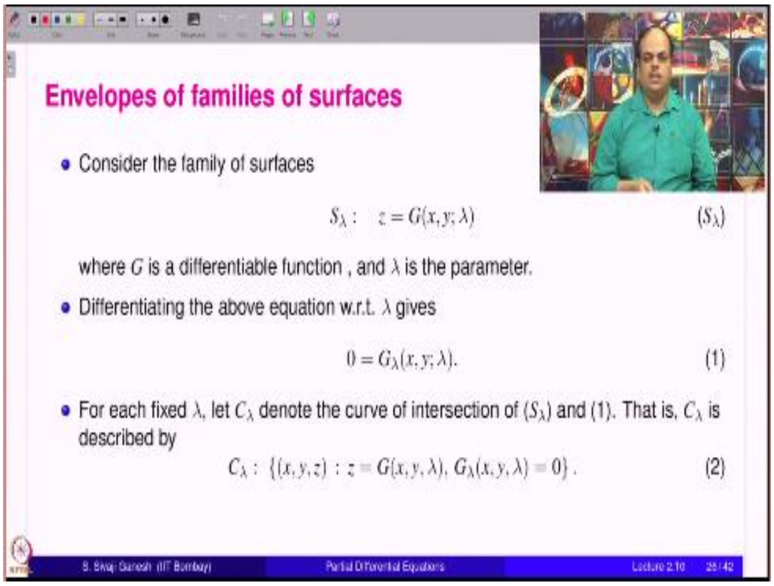

Now, let us have a brief excursion into the theory of envelopes. Envelopes are families of surfaces. So we take the family of surface of this special type actually graphs of functions z = g of x, y, lambda. S lambda is given by this. Lambda is a parameter. Anything you want to do you must assume that functions are differentiable otherwise there is very literally one can do. So, lambda is a parameter, G is a differential function.

So, I have got a family of surfaces. Now, differentiate that with respect to lambda. So, you get, this side is 0. This it is just G lambda of x value. Now, for each fixed lambda let C lambda denote the curve of intersection of S lambda and one main set of all x, y which are common to this and this. This is what we expect whether it be curve, etcetera one has to see. This is a surface, this is another surface.

Intersection to surfaces we think which occur again. That is the background behind calling this. It all depends on how the lambda appears. But what is the envelope? First you look at the family. Differentiate you get some other equation. Look at the common points. Fix lambda. Look at the common points. Set of all x y z, such that this 3 tuple is there, here as well as here. That is what we have written the 2 equations.

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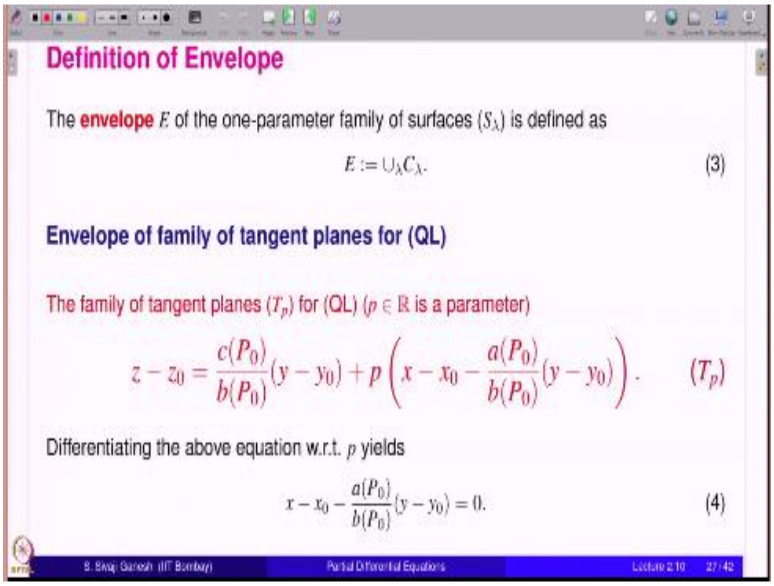

The envelope or the one parameter family of surfaces is defined as a union of C lambdas. Now, let us go back and see what is the QL? We had a family of tangent planes. Possible tangent planes and what is its envelope? Let us see that. So, this is the family. Remember it depends on p. So, instead of lambda we have a p here, so, we need to differentiate this with respect to p.

That means that this will be 0 equal to this quantity, because p appears only here, small p. So, this is 0 as well as this equation is satisfied. What does that mean? This is not there. It is only this. It means it says some relation between z – z 0 and y – y 0.

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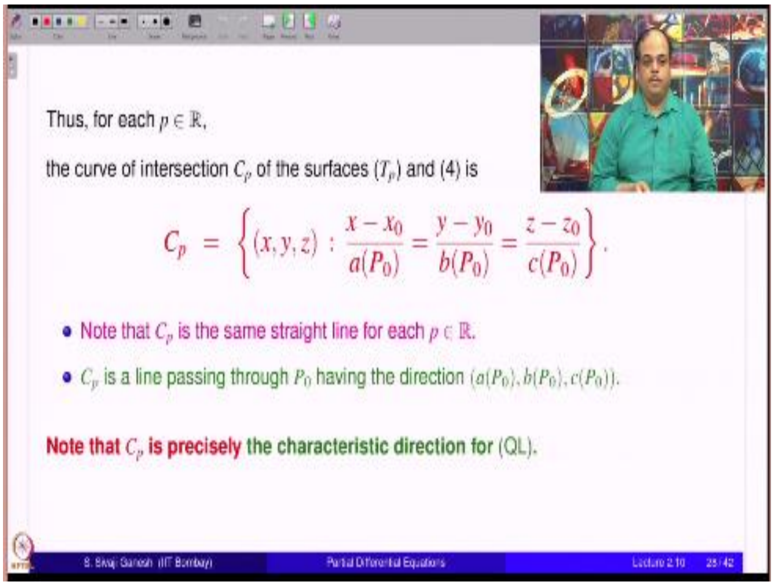

Please do this computation slowly. So, this is what we end up with C p, p is a parameter. Note that C p is the same straight line for each p. It does not depend on small p. It is just a straight line. It all depends in a, b, c at P 0. C p is a line passing through P 0, having this direction abc. What is abc? It is a characteristic direction at the point P 0.

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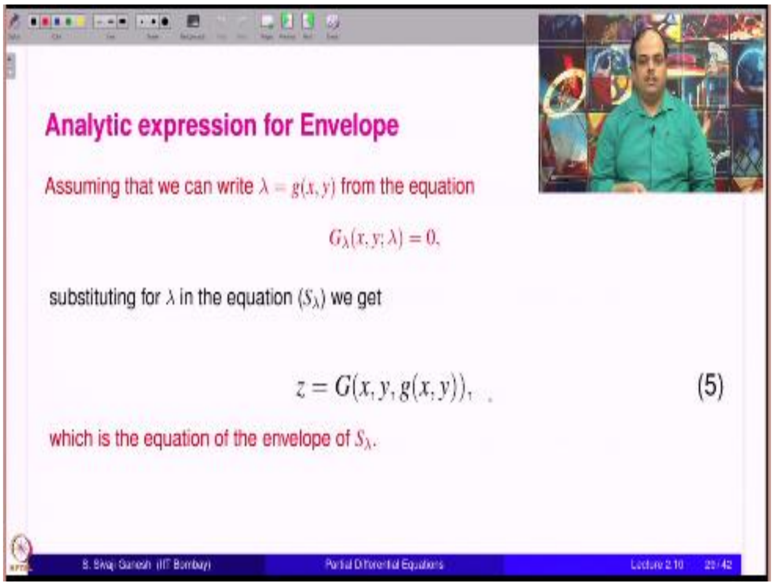

Analytic expression for envelope. So, we assume that we can write lambda as a function of x, y from this equation and then go back and substitute in the equation for the family of surfaces which will give us z = capital G of x, y instead of lambda I have x, y, lambda as a function of x, y, so, I write g x, y. That is assumption. If you can do this, then this is an expression. One single equation for envelope.

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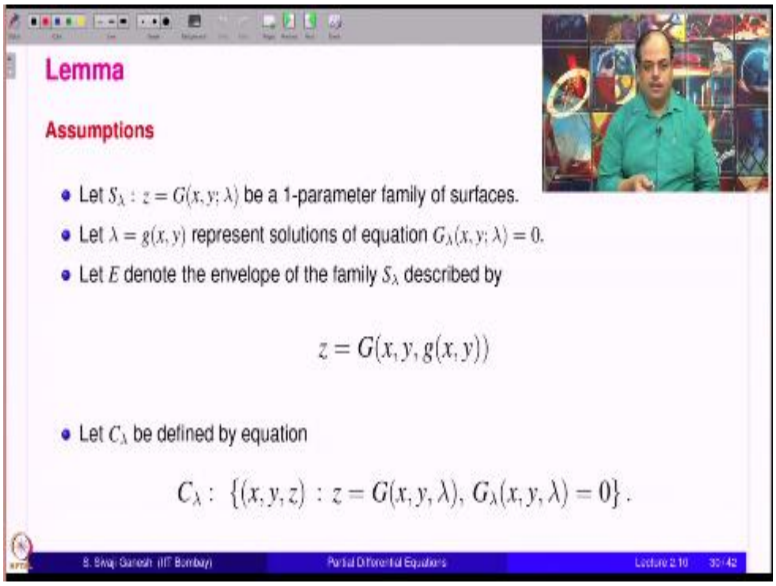

Let S lambda be a one parameter family of surfaces And let lambda represent solutions of this so that I am going to go back and substitute in this. Therefore, E denotes the envelope. We have defined it as a union. Now it is simply this, G of x, y, g x, y. Let C lambda be the curve of intersection of z = G as well as G lambda = 0.

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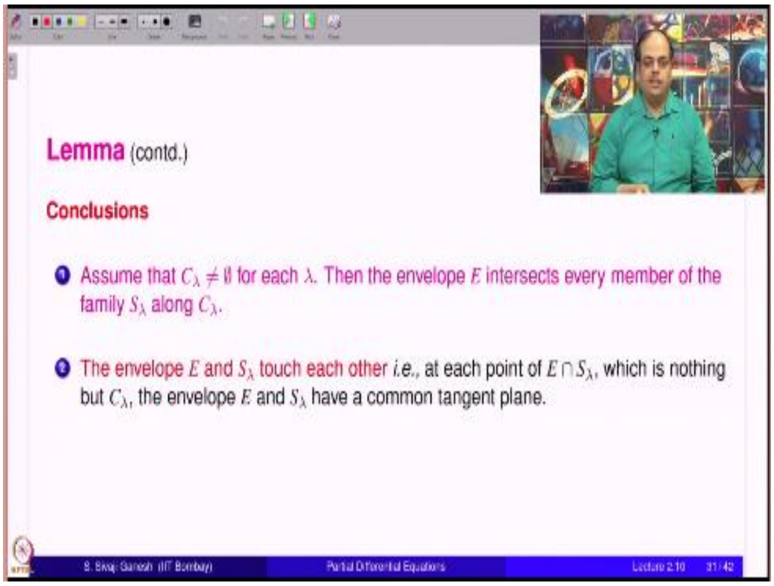

Conclusions: Assume that C lambda is non empty for every lambda. In fact, for some surfaces, C lambda could be empty for some values of lambda. In that case, I do not want to talk about that. Therefore, I assume that C lambda is not empty for every lambda. Then the envelope intersects every member of the family S lambda because I assume C lambda is not empty. Therefore, envelope intersects every member of S lambda along C lambda and that is by definition.

Important thing is the second assertion. The envelope and S lambda touch each other. That means at every point which is common to them, which is nothing but C lambda. By definition, the envelope and S lambda have a common tangent plane.

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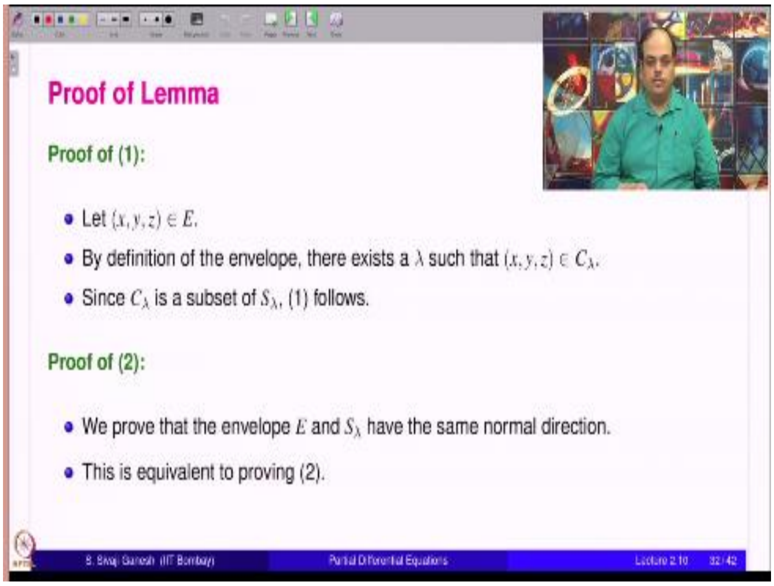

Proof of 1: So, take a point on E. Definition of envelope says that there is a lambda because the union of the lambdas of C lambda over lambda. So there is a lambda such that x, y, z belongs to C lambda. But C lambda means it is there in S lambda also because that is a part of the condition after definition of C lambda. Therefore, one follows. That is simple. Proof 2:

We prove that the envelope and S lambda has the same normal direction at points which are common to both of them. That will establish that the tangent plane is same.

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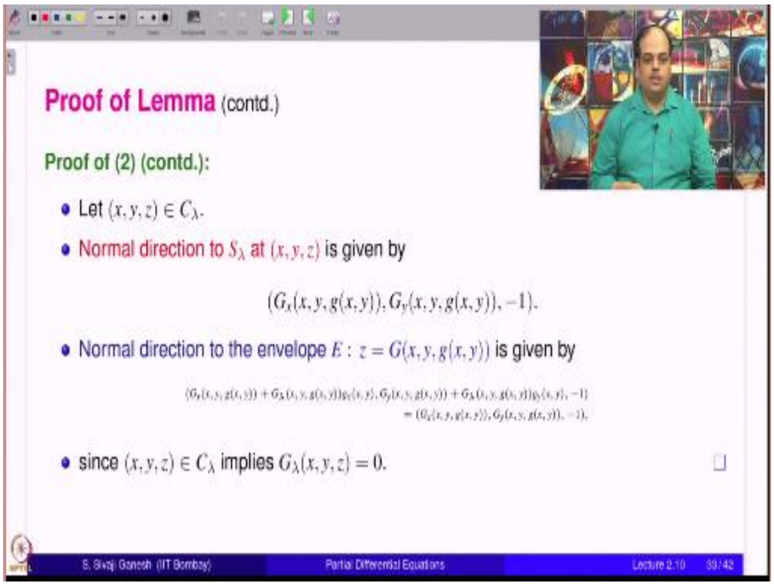

So, we are going to inquire into the normal directions for E as well as S lambda. So, take a point in C lambda. Normal direction to S lambda at that point x, y, z is given by G x, G y – 1. This is a general fact that we have discussed many times. Whenever you have a function phi of x, y, z = 0, the normal direction is phi x, y, phi z. In this case, the envelope was this. z = G x, y, g x, y. Here I have typeset in small font, because it was not filling.

So, normal to the envelope is exactly this. You need to differentiate this with respect to x. x dependence comes in the x as well as in the lambda. That is why x derivative and lambda derivative. Once you take lambda derivative, you have a G there x dependence, so G x. Similarly y derivative and - 1 which on simplification is this. Because this is 0. G lambda of x, y, g x, y is 0.

Therefore, it is simply G x, G y – 1. That is it. So, it showed both are same.

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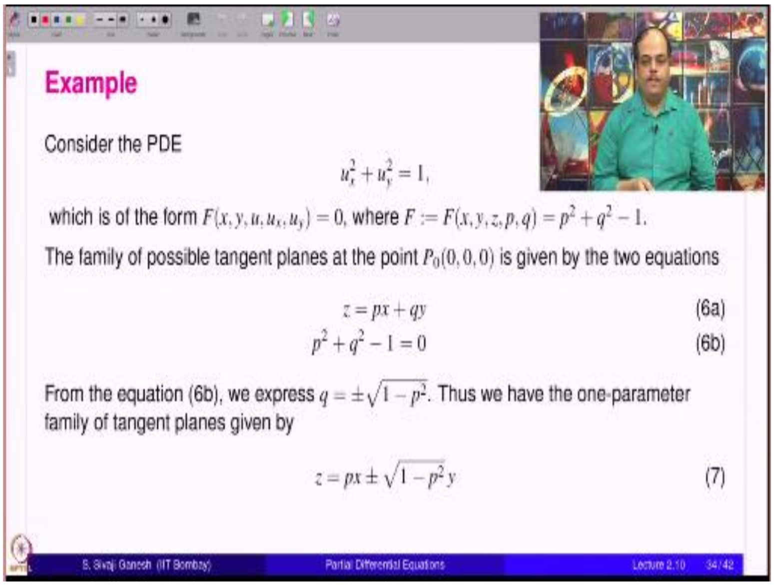

So, therefore, they share a tangent plane. That means, they touch each other. Now, let us look at this PDE. This is a nonlinear PDE. F of x, y, z, p, q is equal to p square + q square - 1. Now, let us write the family of possible tangent planes. Z = px + qy. That is coming from equation of a plane and pq are constraints to satisfy this equation. Therefore, p square + q square - 1 = 0.

From here luckily, like in QL, we could solve for q in terms of p which is root 1 - p square. Of course, this makes sense only when mod p is less than 1. Of course, you have 2 solutions for q. In that we have a one parameter family of tangent planes. I go back and substitute for q in the p. Now, I will find envelope of this.

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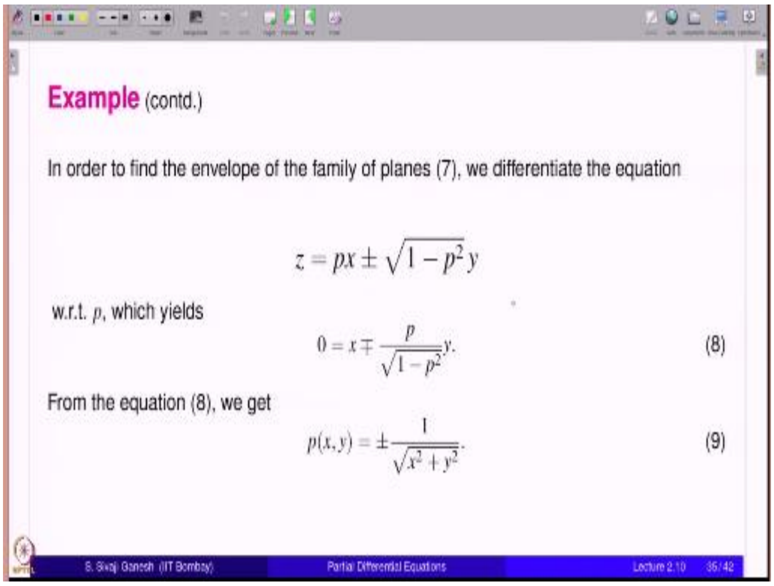

We need to differentiate this with respect to p. That will give us this relation and express p as a function of x and y. We get this. Now go back and substitute in this equation.

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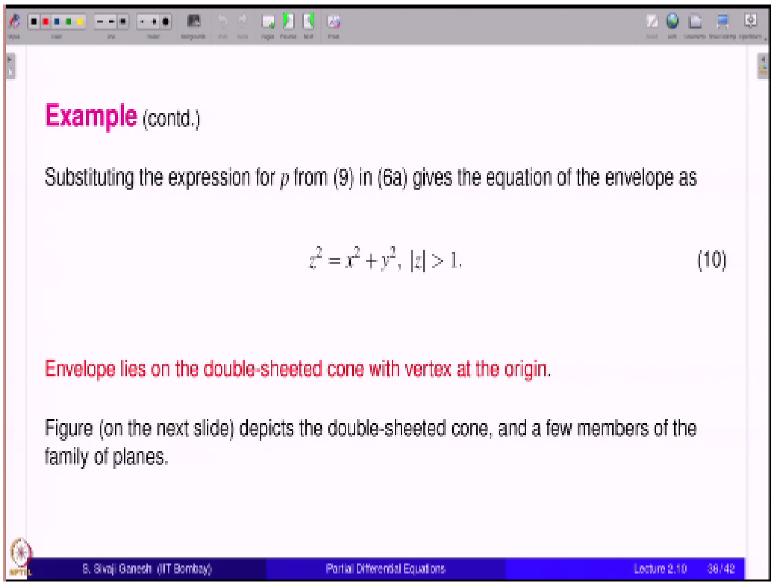

That will be the envelope. Envelope is z square = x square + y square. Mod z bigger than 1. So, envelope lies on the double sheeted cone with the vertex at the origin. The figure is there in the next picture and a few members of the family of possible tangent planes is also there.

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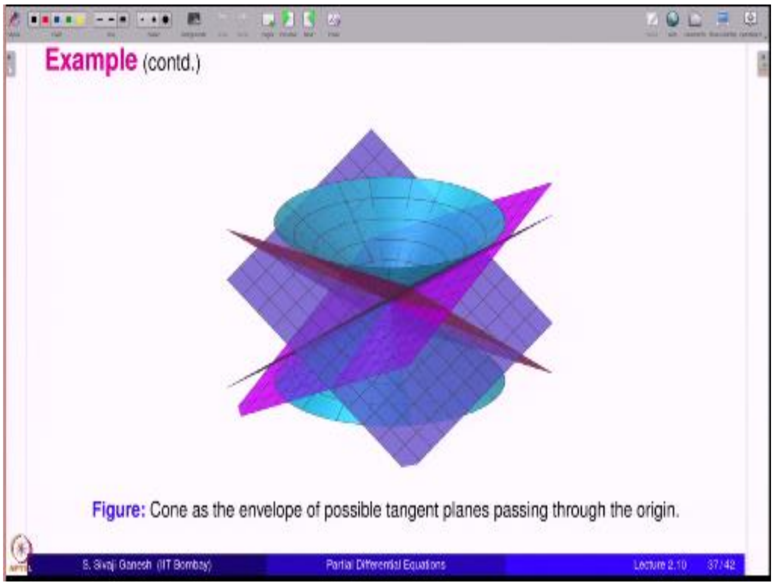

These are various planes. Their envelope is this blue colour, the cone.

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Now, a few misconceptions that are prevalent.

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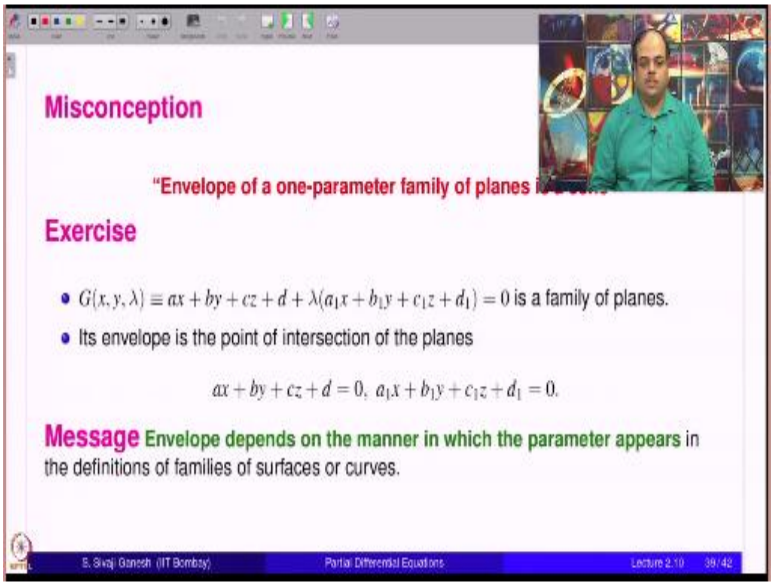

Envelope of one parameter family of planes is a cone is not correct. Look at this example. This is an exercise. G of x, y, lambda = ax+ by + cz + d + lambda times a1x+ b1y + c1z + d1 = 0. It is a family of planes. So, find the envelope. It turns out to be the intersection of these 2 planes. So, message: Envelope depends on the manner in which the parameter appears in the definitions of the families of surfaces or curves.

So, we cannot have a blanket statement like this. One parameter family of planes, envelope will be a cone. No such things are not true.

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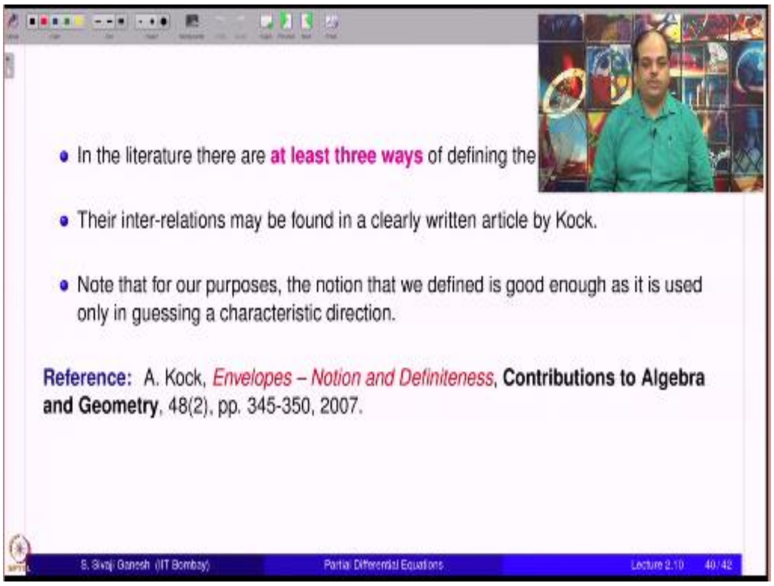

So, in the literature, there are at least 3 ways of defining the notion of envelope. Their interrelations may be found in a beautiful article written by Kock. Note that for our purposes, it does not matter what is the correct envelope that we have to consider, current definition etc.

What all matters is does it serve a purpose whichever you follow? So, the notion that we define is good enough, as it is used only in guessing, a characteristic direction and success that we will see in the next lecture.

This is a reference for Kock's article, It is called Envelopes - Notion and Definiteness, Contribution to Algebra and Geometry. He has written this article in 2007. So a beautiful article.

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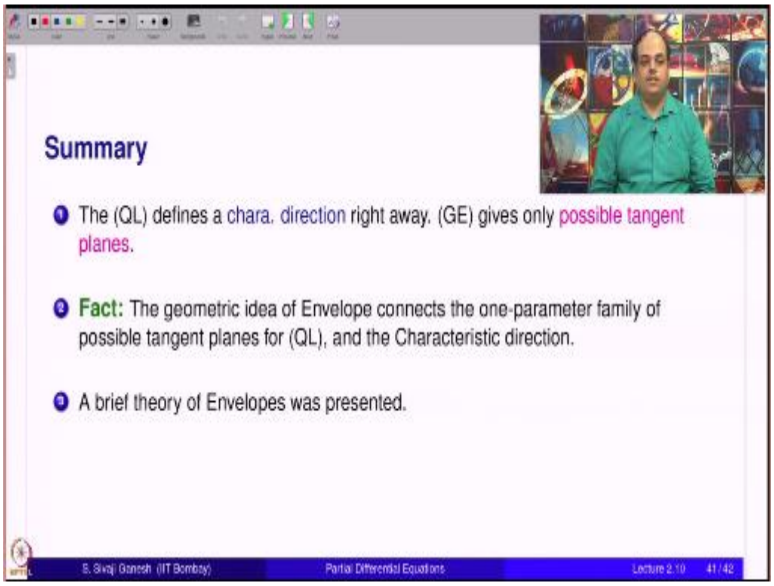

Let us summarise what we did. Note that the Quasilinear equation define the characteristic direction right away. GE gives only possible tangent planes that is all. Fact is that geometric idea of envelope connects the one parameter family of possible tangent planes for QL and the characteristic direction. We have just seen that. We have proved this. A brief theory of envelopes was presented.

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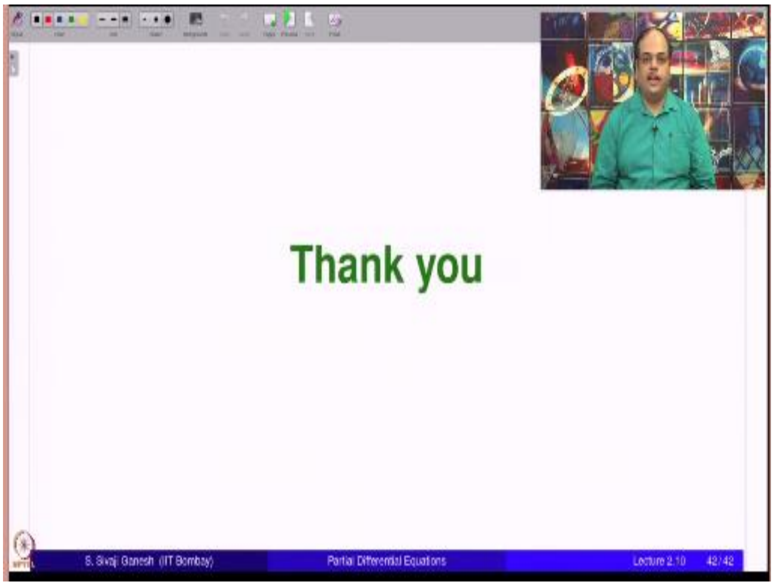

In the next lecture, we are going to work further the next steps. In fact, we have not yet found characteristic direction. We will do that in the next lecture and continue the analysis of Cauchy problem for general nonlinear equations. Thank you.