Decision Making with Spreadsheet

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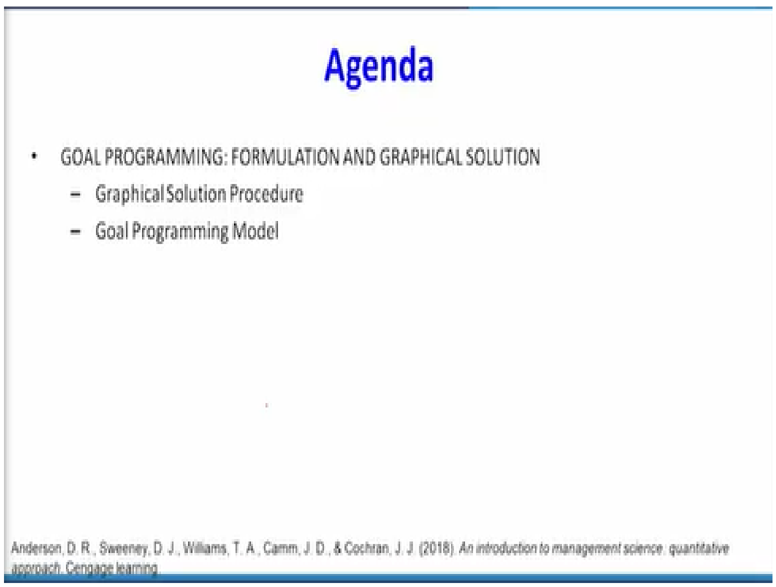
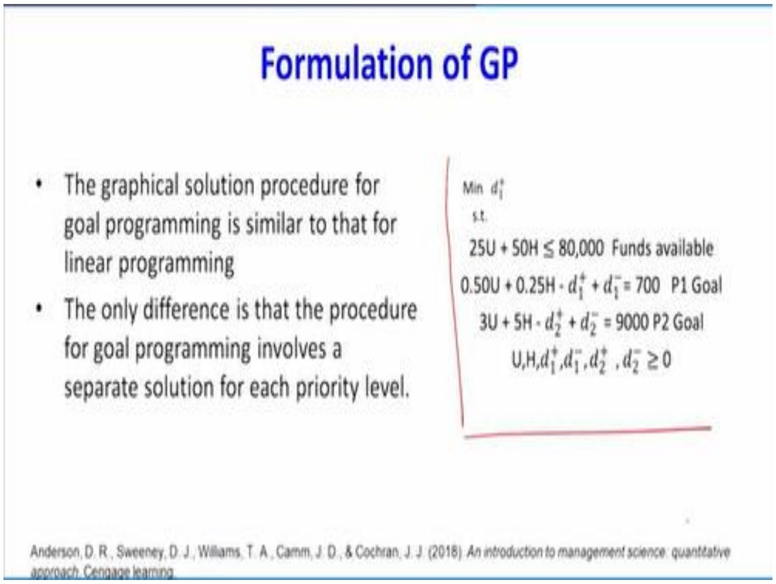
Lecture - 52

Formulation of Goal Programming - II

Dear students, in this lecture, I am going to explain how to solve goal programming problems graphically using the software called Desmos. For the problem that we have formulated in the previous lecture.



So, the agenda for this lecture is goal programming formulation, which you have done in the previous lecture, and also the graphical solution. After that, I am going to explain step-by-step procedure for solving the goal programming. At the end, I will explain what is the goal programming model.



We will continue the formulation of goal programming. I have brought what we formulated in the previous lecture. So, minimize d_1^+. 25U + 50H less than equal to 8000. That is the fund's available constraint. Goal 1 is 0.5U + 0.25H - d_1^+ + d_1^- = 700, which is the goal for achieving the risk index. The next goal is 3U + 5H – d_2^++ d_2^- = 9000, that is for achieving the minimum return of 9000 dollars.

Min d_1^+

s.t.

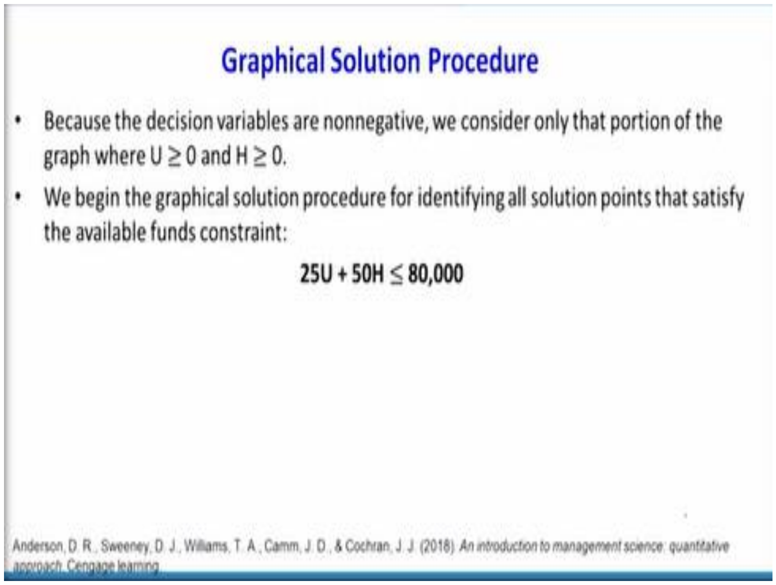
25U + 50H ≤ 80,000 Funds available

0.50U + 0.25H - d_1^+ + d_1^-= 700 P1 Goal

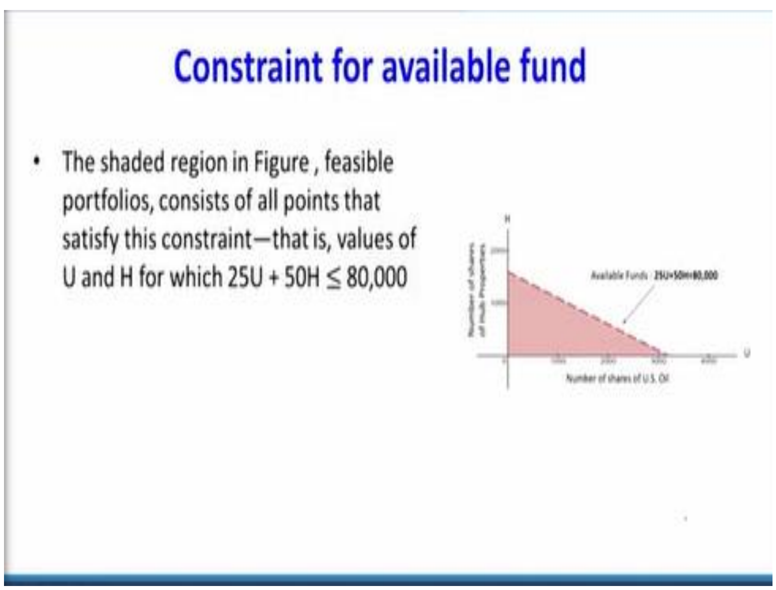
3U + 5H - d_2^+ + d_2^- = 9000 P2 Goal

U,H,d_1^+,d_1^-,d_2^+ , d_2^- ≥ 0

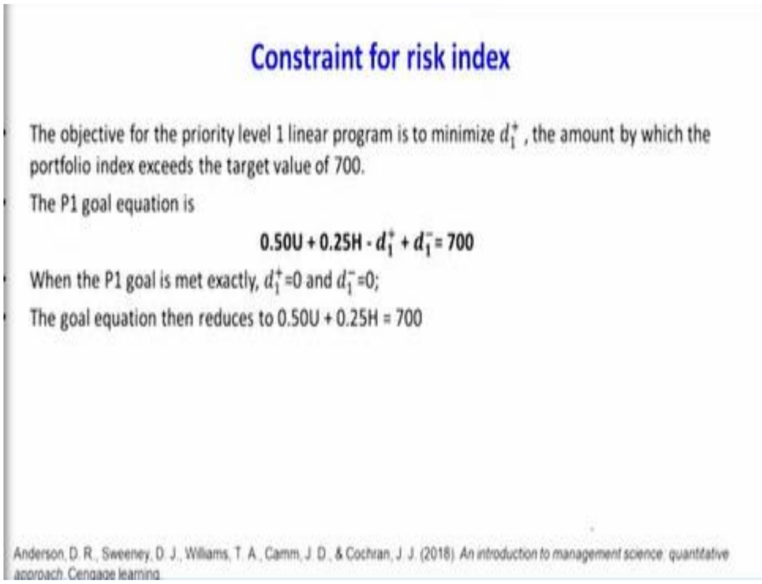
So, the graphical solution procedure for goal programming is similar to that for linear programming. The only difference is that the procedure for goal programming involves separate solutions for each priority level; for example, we have two goals 1 is P1 and P2. So, we need to solve P1 separately and then P2 separately.



Because the decision variables are non-negative, we consider only the portion of the graph where U is greater than equal to 0 and H is greater than equal to 0. We begin the graphical solution procedure for identifying all solution points that satisfy the available fund constraint. So, one of the constraints for the problem is the fund's availability. So, 25U + 50H is less than equal to 80000 dollars.

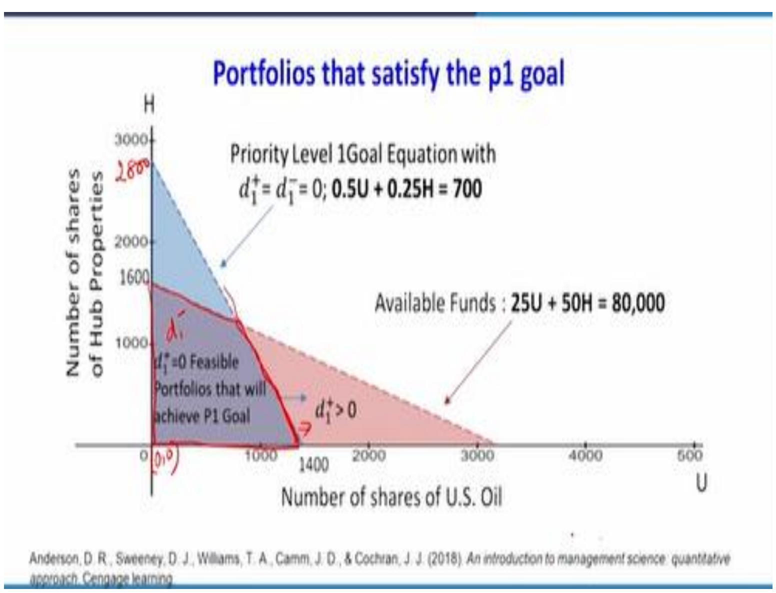


So, we are going to plot that I have plotted in Desmos, which I have brought here, so the shaded region in this figure's feasible portfolios consist of all points that satisfy this constraint. That is the value of U and H for which 25U + 50H is less than equal to 80000.



Now, we plot the first goal, which is a constraint for the risk index. The objective for the priority level one linear program is to minimize d_1^+; that is, positive deviation has to be minimized. The amount by which the; portfolio index exceeds the target value of 700. What is the P 1 goal equation? That is $0.5U + 0.25H - d_1^+ + d_1^- = 700H$. When you look at this equation, you see that there are four variables, but, in a graph, we can plot only two variables.

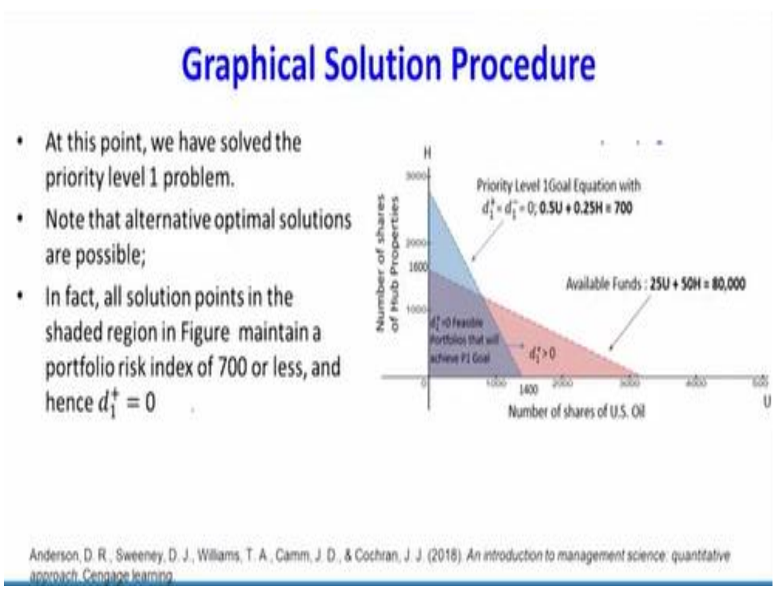
So, when the P1 goal is met, exactly what will happen automatically, the $-d_1^+$ and d_1^- will become 0. Then, there will be only two variables that is 0.5U + 0.25H = 700, which I am going to plot in the graph.



So, I have plotted which one, this one, this line. So, it is meeting the x-axis where U = 1400, and it is meeting the y-axis, so this is 2800. Now, we are going to find out the feasible reason for goal 1. What will happen? You see that when you go see the right-hand side, that means we are going to exceed 700, which means that the value of d_1^+ is going to be positive. On exactly on the line, both - d_1^+ and d_1^- are 0.

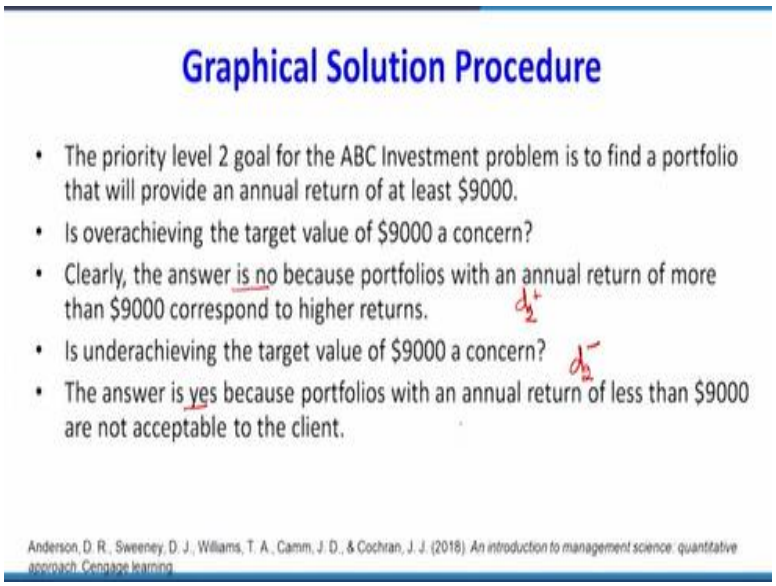
On the left-hand side, what will happen is the value of d_1^- is going to get some positive value. But here achievement is not the problem. The problem will only be the over-achievement. So, what will happen? All the points in this region will satisfy goal 1; for example, you can go for this 0, 0, and your risk index will be 0, but that is not required; that is not expected.

The risk index of 0 means when there is a lesser risk, there is a lesser reward. So, when is the risk 0, and the reward also 0? But this seems to be the extreme point. What will happen to the higher risk? Obviously, when there is a higher risk, there is a possibility of a higher reward that we will find out. What is the value of that? So, now I have plotted two equations. So, all these points in the feasible region for goal 1 will satisfy or will help to achieve goal 1.



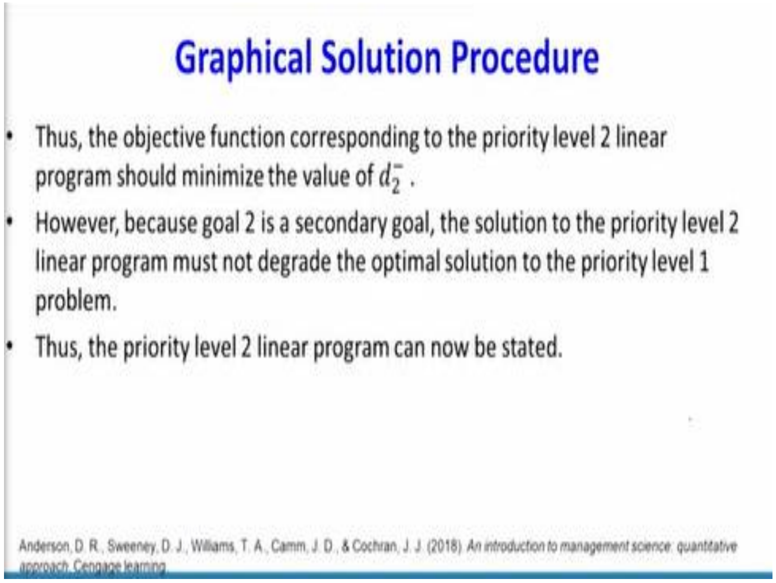
At this point, we have solved the priority level 1 problem. What is the meaning of that? So, all the points in the blue area will satisfy priority level 1. So, you can take any point and substitute it, what will happen? So, the d_1^+ will be minimized the positive deviation will be minimized. We may get some d_1^- but that is not the problem for us. So, what will happen note that the alternative optimal solutions are possible.

There may be different solutions. In fact, all solution points in the shaded region in the figure maintain a portfolio risk index of 700 or less. Hence, the value of $d_1^+ = 0$, so what will happen? All these points will help you to achieve a risk index of 700 or less.



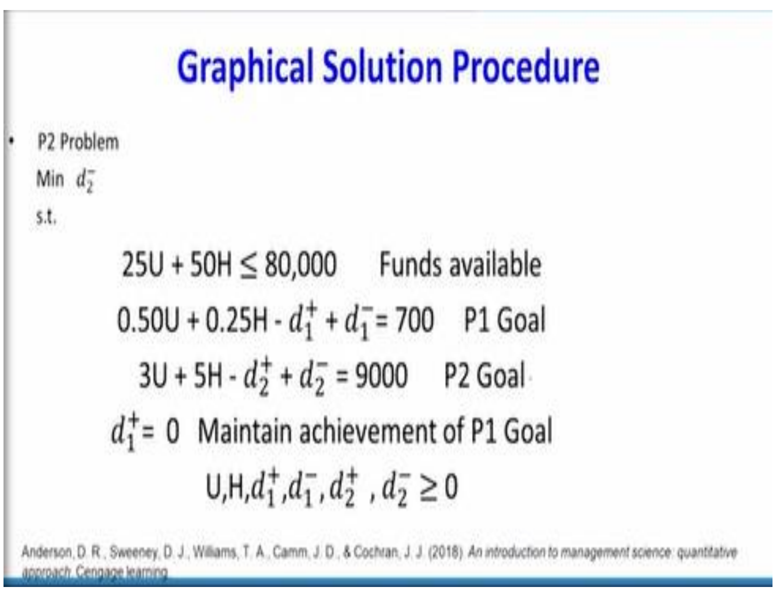
The priority level 2 goal for the ABC investment problem is to find a portfolio that will provide an annual return of at least 9000 dollars. Now we have to see whether over-achievement or under-achievement of this goal is a problem for us or not. The first part is over achieving the target value of 9000 dollars a concern. Clearly, the answer is no because a portfolio with an annual return of more than 9000 dollars corresponds to a higher return.

So, that means the d_1^+ is not a problem for us. The next part is that achieving the target value of 9000 is a concern. The answer is yes, so that means d2 is the second goal, so I am writing d2. So, d_2^- if you underachieve this goal, that is a problem. So, here the answer is yes because portfolios with annual returns of less than 9000 dollars are not acceptable to the client.



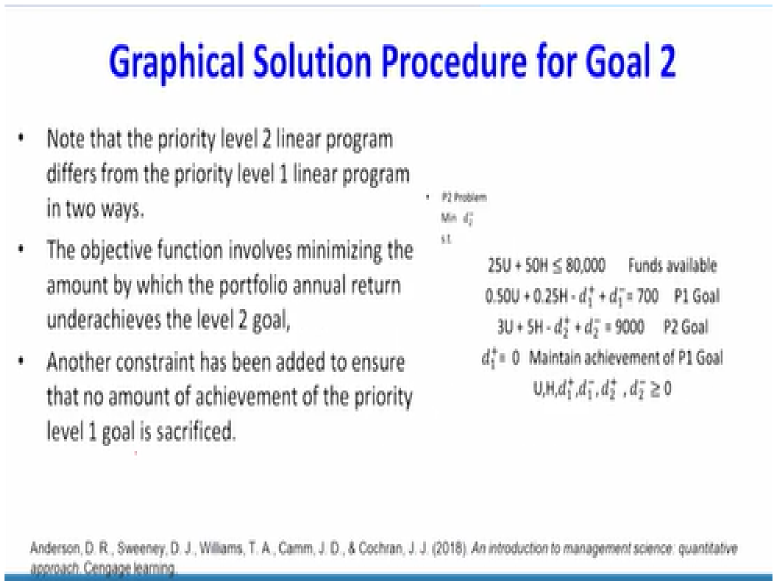
Thus, the objective function corresponding to the priority level two linear program should minimize the value of d_2^-, which means the underachievement for goal 2 has to be minimized. However, because goal 2 is a secondary goal the solution to the priority level two linear program must not degrade the optimal solution to the priority level 1 problem. So, what is the meaning of that? Already we have got a solution space for goal 1.

When you are trying to get solutions for P2 that should not degrade the previous solution. Thus, the priority level 2 linear program can now be stated.



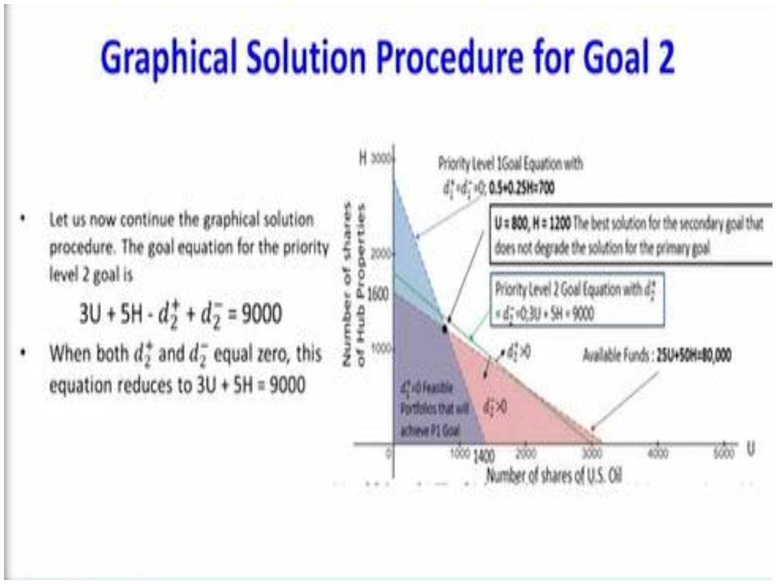
So, what is the priority level 2? So, we have to minimize the underachievement of goal 2 funds available for goal 1 goal 2, and one more thing, we are adding one constraint, d_1- is equal to 0.

This is to maintain the achievement of the P1 goal. What is the meaning of that? So, the P1 goal is the risk index. So, the value of the over-achievement should be equal to 0. That is why we have added d_1^+.

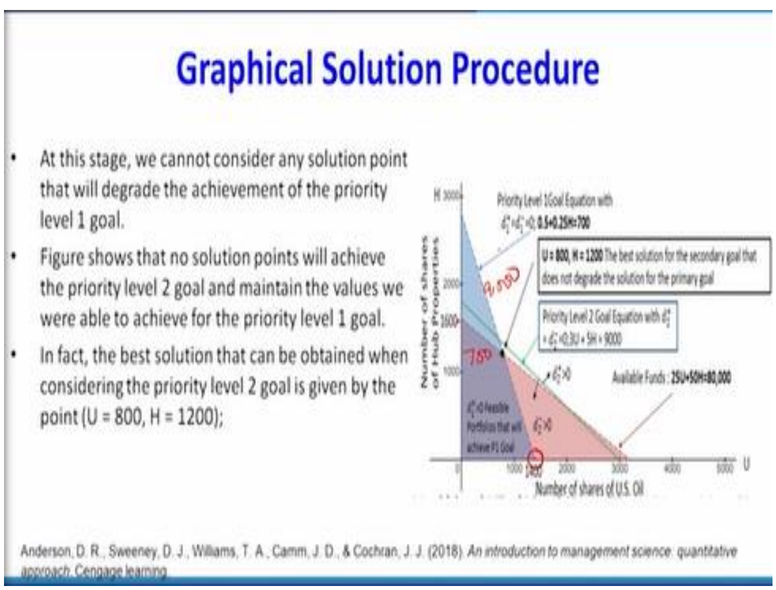


Note that the priority level 2 linear program differs from the priority level 1 linear program in two ways. What are they? The objective function involves minimizing the amount by which the portfolio annual return achieves the level 2 goals, that is, d_2^-. Another constraint has been added to ensure that no amount of achievement of the priority level 1 goal is sacrificed. So, this one.

So, we already have the solution for priority level 1 that should not be sacrificed while getting the solution for goal 2. That is the two differences.



Let us now continue the graphical solution procedure. The goal equation for the priority level 2 goal is 3U + 5H – $d_2^+ + d_2^-$ = 9000. When both $d_2^+ + d_2^-$ are equal to 0, this equation reduces 3U + 5H = 9000. So, I have drawn with the help of this green ink color line. So, when d_2^+ and d_2^- is 0, so the equation is 3U + 5H = 900.

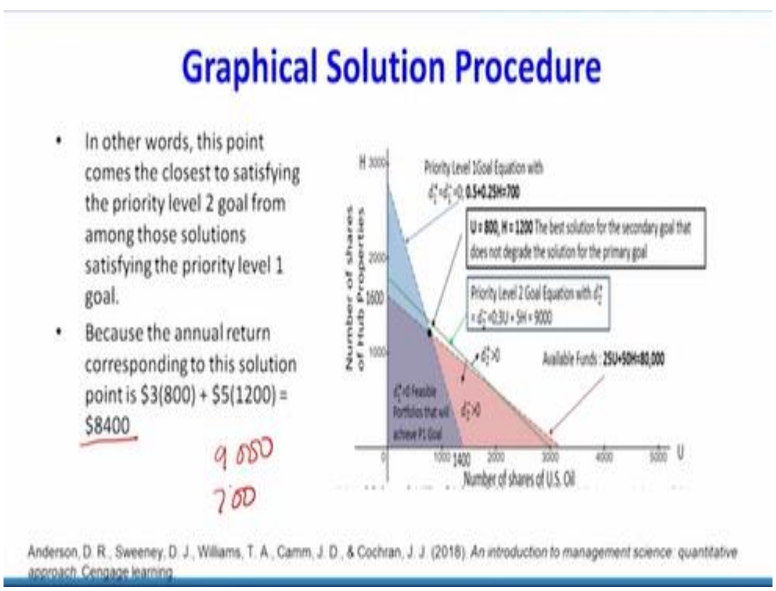


At this stage, we cannot consider any solution point that will degrade the achievement of priority level 1 goals. The figure shows no solution point will achieve the priority level 2 goal and maintain the value we are able to achieve for the priority level 1 goal. You see that for goal 1, the maximum is 700. Goal 2 has to be more than 9000. So, what will happen? It is very difficult to achieve the priority level 2 goal without sacrificing the P1.

But the condition is we should not sacrifice. So, the only possible and closest solution is this point. So, what are the possible? For example, you can go here at 1400, or you can go at 1600, so the figure shows that no solution points will achieve the priority level 2 goal and maintain the value we are able to achieve the priority level 1 goal. In fact, the best solution that can be obtained when considering the priority level 2 goal is given by the points U = 800 and H = 1200.

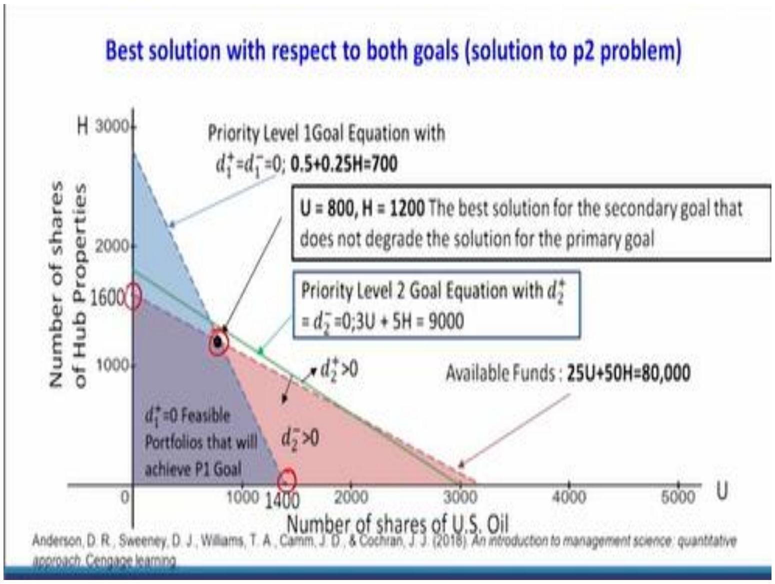
You see what we have to do in the feasible region for goal 1, what are the extreme points? This is one extreme point, 1400; this is another extreme point; this is 1600, another extreme point. So, these regions that are bounded by these points will satisfy goal 1, but goal 2 is green in color, but that should be greater than 9000. So, it is tough to achieve the goal 2 without sacrificing the goal 1.

So, what do we have to do? We have to find out the closest point which is not disturb goal 1. At the same time, we are able to achieve goal 2. So, goal 2 is 9000 dollars, which has to be achieved, but we cannot find a common point that satisfies goal 1 and goal 2. So, what are we going to do? We are going to sacrifice goal 2 to achieve goal 1. So, I will explain the next slide.

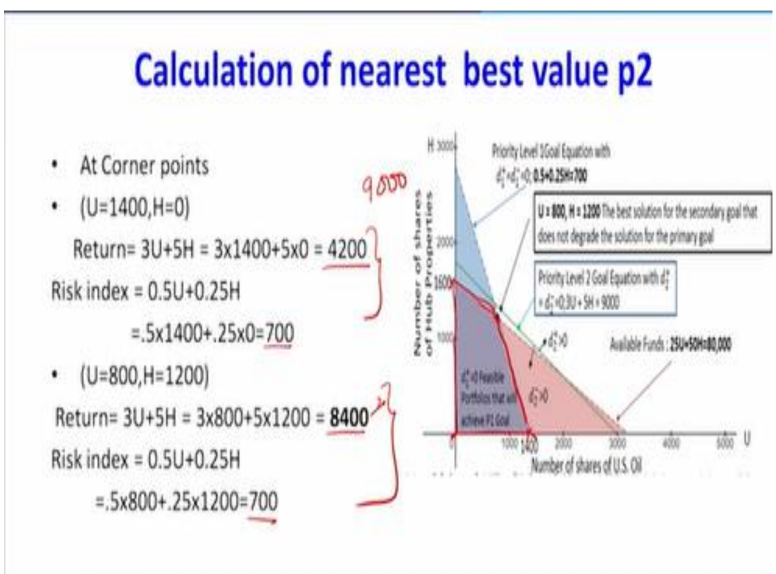


In other words, this point comes closest to satisfying the priority level 2 goal from among those solutions satisfying the priority level 1 goal because the annual return corresponding to this solution point that is 800 and 1200 is 8400 dollars. Actually, we have to achieve 9000 dollars, which cannot be achieved. What will happen? If you achieve 9000 dollars, there is a possibility you can violate this first objective.

That is, the risk index should be less than 700, which I will explain with the help of by plotting all the points.



So, what am I going to do? I am going to take this point, this point, this point. So, out of these three points, I am going to check which point is satisfying goal 1 and which point is going to approximately satisfy goal 2.

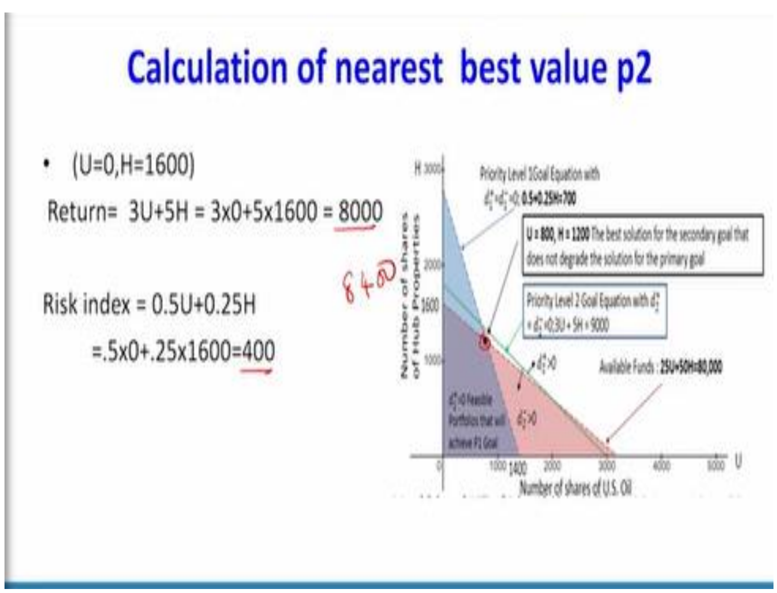


Now, the calculation of the nearest best value for p2. We already know the feasible region for goal 1 is these points, and this region is bound by these lines. So, all the points in this region will satisfy goal 1, but we now have plot goal 2 also with the help of the line, which is green in color. So, what will happen? The logic is we have to achieve goal 2 without violating or sacrificing goal 1. That is the meaning of preemptive goal programming.

So, I am going to consider three points as a feasible reason for the goal: 1. For example, this one 1400 and 0. So, at this point, I am going to find out the return; suppose I consider this point as the solution. What will happen? The return will be 3U + 5H, so 3 into 1400, it will be 4200. But I want 9000. What will happen to the risk index? The risk index will be 0.5U + 0.2H, so you will get 700. So, what will happen? I am going to achieve goal 1, but I am not able to achieve goal 2.

What was the minimum? I need 9000 dollars, but if I choose 1400, this point is my solution, so I am able to get only 4200. Next, I am going to find out this point. I got this point by solving the corresponding two lines, so U = 800 and H = 1200. So, first, I will find the return for this point, so 3U + 5H when I substitute, I am getting 8400, and the risk index is 700. So, now what happened for the same risk index? At this point, 800 and 1200 are giving higher returns.

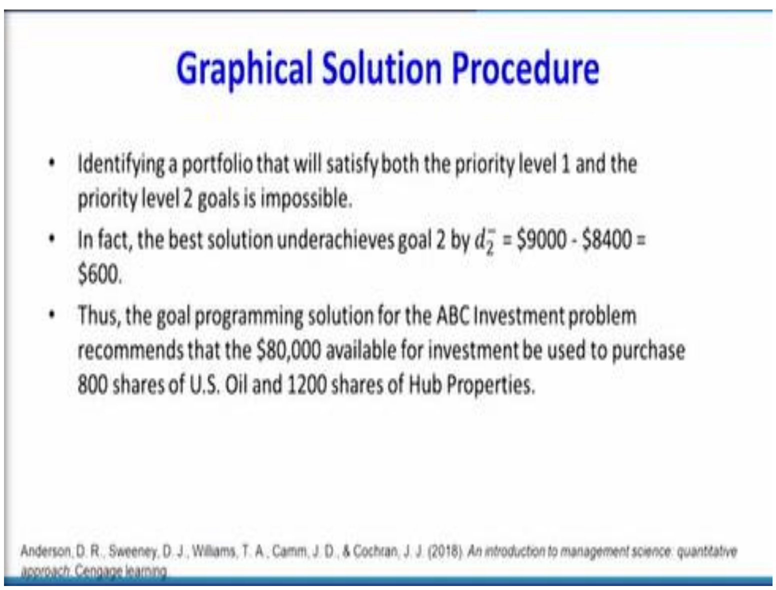
So, when compared to this first one, this is the better solution. What is another possibility? We can go to this point, the third point.



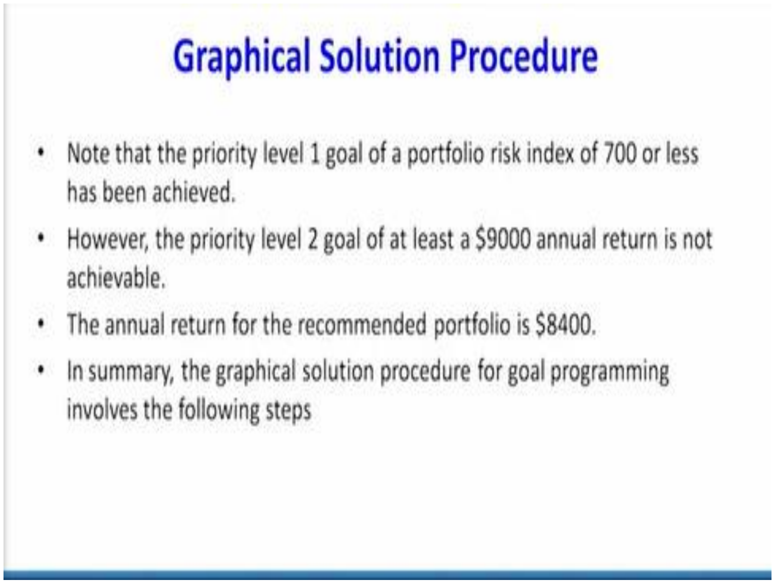
So, the third point is 0, 1600. For this reason, the return is 8000, but the risk index is 400, so I can go to a maximum of 700. The return is also less than 900, but here, there is no problem with the lesser risk index. But out of these three points, so this point where U = 800 and 1200 what I am able to get, I am able to get a risk index of 700, and at the same time I am going, I am able to get my return of 8400.

So, this point is the nearest best solution, which satisfies goal 1. At the same time, it also satisfies goal 2. It is not exactly satisfying; we need goal 2 to be 9000, but we are able to achieve closer to 9000, which is 8400. The only points that satisfy the need to get 8400 are U = 800 and 1200. So, the solution for this problem is U = 800 and 1200. So, in another way, if you buy 800 stocks of U.S. oil and 1200 stocks of Hub Properties oil.

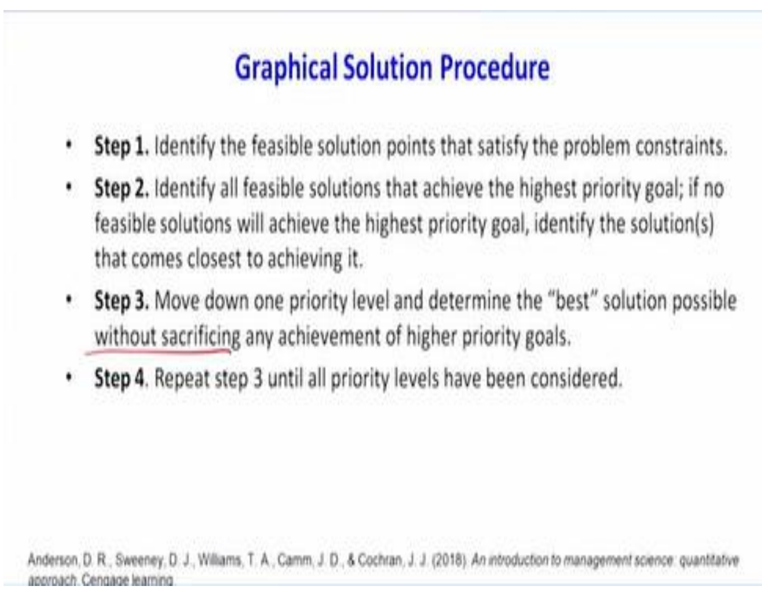
You can achieve the minimum risk index of 700, and at the same time, you can achieve a return of 8400.



Identifying a portfolio that will satisfy both the priority level 1 and the priority level 2 goals is impossible. We have seen there is no overlapping. In fact, the best solution to achieve goal 2 is d_2^- = 9000 - 8400 = 600. So, what are we getting? We are underachieving our second goal, but we are exactly able to achieve goal 1. Thus, the goal programming solution for the ABC investment problem recommends that the 80000 available for investment be used to purchase 800 shares of U.S Oil and 1200 shares of Hub Properties.

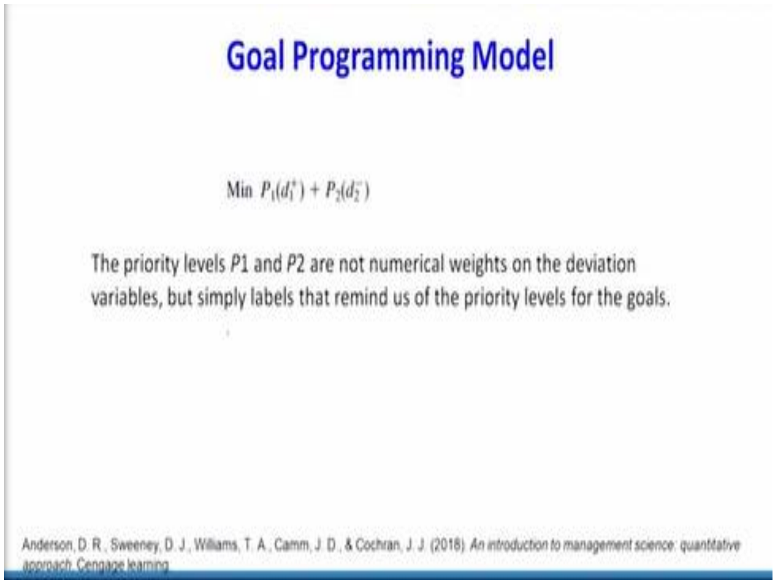


Note that the priority level 1 goal of a portfolio risk index of 700 or less has been achieved perfectly there is no problem. However, the priority level 2 goal of at least 9000 annual return is not achievable because there is no common area. So, the annual return for the recommended portfolio is only 8400. The graphical solution procedure for goal programming involves the following steps.

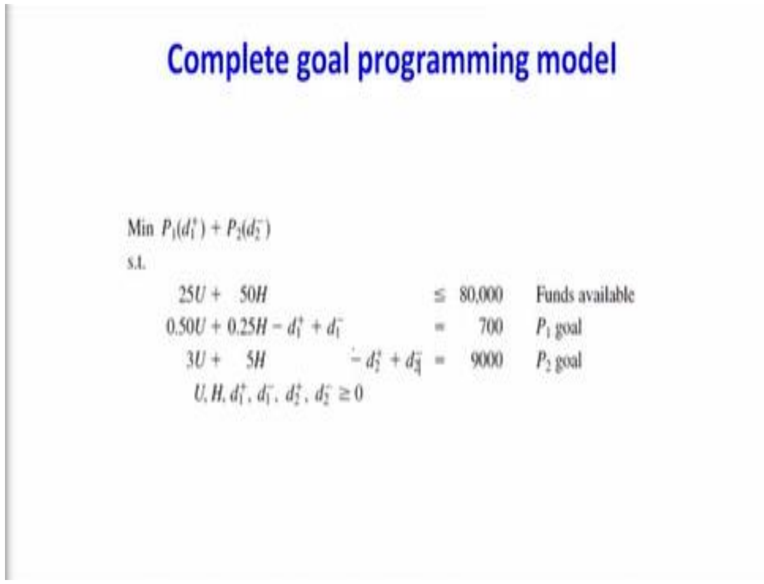


What are the steps? Step 1: identify the feasible solution points that satisfy the problem constraint. For example, you are done for P1, and we have found the feasible region. Step 2: identify all feasible solutions that achieve the highest priority goal. If no feasible solution will achieve the highest priority goal, identify the solutions that come closest to achieving it. That also we have done. We have taken three points.

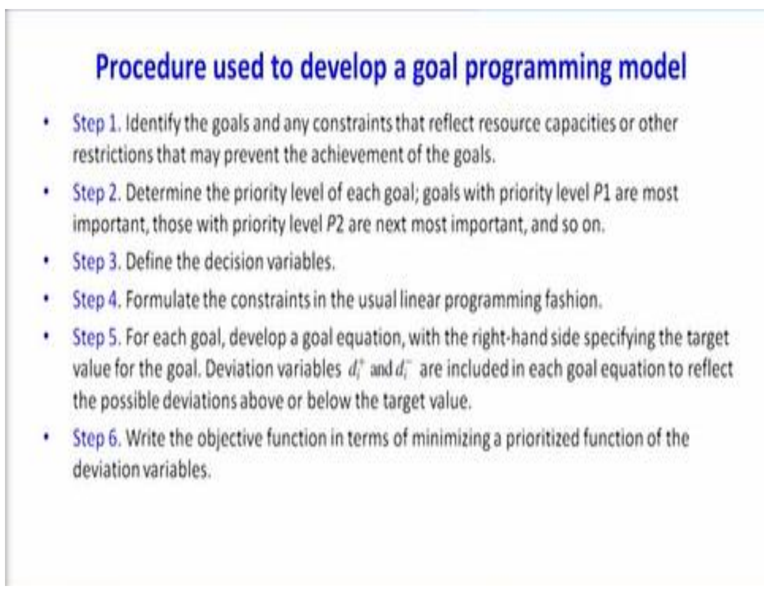
Out of these three points, we have identified which satisfies Goal 1 and Goal 2. Then, step 3, move down one priority level and determine the best solution possible without sacrificing any achievement of higher priority goals that also we have done. We have seen the points that do not sacrifice the P1 goal and found which is closest to achieving goal 2. So, step 4, repeat step 3 until all priority levels have been considered. These are the four essential steps.



Next is the general formulation of the goal programming model: So, how can we write it? We can write minimize $P_1(d_1^+)$, which means the overachievement of goal 1 has to be minimized, plus $P_2(d_2^-)$ in goal 2, and the underachievement has to be minimized. Here, P1 and P2 are priority levels. The priority levels P1 and P2 are not numerical weights on the deviation variables. But simply labels that remind us of the priority levels for the goals. So, the priority is goal 1, and the second priority is goal 2.

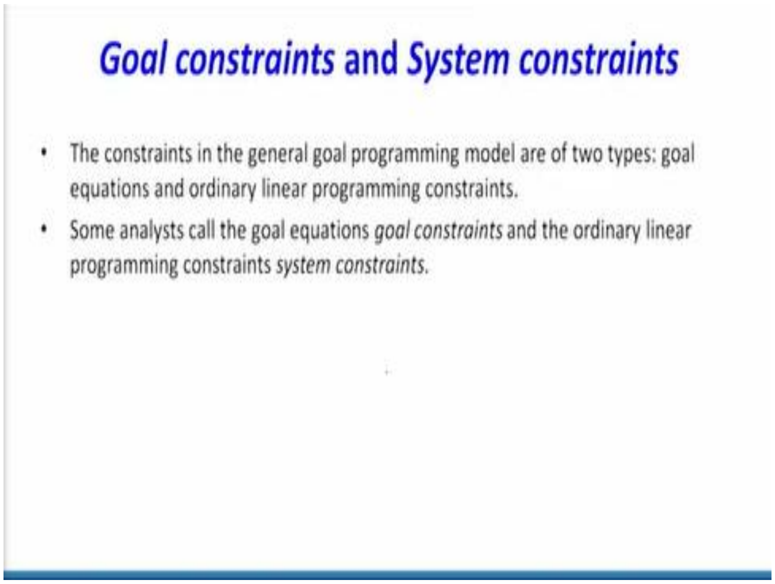


Next, we can present all the constraints, one being the funds available constraint, goal constraint for goal 1, and goal constraint for goal 2. Then, we have to mention all the decision variables.

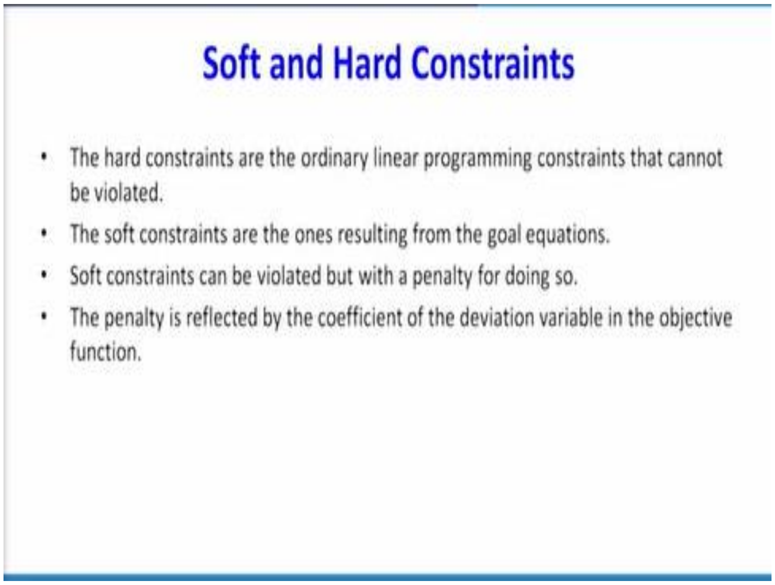


The procedures used to develop a goal programming model. Step 1: identify the goals and any constraints that reflect resource capacities or other restrictions that may prevent the achievement of the goals. Step 2: Determine the priority level of each goal. Goals with the priority level P1 are most important, those with the priority level P2 are next most important, and so on. Then define the decision variables. Then, formulate the constraint in the usual linear programming fashion.

For each goal, develop a goal equation with the right-hand side specifying the target value of the goal; the deviation variable d_1^+ here I represent goal 1, goal 2, and goal 3. And d_1^- so d_1^++ is over achievement d_1^- is under achievement are included in each goal equation to reflect the possible deviations above or below the target value. Then write the objective function in terms of minimizing a prioritized function of deviation variables.

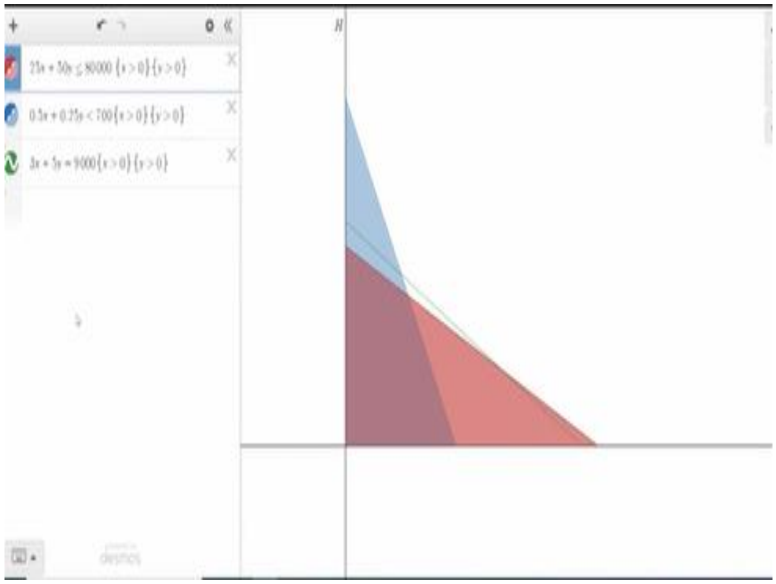


Now, I am going to explain goal constraints and system constraints. In some of the terminology, the constraints in the general goal programming model are of two types: goal equations and ordinary linear programming constraints. Some analysis calls goal equations goal constraints and ordinary linear programming constraints system constraints. In our problem, the funds available are linear programming constraints. There are two goals that and all there are two goals that two are called goal constraints, P1 and P2.



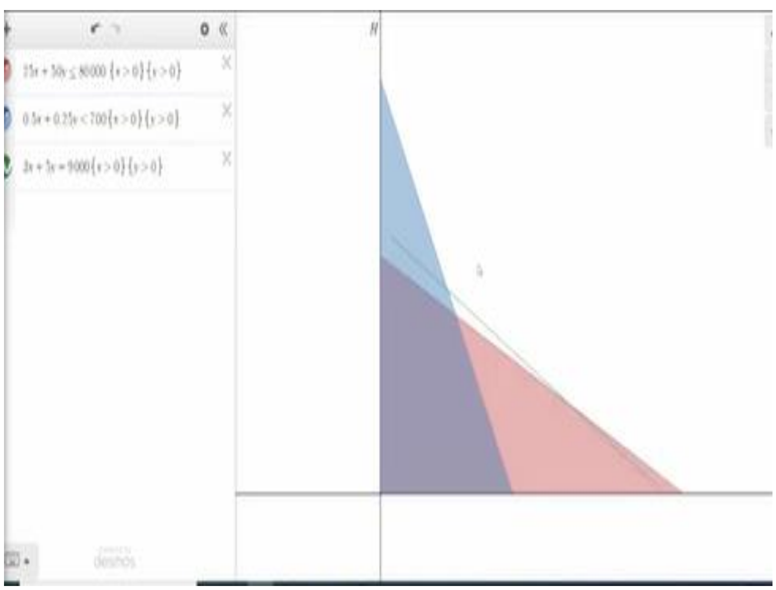
The other type of constraint we can say is a soft and hard constraint. This is a different terminology. Hard constraints are the ordinary linear programming constraint that cannot be violated. In our problem the funds available that constraint is a hard constraint that cannot be violated. The soft constraint is one resulting from the goal equation. Soft constraint can be violated, but with a penalty for doing so, for example, in our problem also, we have violated the second goal.

What is that? The second goal is we have to achieve a minimum of 9000 dollars, but we are not able to achieve that. So, we can add some penalty for not achieving so the penalty is reflected by the coefficient of deviation variable in the objective function. So, what we can do? We can add some coefficients as a penalty for not achieving that target.



Dear students, I will now explain how to use Desmos to solve the goal programming. First, I have written 25x + 50y less than equal to 80000, which is a hard constraint. You see, I need to get only the positive value of x, so I have included x greater than equal to 0 and y greater than 0. The second one is for goal 1 which is the area in the blue color. The overlapping area, for example this overlapping area is the region that satisfies the requirement for achieving goal 1.

The second goal is 3x + 5y = 9000. See that it is green in color. So, what have we seen? But we are because that is 3x + 5y >= 9000, but we are not able to get any common area. So, the best possible area is where the two points intersect.



So, the solution to the problem is where the point where the pink line and the blue line intersect. Dear students, in this lecture, I have discussed solving the goal programming graphically. The important point that I want to emphasize here is that the priority levels of the goals have to be maintained. So, after that, I have explained the constraints like soft and hard constraints. Thank you very much.