Decision Making with Spreadsheet
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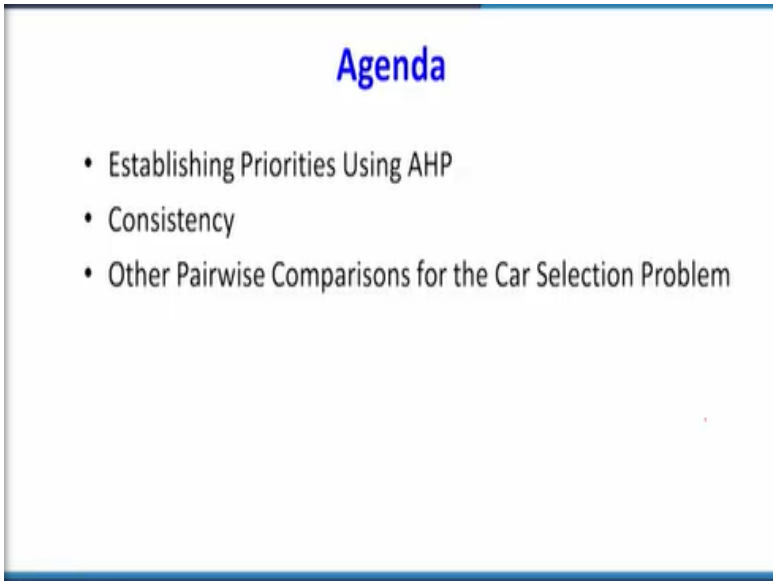
Lecture - 55
Multicriteria Decision - III

Dear students, we will continue with the previous lecture on the analytic hierarchy process. So far, we have discussed how to do pairwise comparisons. In this lecture, I will explain how to achieve a priority level for criteria and an overall priority level for the three alternatives.



The agenda for this lecture is to establish priorities for criteria and alternatives using AHP. Next concept, I am going to explain a very important concept in AHP called consistency, then other pairwise comparisons for the car selection problem.

Ranking of priorities

- Consider [$Ax = \lambda_{max} x$] where
  - A is the comparison matrix of size n×n, for 'n' criteria, also called the priority matrix.
  - x is the Eigenvector of size n×1, also called the priority vector.
  - λ_{max} is the Eigenvalue, $\lambda_{max} \in \Re > n$.
- To find the ranking of priorities, namely the Eigen Vector X:
  1) Normalize the column entries by dividing each entry by the sum of the column.
  2) Take the overall row averages.

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018) *An introduction to management science: quantitative approach*. Cengage learning.

First, the ranking of priorities considers $Ax = \lambda_{max} x$. Here, A is the comparison matrix for size n cross n; for the priority level, we have a four priority, so it is a four cross 4, also called the priority matrix. So, A is the priority matrix, and x is the eigenvector of size n cross 1, also called the priority vector. Lambda max is the eigenvalue; the lambda max belongs to a real number, which should be greater than n, and n is the size of the matrix.

To find the ranking of priorities, namely the eigenvector X, there are two steps. first, we must normalize the column entries by dividing each entry by some of the columns. Then, we must take the overall row averages. So far, we have got only pairwise comparisons. We will perform these two steps to get the priority vector x.

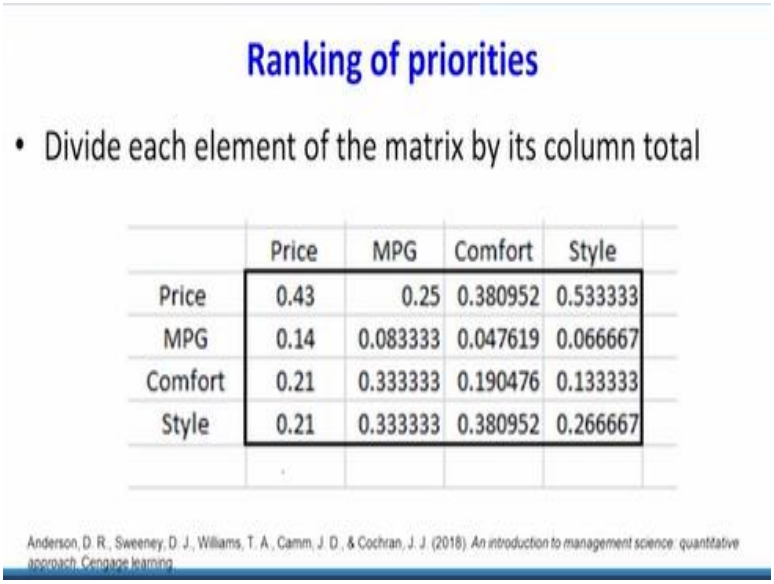
Ranking of priorities

- Sum the values in each column.

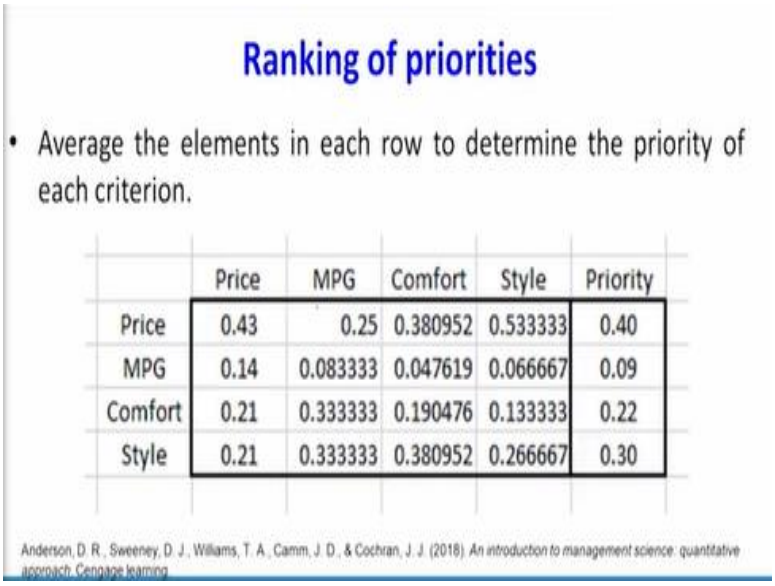
	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	1/3	1	1/4	1/4
Comfort	1/2	4	1	1/2
Style	1/2	4	2	1
Column Sum	2.333333	12	5.25	3.75

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018) *An introduction to management science: quantitative approach*. Cengage learning.

So, this slide has come from this picture has come from our previous lecture. Here there are four criteria prices, miles per gallon, comfort and style. We have got the pairwise comparison. The first step is to sum the values in each column. For example, here we have to sum this 1 + (1/3) + (1/2) + (1/2). So, you will be getting 2.33. Similarly, for the comfort column, also do the row sum.

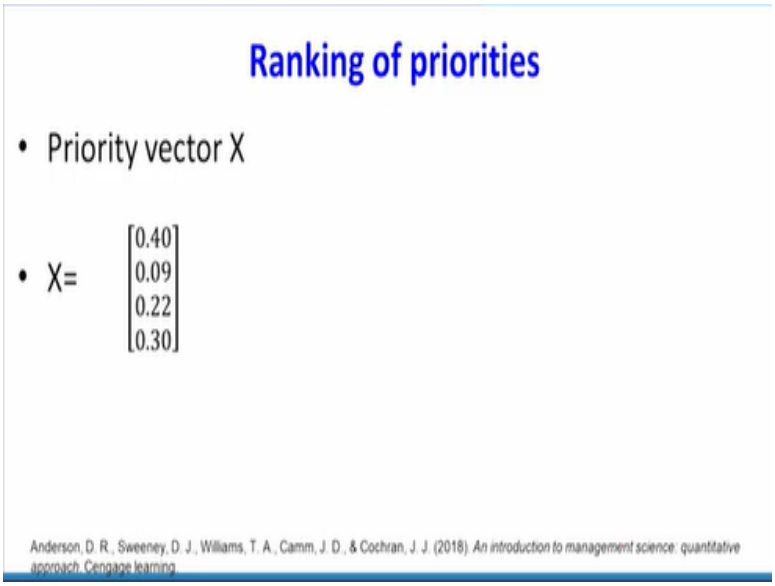


After doing this row sum, for example, the price is 2.33 seconds, and one divides each element of the matrix by its column total. So, we got the row sum, so how did we get these four points? Sorry, 0.43? So, that element is divided by column sum. For example, you see it is one upon 2, 0.33, so this value is one upon 2.33.

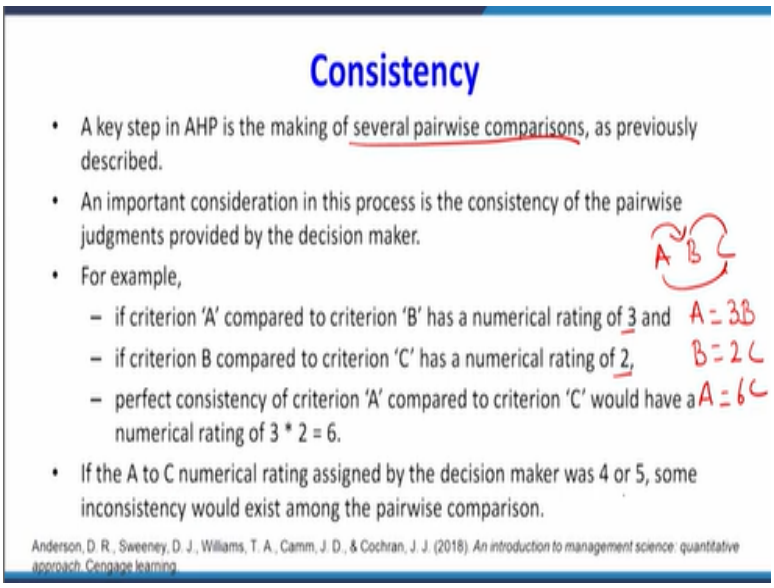


Similarly, we have to get for each criterion. The third step is to average the elements in each row to determine the priority level of each criterion. So, after dividing each value by its corresponding column sum, you have to find the average of this row. So, there are four elements you submit by four, you will be getting 0.40; for miles per gallon, it is 0.09. For comfort, it is 0.22. For style, it is 0.30, so this matrix is the x matrix called priority.

So, that decision maker has given weightage 40% for the price, 9% for miles per gallon, 22% for comfort, and 30% for the style. So, for the decision maker, the important criterion is the price because it is 40%. The next one is the style, and the next one is the comfort. In the end, it is miles per gallon.



So this is called priority vector X.



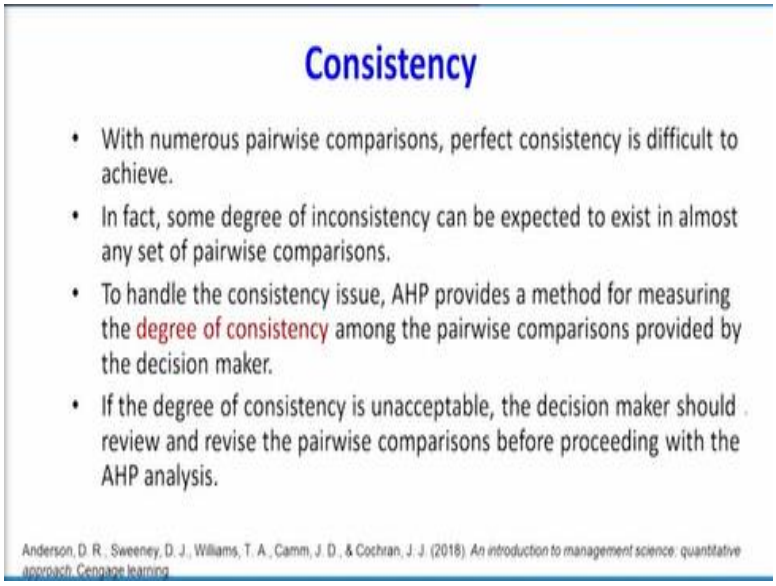
Now, I am going to explain a concept called consistency. Here, consistency means consistency in giving the weightage for two criteria when we do the pairwise comparison. A key step in AHP is the making of several pairwise comparisons, as previously described. An important consideration in this process is the consistency of the pairwise judgments provided by the decision-makers.

For example, if criterion A compared to criterion B has a numerical rating of 3, if criterion B compared to criterion C B and C has a numerical rating of 2, the perfect consistency of criterion A compared to C would have a numerical rating of 3 multiplied by 2 = 6. So, when you say A = 3B, B = 2C, so there are A, B, and C, so when you compare A and B, A = 3B. When you compare B and C, it is B = 2C, so when you compare A and C it should be A = 6C.

If the A to C numerical rating assigned by the condition maker was 4 or 5, suppose the decision maker is giving less than 6, 4, or 5. Some inconsistency would exist among the pairwise comparison. The reason for this inconsistency is that in AHP, we do the pairwise comparison by considering only two criteria at a time. So, as a decision maker, A and B you can remember what the weightage you were given was, and B and C. also you can remember what weightage you were given.

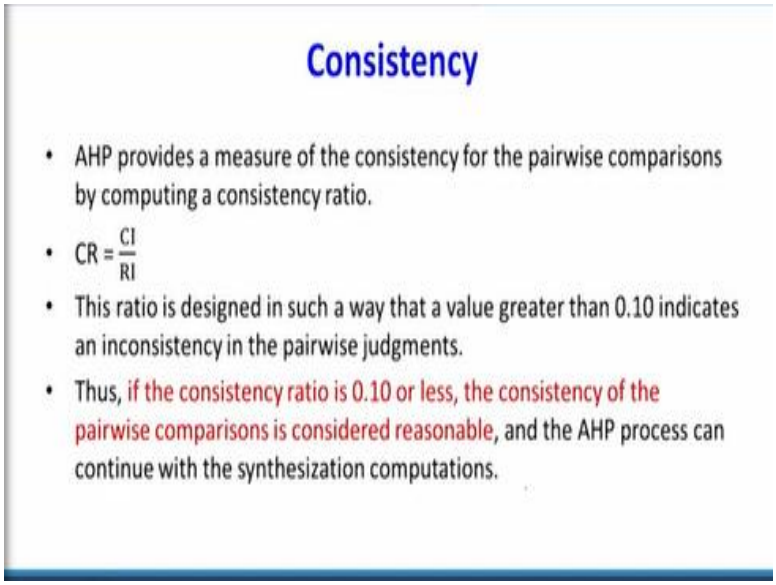
However, when you compare A and C, there is a possibility you may not maintain consistency while giving the rating. That consistency concept is explained by this one: A = 3B, B = 2C, then

A should equal 6C. If the weightage is less than 6, there is an inconsistency. They should not be there.



With the numerous pairwise comparisons, perfect consistency is difficult because, as a decision-maker, you cannot remember. In fact, some degree of inconsistency can be expected to exist in almost any set of pairwise comparisons. To handle the consistency issue, AHP provides a method for measuring the degree of consistency among the pairwise comparisons offered by the decision maker.

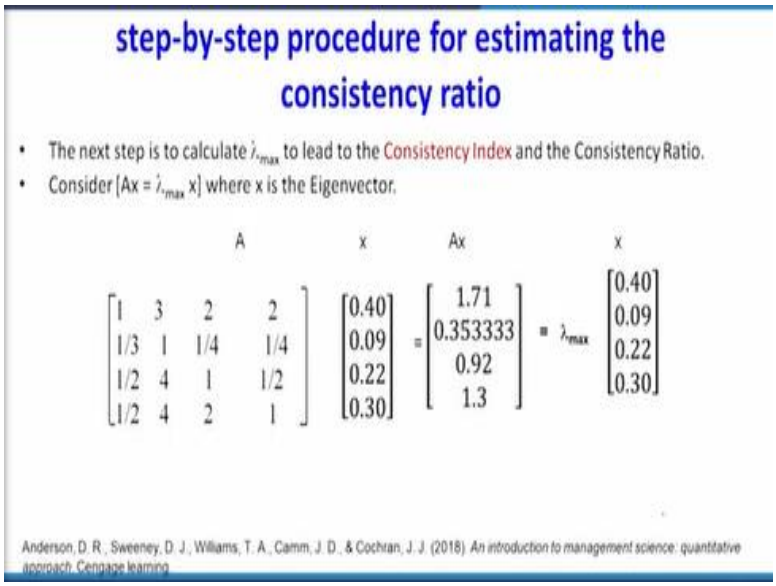
If the degree of consistency is unacceptable, then the decision maker should review and revise the pairwise comparisons before proceeding with the AHP analysis. So, what the decision maker has to do? For every matrix, every pairwise comparison matrix, you have to check the consistency accepted level is there. If it is not up to the accepted level, he has to revise these pairwise comparisons to which the decision maker was given a different weightage.



AHP provides a measure of consistency for pairwise comparisons by computing the term called the consistency ratio. How are we going to get a consistency ratio? So, the consistency ratio is nothing but a consistency index divided by a random index. So, first, we will get the consistency index, and then we will divide that value by this random index. This random index is provided by the sortie, which developed this algorithm.

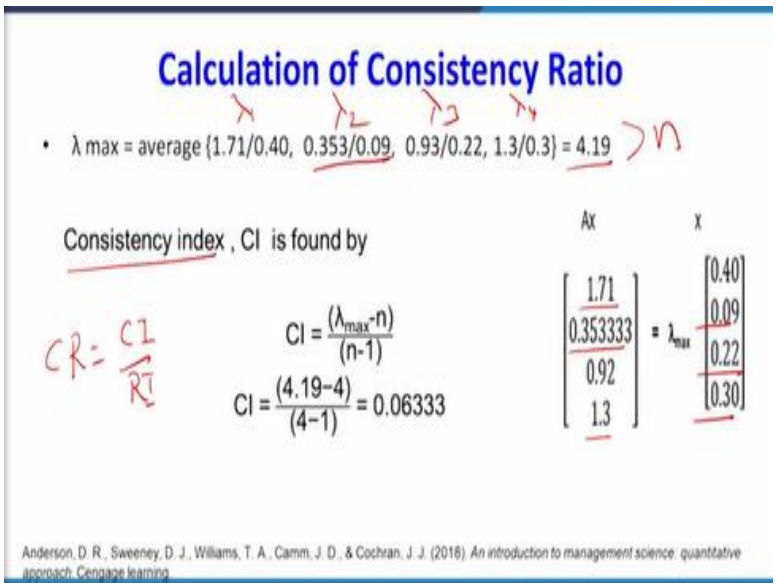
This ratio consistency ratio is designed in such a way that a value greater than 0.10, if the consistency ratio is greater than 10%, indicates there is an inconsistency in the pairwise judgment. If the consistency ratio is 0.10 or less, the consistency of the pairwise comparison is considered reasonable. If it is less than 10%, it is acceptable, and the AHP process can continue with the synthesization computations.

If it goes more than 10% we have to ask the decision maker to revise the value which has provided for the pairwise comparisons.



Next, I am going to explain the step-by-step procedure for estimating the consistency ratio. Remember to get the consistency ratio first. You should know the consistency index. So, the next step is to calculate lambda max to lead to the consistency index and then the consistency ratio. So, first, we have to find out the consistency index. For that, you need the lambda max, and then the consistency index will help you to get the value of the consistency ratio.

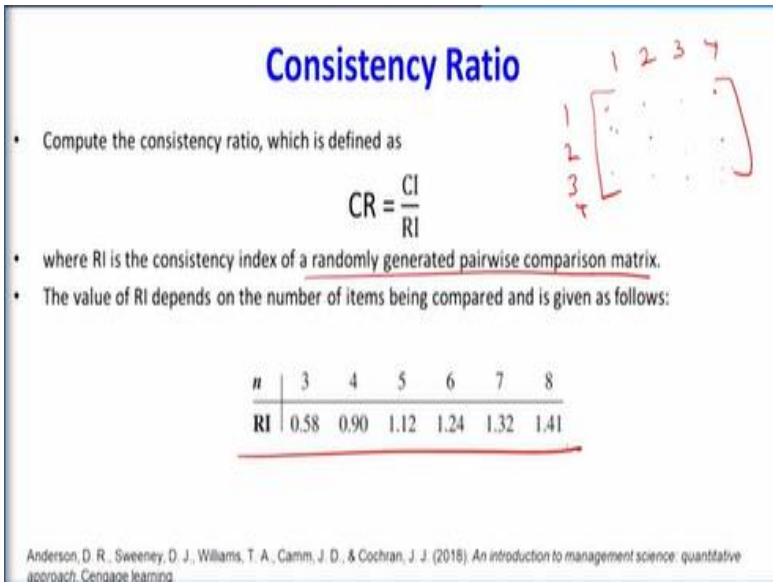
So, we know $Ax = \lambda_{max}\ x$, so I have taken the first pairwise comparison, which we have used for our comparing the different criteria. There are four criteria: price, miles per gallon, comfort, and style. This priority matrix also we have got it so when you multiply, you will get Ax. You know that $Ax = \lambda max\ x$, so lambda max this x also you know. So, first, what do we have to do? We have to find out what the lambda max is.



The lambda max is how we are going to do how we are going to calculate. So, you see this lambda max, so we are going to divide this 1.71 by 0.40, so you will get lambda 1. Then, when you divide 0.353 by 0.09 this value you will get lambda 2. So, this is your lambda 1 lambda 2 lambda 3 lambda 4. So, the lambda 3 is 0.92 divided by 0.22 lambda 4, 1. 3 upon 0.3. So, out of this, which is the highest?

So, after finding lambda 1, lambda 2, lambda 3, lambda 4 you have to find the average of this lambda 1, lambda 2, lambda 3, lambda 4 that average is 4.19. You see that this is greater than our n that was our condition. So, once we know the lambda max, lambda max is used to find the consistency index. So, you know that what is the consistency ratio = consistency index divided by random index? I will explain the concept of a random index.

So, first, we have to find out the consistency index. So, the consistency index is lambda max – n, and n is the size of the matrix divided by (n -1). So, we got lambda max (4.19 – 4) / (4- 1), which will be getting 0.063, so we get a consistency index.

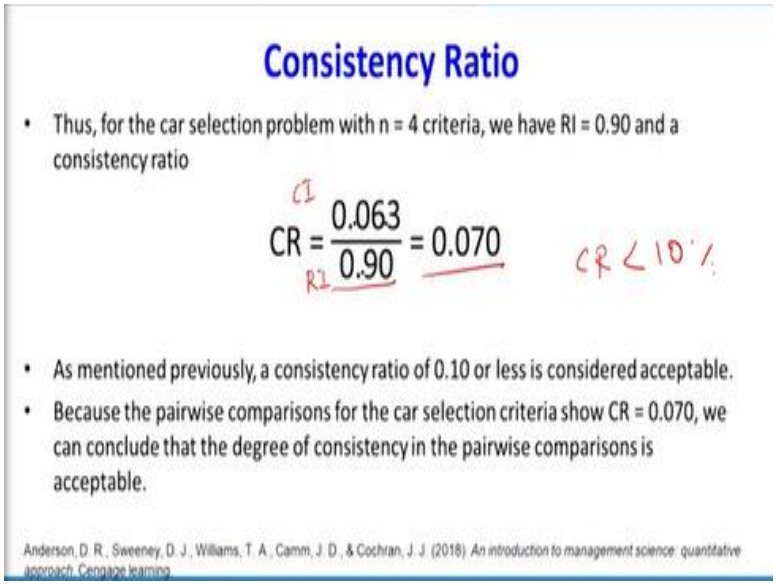


The next term is a random index. So, to compute the consistency ratio, which is defined as CR, it is a consistency index derived by random index. Here, RI is the consistency index of a randomly generated pairwise comparison matrix. What is the meaning of this randomly generated pairwise comparison matrix? Remember there was 1, 2, 3, 4; 1, 2, 3, 4. We have got different pairwise comparison values after discussing the expert.

Assume that you are not discussing with an expert. You randomly provide some number. If you provide some numbers, this RI gives the consistency index for the randomly generated pairwise comparison matrix. So, if you randomly put different numbers, then you find the consistency index. We know how to find out the constants index. We have to find lambda 1 lambda 3, and then we have to find the average of this lambda.

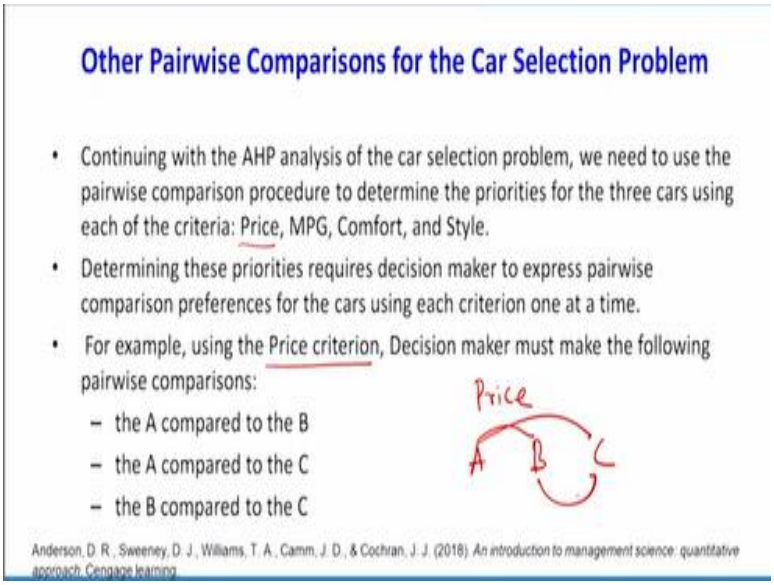
So, that value is given in the form of this table, which Professor Sarty has provided. Suppose the size of the matrix is 3 and the random index is 0.58. How did we get 0.58? He got some experiments, so he has given an approximate value by considering all by considering all possibilities of different values in the pairwise comparison matrix. That means by randomly giving some numbers and then finding the index of the constraint. That index is called your random index.

So, in our problem, the size of the n matrix is 4, so we are picking this random index from this table as 0.90.



So, in the next slide what we have done this value is our random index. The top one, the numerator, is we know already the consistency index. When you divide the consistency index by the random index, we get 0.070. Remember that I have already explained if the CR is less than 10%, it is acceptable. Now, it is coming to only 7%. So, the pairwise comparison matrix, which you have considered the consistency, is maintained.

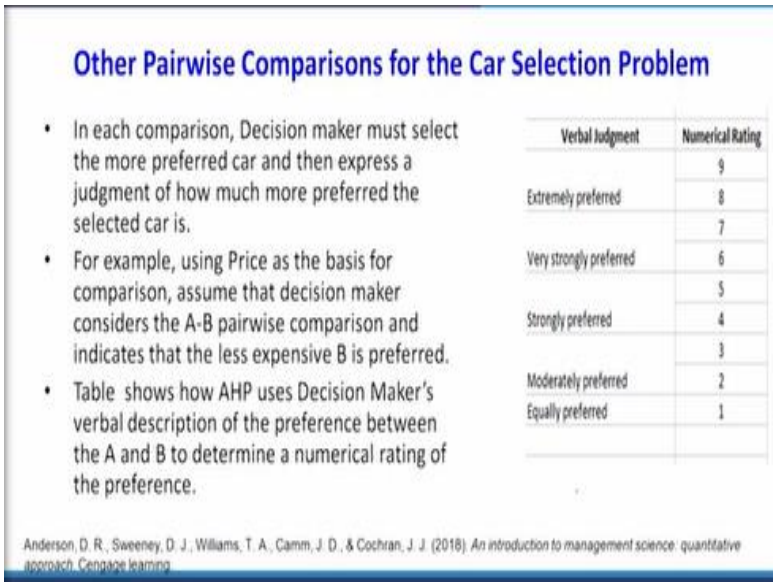
Otherwise, the inconsistency is within the accepted level. As mentioned previously, a consistency ratio of 0.10 or less is considered acceptable. Because the pairwise comparison for the car selection criteria shows that the constraints ratio is 7%, we can conclude that the degree of consistency in the pairwise comparison is acceptable, and we can proceed.



So far, I have explained how to get pairwise comparisons and how to get to the priority level for the criteria then. I also have explained how to check the consistency. Now, we are going to do another pairwise comparison for the car selection problem. So, continuing with the AHP analysis for the car selection problem, we need to use a pairwise comparison procedure to determine the priorities for the three cars using each of the criteria.

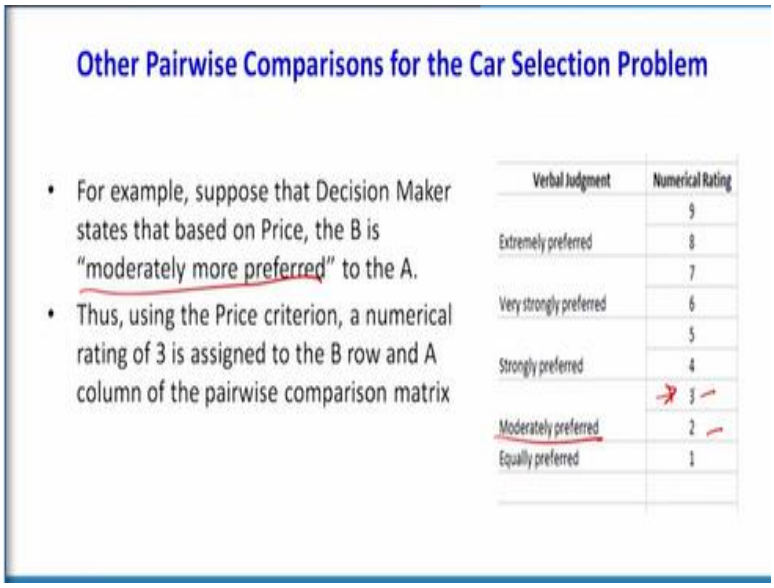
So, by considering price as the criteria, we are going to do a pairwise comparison for the three cars similarly for miles per gallon, comfort, and style. Determining these priorities requires the decision maker to express pairwise comparison preferences for the cars using each criterion one at a time. For example, using price criterion the decision maker must make the following pairwise comparison.

So, for price, he has to compare A and B, A and C, and B and C, so there will be three matrix three pairwise comparison matrices. Similarly, by considering miles per gallon there will be another three pairwise comparison matrix. Similarly, by considering comfort as the criteria you will get another three matrices. Similarly, style is the criteria to do another three pairwise comparisons for the three alternatives there is a car.



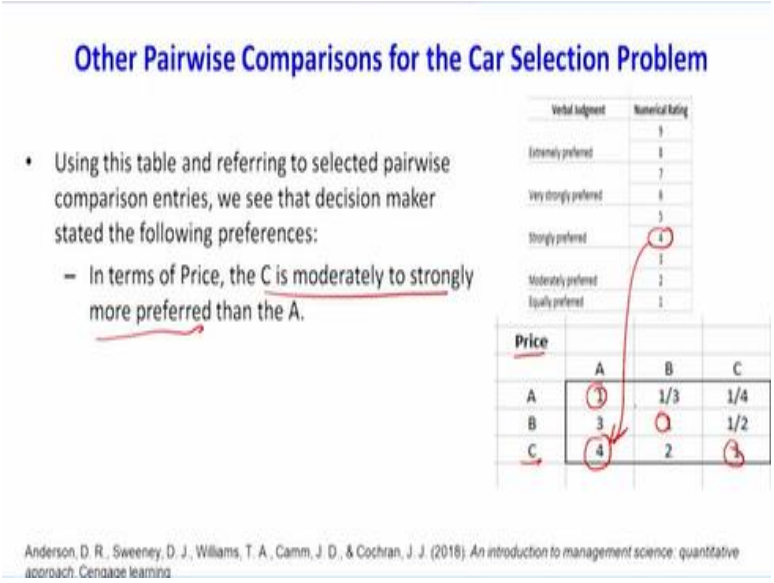
In each comparison, the decision maker must select the more preferred car and then express judgment of how much more preferred the selected car is. This is similar to what we have done for our priority matrix. For example, using price as the basis for the comparison, assume that the decision-maker considers car A and B comparison and indicates that the less expensive B is preferred.

So, the table shows how AHP use uses the decision maker's verbal description of the preference between A and B to determine a numerical rating for the preference. Suppose we compare if he has preferred A and B as equally preferred one when compare suppose when comparing A and B suppose he prefers B moderately preferred. So, it can be 3 or 2 strongly preferred, it can be 4 or 5. So, this scale is used for numerical ratings for each verbal judgment.



For example, the decision maker states that based on the price, B is moderately more preferred to A. See that this term is moderately more preferred. Thus, using the price criterion, the numerical rating of 3 is assigned; why 3? Because it is moderately preferred. So, you can have 3 or 2, but it should be consistent when you say moderately preferred. It is up to you to decide if you can have 3. So, for every pairwise comparison, if it is moderately preferred, you should use 3.

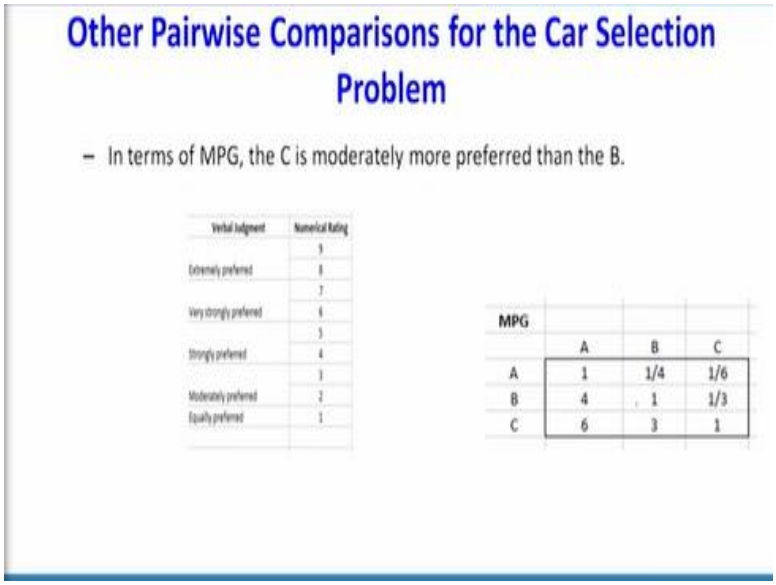
So, a numerical value of 3 is assigned to the B row and A column of the pairwise comparison matrix.



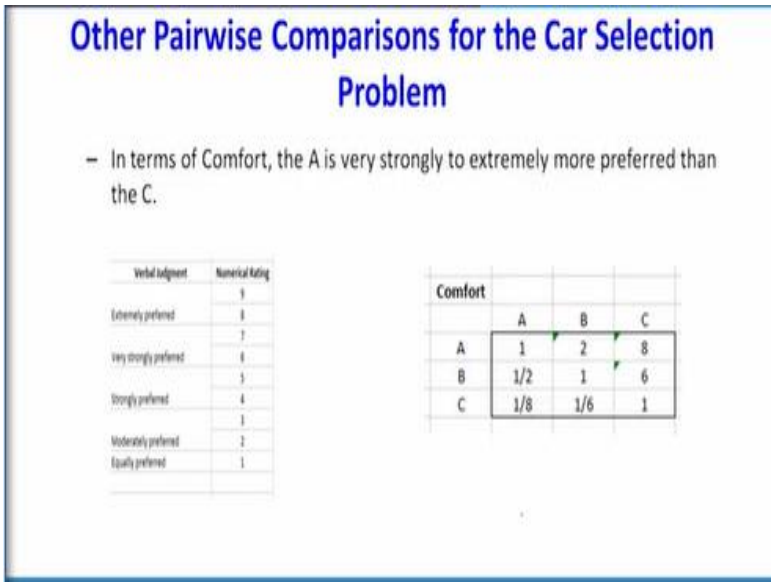
Other pairwise comparisons for the car selection problem. So, we got one table; for example, with respect to price, we asked the decision maker what is their preference. We know that the

diagonal value is 1, 1, 1, 1. Suppose here it is a 4. What is the meaning of this 4? So, with respect to the price, we asked the decision makers C and A which you prefer, so he answered I prefer C. How much more do you prefer C than A? So, he answered C is moderately too strongly preferred.

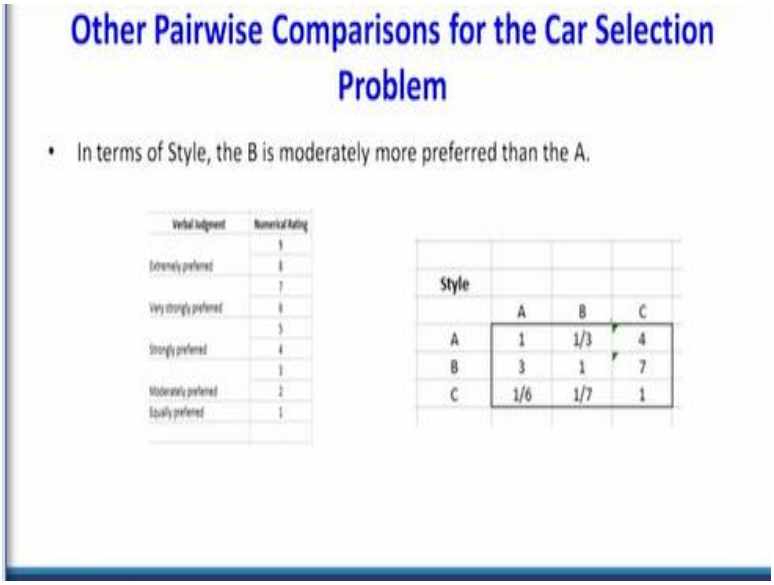
So, for this verbal term, moderately too strongly so here this is the 4, so 4 I have entered here. So, every numerical value is entered by looking at the decision maker's verbal statement and corresponding numerical rating.



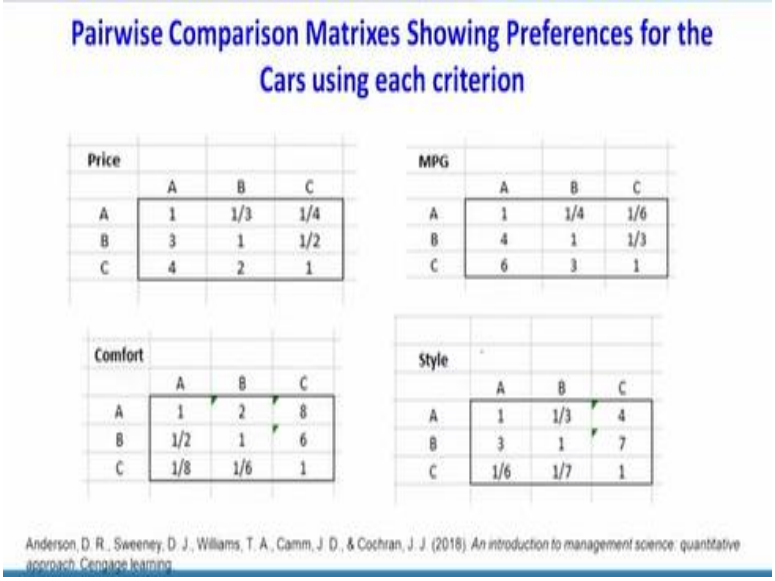
Similarly, I have asked the decision maker with respect to miles per gallon if you compare cars C and B, and he has answered he prefers C. How much more does he prefer C than B? So, his answer C is moderately more preferred than I have checked for this term moderately more preferred, which means moderately more preferred means this 3. So, the value is 3.



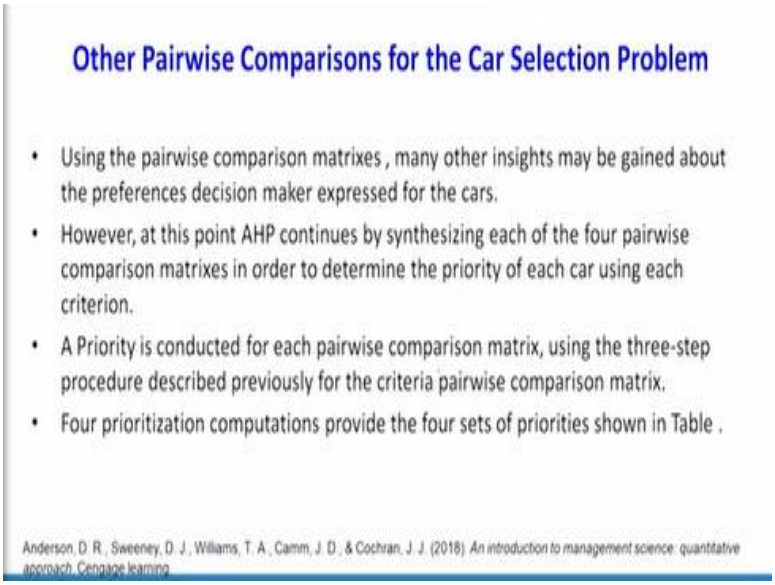
In terms of comfort, the respondent, the decision makers, answered A is very strongly to extremely preferred. So, he prefers A strongly to extremely preferred, so that means this value 8, so here we have entered 8. Likewise, I have filled in all pairwise comparisons in terms of style.



So, he answered he likes B, but B is moderately more preferred than A, moderately more preferred so the rating is 3, so I have entered 3. Likewise, I filled all other cells.

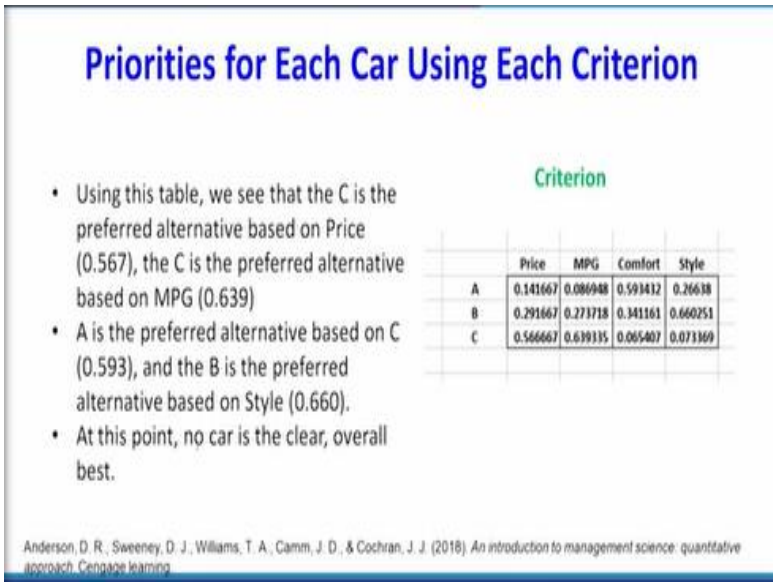


Now I brought the summary. So, this pairwise comparison matrix is only for the price by considering all three alternatives. This is with respect to miles per gallon comfort and style.



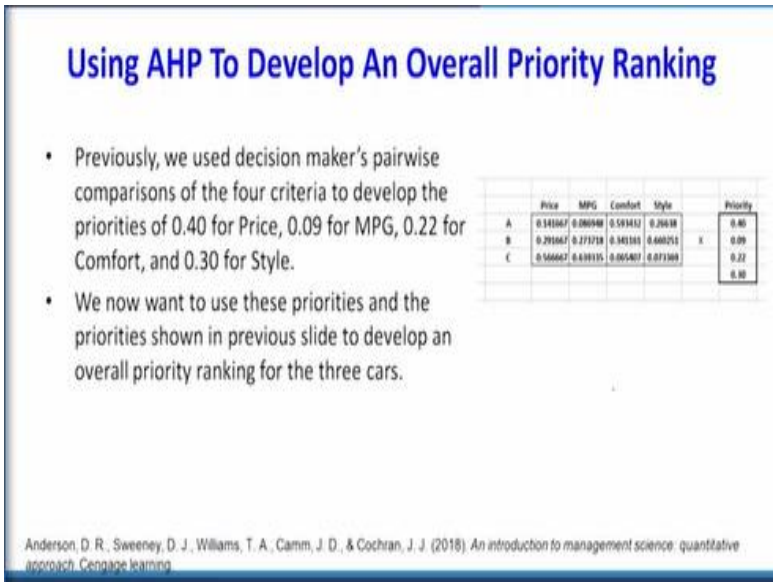
Using the pairwise comparison matrix, many other insights may be gained about the preferences decision makers expressed for the cars. However, at this point AHP continues by synthesizing each of the four pairwise comparison matrices in order to determine the priority of each car using each criterion. So, we are going to find out the priority of each car using each criterion. So, a priority is conducted for each pairwise comparison matrix using three step procedures described previously for the criteria pairwise comparison matrix.

What are the three sub-procedures? First, we have to find out the column sum, and then each element has to be divided by the corresponding column sum. In the end, we have to find out the row average. That is what we are going to do. So, four prioritization computations are provided for four sets of priorities, as shown in the table.

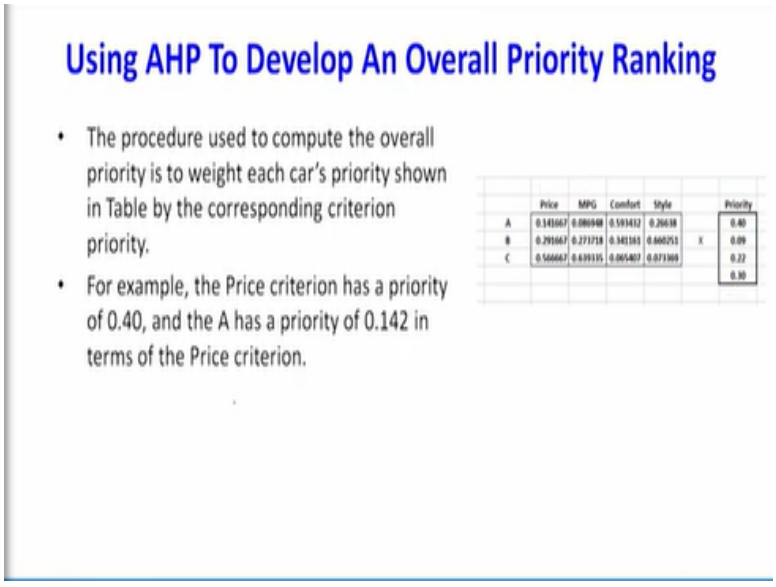


So, with respect to price, we got the priority level for three alternatives. With respect to matrix miles per gallon, we got a priority matrix. How did we get it? So, we have to go to the price of the pairwise matrix of price with respect to three alternatives. We have to do column sum. Each element has to be determined by the corresponding column sum, and then you have to find the row average. So, that row average is this value 0.14, 0.29, 0.56 like that we have to do for all four criteria.

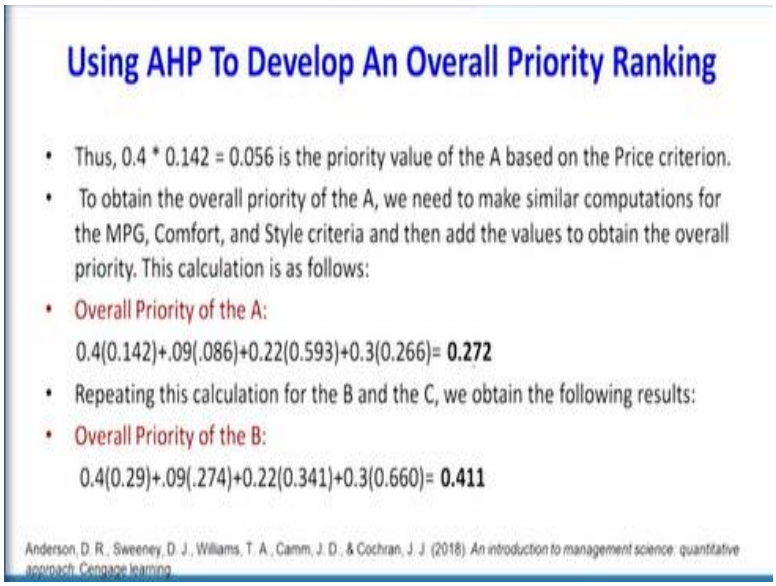
So, using this table, we see that C is the most preferred alternative based on price because the value of C is 0.56, so C is preferred alternatively based on miles per gallon. Again, C 0.63 is the highest value when we compare with respect to miles per gallon. So, car C is preferred with respect to comfort, A is most preferable with respect to style, and B is most preferable. So, at this point, no car is the clear. We are not able to find out overall which is best. So, now we are going to do small mathematical calculations.



So, what we have done? Previously, we used decision-makers pairwise comparisons of the four criteria to develop the priorities. That priority matrix I have brought it here; this is the priority matrix for different alternatives with respect to different criteria. So, now we want to use these priorities and the priority shown in the previous slide this one to develop an overall priority ranking. So, what I am going to do? I am going to do a matrix multiplication.



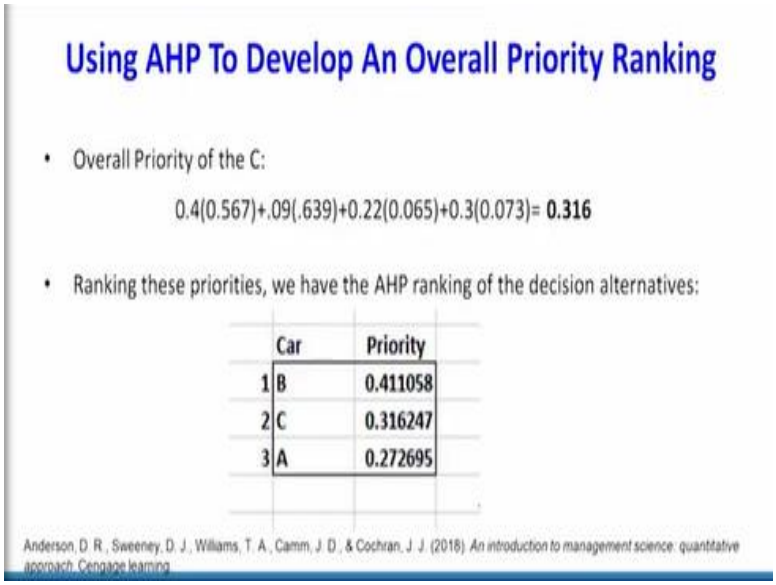
The procedure used to compute the overall priority is to weigh each car's priority shown in the table by corresponding criterion priority. For example, the price criterion has the priority of 0.40 this one and A has the priority of 0.142 in terms of price criterion here. So, I am going to multiply these two.



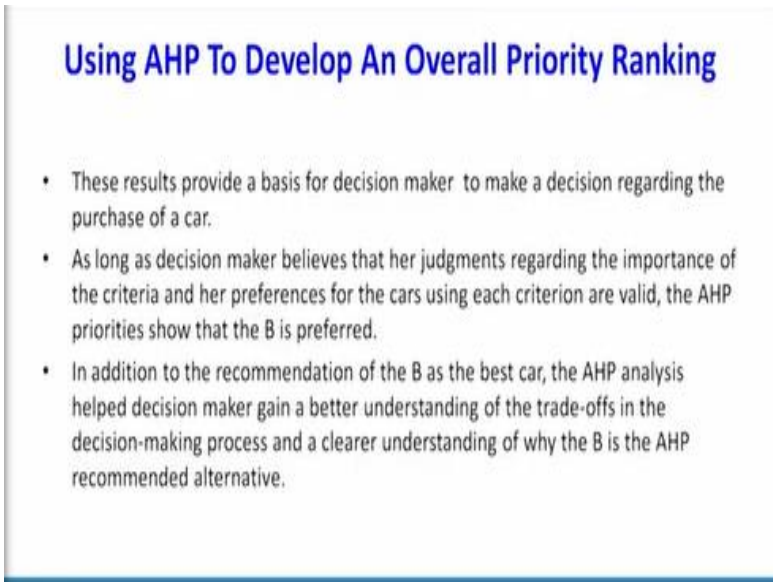
So, when I multiply 0.4 and 0.142, I am getting 0.056 is the priority value of the A based on the price criterion. So, to obtain the overall priority of the A, we need to make similar computations for miles per gallon comfort and style criteria and then add the values to obtain the overall priority. So, simply, it is matrix multiplication. So,

0.4(0.142)+.09(.086)+0.22(0.593)+0.3(0.266)= **0.272**

you will get 0.272. Similarly, for B you will be getting 0.11.



Similarly, for C, you will be getting 0.316, so when I arrange it in descending order, the first priority is car B because it is 41%. The second priority is car C, and the third priority is car A.



So, you see, these results provide a basis for decision-makers to make decisions regarding the purchase of a car. As long as the decision maker believes that her judgment regarding the importance of the criteria and her preference for the cars using each criterion is valid, the AHP priorities show that B is the preferred car. In addition to the recommendation of car B as the best car, the AHP analysis helped the decision-maker gain a better understanding of trade in the decision-making process.

And a clearer understanding of why B is the AHP-recommended alternative. Now, I am going to open Excel for the whole process of AHP; I am going to explain with the help of Excel.
Now I am going to do how to get the priority level for criteria. The first step is to do the priority level. Now look at the matrix here, which is J19, J19 to M22. So, this pairwise comparison was given by the decision maker. So, what have I done? I have found the column sum 2.33 for price miles per gallon comfort and style. So, how did I get 0.4285? I have divided this one by 2.33 we got 0.43. The next one is one upon three divided by 2.33.

So, it is nothing but that element delta by the corresponding column sum. So, I have got the matrix step two in step 3. I found the row average to be 0.4, so now we have the weightage for the price of 40% miles per gallon 9 Comfort 22 and 30. Now, I am going to explain how I got the lambda max. We know $Ax = \lambda_{max}$ x then first I have to find out the lambda 1. Lambda 1 is 1.71 divided by 0.4, 1.71 divided by 0.4, and then 0.355 divided by 0.09.

So, I found the average was 4.19. Now we have to do the pairwise comparison for each alternative with respect to price. I have got the pairwise comparison, the same procedure I have adapted. First, I found the column sum, then each element was divided by the corresponding column sum, and then I found the row average. So, we got 0.14, 0.29, 0.56. Similarly, I have done it for miles per gallon. The miles per gallon also had the same procedure, and then I got the priority level for A, B, and C.

The next criterion is stale style. I have done the same procedure. The next one is comfort the comfort also I have done the same procedure. Now you see that J102, I have entered all the priority levels for price for all three cars, miles per gallon, and you see O103. There, I entered our priority matrix for our criteria, and then I did the matrix multiplication. Dear students, I am going to explain how to do matrix multiplication in Excel.

So, select three cells equal to matrix multiplication, select this matrix, and select the priority matrix. Then you have to press control shift enter. So, now we got the value for car A, weightage is 27% for car B, it is 41 car C, it is 31. So, we are recommending that car B is the best car for this decision-maker.

Dear students, in the last two lectures, I have explained the multi-criteria decision-making technique called the AHP analytic hierarchy process. I have taken the problem of selecting a car by considering multiple criteria. I have explained the step-by-step procedure of AHP, and I also explained the concept of consistency, and I have explained how to check that. Thank you very much.