Decision Making With Spreadsheet
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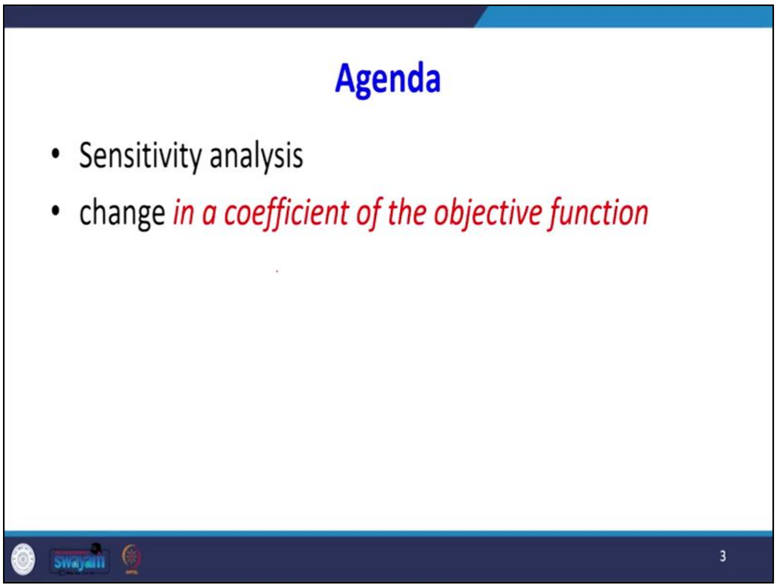
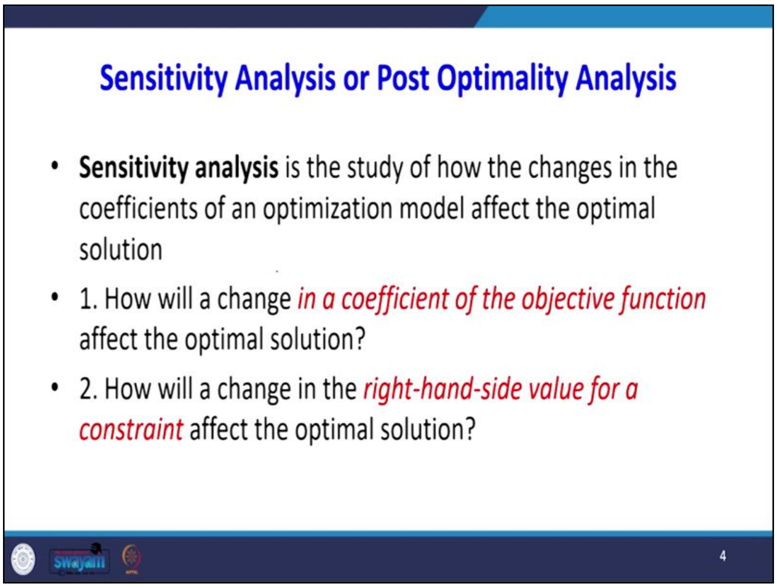
Lecture-06
Sensitivity Analysis - 1

Dear student, in the previous lecture, I discussed graphical solutions using a graphical calculator and how to use a solver. In this lecture, we discuss sensitivity analysis.

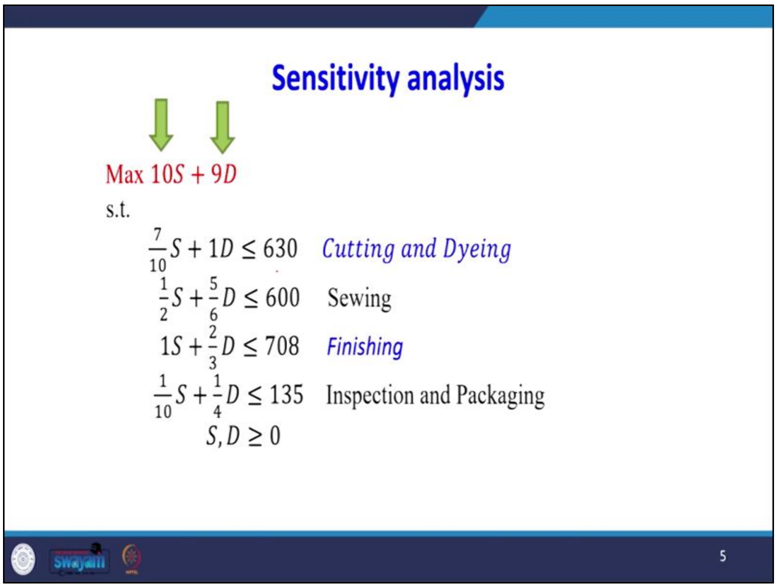


The agenda for this lecture is sensitivity analysis in that if there is any change in the coefficient of objective functions, how will that affect our result?



So, another name for sensitivity analysis is post-optimality analysis. So, sensitivity analysis is the study of how the changes in the coefficient of an optimization model affect the optimal solution. There are 2 ways it will affect how a change in a coefficient of objective function affects the optimal solution. The other one is how a change in the right-hand side of a constraint affects the optimal solution.

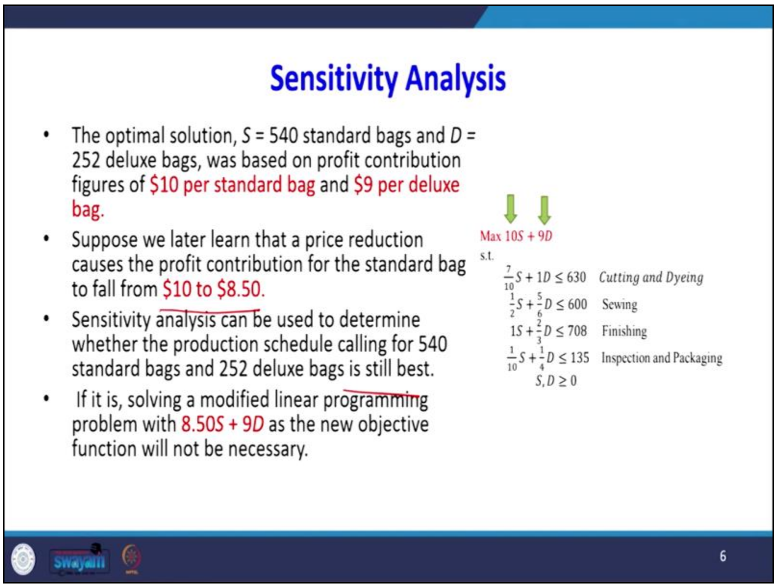
In this lecture, we will discuss the first case: if there is any change in the coefficient of the objective function, how does our solution optimal solution get affected?



This was the problem that we used right from the beginning

Maximizing 10S + 9D,

There are four constraints. So, I will explain how to do sensitivity analysis for this problem. What is the center of analysis we are talking about if there are any changes in the coefficient of this objective function? For example, if the 10 may be changed, the ten is the profit contribution. Obviously, the profit contribution will not be the same. Sometimes, the profit contribution will be more, and sometimes, the profit contribution will be less. But what will the effect of this profit contribution be on our final solution?



I have brought up the same problem here. The optimal solution for the given problem, which is on the right-hand side,

S = 540 standard bags,

and D = 252. deluxe bags,

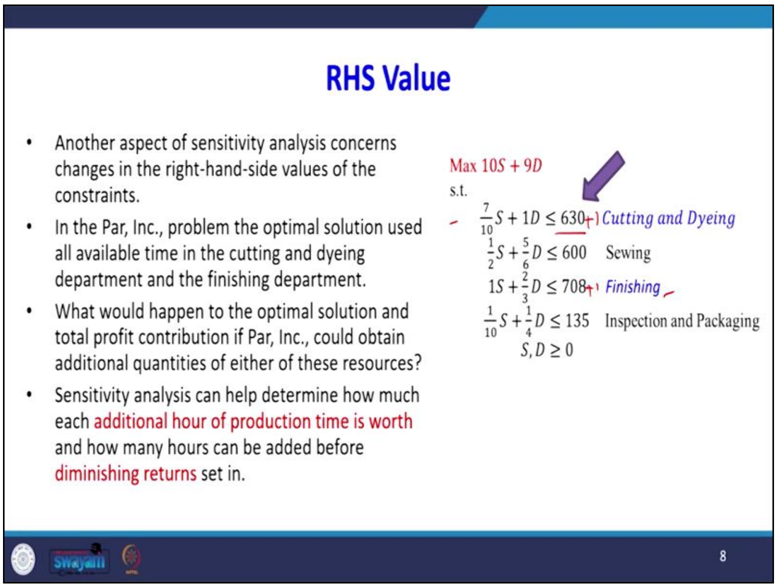
So, based on the profit contribution, figures are 10 dollars per standard bag and 9 dollars per deluxe bag. Suppose we later learned that a price reduction causes the profit contribution of the standard to fall from 10 dollars to, say, 8.5 dollars.

So, sensitivity analysis can be used to determine whether the production schedule calling for 540 standard bags and 252 deluxe bags is still best. If there is a drop in price, our solution which S = 540 standard bags,

and D = 252. deluxe bags,

Is it still the best, or where is it? A drop in profit contribution will affect our result, and that is the purpose of sensitivity analysis. If it solves a modified linear programming problem by changing the coefficient of the objective function, the new objective function will not be necessary.

So, obviously, you may ask this question: if the profit contribution is decreasing, we can change that coefficient value in Excel, and then we can get the final answer, but the recall redoing of the whole process is not required.



So, management would especially want to consider how the optimal production quantity should be revised if the profit contribution per deluxe bag were to drop. So, if the price of the product drops, how will that affect our production quantities? There is another situation where sensitivity analysis can be done. Suppose look at the right-hand side of constraint 630 in the cutting and dying department. The total available hours are 630.

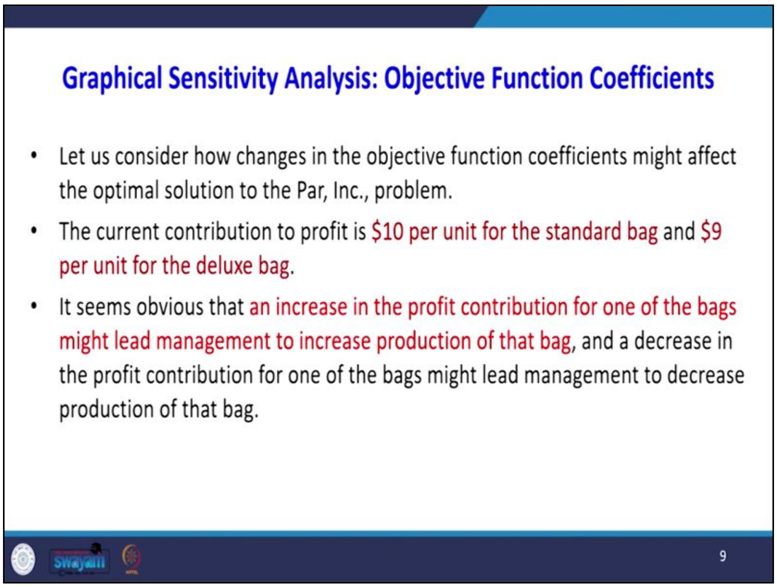
That is what I have brought here. Another aspect of sensitivity analysis concerns changes in the right-hand side value of the constraint. In the problem, the optimal solution used all available time in the cutting, dying, and finishing departments. You might have recollected from our previous lectures. So, this 630-hour constraint is our binding constraint. What is the meaning of binding constraint?

We have utilized all 630 hours, and this is happening for the finishing department. also, that is how I brought in different colors. So, what would happen to the optimal solution and the total profit contribution if that company could obtain additional quantities of either of the resources? So, what is the meaning here? Suppose the company is trying to bring some additional resources, they say, plus one something.

By bringing these additional resources, we need to check whether that will affect our final solution that will maximize or that will affect our objective function, whether it is a maximization or minimization type. This is another way of doing sensitive analysis, just as I am recollecting. There are 2 ways to do the sensitivity analysis: one is by changing the coefficient of the objective function, and the second one is by changing the value on the right-hand side of our constraints.

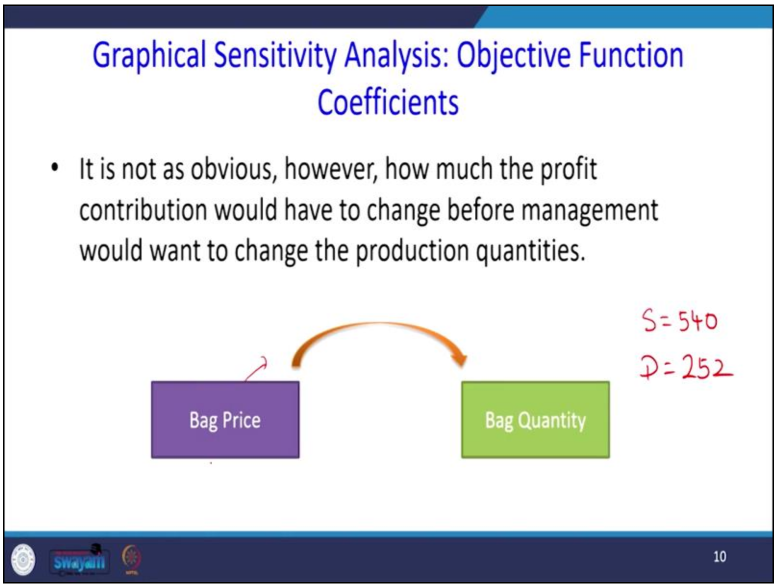
So, sensitivity analysis can help determine how much each additional production time is worth and how many hours can be added before diminishing return certain. So, what would the meaning be if I added one extra resource? Is adding this extra resource worth it or not, and how much is it worth? How much it is worth means how that will affect my objective function.

And sometimes, what will happen when you keep on adding resources instead of it is positively affecting our objective function? Sometimes, it will decrease. So, adding more resources will not help our objective function. So, that is the case of these diminishing returns.



But in this class, we discuss the first case, which is if there are any changes in the coefficient of the objective function, how that will affect our final result. Let us consider how the changes in the objective function coefficient might affect the optimal solution. The current contribution to the profit is 10 dollars per unit for the standard bag and 9 dollars per unit for the deluxe bag. It seems obvious that an increase in the profit contribution for one of the bags might lead management to increase the production of that bag.

And a decrease in the profit contribution for one of the bags might lead management to decrease the production of that bag. So, the profit contribution increases, obviously, will the company go for manufacturing more bags. If it decreases, they will try to reduce the number of quantities manufactured.



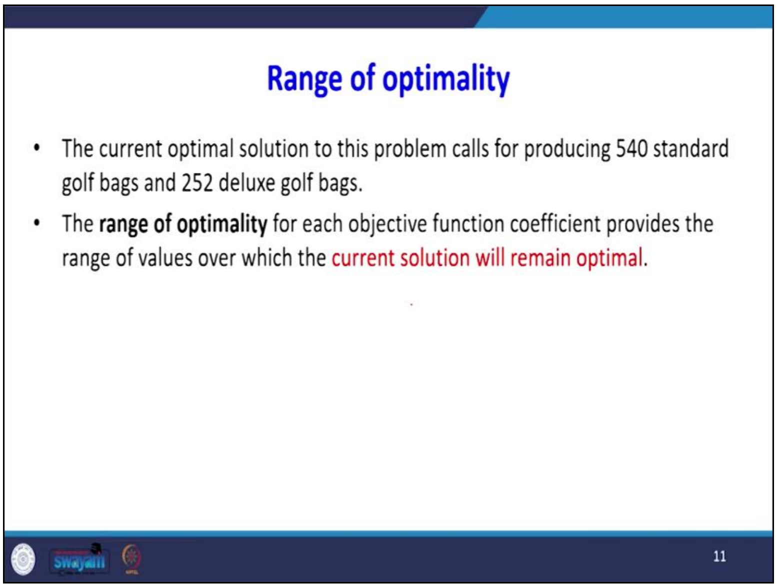
It is not as obvious, however, how much the profit contribution would have to change before the management would want to change the production quantities. So, what is the meaning of this?

We know that we already got the result that is S = 540 standard bags,

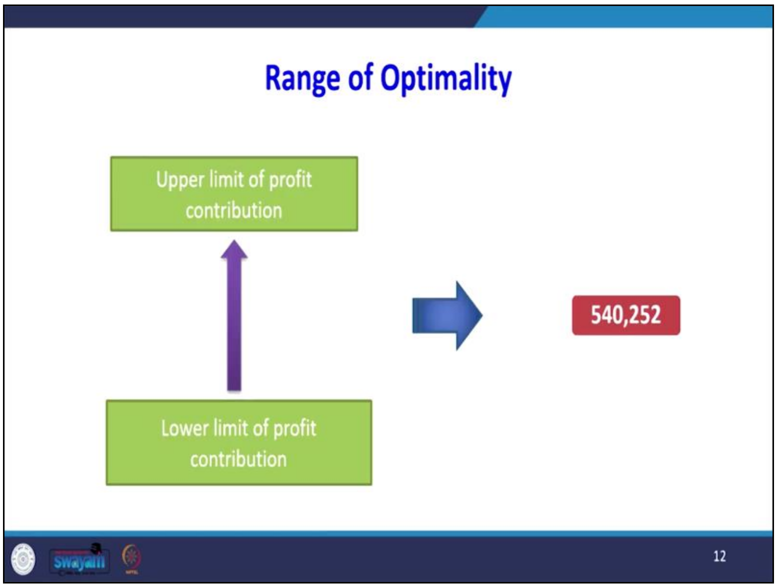
and D = 252. deluxe bags,

If there is any small change in bag price right, sometimes the small change in bag price will not affect your production quantities, but if the bag price is significantly changing.

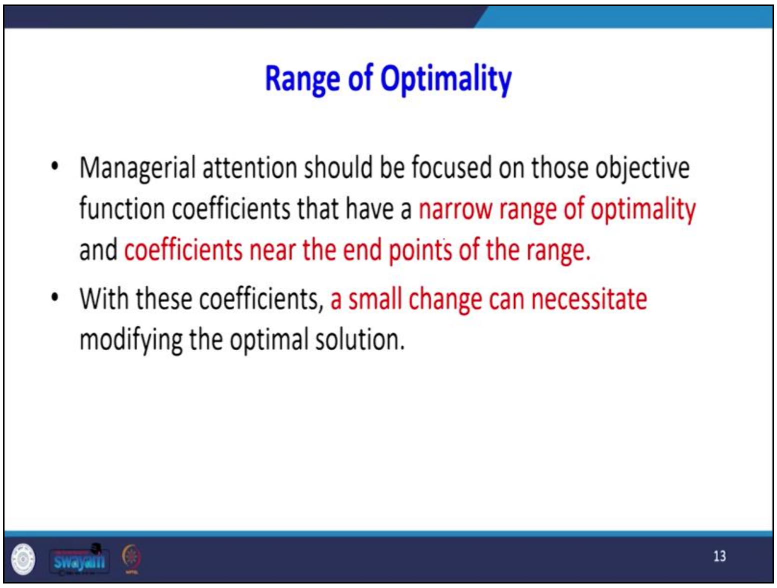
So, what is required is that sometimes you also have to change these production quantities. So, what are we going to do in this one? How much allowable change is permitted? How much change in profit contribution is permitted so that our solution remains changed.



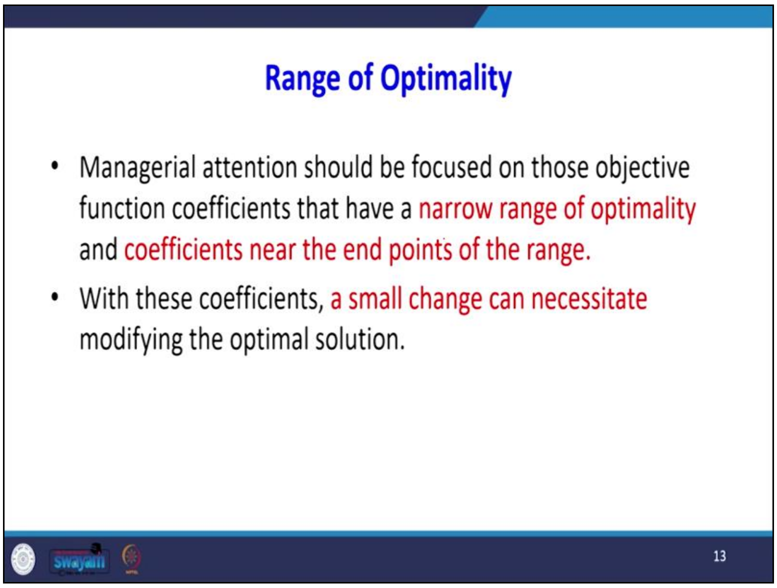
So, I am going to introduce the term called range of optimality. The current optimal solution to this problem calls for producing 540 standard golf bags and 252 deluxe golf bags. The range of optimality for each objective function coefficient provides the range of values over which the current solution remains optimal.



I think this picture will give you a clear idea. You see, this is the coefficient of our objective function. So, there is a lower limit for profit contribution than the upper limit for profit contribution. So, this much change is permitted. So, this change will not affect your final solution, which is called the range of optimality. Even though there is a variation in the coefficient of your objective function, that variation will not disturb your solutions. That allowable variation, the range of variation, is nothing but your range of optimality.

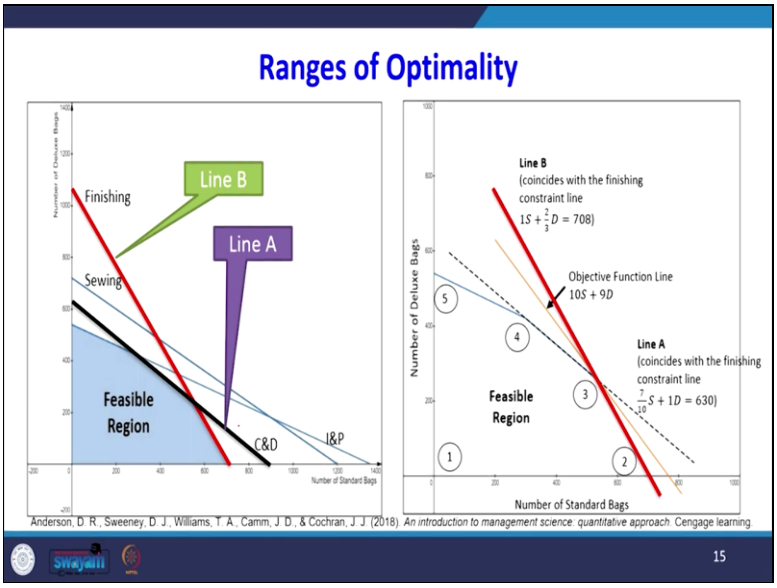


The managerial attention should be focused on those objective function coefficients that have a narrow range of optimality and the coefficient near the end point of the range. obviously, when the range is very narrow, the management should pay more attention to that because immediately your solution needs to be changed. If there is a slight variation in the product, the product price may affect your scheduling. So, with this coefficient, a small change can necessitate the modification of the optimal solution.



Now, I will explain this range of optimality graphically. We know that this region is our feasible region, which we discussed in our previous lectures. This point is where we got the optimal solution. So, now we will go back to the problem. The graphical solution of the problem with the slope of the objective function line between the slope of lines A and B, the extreme point 3, is optimal.

So, this point is the optimal point for the given problem. See there I am writing what line A and line B.

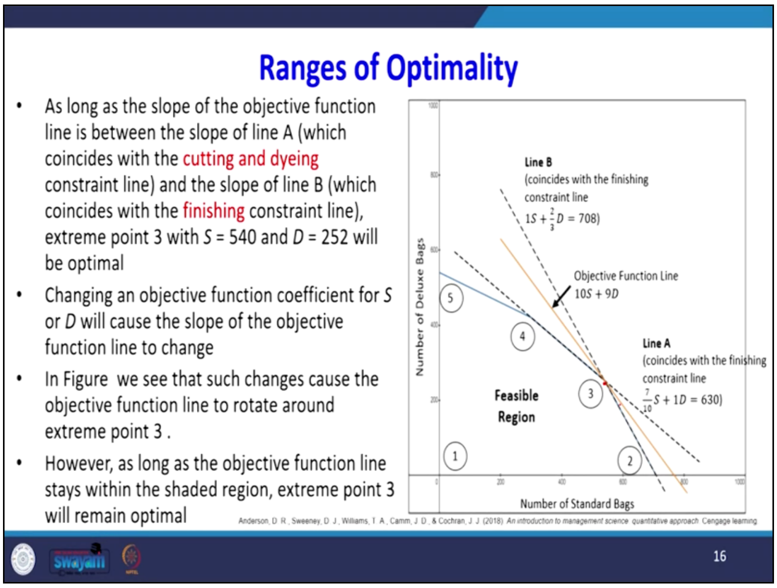


The cutting and dying department is called that line, which I am calling line A, and the finishing department's corresponding constraint is called line B. So, in the range of optimality, we have the objective function in this line, which is mentioned in orange.

This is 10S + 90.

So, as long as the slope of the objective function is between lines A and B, it is correct.

If the slope of our objective function is between line and b, that will not affect our result. So, that will provide your range of optimality.



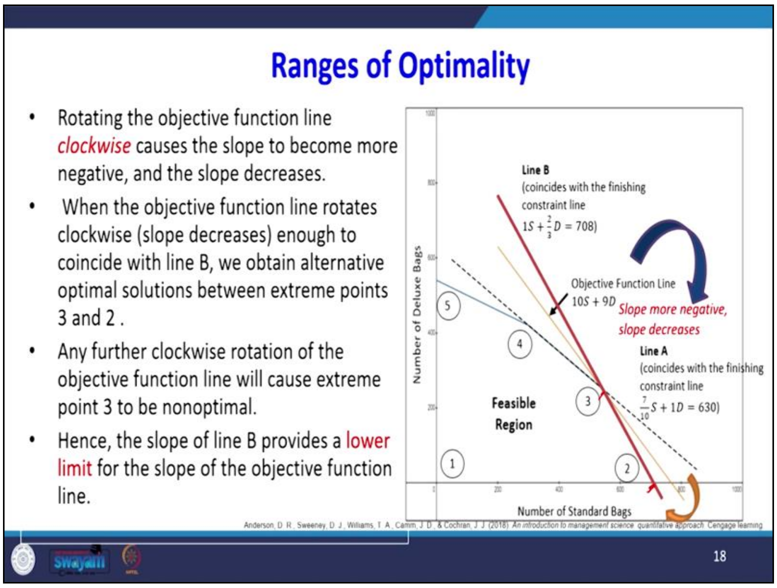
As long as the slopes of the objective function line are between the slope of line A, see if the slope of line A is the one that coincides with the cutting and dying constraint line and the slope of line B, which coincides with the finishing constraint line.
The extreme point 3; S = 540 and D = 252
So, changing an objective function coefficient for S or D will cause the slope of the objective function line to change.

Suppose any change in the coefficient of S and D will affect the slope of that objective function. In the figure, we see that such changes cause the objective function line to rotate around the extreme point three. So, if any changes in the coefficient of the objective function will cause the objective function, however, as long as the objective function line stays within the shaded region, extreme point 3 will remain optimal.

What is the shaded region? This region is rotating the objective function counterclockwise. Suppose the objective function is orange. Suppose if you rotate this objective function counterclockwise, the slope becomes less negative, then the slope increases ok. So, when you rotate this way, the objective function line rotates counterclockwise because the slope is increased enough to coincide with line A.

So suppose this line coincides with line A. We obtain alternative optimal solutions between extreme points 3 and four. What will happen if this orange line coincides with this red line? So, what will happen in this region? Ok, seeing all the points in this region will be your optimal solution. So, we will get an alternate optimal solution. Any further counterclockwise rotation of the objective function line will cause extreme point 3 to be nonoptimal.

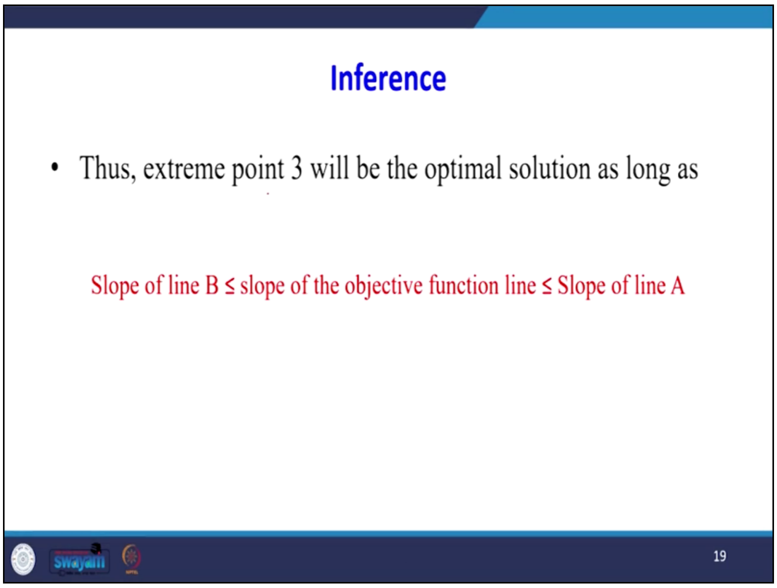
So, beyond this red line, if you rotate this orange line beyond that what will, what will become then the solution point 3 will not be the optimal solution; hence, the slope of line A right the slope of line A provides an upper limit for the slope of the objective function line.



Now we will see the other case rotating the objective function line clockwise ok. Now, the objective function line is going to be rotated clockwise, causing the slope to become more negative and the slope to decrease. When the objective function line rotates clockwise, then the slope will decrease enough to coincide with line B. What is line B, which is in red color? Line B, we obtain an alternate optimal solution between 3 and two. So, what will happen in this region? Your objective function line will sit there.

So, when it is coinciding, so, all these points between 2 and 3 will become your optimal solution. You will get multiple optimal solutions. Any further clockwise rotation of the objective function line will cause extreme point 3 to be nonoptimal. So, if you rotate beyond this so 3 will not be your optimal solution. Hence, the slope of line B provides a lower limit for the slope of the objective function line.

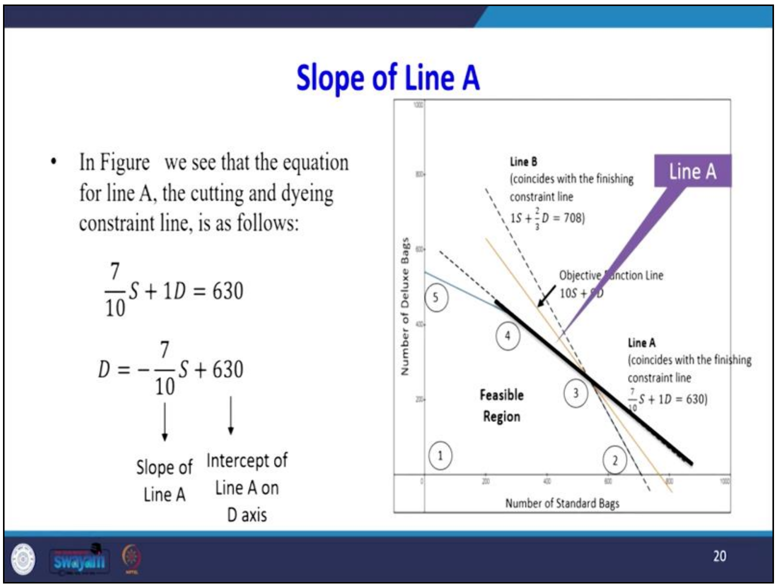
So, now we have the upper limit and lower limit of your objective function line so that point 3 will remain optimal.



So, extreme point 3 will be the optimal solution as long as

the slope of line B <= the slope of the objective function line is <= the slope of line A

It says that if the slope of your objective function line is between the slope of lines B and A, your optimal solution will remain the same, which will not disturb your optimal solution.



In the figure given on the right-hand side, We see that the equation for line A is this one line A that is the cutting and dying constraint line is like this what is that line because it is equal to write

(7 /10)S+ 1D = 630.

So, I want to know the slope of this equation. So, I am going to write rearrange. We know that very,

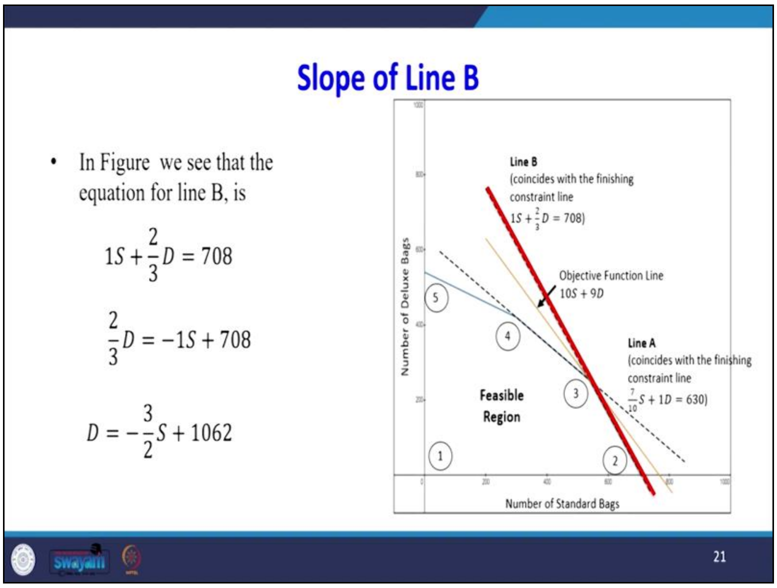
region y = m x + b , where m is the slope here is nothing but your d number of deluxe bags.

So, I am going to write this equation to bring in this form

y = m x + b. So, I can find out what the slope is. So, when you simplify D ok, you will get

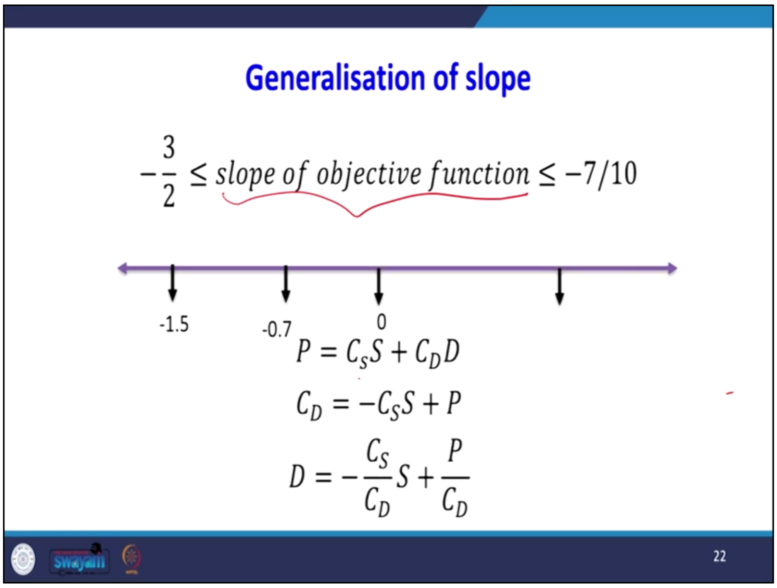
D= -(7 /10)S+ 630.

-(7 /10)= the slope of your line A, and the 630 is the intercept of line A on the D axis.



Similarly, we will find the slope of line B. In the figure,

We see that the equation for line B: 1S + (2/3) D = 708.



So, P = C_SS + C_DD,

C_S means coefficient of a standard back

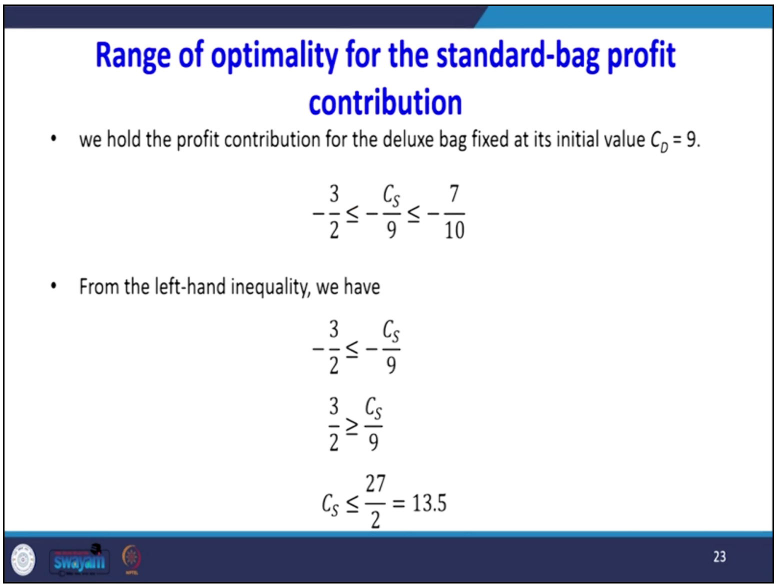
C_D means the coefficient of the deluxe back.

So, when I simplify

C_D = - C_SS + P.

So, if I further simplify, I get

D = - (C_S/C_D) S + P / C_D.



We hold the profit contribution for the deluxe bag fixed at an initial value C_D = 9.

We know that it is a 10S + 9D.

So, I will fix this value of C_D = 9.

We know that

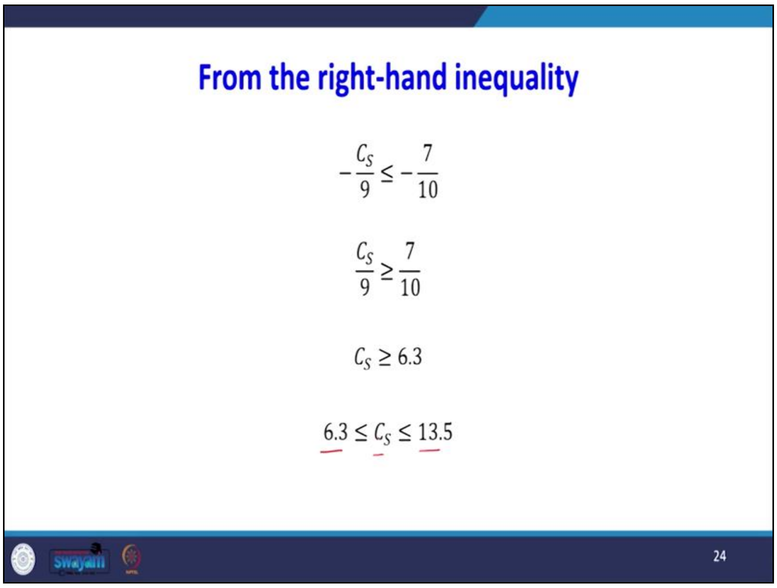
-3/2 <= - (C_S/C_D).

So, instead of C_D, I have substituted the value 9 <= - (7/10).

So, when I simplify, suppose I take only on the left-hand side.

When I simplify this, -3/2 <= - (C_S/9). So, when minus gets canceled.

So, the value of C_S =13.5.



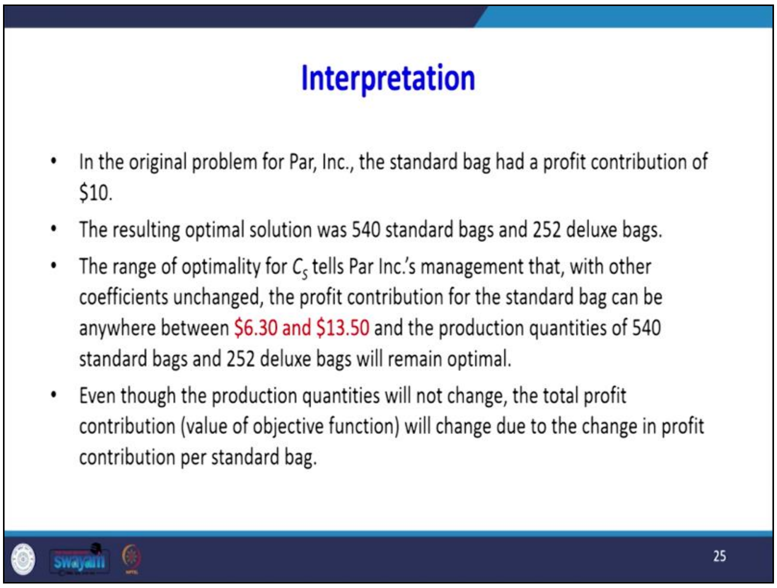
Similarly, if I take the right-hand side inequality

$-(C_S/9) <= -(7/10)$

$(C_S/9) >= (7/10)$

So, I am getting the $C_S = 6.3$.

So, what we got, I have got the coefficient value for the standard bag between 6.3 to 13.5. So, what this number says is that as long as the coefficient of the standard bag is within this range, that will not disturb our final answer. There is a final solution. So, that will not affect our optimality.

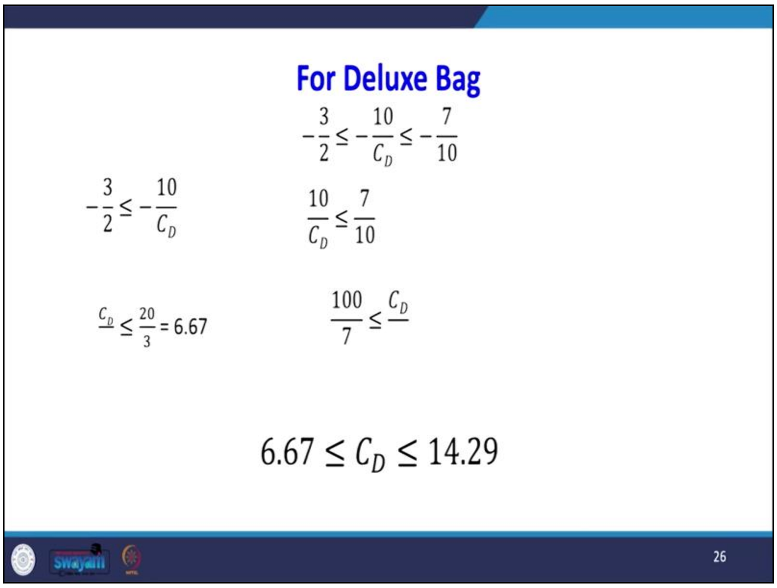


So, as we interpret in the original problem, the standard back had a profit contribution of 10 dollars. So, the optimal solution was S = 540, D = 252. So, the range of optimality for C_S tells for the given problem with other coefficients unchanged. What is the another coefficient of C_D? The CD is unchanged. That is why we have substituted $C_D = 9$; the profit contribution for the standard bag can be anywhere between 6.3 dollars and 13.5 dollars.

And the production quantities of 540 standard bags and 252 deluxe bags will remain optimal. That is the interpretation of the slope that I wrote in the previous slide. Even though the production quantities will not change the total profit contribution, the value of the objective function will change due to the change in the profit contribution of standard backs.

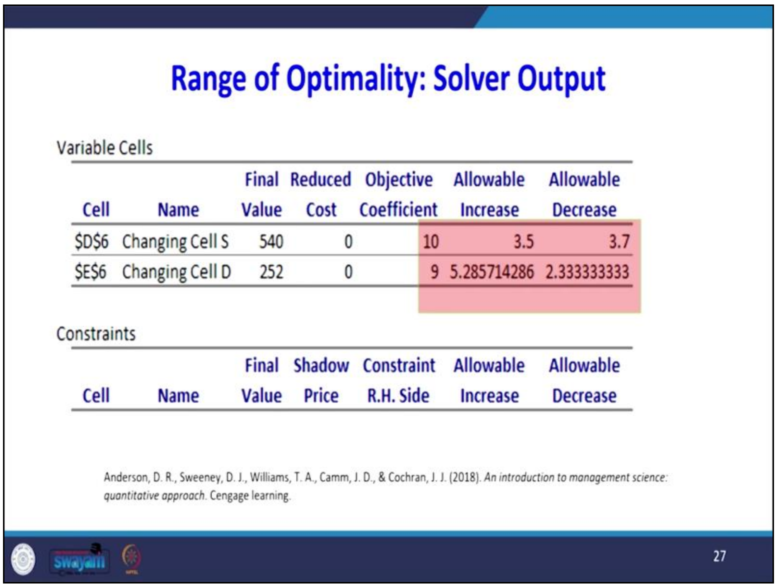
So, what is our objective function $z = 10S + 9D$?

So, the value of this coefficient of standard bags, even though it goes up to 13.5 and 6.3, our solution, which we got S = 540, D = 252, will remain the same. That is the interpretation of this range of optimality. So, range of optimality for standard bags. Similarly, we can find the range of optimality for the deluxe bag; also, that is what we have done here.



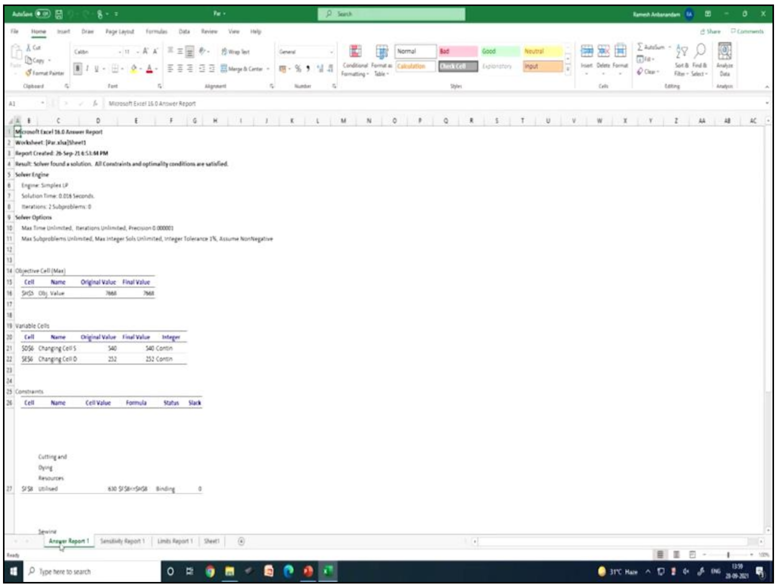
.So, the C_S value I have substituted is equal to 10 because
our objective function is z = 10S + 9D. So, I am going to find out what is the allowable range for the coefficient of deluxe bags by assuming that the coefficient of the standard bag is 10. So, 10 I have substituted, and then I have readjusted on the left-hand side.

So, I got 6.67 as the lower limit, and the upper limit is 14.29 so, what this number implies that as long as the coefficient of deluxe bags this one goes up to 6.97 and 14.29, our solution that is, our S = 540, D = 252 remain same.

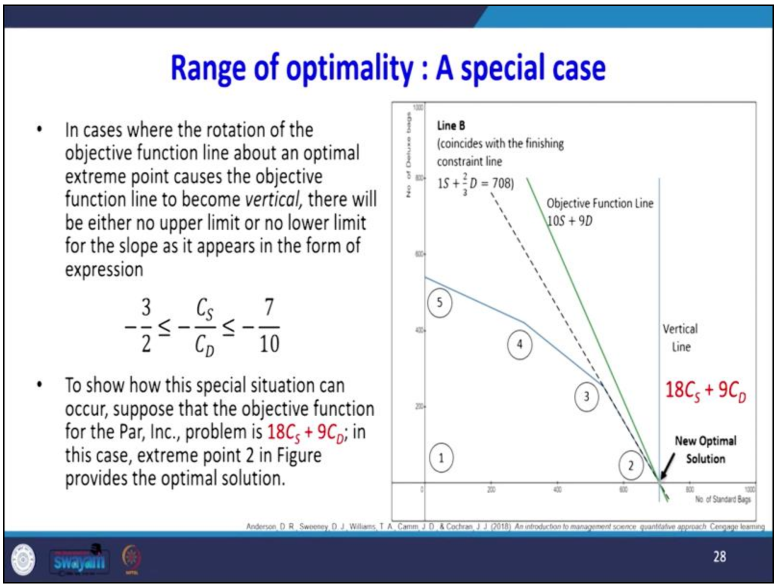


Now, how do we see this range of optimality in our solver output? So, I have brought the output of the solver in the variable cell you see that the final value that is our answer is objective function coefficient 10 and 9. So, here, the allowable increase is 3.5, which is 13.5, and the allowed decrease is 3.7. Similarly, for the deluxe bag, currently, it is 9; it can be increased by (9 + 5.2) and decreased by (9 - 2.3).

So, this is the place where you can get the range of optimality. So, the range of optimality provides flexibility in price changes, even though the price changes we need not bother about that because that will not affect our optimal solution. Now, I will show this output in our Excel sheet.



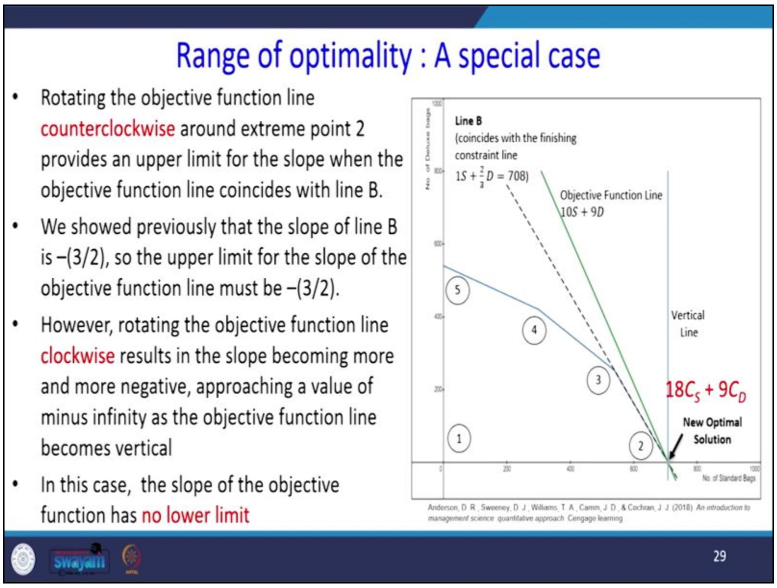
Now, this problem which we have solved in the previous class, we got the answers 540 and 252. When I go to this answer report, I have to go to the sensitivity report and look at this location. This location is the final value of our answer. There is an objective function that is allowable increase and allowable decrease. So, this says the range of optimality.



Now we can see the special case in the range of optimality. In cases where the rotation of the objective function line about an optimal extreme point causes the objective function line is become vertical. See, this green color saves our given objective function sometimes, if the objective function is vertical, there will be either no upper limit or no lower limit for the slope as it appears in the form of expression.

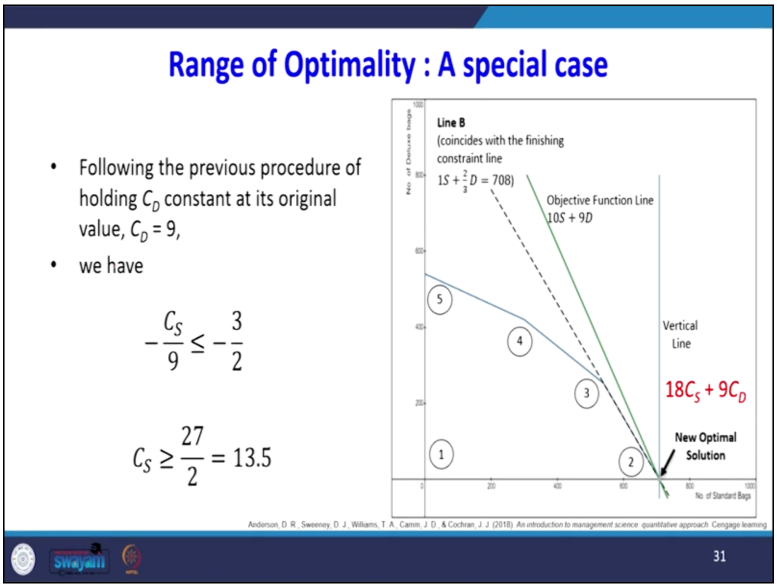
So, we know that

- (3/2) <= - (C_S /C_D) <= -(7/10). To show this special situation can occur suppose the objective function for the problem is (18 C_S+ 9C_D). Say this is the new objective function. In this case, the extreme point 2, so now this point 2 provides the optimal solution.



So, rotating this objective function line counterclockwise this way rotating in counterclockwise around extreme point 2 provides an upper limit for the slope when the objective function line coincides with line B. So, rotating the objective function line counterclockwise around extreme point 2 provides an upper limit for the slope when the objective function line coincides with the line B.

This is our line B. We showed previously that the slope of line B is - (3/2).
So, the upper limit for the slope of the objective function line must be minus - (3/2). There is an upper limit. There is no problem. However, rotating the objective functions clockwise, suppose you rotate this objective function this objective function clockwise. Clockwise results in the slope becoming more and more negative, approaching a value of minus infinity as the objective function line becomes vertical. In this case, the slope of the objective function has no lower limit.



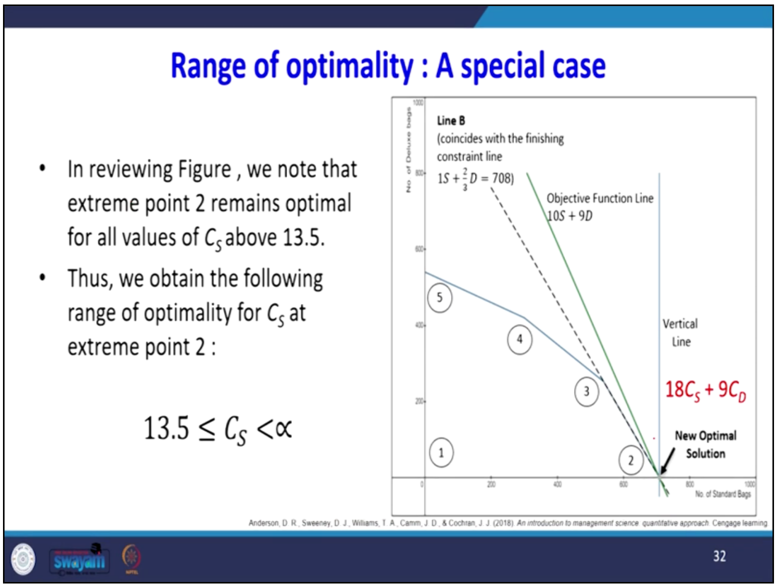
Using the upper limit of - (3/2)

we can write minus - (C_S /C_D) <= - (3/2), which is the slope of our objective function.

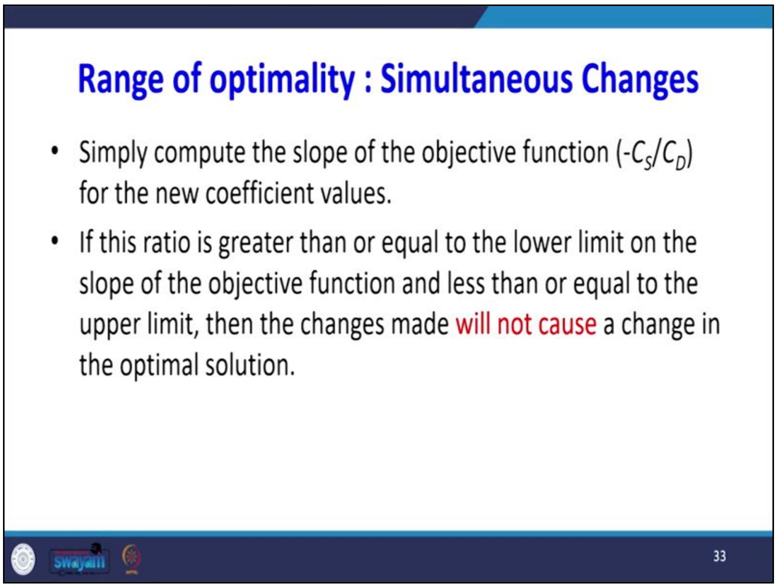
Following the previous procedure of holding C_D constant, the original value is C_D = 9.

So, we have - (C_S /C_D) <= - (3/2).

So, when you simplify this, we get C_S, which is 13.5.



In reviewing this figure, we note that extreme point 2 remains optimal for all values of C_S at about 13.5; thus, we obtain the following range of optimality for C S at the extreme point. So, the lower limit for this coefficient of a standard bag is 13.5, and there is no upper limit. This is the special case of optimality when the objective function is vertical.

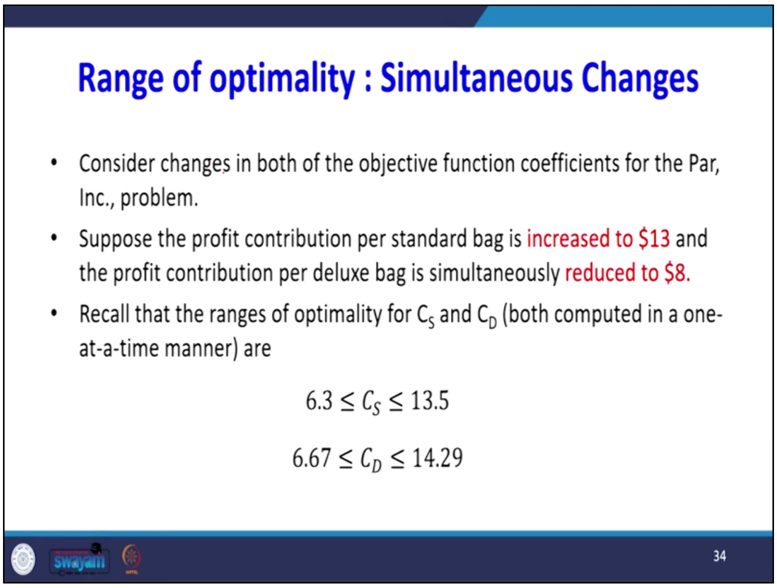


Now, there may be another situation: simultaneous changes. What is the meaning of simultaneous changes coefficient of standard bags and the coefficient of deluxe bags?

So, simply compute the slope of the objective function - (C_S /C_D) for the new coefficient values.

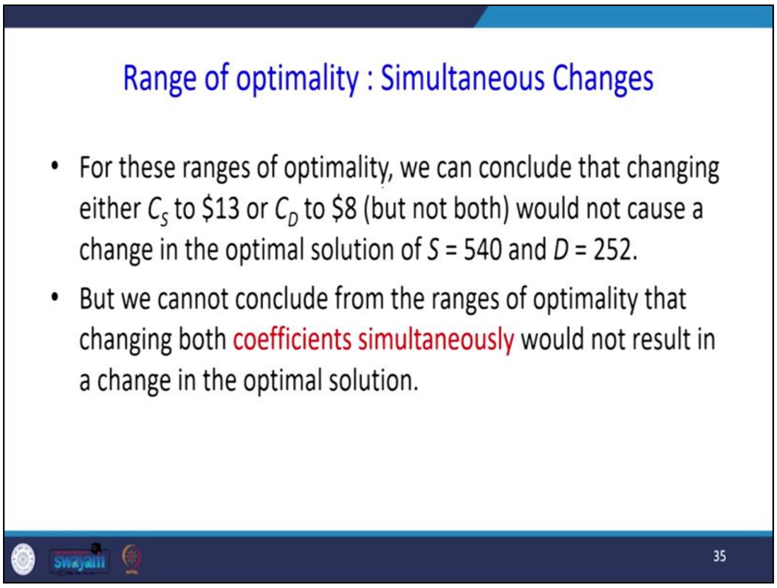
If this ratio is greater than or equal to the lower limit on the slope of the objective function or less than or equal to the upper limit, then the changes made will not cause a change in the optimal solution.

If there is a simultaneous change, you have to find the ratio of - (C_S /C_D) if that ratio is within the limit. So, our results will not change.



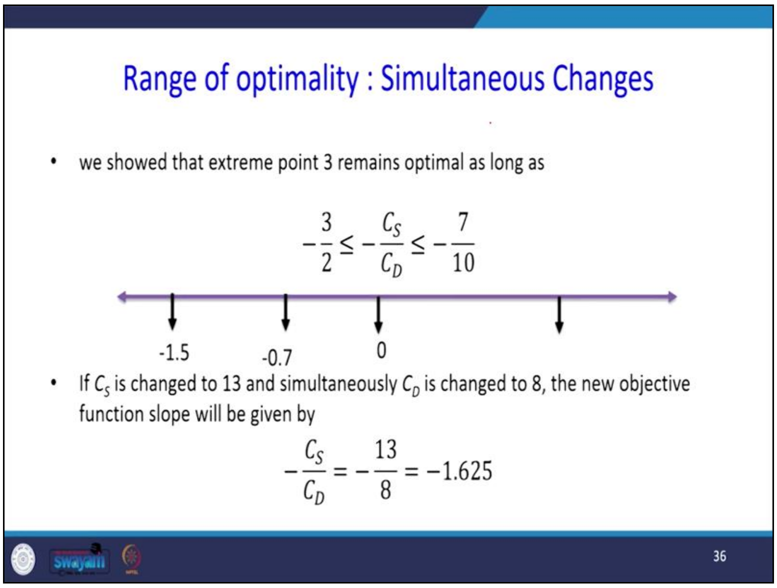
So, changes in both of the objective function coefficients for the problem we are discussing should be considered. Suppose the profit contribution for the standard bag is increased to 13, and the profit contribution per deluxe bag is simultaneously reduced to 8. Now, both things are happening together. Recall that changes for optimality for C_S and C_D both computed in one at a time manner the way we have arrived C_S the range of optimality because we have kept the coefficient of one.

For example, CS is kept constant, we found the answer for D. Similarly, then D kept constant, so we got the answer for S. But now there are simultaneous changes. So, C_S is 6.3 to 13.5. Now C_D is 6.67 to 14.29.



So, for these ranges of optimality, we can conclude that changing either C_S to 13 dollars or C_D to 8 but not both changes that can take place would not cause a change in optimal solution S = 540, D = 252. Even though there are 2 changes taking place we are considering only one change at a time. If there is any one change taking place at a time, both changes are within the limit and will not affect our optimal solution.

However, we cannot conclude from the ranges of optimality that changing both coefficients simultaneously would not result in a change in the objective function. So, what will happen if there are simultaneous changes that will affect our range of optimality simultaneous changes.

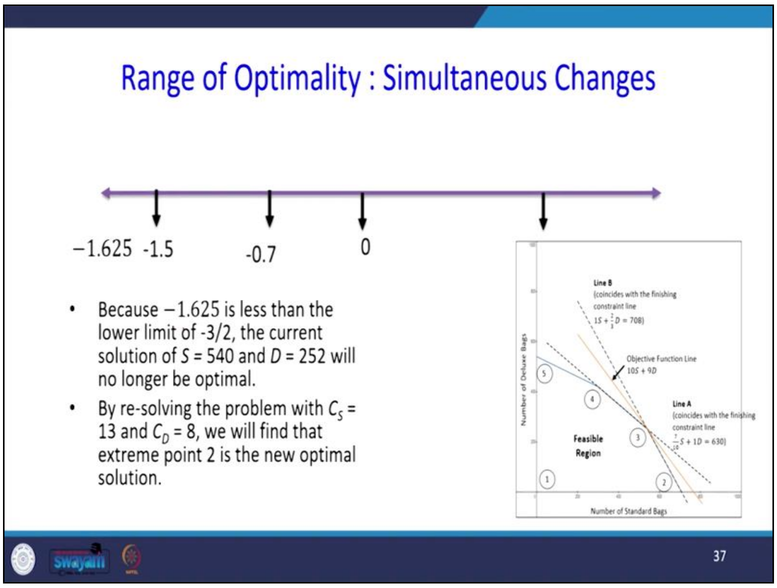


We showed that the extreme point 3 remains optimal as long as

$-(3/2) <= -(C_S/C_D) <= (7/10)$.

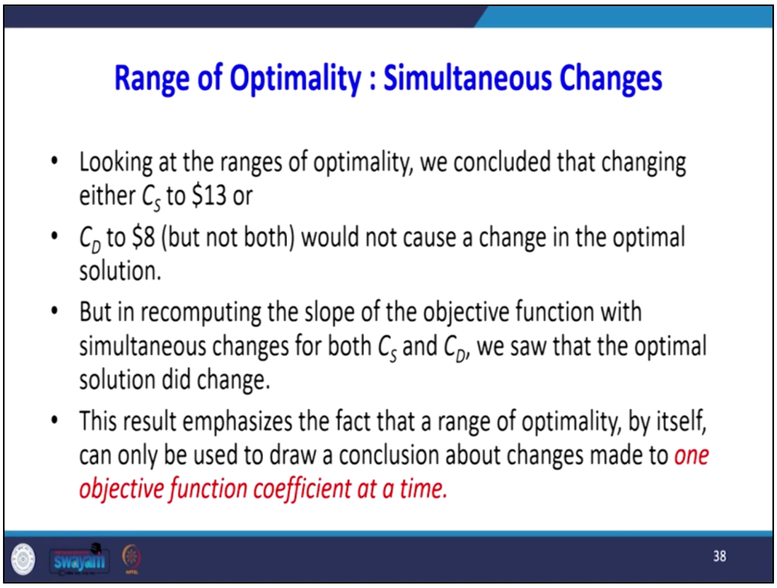
So, I brought in the number line 0 minus. So, as long as this ratio is within this limit that will not disturb our optimal solution if the C_S is changed to 13 and simultaneously C_D is changed to 8 the new objective function slope will be given by minus C_S by C_D.

So, this - 1.62. So, what will happen if both the things are changes are taking place simultaneously so you will not get the optimal suit that is you will get optimal solution solution you already have that will be disturbed because it is going beyond the ranges yeah - 1.625?



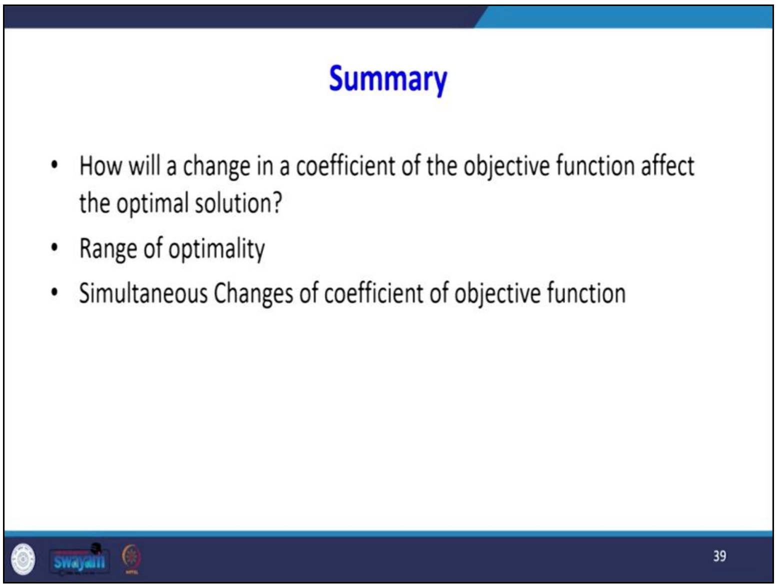
Because -1.65 is less than the lower limit of - (3/2) the current solution S = 540 and D = 252 will now will no longer be optimal. So, in this case, by resolving the problem with the C_S = 13 and C_D = 8, we will find that extreme point 2 is the new optimal solution. Now, this is 2 is the new optimal solution. So, what we are learning from here if there is a simultaneous change, then we have to resolve the problem.

So, in sensitivity analysis, what we have discussed so far is any one value changes at a time, not both. If both the coefficients are changing at a time we have to resolve the problem.



Range of optimality simultaneous changes looking at the ranges of optimality we conclude that changing either C_S = 13 or C_D = 8 but not both would not cause change in the optimal solution. But in recomputing the slope of the objective function with simultaneous changes for both C_S and C_D, we saw that the optimal solution did change. So, what we are learning if there is a simultaneous change, our optimal solution will change.

So, the result emphasizes that the range of optimality by itself can only be used to conclude changes made to one objective function coefficient at a time. If both the coefficients are changing simultaneously, the range of optimality, the answer we got will not be valid.



Now we will summarize what we have learned in this class we have learned how. Will a change in the coefficient of an objective function affect the optimal solution? Then we learned a new term called the range of optimality. Then, we saw if there were simultaneous changes in the coefficient of the objective function and how that would affect our optimal solution. Ok, student, in the next class, we will learn how a change in the right side value of constraint affects the optimal solution.

So far, we have seen if any changes in the coefficient of objective function how data will affect our optimal solution, but in the next lecture, if the right-hand side of the constraint is changing, how that will affect our optimal solution? Thank you.