Mathematics in India: From Vedic Period to Modern Times
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Lecture-15
Mahaviras Ganitasarasangraha

Okay, so today I will be talking about Mahaviras Ganitasarasangraha is the outline is something like this.

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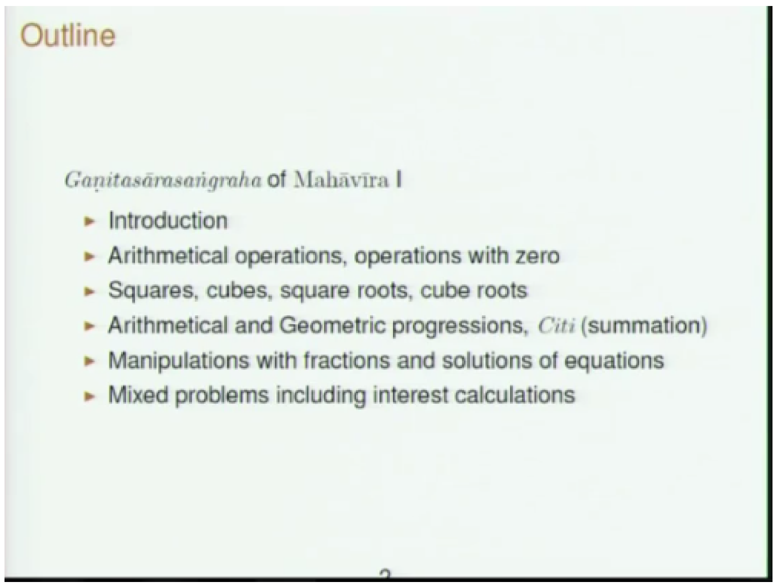

First the introduction then the arithmetical operations and operation with zero, squares, cubes, square roots, cube roots, arithmetical and geometric progressions citi summation, manipulations with fractions and solutions of equations and mixed problems including interest calculations. So, this is the outline of the first lecture on Mahaviras Ganitasarasangraha.

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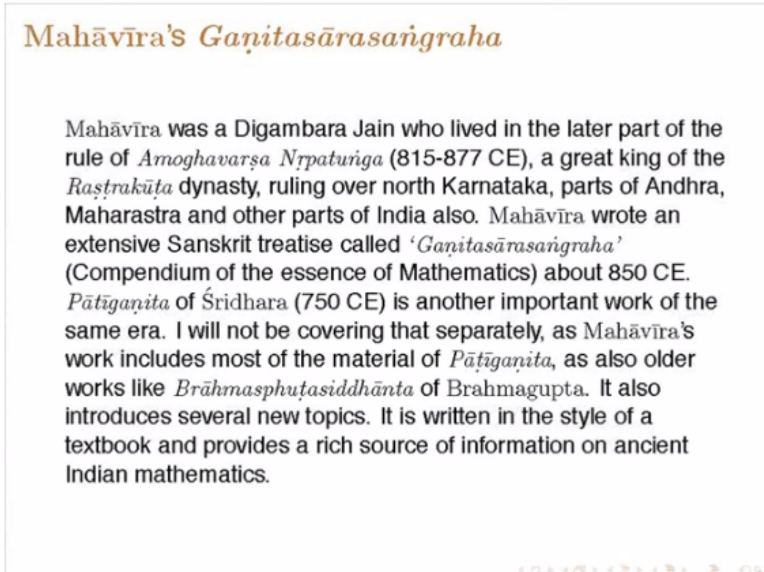

So, mahavira was a Digambar jain who lived in the later part of the rule of (FL) 1815 to 1877 CE you know a king who live avail who ruled of for a long period a great king of (FL) dynasty ruling over north Karnataka parts of Andhra, Maharashtra and other parts of India also. Mahavira wrote an extensive Sanskrit treatise called (FL), so, whose the I mean the literal translation is compendium of the essence of mathematics about 850 CE.

(FL) which is around 750 Christian era is another important work of the same era but I will not be covering that separately as mahavira's work will include most of the results in (FL) as well as another work by him called (FL) it also introduce several new topics and it is written in the style of a textbook and provides a rich source of information on ancient Indian mathematics what is very significant it is a earlier works we consider Aryabhata.

Aryabhatiya and Brahmaputhasiddhanta there mainly works an astronomy and mathematics for the part of it (FL) in Aryabhatiya, so one about 33 verses also and in (FL) though very major things and mathematics are being done it is a small part of the treatise as such. So, that is understandable because much of the development of mathematics in the earlier times both you know in the Indian tradition and greek tradition and many other traditions went hand in hand with astronomy.

Because astronomy was the exact sciences and they required several tools you know 2 excess things in the mathematical manner to for the precise description, so it is not surprising that you know many of the mathematical developments are very closely associated with astronomy, if for an trigonometry, trigonometry was basically developed in the context of astronomy and later you know it is a concept of instantaneous velocity all these things was developed in the context of astronomy.

So, this astronomy and mathematics went hand in hand and in the in India also lot of works are containing both and typically mathematics will the part of the any some treatise and astronomy. Now Mahavira's (FL) is one of the important works first work to be entirely devoted to mathematics he does not talks about astronomy at all I mean there maybe some application of the result in his book on astronomy.

But he does did if astronomy it not it prime concern mathematics is the prime concern and you know that is what it develops and he develops in a very very you know elaborate manner is a very elaborate work it is can be stay u straight away could have been used must have use as the textbook at that time it is very large work more than 1000 verses and lots of examples of various kinds and any mathematical operation any of this method new method he will have lot of examples you know illustrating various points you know. So, that why it is an extremely important work.

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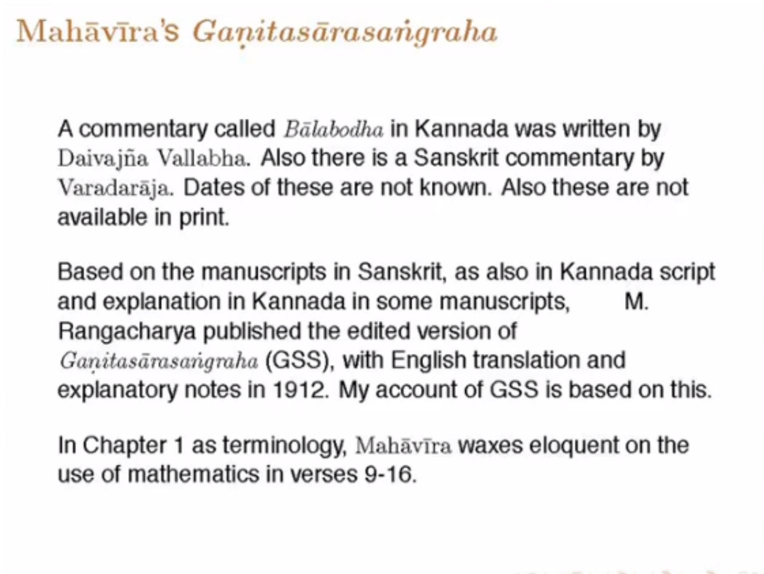

So, we will discuss some parts of it you know we will not able to cover all the things some parts of it in the 3 lectures, see a commentary called (FL) in kannada was written by (FL) and there a sanskrit commentary by Varadaraja on this work. Dates of these are not known also these are not available in print based on the manuscripts in Sanskrit as also in Kannada script and explanation in Kannada in some manuscripts.

M.Rangacharya published the edited version of (FL) with English translation and explanatory notes in 1912 about nearly 100 years back. So, my account of (FL) is entirely based on his Rangacharya's edited work with translation yes translated and he actually he is done a very good job you know he has experience very patiently explained all the verses examples and all that.

And in the modern notation and why various manuscripts in Kannada and already it is understandable because Mahavira was belonging to the Karnataka region, so it became very popular there, so naturally many of the manuscripts are there in Kannada. And but there are some Telugu manuscripts also and whatever is available Ragacharrya used and published this edition. Now in chapter 1 which is terminology basically Mahavira waxes eloquent and use of mathematics in verses.

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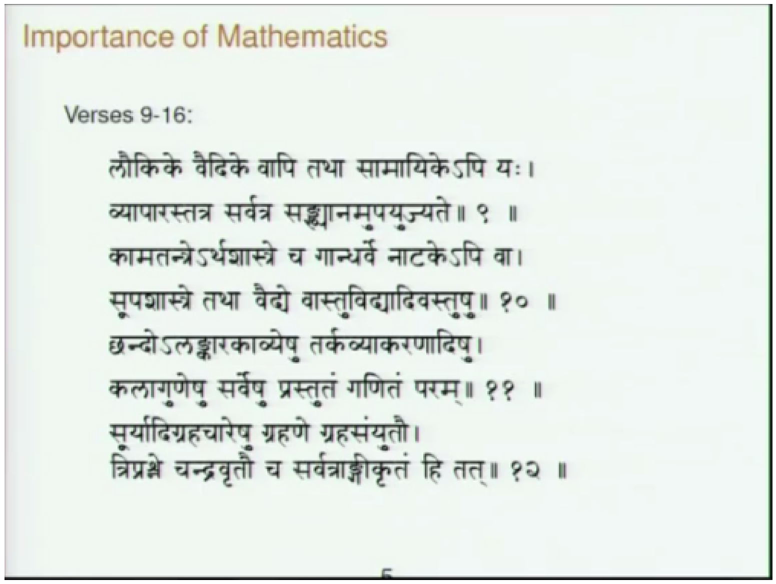

(FL) he already have this recent extensively about the gives the mathematics in (FL) so, in astronomy naturally it is I have told let the most mathematics developed in this context were not surprisingly lot of mathematics in there (FL) the problem of direction, time and space you know which is important chapter of astronomy in all Indian text (FL).

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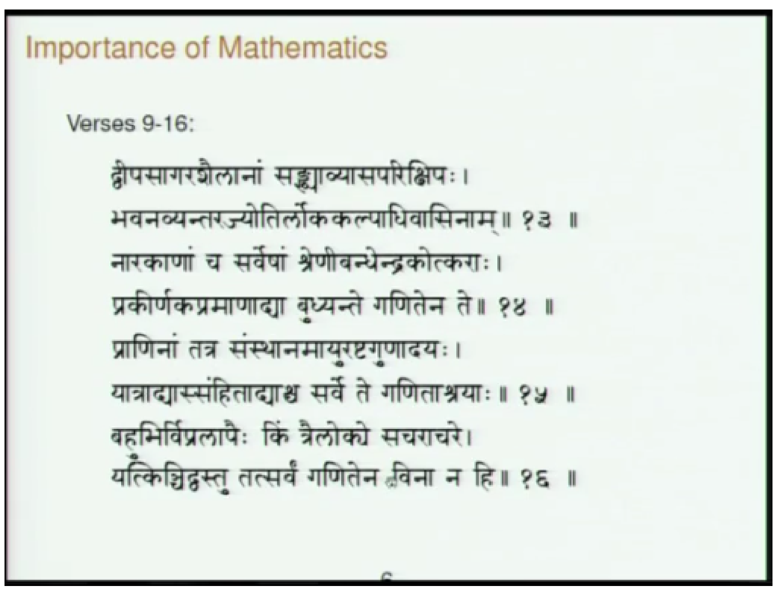

(FL), so nothing happen without it.

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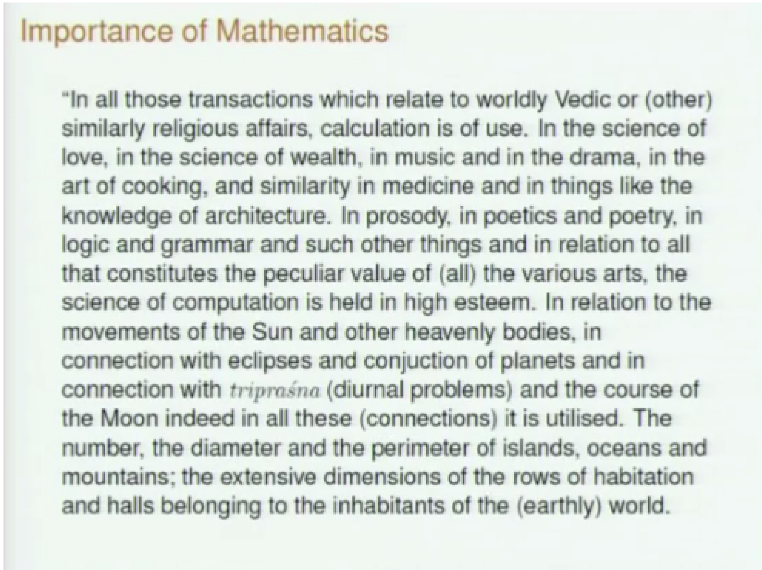

You must all have noticed that you know the verses themselves or somewhat you know as have an easy flow compare (FL) and all that he is actually very great poet also and verses are extremely nice and if one more Sanskrit one will be able to appreciated very well. So, in all those transactions which will relate to only vedic or other similarly religious affairs, calculation is of use.

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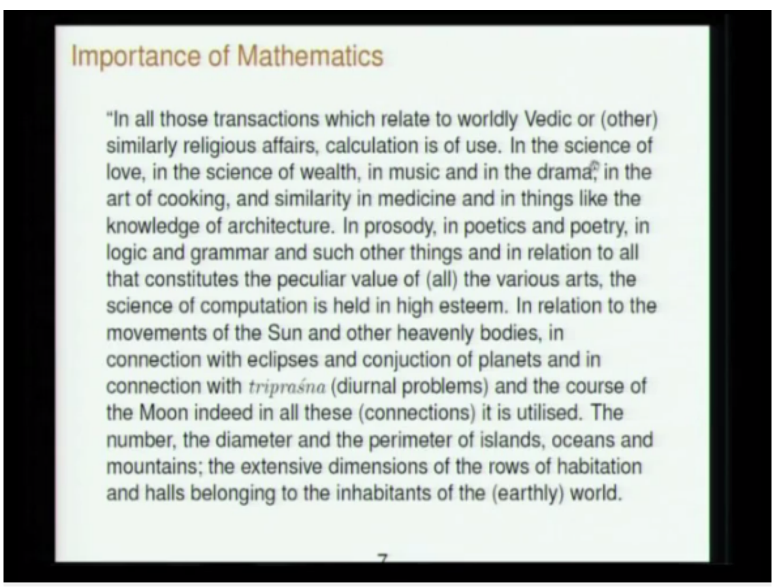

In the science of love, in the science of wealth in music, in drama, is a art of cooking, prosody, poetics, poetry all these things the science of computation is held in high esteem relation a movements of sun, moon other heavenly bodies **diurnal** diurnal problems the course of moon the number diameter all the numbers associated with islands oceans, mountains and we already see that revolution numbers and all that you know over various things can be express. And of course later many more interesting aspects of these mathematics will come in this revolution numbers. So, all these numbers are important and mathematics these to them.

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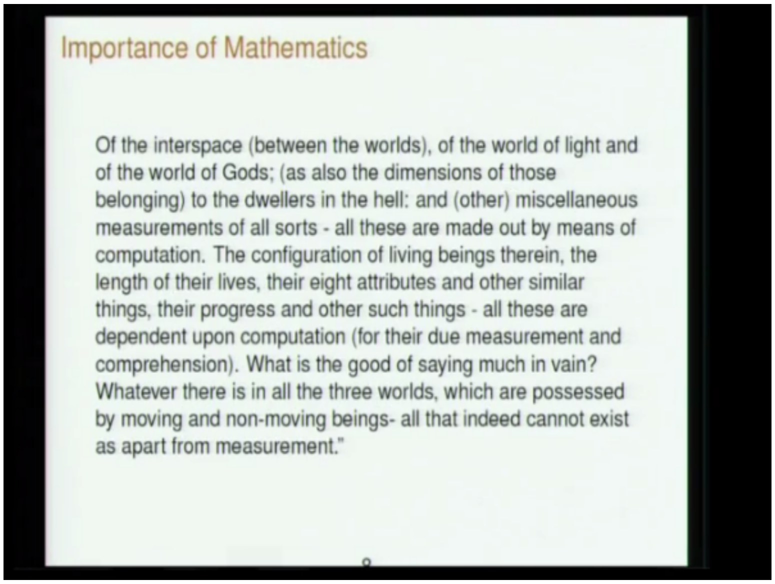

Of the inter space between the worlds of the world of light and the world of gods to the dwellers in hell it does not for forget the hell dwellers miscellaneous measurements of all sorts. All these are made out by computation the configuration of living beings, length of their lives, eight attributes similar things, their progress and such other things. All these are dependent upon computation what is the good of saying much in vain?. Whatever there is in all the three worlds which are possessed by moving and non-moving beings all that indeed cannot exist as a part from measurement.

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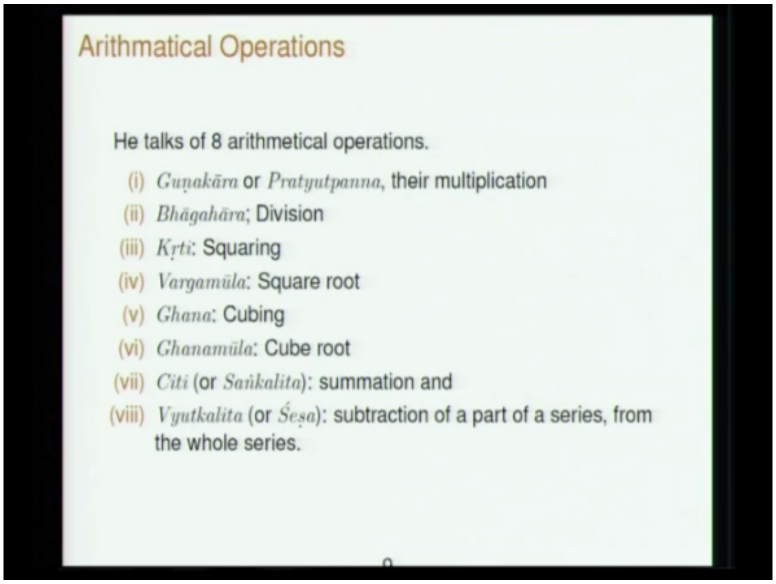

So, he talks of a 8 arithmetical operations, (FL) then (FL) summation (FL) subtraction of a part of a series from the whole series. It will not be dealing you know with whatever has been done into much detail.

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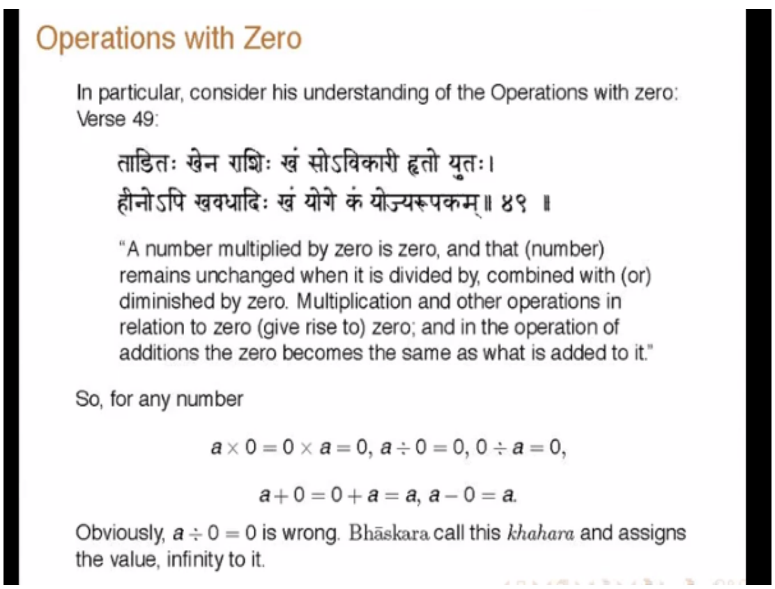

So, do not get you know bold expects something straightly more you know **moo** newer things okay see in particular consider is understanding of the operation be 0 one at the first time square the operations at be 0 has been describe (FL) a number multiplied by 0 is 0 and that number remains unchanged when it is divided by combined with (or) diminished by 0 multiplication other operations in relation to 0, give rise to 0. And in the operation of additions the zero becomes the same as what is added to it.

So, for any number a*0 is 0*a is equal to 0, a/0 is 0, 0/a is 0, a+0 is equal to 0+a is equal to a and the does not remains unchanged a-0 is a, so obviously a/0 is the wrong so, when the first time it is happening so, some mistakes are bound to arise because the concepts will become clearer. And very crystallized later only in any mathematical tradition baskara called this khahara operations and assigns the value infinity to it. I think baskara was one of the first to understand the significance of this infinity.

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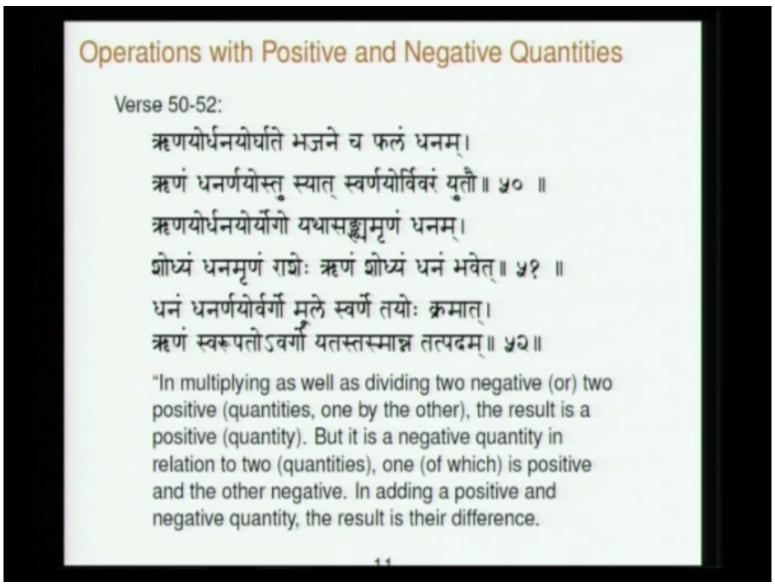

And similarly it talks about the operations with positive and negative quantities (FL) in multiplying as well as dividing two negative (or) two positive (quantities, one by the other), the result is a positive (quantity), but it is a negative quantity in relation to two quantities one (of which) is positive and the other negative. In adding a positive and negative quantity the result is difference, so this is understanding you know.

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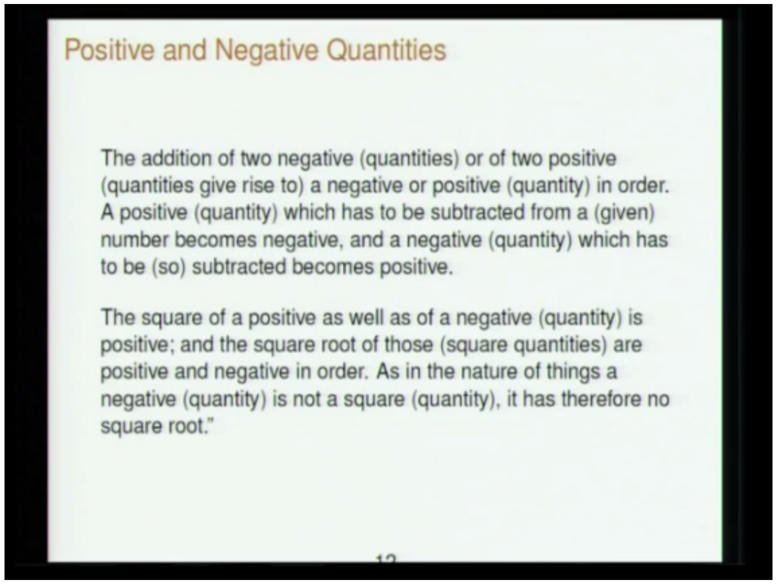

Already very clear understanding of positive and negative numbers the addition of according to them the addition of two negative (quantities) or of two positive (quantities give rise to) a negative or positive (quantity) in order. So, +, +, + is a+, -+, - is –a positive quantity which has to be subtracted from a given number become negative. So, clearly a okay - of +a will become a +b is a-b.

And then negative quantity which has to be (so) subtracted becomes positive. So, that is a- of -b is a+b and a square of a positive as well as the negative quantities positive and a square root of those square root of those (square quantities) are positive and negative in order. As in the nature of things a negative(quantity) is not a square (quantity), it has therefore no square root.

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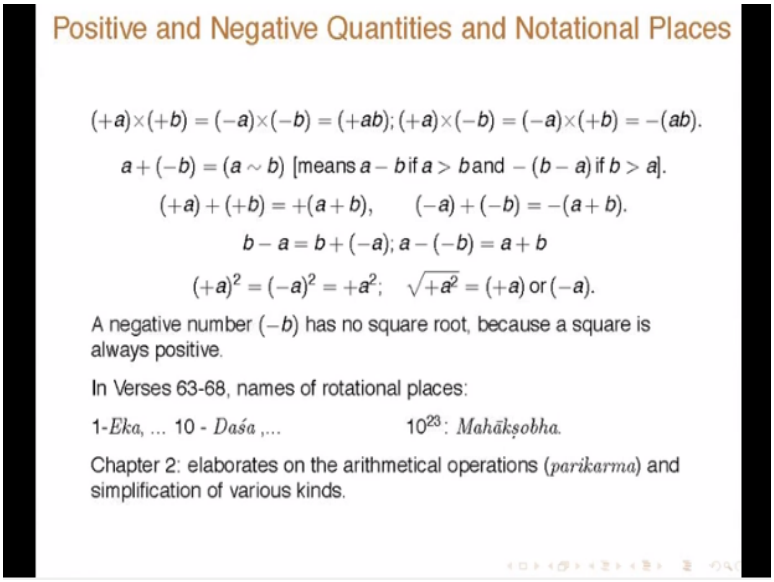

So, what is saying is +a*+b is –a*-b so, is + of ab then you have to multiply naturally and a*-b is –ab and a+-b is a difference b which means a-b if a is greater than b and -b-a is b greater than a +a ++b is equal to + of a+b like that b-a is b+-a a- of-b is a+b and +a whole square is -a whole square is +a square and square root of + a square is +a or -a that also he clearly recognizes a negative has –b have has no square root, because a square is always positive.

See these all look somewhat in a very trivial to us already we all start with that you know in your high school onwards this operations be + and - but it took a longtime for this to be digested okay. In fact there one historian of mathematics from the west world come and you know he had given at giving the talk on this use of this negative quantities and all that. So, he was saying that the using of negative quantities in a people were very uncomfortable with it even in 19^{th} century.

So, I mean it is not what we take for granted many things you know it is not that trivial you see when it started because something you know some negative dimension see is there and then how do define it how to define operations with it, so that is very important. And of course says you know a negative quantity does not have a square root because if you take a positive quantity it is square is positive.

If you take a negative quantity it is square is again positive, so both positive, negative quantities have positive squares, so how can you have a negative square. So, now of course we are saying that you know there is a -1 square of -1 we denote it at I and use it in extensively but at depends upon what we are using see what we are dealing with, so we are using only real quantities certainly it is very hard to imagine a quantity with a negative square yeah.

So, and then in a in the history verses later verses see please you know in this this verses means it does not mean 63-68 it is actually is various chapters are there and each of them verses are number like this. So, this is not verse number in the text some chapter only chapter that is all you know, so you should be remember that, so he gives the names of notational it is not notational places up to (FL)like that.

So, I do not have to go to all that you have already heard about in a how various powers of 10 were considered you know from vedic times in India then chapter 2elaborates on the arithmetical operations parikarma and simplification of various kinds.

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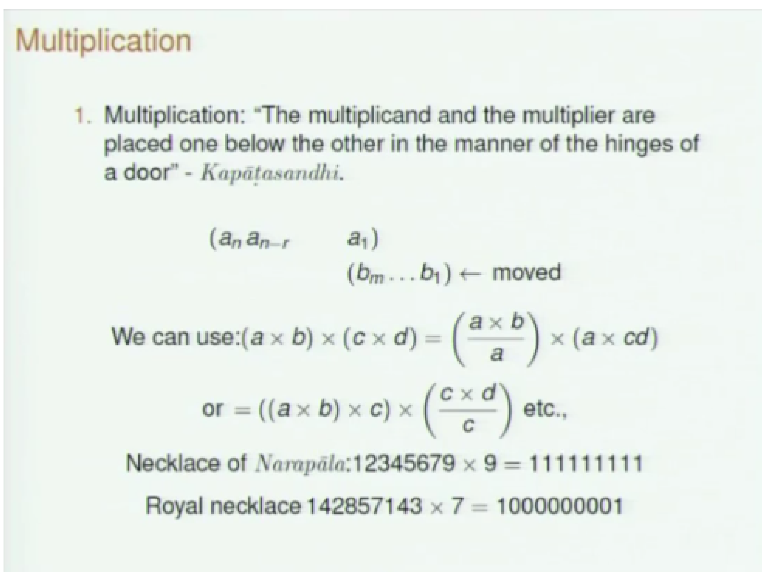

So, multiplication all this things you know he earlier was always deal with various you know the simplest operations they all are you know discuss in great detail. Because multiplication, addition, etc. are very important and multiplication it can be done in different ways obviously people you know who are very, very would do like a shakuthala devi you see who could compute very you know product two huge amount**.**

She would have some method in you see so, the some very tricky distinction will be methods will be there interesting methods.

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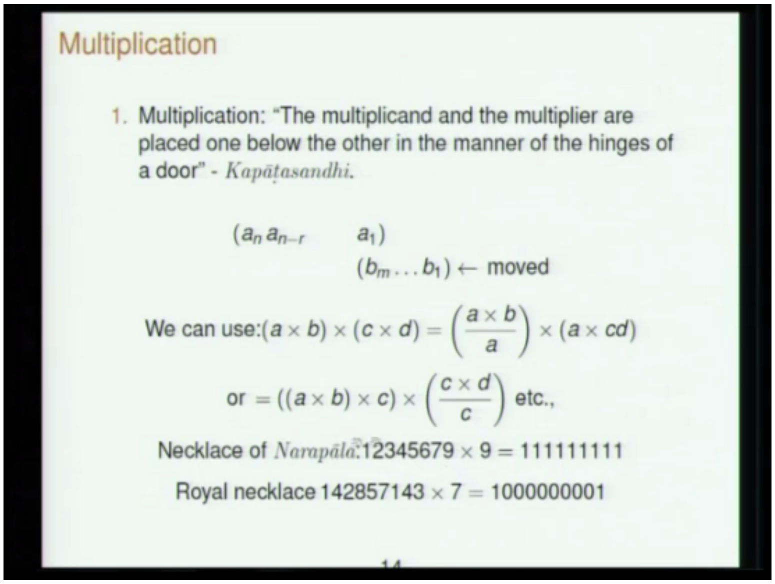

So, there many innumerable method but the common methods are what are discussed in the books. So, it is called (FL) so, this you write the number like this and other the other multiplier and a multiplicand and you keep multiplying each of the digits of the one by the other and keep moving. So, I do not want to explain these things is very simple then a*b *c*d +a*b/a*a*cd I mean in some applications some multiplications particular examples. It may be convenient to divide by something and then afterwards multiplied by the same thing.

So, that is all it means. So similarly and then he gives some interesting products like 123456789*9 1111111111 is called (FL) so, similarly 142857143*7 is 1 into all the various zeroes into 1 etc… in fact one can see if you read brook work you see the go through the work a really had a very good sense a humor also you know. So, all these things it does things many things through attract students.

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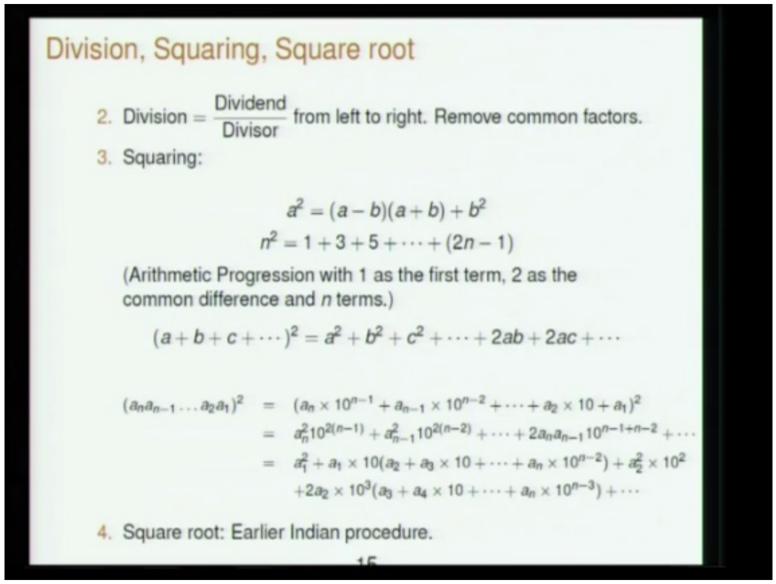

Similarly division is Dividend/ Divisor, some left to right remove common factors. So, all those things are done and squaring see it does not many they get me some implication in you know in splitting into various parts. So, that is what he will discuss the square is a-b into a+b+b square. So, in some set up if you want to has some 998 whole square you see is very convenient to write it 1000-2 whole square and use this yes like that.

So, similarly n square if you want to take the n square it is 1+3+5 into 2 upto 2n-1 arithmetic progression with one as the first term and 2n-1 and the last. So, that the sum is n square and similarly he very clearly say that a+b+c etc… whole square is a square+b square+c square etc… +2ab+2ac+etc… all the products twice of that you see various products which will come n*n-1/2 products will come.

So, all these so, in particular if you have ths number like this in decimal notation. So, then you have this number like this in decimal notation. So, then you have this finally you get a1 square units place what a a1 square in 10 lace you get a1*10 into all these a2+a3 into 10 etc… so, that is a very I mean he explains all these things in detail you know. Because either I told you nothing is taken for granted from the scratch is building up the whole mathematical vocabulary of those times that is very important.

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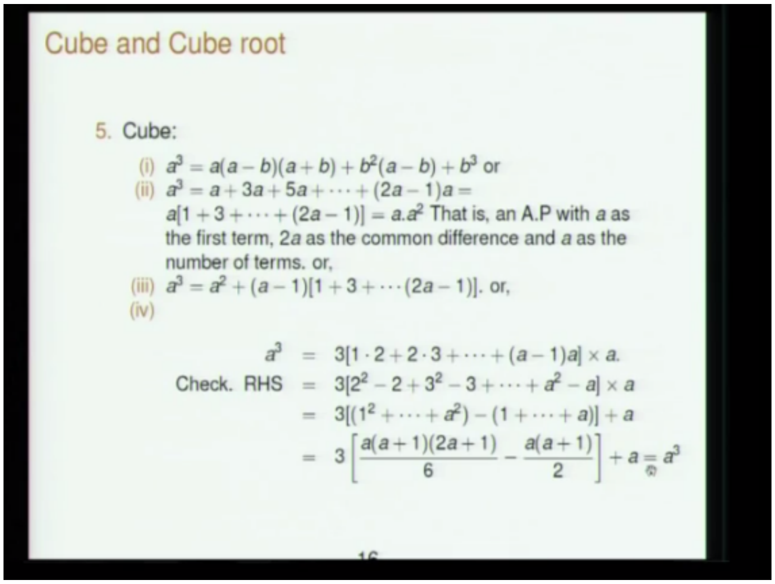

And the 100 place it is a2 square so, like that square root is earlier Indian procedure I do not have to go through that cube is sometimes it is you know convenient to write it like this a*a-b*a+b+b square into a-b+b cube or interestingly it cannot also by written has a cube it is equal to a+3a+etc., it is also a sum of a arithmetic expression with a as the first term and 2a as the common difference.

And similarly a cube a cube can also be written like this a square+a-1 into this arithmetic series progression and similarly write a cube it is equal to 3 into 1 into 2+2 into 3+ etc… like this. So, is very, if you see the, a square it is clear that, 3 into 2 square-2+3 square-3 etc… so, it is very cleverly it has been done. You see because this will be 1 square + etc…+a square-1+ etc… +a 2 progressions okay. So, then we know this we already know this results, so you get the a cube.

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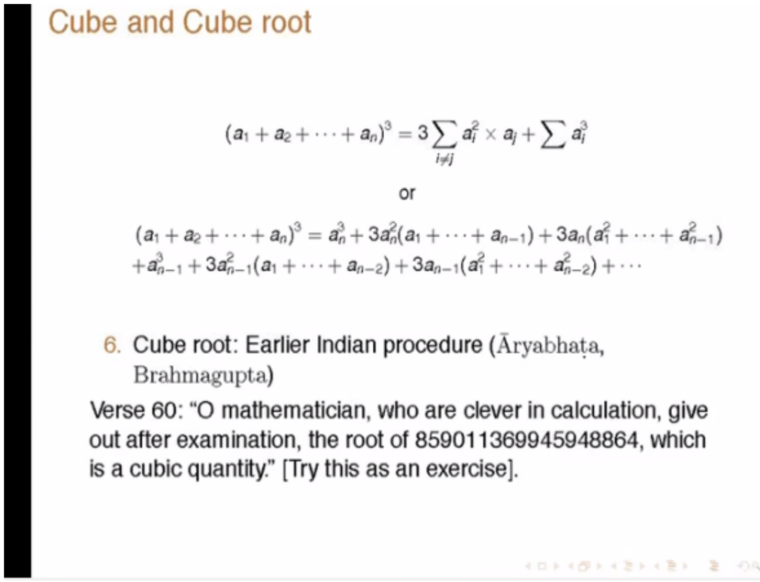

So, similarly it talks about cube so, various the essentially based on the same principle as for the square but of the one more distinguish is there see you got a three so, you will get various terms like this all these things you will explain. And cube root is and I do not want to talk about it all these has been talked about enough whereas same as the earlier Indian procedure the Aryabhatta and Brahmagupta.

So, but I think you should try this example O mathematician who are clever in calculation give out after examination the root of 859011369945948864 which is the cubic quantity okay. So, here already given a cubic quantity. So, please do this.

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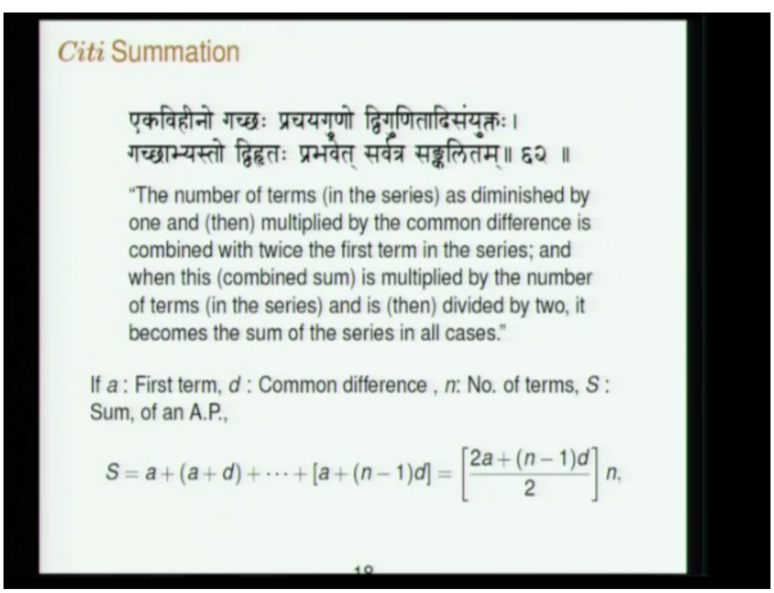

So, then a citi or summation it talks about that that is the one of the arithmetical operations so, if your sum is a this a+ etc. a+n-d is (FL) is the number of term (FL) like that (FL) distinct d the difference (FL) is the first term so, like that so, that is the we already discussed in detail earlier lectures.

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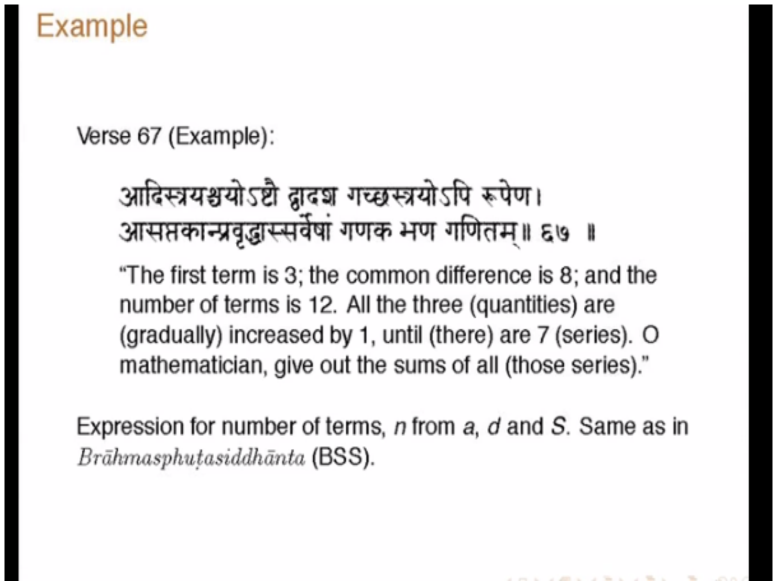

So, he gives an example the first term is 3: the common difference is 8: the number of terms is 12. All the three (quantities) are (gradually) increased by 1, until (there) are 7 (series) okay. So, not 1 7 small sub problems are there in this okay. O mathematician towards the sums of all those series so, this is the verse then expression for number of terms n from a, d and S same as in Brahmasphutasiddhanta.

We had seen that further and of course Aryabhattiya also gives that so, this is essentially you got solve a quadratic equation right n from ad and S.

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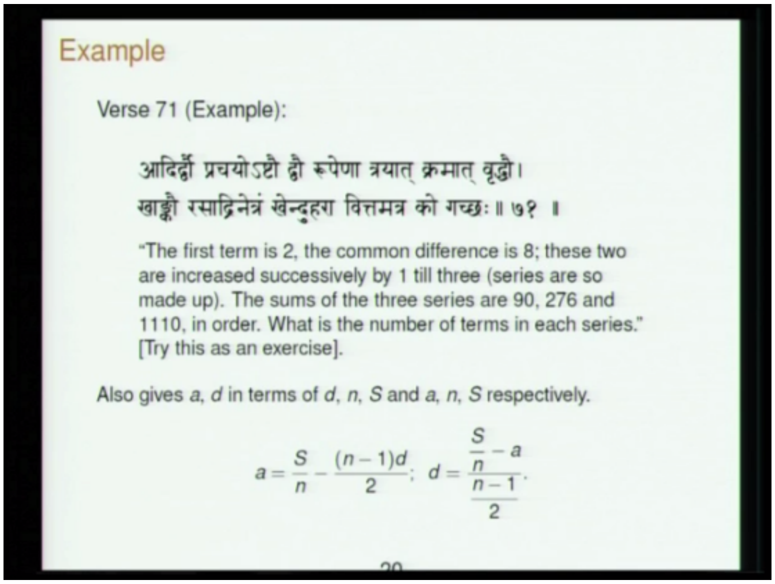

So, and then he gives various examples (FL) okay the first term is 2, the common difference is 8: these two are increased successively by 1 till three(series are so made up) the sums of the three series are 90, 276 and 1110 in order. What is the number of terms in each series so, you have to do this I think you have to copy this and try this exercises. And similarly you can find a and a in terms of d n and s and d in terms of a and n and S. So, there are quite trivial

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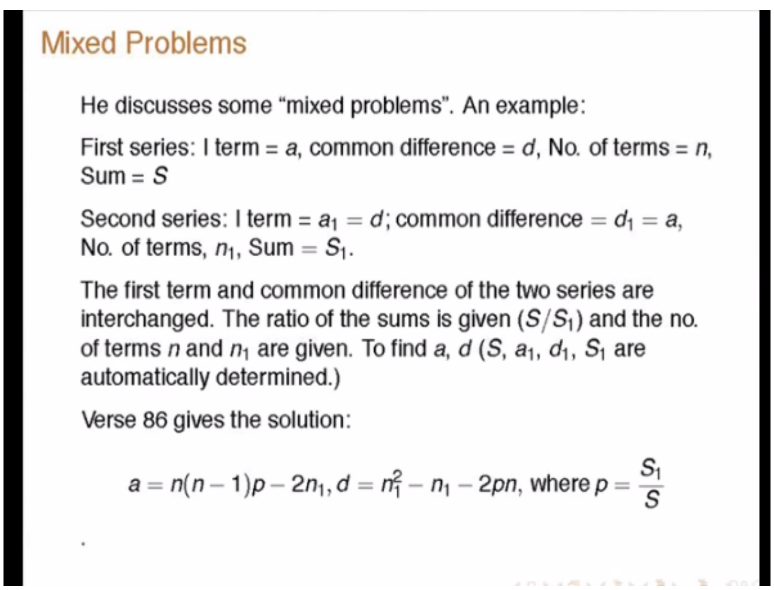

Then it discusses some mixture problems so, you consider 2 series the first series I mean the 2 series is the number of the first term and a common difference they are interchange. In number of the first term and the common difference they are interchange in the first series first term is a, common difference is d number of terms is n, sum is s in the second series the first term is d and the common difference is a and the number of terms is n1, sum maybe s1. So, then one can the first term in the common difference of the 2 series of the interchange as I told you the ratio of the sums is given.

And the number of terms n and n1 are given you have to find a, d and other things and he has gives a given a solution you understand in the first series the first term is a common difference is d, number of terms is n, sum is s and the second series the first term is d the common difference is a, the number of terms is n1 this equal to n1and sum is s1. So, gives a solution like this n into n-1 by into p-2n1and d is equal to n1 square-n1-2pn, where is p is S1/S essentially in fact what is given is only the ratio of the sums okay.

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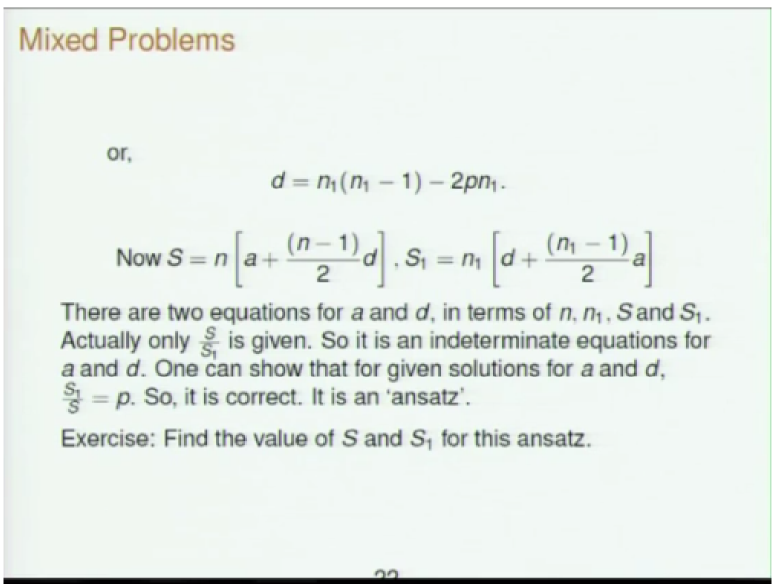

So, here you see actually S is this and S1 is this right sum of the arithmetic series, so they are 2 equations for unknowns you have to solve for a and d, in terms of this but only S/S1 is given okay, there is only one essentially there will be you have to divide this 2 equations there will be only one equation S/S1 okay. So, the one equation for 2 quantities a and d, so indeterminate equation actually, so you can show that actually if you take S1/S is equal to p.

So, then a and d that will be correct, in fact in the modern language is what is known as an ansatz kind of a **non** some kind of a we start with some kind of a you know from some intelligent guess you assume a form for the solution okay and then carry on because it is actually indeterminate and you do not have a you know unique solution.

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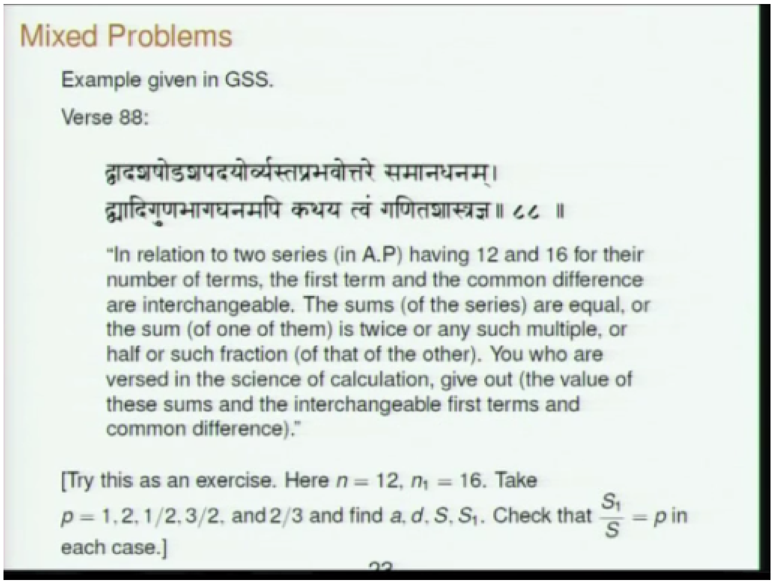

So, he has given some examples (FL), so in relation to 2 series in A.P having 12 and 16 for their number of terms first and common difference are interchangeable. The sum of the series are equal or some of one of them is twice or any such multiple or half or such fractions okay, see is always you know our man is slightly do small problems are embedded you know a small set of problems will involved embedded in each problem okay.

So, here you have to n is equal to 12 n1 is equal to 16 and take p is equal to 1, 2, half, 3/2 etc., there is not so in this ratio of the sums and you have to find this a, d, S, S1 like that okay.

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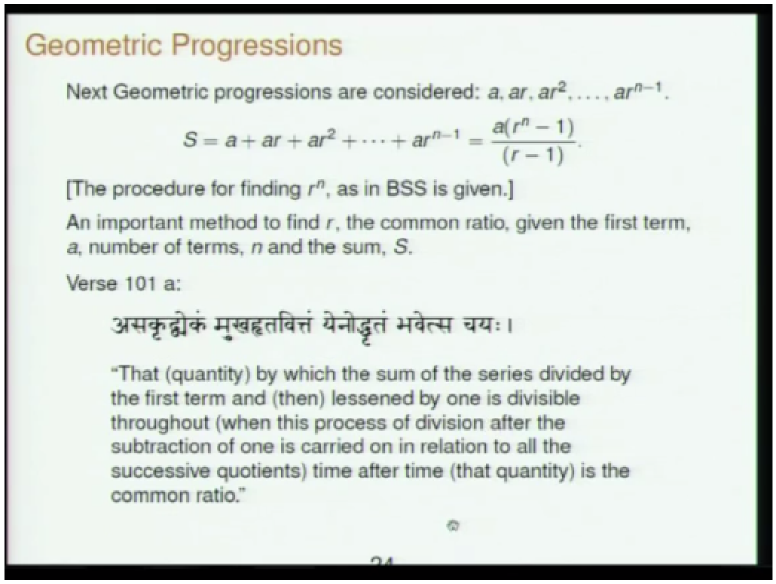

So, now he will come to the geometric progression the important thing as this is the sum whatever is there the earlier thing there is the quote stated but you please notice that there is some improvement in everything there is some small improvement you know some advancement and that is a characteristic of you know Indian mathematics Mahavira you know has some various new things you know from compare to Brahmagupta. Brahmagupta himself had various new results apart from what Aryabhatta is stated.

Aryabhatta himself of course got some results from maybe earlier tradition and got something see this thing and similarly one can see that you know Bhaskara will give a lot more this things the advance is the mathematical knowledge in various ways and narayapunita much more sophisticated ways you know very important departures and Kerala school of mathematics you know of course the almost a new face you enter you know with some well sophisticated discussion on analysis, infinite series and all that.

So, that is the thing, so though you are toughing with this things whatever was known from the earlier thing is adding up, so that is happening, so this sum of this geometric progressions is given, so this all this is I do not have to explain this result now an important measure to find r the common ratio is to given the first term number of terms n and the sum is this is not a you know given a, r and n you can find S.

But given S, a and n to find r is it nth degree equation, so it is not easy, so he gives a very interesting method (FL) that quantity by which sum of the series divided by the first term and then lessened by one is divisible throughout when this process of division after the subtraction of one is carried on in relation to all the successive quantities time after time that quantity is the common ratio.

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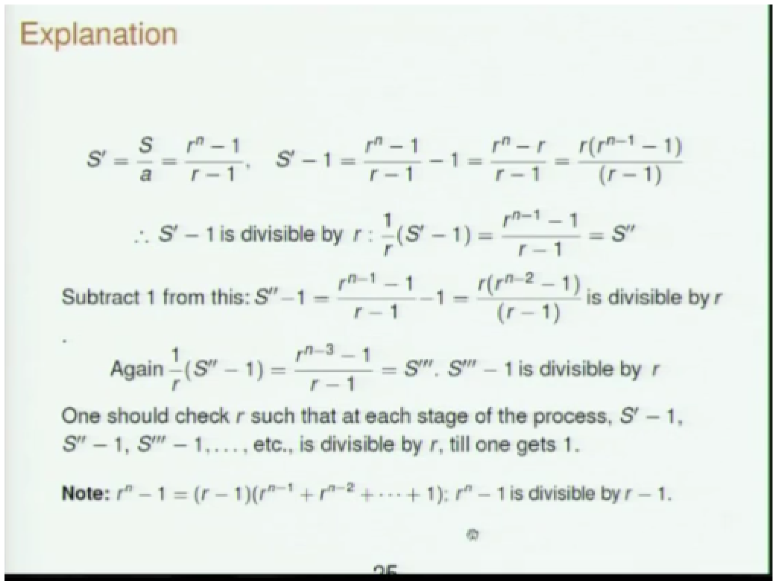

So, what he is trying to say is, so the S is this S/a is r to the power of n-1 divided by r-1 and S prime-1 is this r to the power of n-1 by this -1, so which is this, so this is divisible by r clearly. So, now take this you take 1/r into S prime-1, so this is call it as S double prime subtract 1 some this again one can read that this is divisible by r, S double prime-1 that is r to the power of n-1-1 you found this ratio then subtract 1, that is also divisible by r.

So, you go on you see and one should check the r said that at each stage of the process S prime-1, S double prime -1 triple prime -1 etc., it is divisible by r till one get 1 okay. So, it is tricky whereas what you sort with you see and sometimes style and error you have to use you see. In a first step it maybe divisible by many numbers and afterwards it subtract 1 and this thing then you know you will be only a few things, so it will get narrowed down you know. Here of course I use the result that r to the power of n-1 is this, so it is always divisible by r-1.

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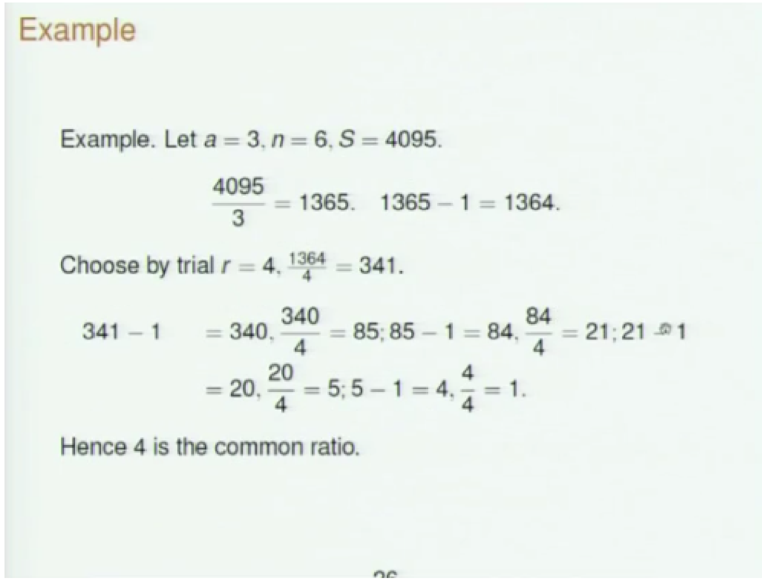

Yeah for instance if let a is 3 b3, n is 6, S is 4095 geometric progression first term is 3 and number of terms is 6 and common ratio you have find out, so what you do is it divide the sum by the first get 1365 subtract 1 you get 1364, so now choose by trail r is equal to 4, so then 1364/4 is 341 then subtract 1, 340 is divisible by4 you get 85, 85 you subtract 1 you get 21 is subtract 1 again that is again divisible by 4, 5-1 is 4 that is again divisible by 4, so 4 is the common ratio, so this how you get.

As I told you sometimes it may be tricky so, you have to watch out.

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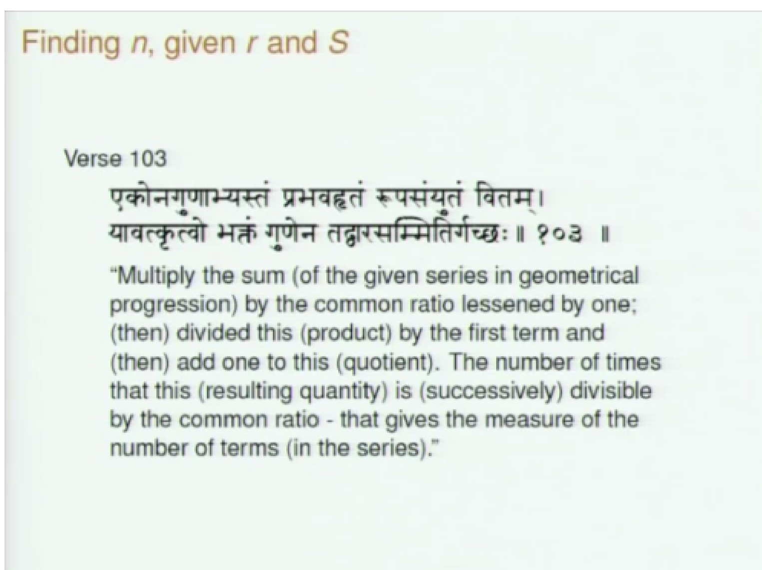

And finding n given r and S that also he gives.

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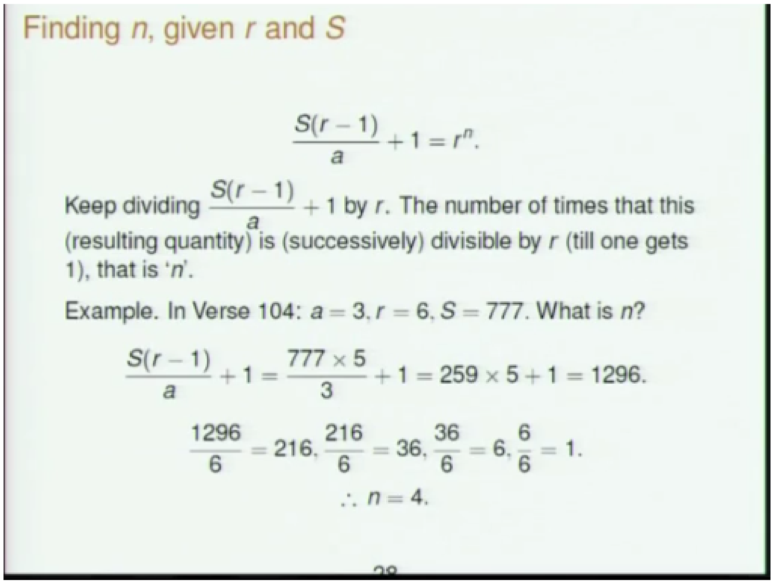

So, for instance S into r-1/a+1 is r to the power of n, you can expect that sum like this, so keep dividing by r this quantity the number of time that is resulting quantity successively divisible by r till one gets 1 that is n, so this somewhat little more simpler okay. Suppose a is 3, r is 6, S is 777 what is n?. So, this quantity is crucial right, so this is 1296 and 1296 this is more straight forward you know 1296/6 is this 6 again is this by 6 again is this, so n is 4, so it is.

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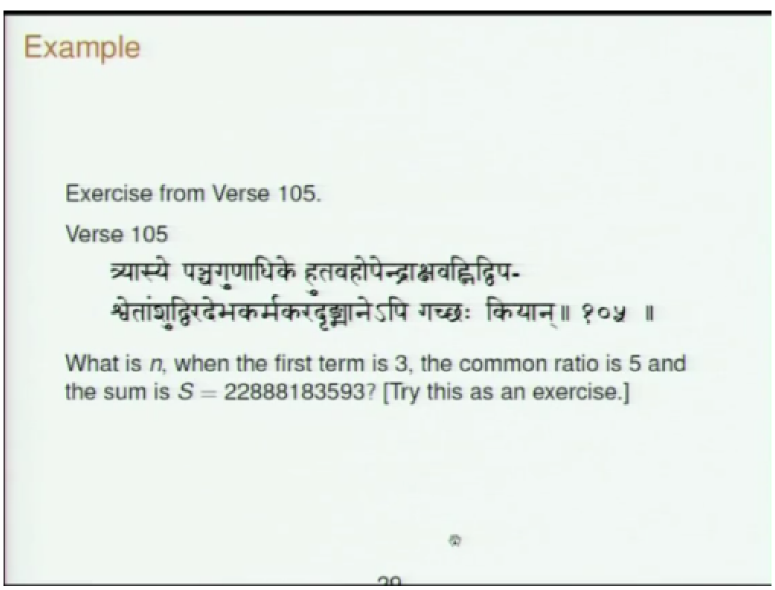

So, exercise is, what is n in the first term is 3 the common ratio is 5 and the sum is 22888183593, so please try this is an exercise okay.

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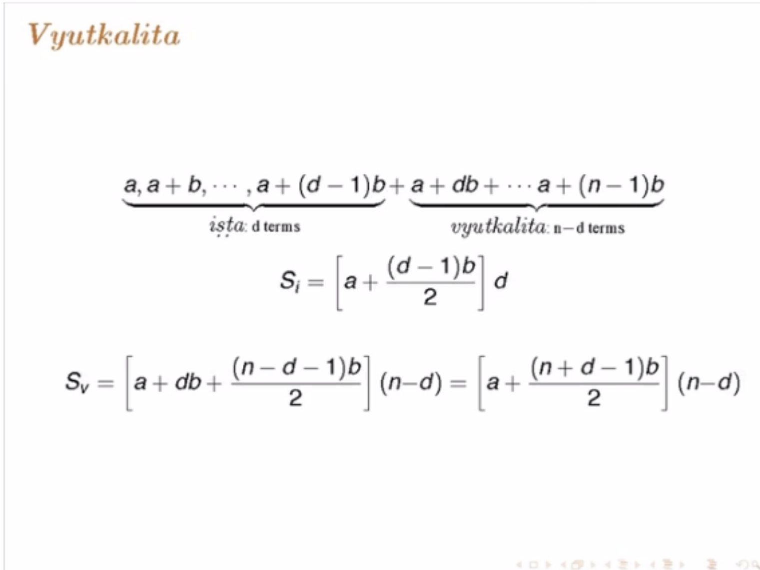

So, then he talks about you know as I told you some improvements you know some advancements are made, so consider this series is you know it goes on up to n-1 but it is split into 2 parts where 1 first part of d term this is n- d terms, so then this is called (FL) that is the later part of a series where the initial part is called (FL), so this (FL) is actually total sum-ista basically, so this what you get.

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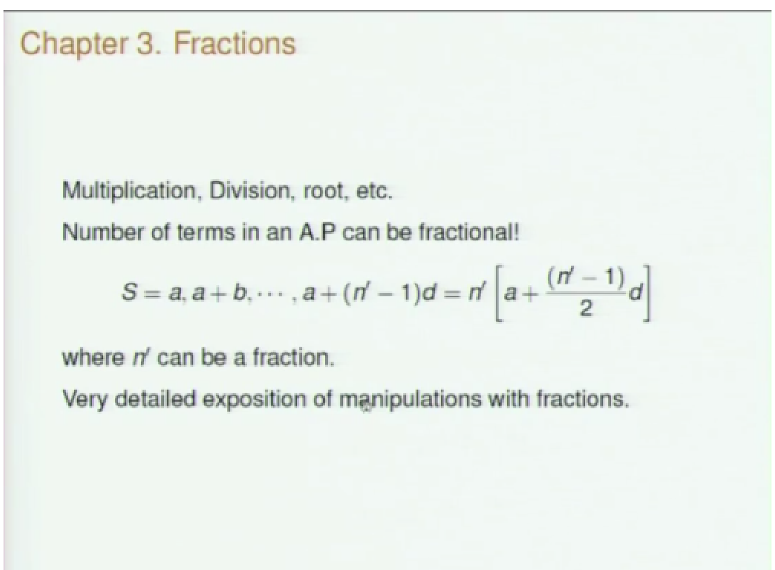

Now multiplication, division, root etc… of fraction she will discuss in rate the various kinds of fractions are there you know. And you know really gives large number of examples to illustrate various kinds of you know manipulations, with fractions, multiplications divisions and so on so both, so that you know things will be fixed firmly in one mind and number of terms in an A.P very interesting you know that he takes this n prime is this where n prime can be fraction the number of terms can be fraction you see, see this kind of generalization all that.

We are keeping doing only doing the past 2 centuries you know so, he did not mind you know that the number of terms can be a fractional, so it is define like that you see up to sometime and then you know go a little further not the next term you know next integer but fractional thing, as I told you very detailed exploitation of manipulation this fractions.

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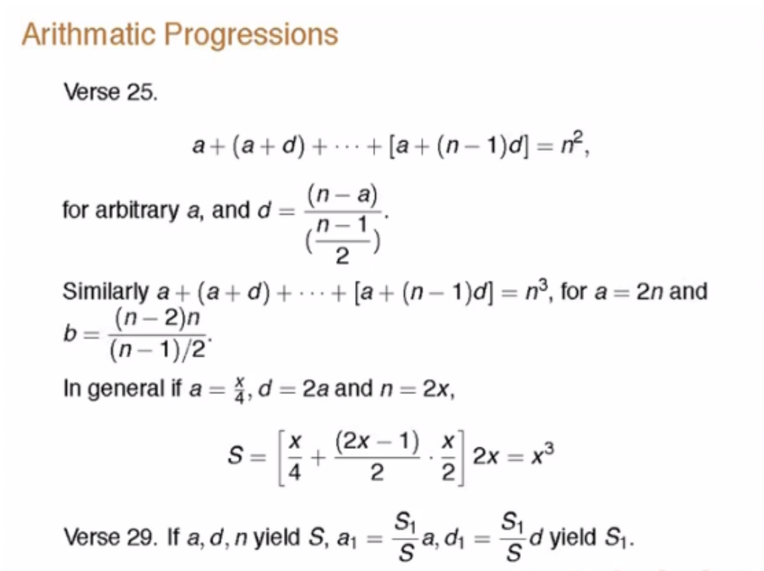

Then arithmetic progressions I will not all this things are known, so for instance if this if you want this to be square of n, a square of n can be got from an arithmetical progression but arbitrary a and d is this if you want to express this square by a arithmetic series some were of some number integer, so similarly cube also can be express like this in fact any number x is cube can be expressed as a sum of kind arithmetic series, where the first term is x/4, the difference is 2a and n is 2x.

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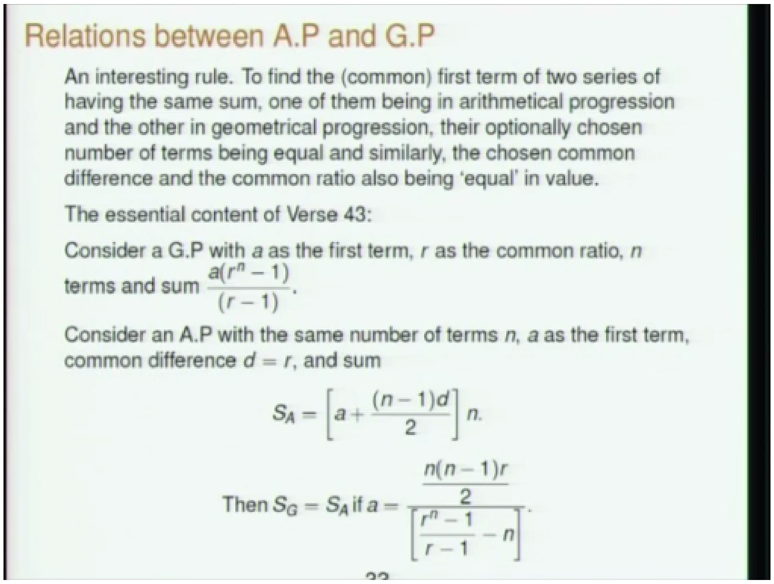

And he talks about various relations between arithmetical and geometrical progressions for instance you can have an arithmetical progression and geometrical progression suppose you want the sums to be equal, so then you can find the first term and all that. If the first term is taken to or the arithmetical progression is taken to be this then the 2 this sums will be equal.

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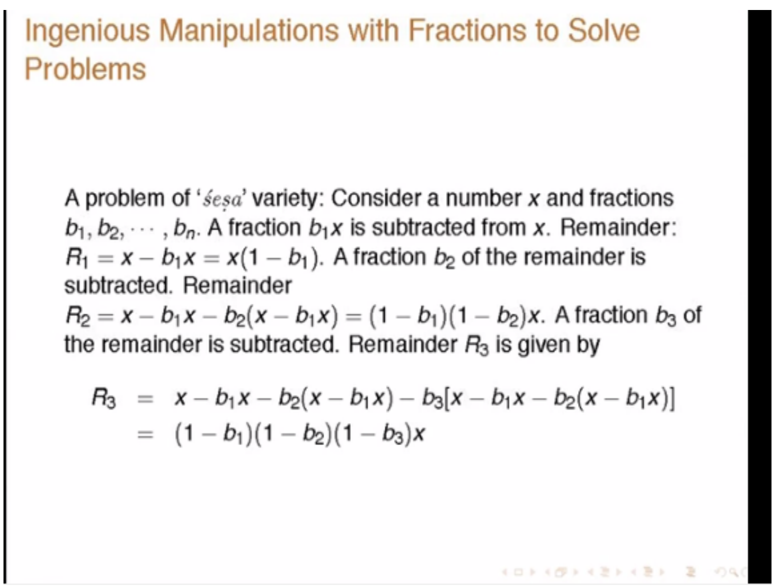

Then it talks about in a various manipulation with fractions to solve problems. So, a problem was sesa variety I will just go through for an example. Consider a number x and fractions you know b1, b2 etc… b and suppose a fraction b1x I subtracted from x. So, remainder is x-b1x right which is written as x into 1-b1x. A fraction b2 of the remainder is subtracted so, x-b1x a fraction b2 of this I subtracted from this. So, it will be this quantity which is 1-b1 into 1-b2x.

And a fraction b3 of the remainder I subtracted so, -b3 into this whole thing you have to write so, which this.

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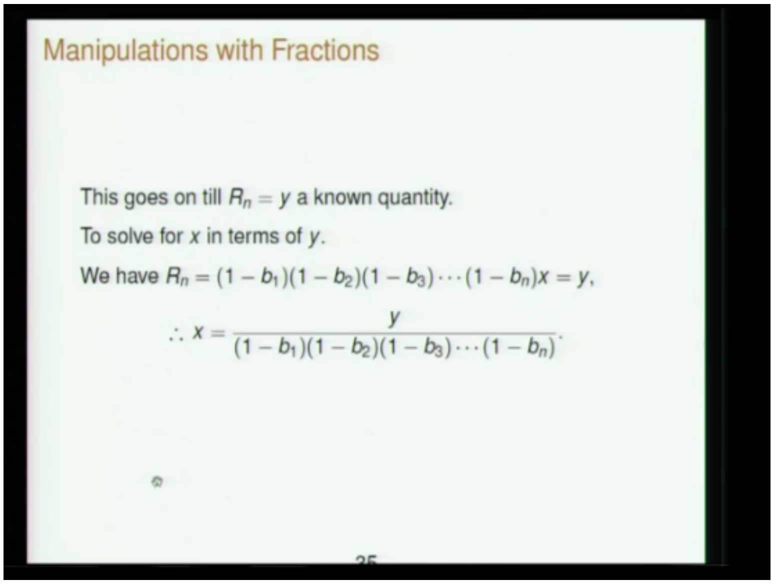

So, you can go on like that so, this goes on till this nth remainder is ya unknown quantity to solve for x in terms of y. So, Rn is this suppose this y the clearly x is y divided by all these.

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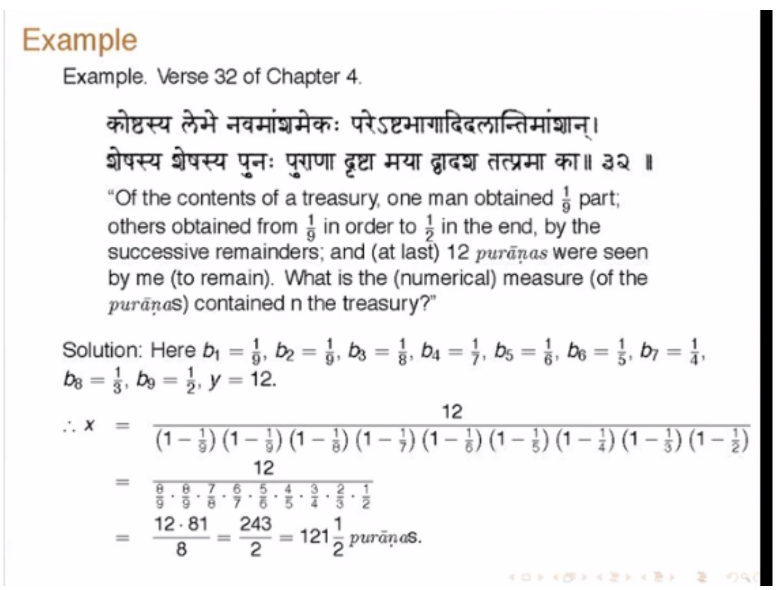

The furnishing gives the example (FL) name of the monitory unit (FL) 12 puranas for seen and the contents of the treasury one man obtained one nine part others obtained from 1/9 in order to 1/2 in the n by successive remainders. And at least 12 puranas were seen. What is the numerical measure of the puranas contained in the treasury?. So, here straight away you just method b1 is 1/9, b2 I s1/9, 1/8 etc etc. and then well is the, what is found so, x is 12 divided by all these so, this will be equal to 243/2 are 121 and a half puranas is solution.

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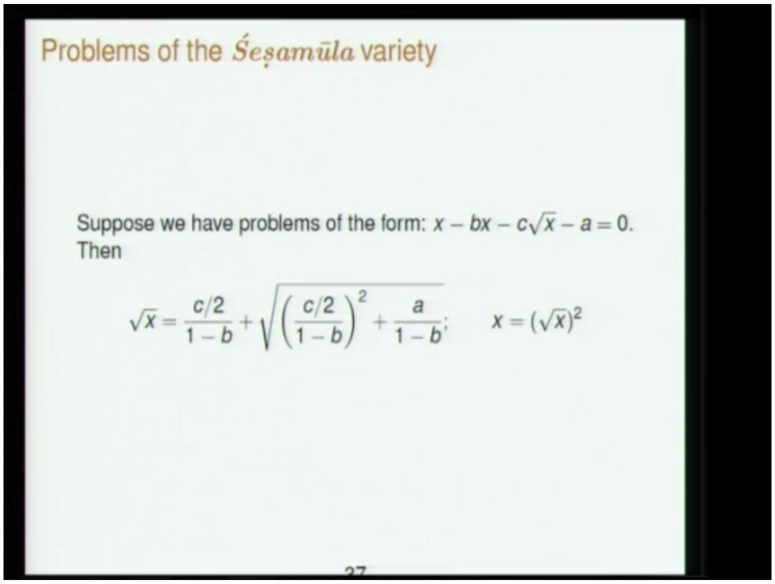

So, this of course will be (FL) so, moola will also come root x kind of a thing suppose your problems is form x-bx-c, root x-a is equal to 0. So, then you have the quadratic equation to write this solution like this one of the roots and then x is root x by this quadratic equation solution.

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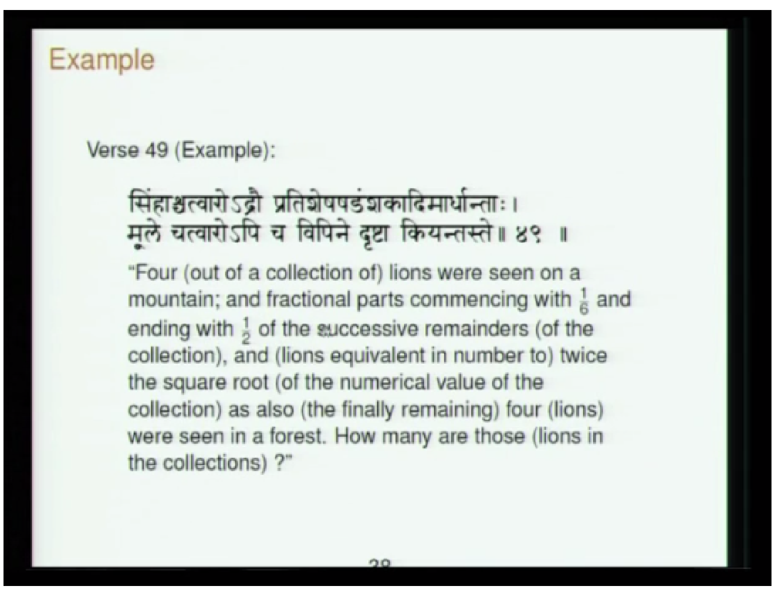

So, he gives an example (FL) four (out of a collection) of lions were seen on a mountain: okay(FL) mountain okay and fractional parts commencing with 1/6 (FL) right (FL) starting from that commencing with that and (FL) ending with half as you remind as and (lions equivalent in number to) twice is square root (FL) twice the (of the numerical value of the collection) as also finally remaining four lions were seen (FL). How many are those (lions in the collections) ok.

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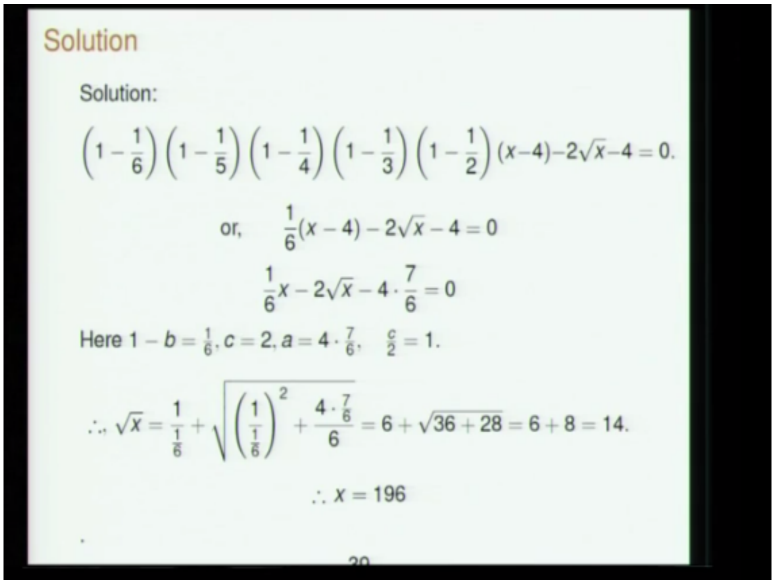

So, then the essentially is the problem is this so, four of them were seen on the mountains you subtract that of the remaining you know 1/6 went away like that. So, it is 1-1/6 of thus for remaining out of that 1/5 went so, like that to this x-4 and then twice the moola okay – so, that is equal to twice the moola. So, then if you subtract that all also 4 was finally remaining if you subtract twice the moola of the original number. So, what to get is a solution sorry equation like this x-4 into this so, this is this thing so, one can and x is equal to 196.

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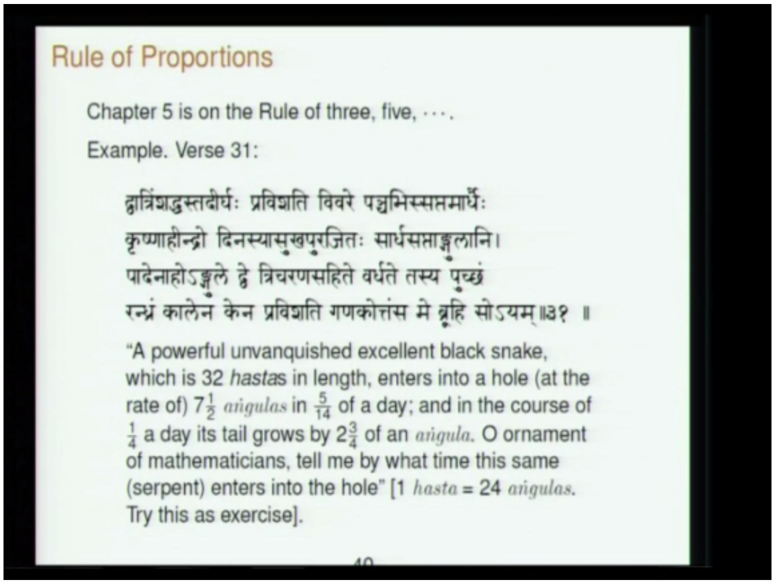

So, the chapter 5 in a rule of three, five etc… again in order going to detail but just mention in interesting problem (FL) A powerful unvanquished excellent black snake which is 32 hastas in length okay(FL) enters into a hole okay enters into a hole at the rate of 71/2 angula okay (FL) in 5/14 of a day; and in the course of ¼ of a day it is tail grows by 23/4 of an angula okay (FL) okay (FL) so, 23/4 right (FL) I s1/4 the of the day (FL) okay tail. So, (FL) tell me by what time this same (serpent) enters into the hole (FL)) okay expert mathematician please tell the solution.

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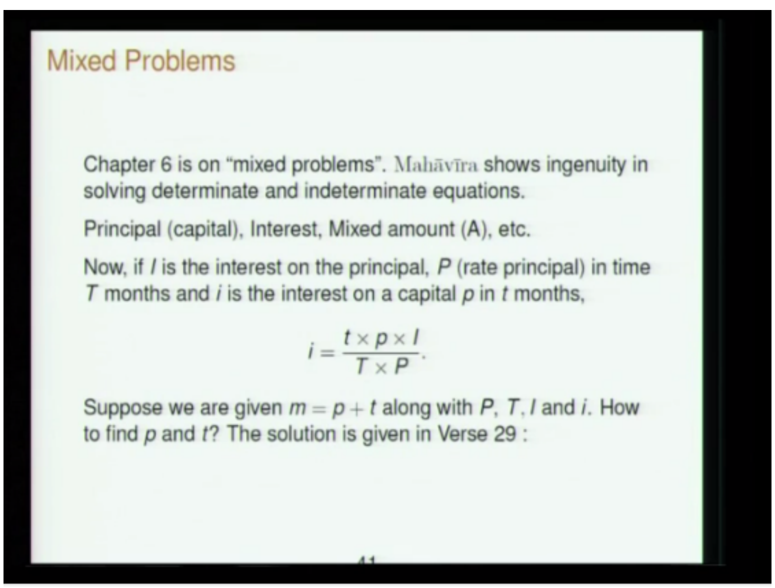

So, that is the (FL) little careful it is direct okay so, then interactive discusses various varieties have mixed problems okay. So, I will come into that in fact that is the lot of much as Varghese on the so, called mixed problems. So, for instance typically if capital is a interest on principal capital p and time capital T months and I is a interest on a capital small p in small t months okay then I is this p*p*I/T*p okay.

I may if the rate of interest is the same I am saying if the rate of interest is the same so, then one can you know you can easily see that I/I will be t*p/T*p so, is the rate of interest is the same. So, this equation will be true but suppose there given m+p+t along with P,T, I and i okay. So, then how to find outs small p and t so, these a slightly invert problem. So, using this small principal and the time okay only the sum is given.

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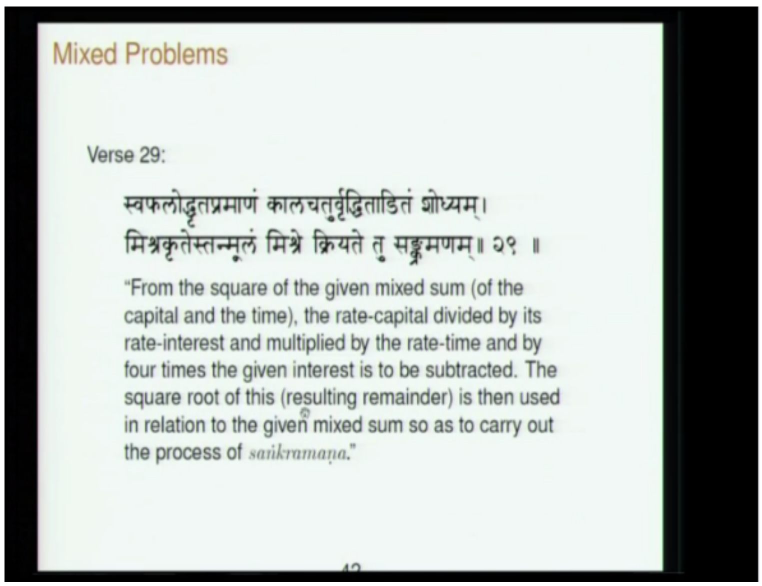

Then one show that

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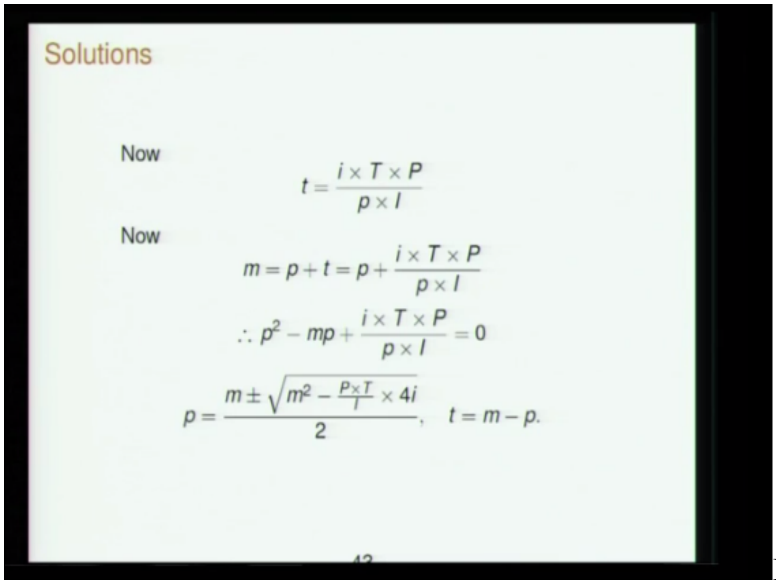

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I will not so, t is this so, m is equal to p+t is you know this so, p squared-mp you will get a quadratic equation so, p will be this actually and t is equal to m-p essentially somewhat similar to brahma what Brahmagupta at consider as a one of the problems that I told so, told you straightly different kind of a problem. But same technique is used.

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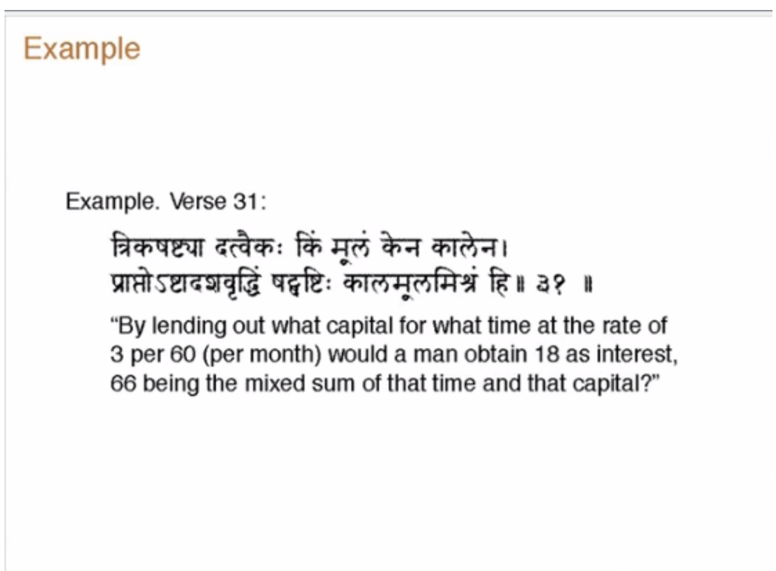

And finally for instance by lending out what capital for what time at the rate of 3 per 60 (per month) would a man obtain 18 as interest, 66 being the mixed sum of that time and that capital? Okay

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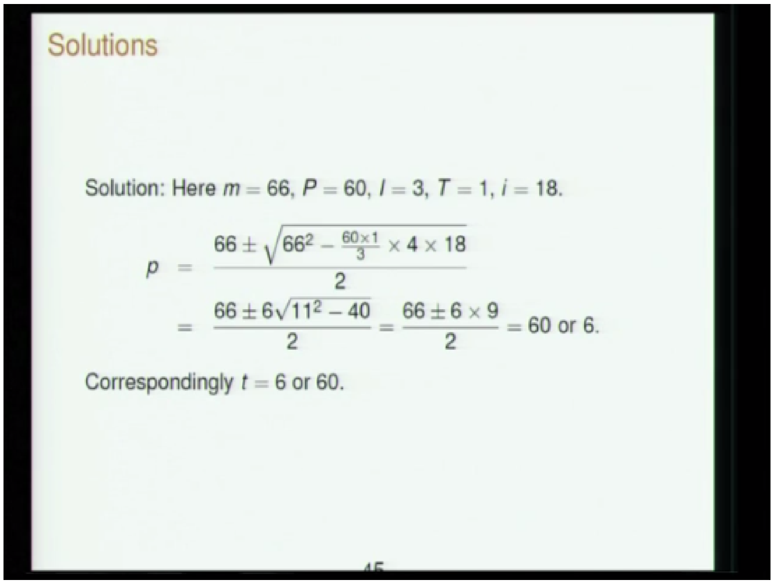

So, here what is given is m is 66 see for a capital p is equal to 60 for one month the interest I s3 okay. So, no, what is the interest suppose the interest is given to be 18 okay. It is some percent some other lending. The interest generated is given to be 18 and the time+the principal is given to be 66 so, is in what is that principal and what is that time. So, for that you have to use the quadratic equation you get the solution 60 or 6 okay.

So, is there you know t is equal to 6 or 60 so, principal is 60 okay so, the interest will be 18 if it is 6 months. But if the principal is 6 the interest is 18 if the time is 60 months so, this is the.

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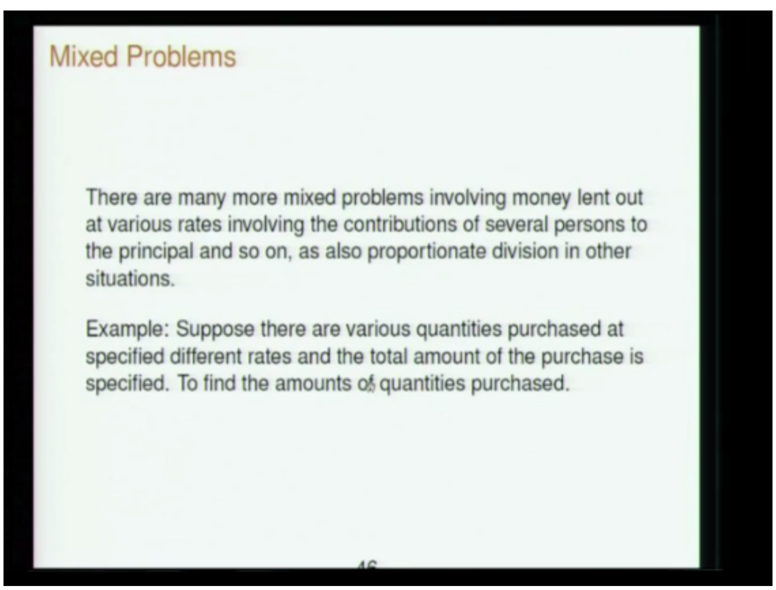

So, there are many more mixed problems involving money lent out at various rates involving contributions the several to the principal and so on as also proportionate division in other situations okay. Suppose there are various quantities purchased at specified different rates and the total amount is the purchase is specified to find the amounts of quantities purchased here all very simple. But you know (FL) more general you know a readily usable form kind of everything.

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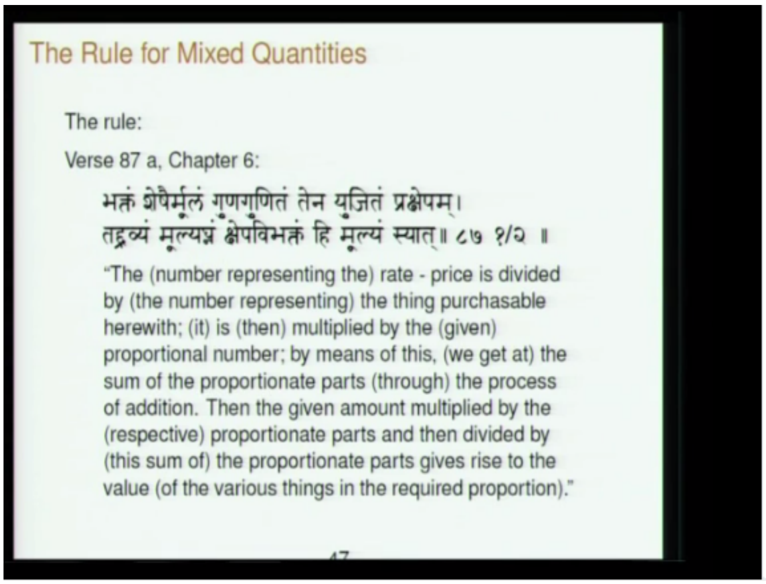

For instance okay I will tell what it is and then may be.

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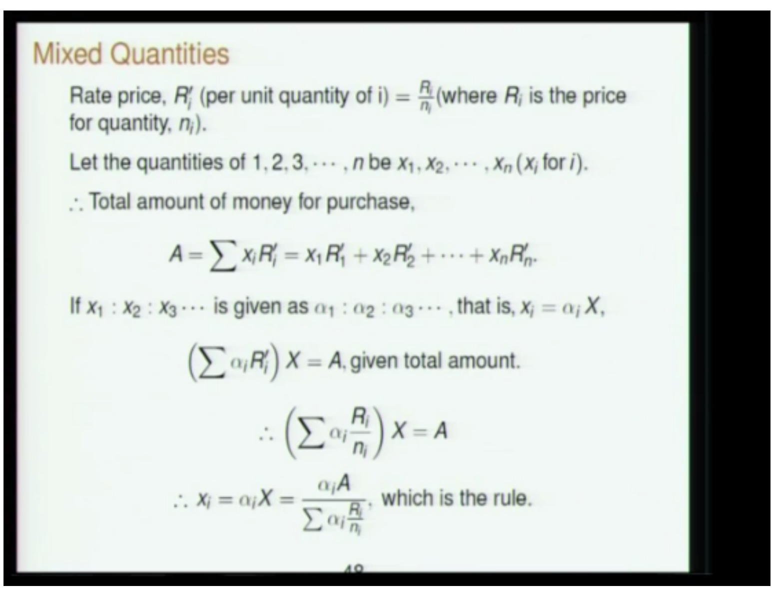

You know because you do not have a too much time see suppose various quantities are bought okay. The rate price is R prime i, I will edge a you will know why I am writing R prime here Ri /ni Ri is the price of the quantity ni okay. Some quantity is there some item is there the quantity of that is ni. The price is Ri for that quantity that is the rate is Ri/ni and a write that the quantity is the 1, 2, 3, n be x 1 to xn.

So, the total amount for purchase is clearly the quantity of each and then multiplied by the rate price. So, this will be the this now suppose you are given the ratios of this quantities you know x1:x2 etc… is equal to alpha1:alpha2 etc. So that is essentially xi is equal to alpha I into x okay. One quantity is there so, you get if you substitute this here you get this so, you get capital X to be you know this.

So, xi is alpha iA/sigma alpha I Ri/ni okay so, you can directly work with Ri/ ni you do not have to use a rate and do all these things.

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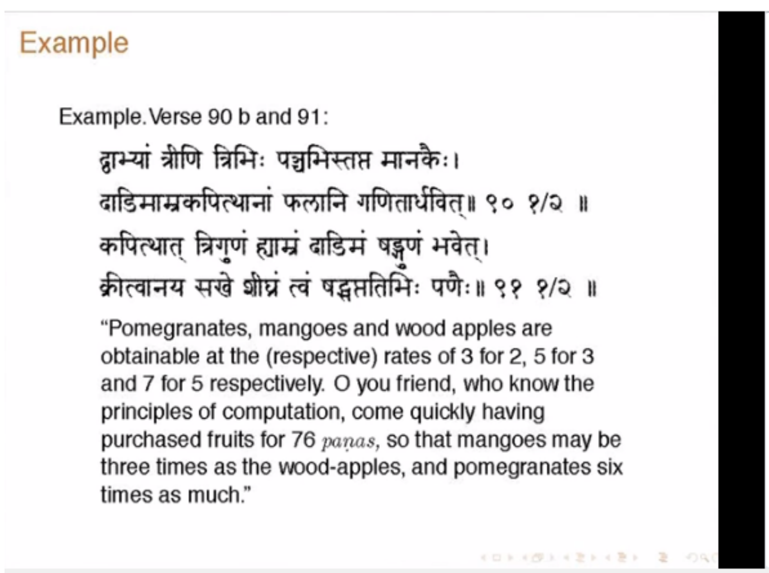

The for a example you have one example is given (FL) is pomegranate (FL) mango (FL) wood apple (FL) okay the number of a mangoes are three times as (FL) wood apples okay. And pomegranate (FL) 6 times this is things. So, and the rates are given (FL) you see for pomegranate the rate is 3 for 2 and similarly for the other mangoes it is 5 for 3 okay (FL) like that you know.

So, and for the wood apples it is 7 for 5 so, you have to find out how many quantities so, then here.

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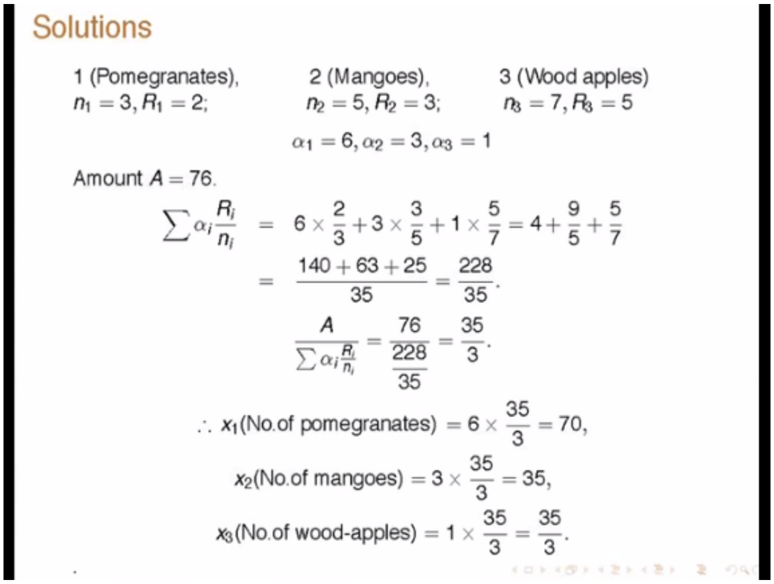

One has to just straight away use this formula so, here for a pomegranates the rate is I mean you get 2 pomegranates per 3 pomegranates for two 2 units of money per mangoes so, the 3 for 5 you get 3 and similarly for wood apple you 5 you get for 7. And the ratios are given to be this 6:3:1. So that is the pomegranates, mangoes and wood apples. So, then amount is given to be 76 so, total amount is this.

What each individual thing you will get so, you will edit to be you have to calculate all these things and finally you get the number of pomegranates to be 70, number of mangoes to be 35, and number of wood apples is equal to 35 units. So, these what you will get for 76 (FL) okay 76 (FL) when it is mention that the number of mangoes is thrice the number of wood apples and number of pomegranates is 6 times the number of wood apples. So, this is one typical problem.

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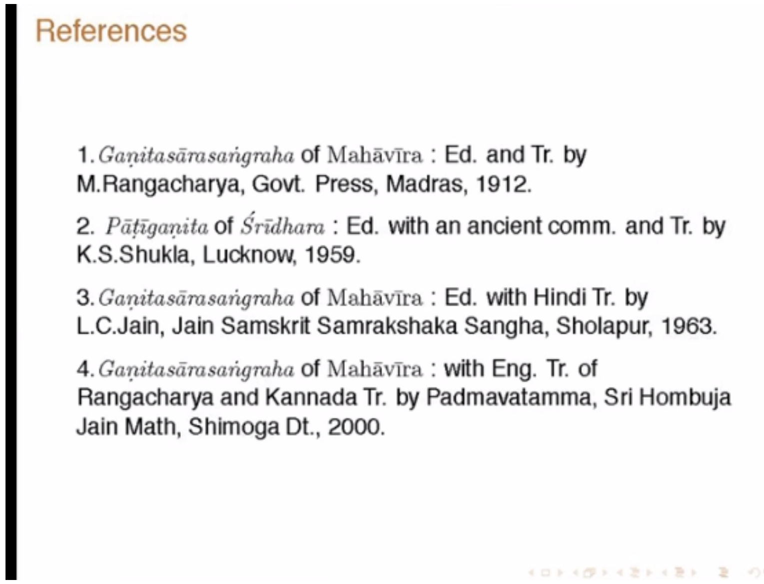

The references are given here so, thank you so, we will discuss some more problems of this solve you know various fixed quantities you know that which are of direct interest in daily life. I will stop here.