Decision Making with Spreadsheet

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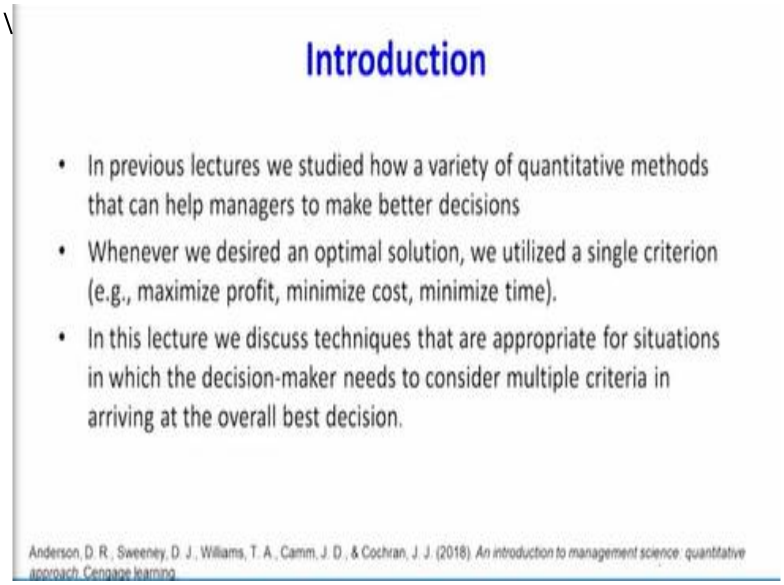
Lecture - 51
Formulation of Goal Programming - I

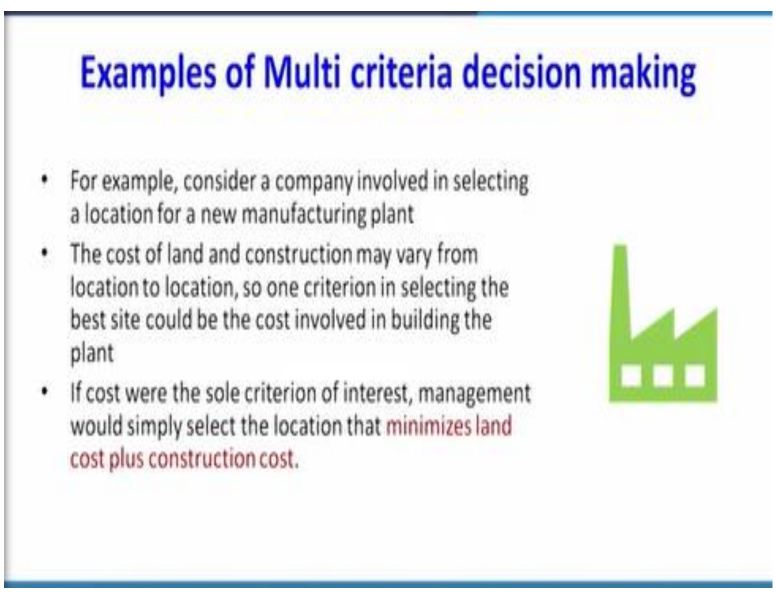
Dear students, so far, we have discussed linear programming with only one objective, either maximizing or minimizing the objective function. In this lecture, I am going to discuss how to handle linear programming, which has more than one objective. The topic that I am going to cover is called goal programming, and I will discuss the formulation of goal programming. In the following lecture, I will solve the problem graphically with the help of software called Desmos.

The agenda for this lecture is goal programming, formulation, developing constraint and the goal equations, and developing an objective function with pre-emptive priorities. What is the meaning of this pre-emptive? Pre-emptive means that priority is specified in advance which cannot be sacrificed. The priority of the objective cannot be sacrificed. That is the meaning of pre-emptive priorities.



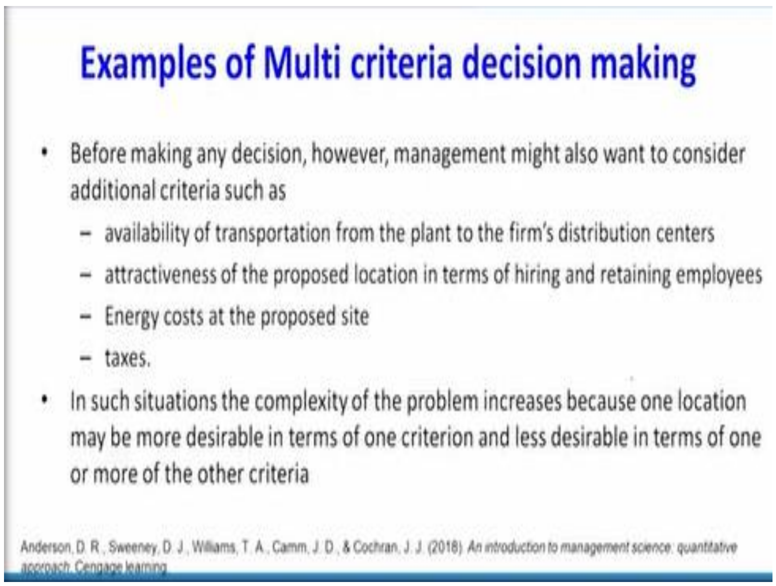
So, in the previous lecture, we studied how a variety of quantitative methods can help managers make better decisions. Whenever we desired an optimal solution, we utilized a single criterion example: maximize the profit, minimize the cost, or minimize the time. In this lecture, we discuss techniques that are appropriate for situations in which the decision maker needs to consider multiple criteria in arriving at the overall best decisions.

Dear students, in real life the decision-making is not done with only one criterion. So, we may come across multiple criteria. If there are various criteria, how to make the decision? That is what we are going to cover in this lecture.



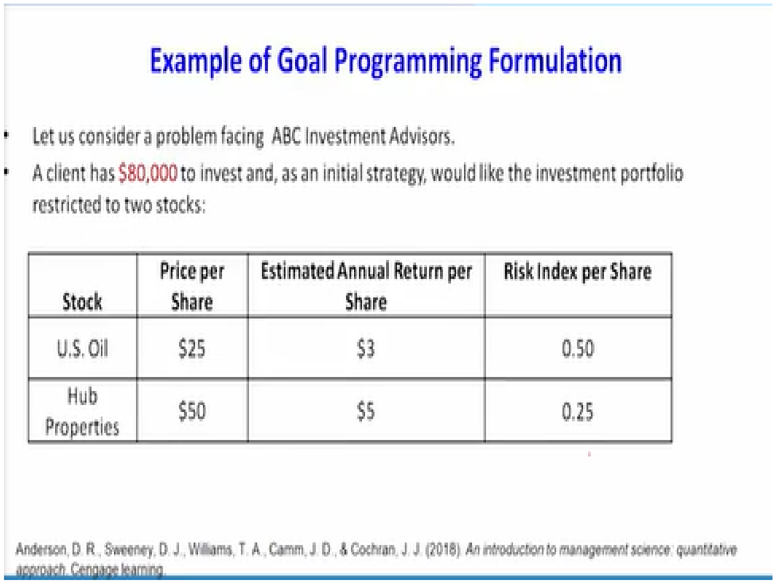
I will explain the examples of multiple criteria decision-making. For example, consider a company involved in selecting a location for a new manufacturing plant. A company is looking for a new manufacturing plant. The cost of land and construction may vary from location to location. So, one criterion in selecting the best site could be the cost involved in building the plant. So, what are we saying?

That one of the criteria for choosing the location is the cost of building and the plant. If cost were the sole criterion of interest, management would simply select the area that minimizes *land cost plus construction cost*.



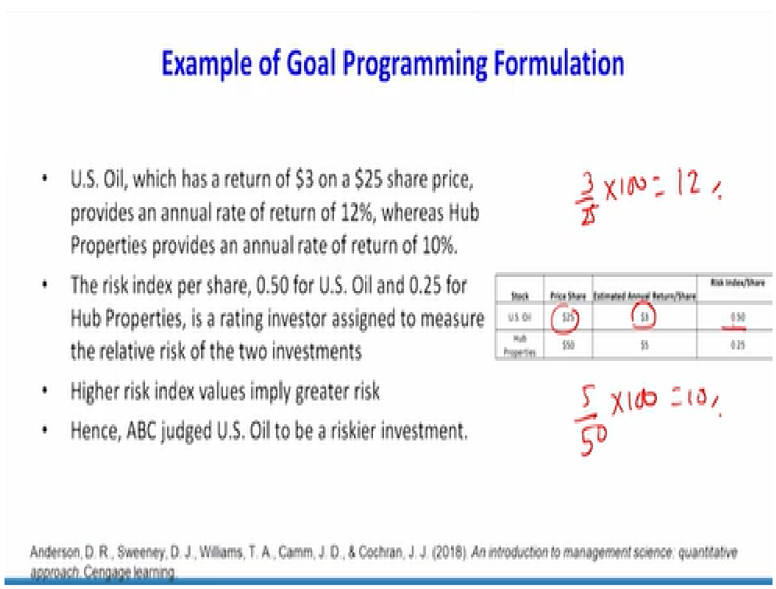
Before making any decision, however, management might also want to consider additional criteria apart from cost they want to consider additional criteria. What are they? Availability of transportation from the plant to the firm's distribution center. The attractiveness of the proposed location in terms of hiring and retaining employees. Energy costs at the proposed site and local taxes are also included.

These are the other criteria for choosing the location for constructing a new plant. In such situations, the complexity of the problem increases because one location may be more desirable in terms of one criterion and less desirable in terms of one or more of the other criteria. So, we have seen cost is one of the criteria. So, some locations may be good at a lesser cost, but that location may not satisfy other criteria.



Example of goal programming formulation. Let us consider a problem facing ABC investment advisors. A client has 80,000 dollars to invest and, as an initial strategy, would like the investment portfolio restricted to two stocks. So, they have 80,000 dollars, and they are planning to invest in two stocks. So, stock number 1, U.S. Oil. Price per share is 25 dollars. The estimated annual return per share is 3 dollars, and the risk index per share is 0.5.

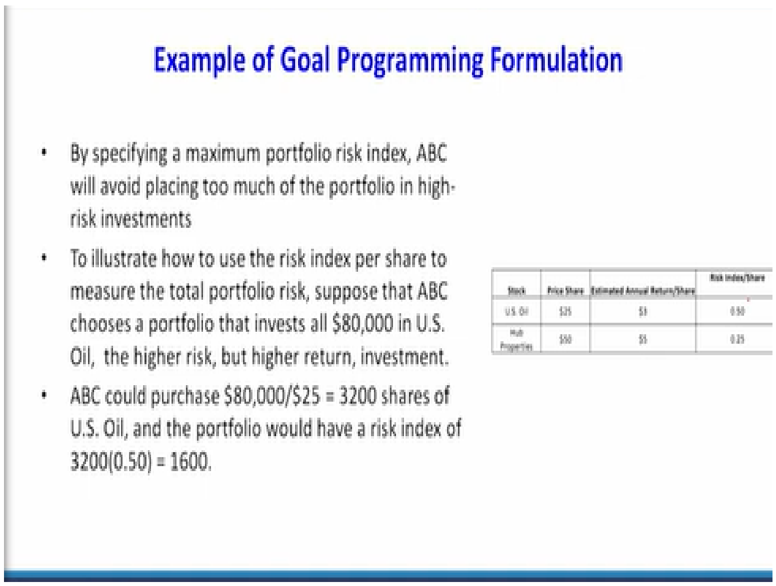
So, another stock is Hub Properties. The price per share is 50 dollars. The estimated annual return per share is 5 dollars, and the risk index per share is 0.25.



U.S. Oil, which has a return of 3 dollars, sees these 3 dollars. A share price of 25 dollars provides an annual rate of return of 12%. How did we get to 12%? 3 upon 25 multiply 100. So, it is 12%.

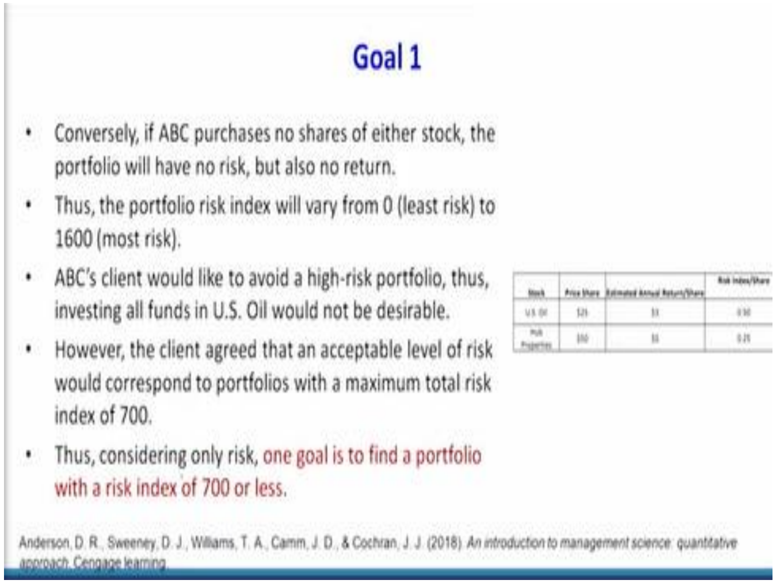
Whereas hub properties provide an annual rate of return of 10%. So, it is five upon 50 multiplied by 100. That is 10%. The risk index per share of 0.5 for U.S. Oil and 0.25 for hub properties is a rating investor assigned to measure the relative risk of two investments.

Higher risk index values imply greater risk. For example, the U.S. Oil is 0.5. So, the U.S. Oil stock is riskier than the Hub properties. Hence ABC judged U.S. Oil to be a riskier investment.



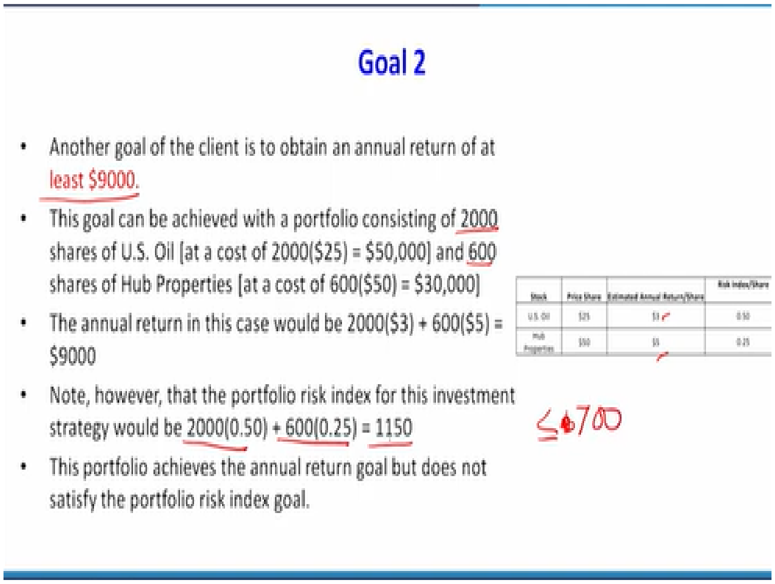
By specifying a maximum portfolio risk index, ABC will avoid placing too much of the portfolio in high-risk investments. Here, that company will not invest all the money in the U.S. because that has a higher risk. To illustrate how to use the risk index per share to measure the total portfolio risk. Suppose that ABC chooses your portfolio that invests all 80,000 dollars in U.S. Oil, which is a higher risk but higher return investment.

So, how could ABC purchase 80,000 dollars divided by 25 dollars 3200 shares of U.S. Oil? And the portfolio would have a risk index of 3200 multiplied by 0.5 = 1600. So, if the company invests all the money in U.S. Oil, the risk is 1600.



Conversely, if ABC purchases no shares of either stock, the portfolio will have no risk but also no return. Thus, the portfolio risk index will vary from 0 for the least risk to 1600 for the most risk. Now we will formulate the goal 1. So, ABC's client would like to avoid a high-risk portfolio; thus, investing all funds in U.S. Oil would not be desirable. However, the client agreed that an acceptable level of risk would correspond to a portfolio with a maximum total risk of total risk index of 700.

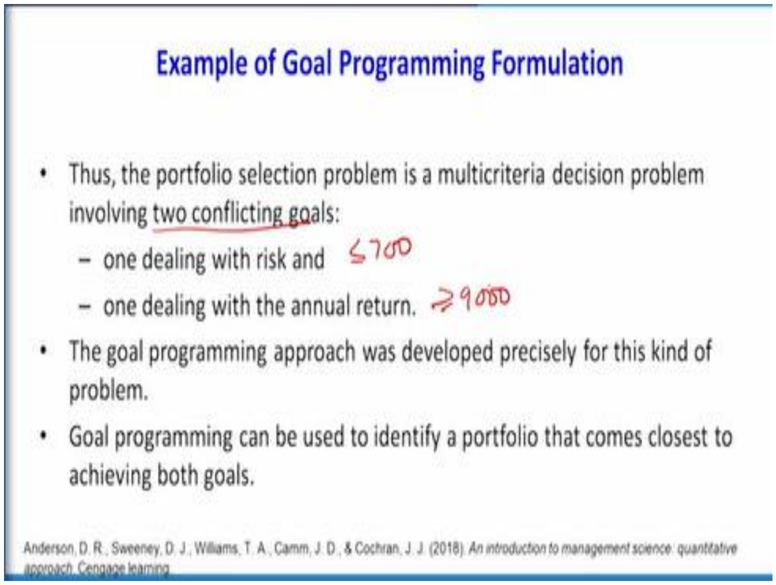
So, that client is agreeing to bear the risk of 700. Thus, considering only risk, one goal is to find a portfolio with a risk index of 700 or less. That is the first goal. What is the first goal? The client says that I can accept a risk index of less than or equal to 700. If the risk index goes beyond 700, I am not interested. That is the expectation of the client.



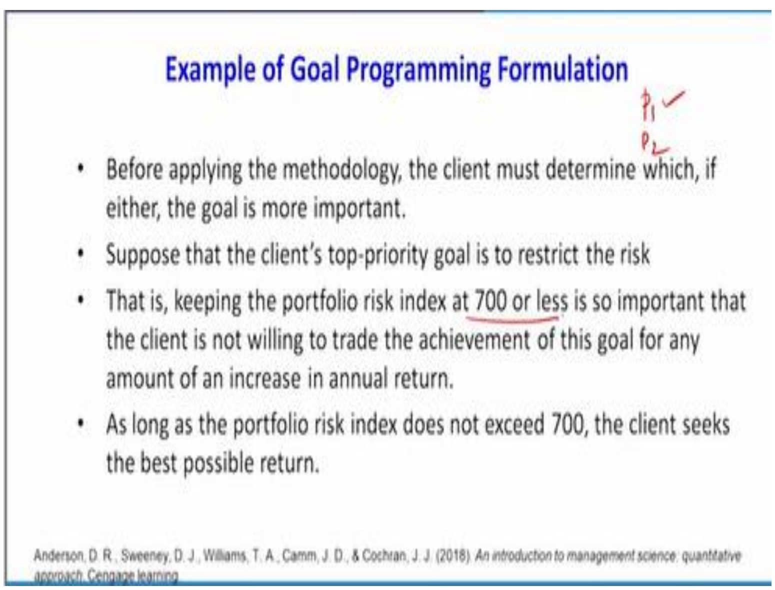
Now we will go for another goal. Why we are talking about goal 2? Remember, we are talking about goal programming. There may be different goals. So, we have to solve goal 1, then goal 2 and goal 3, and so on. So, we have discussed about the goal 1. Now, we will go to goal 2. It is like a second objective. The first objective is that the risk should not exceed 700 units. Similarly, we will discuss about the goal 2.

So, another goal of the client is to obtain an annual return of at least 9,000 dollars. This goal can be achieved with your portfolio consisting of 2,000 shares of U.S. Oil. Suppose you buy 2,000 shares of U.S. Oil. What will be the cost? What will be the cost of that? 2,000 multiplied by 25 50,000 dollars and 600 shares of Hub properties. How much cost will it cost? 600($50) = $30,000. 50,000 plus 30,000, so it is a total of 80,000 dollars.

So, out of 80,000 dollars, 200 shares if we invest in U.S. Oil and 600 shares in Hub properties, the annual rate return in this case would be 2000 multiplied by 3 plus 600 multiplied by 5, which is 9,000. Note, however, that the portfolio risk index for this investment strategy would be if you go for 2000 in U.S. Oil, the risk index will be 2000(0.50) + 600(0.25) = 1150.
You see that in the previous slide the goal 1 is that it should be less than 700. But it is going beyond 700 that is 1150. So, this portfolio achieves the annual return goal but does not satisfy the portfolio risk index goal.

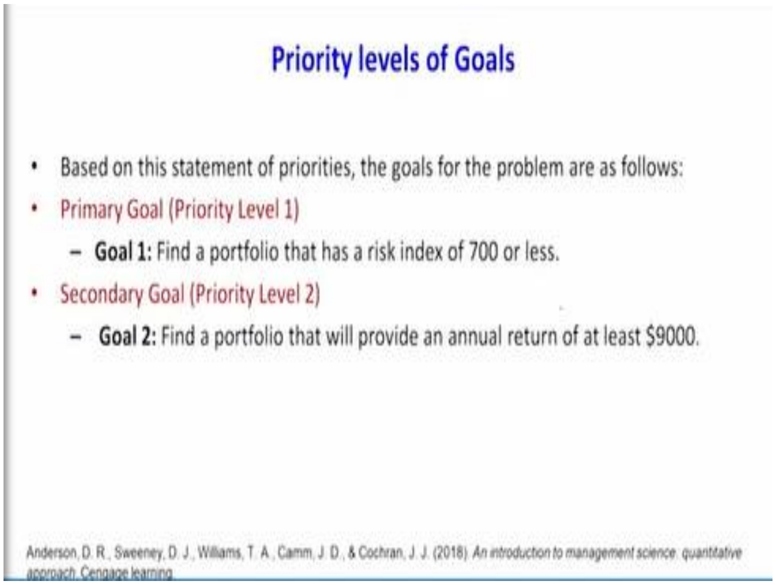


Thus, the portfolio selection problem is a multi-criteria decision problem involving two conflicting goals. One dealing with risk that should be less than or equal to 700. One dealing with annual return that should be greater than or equal to 9000 dollars. The goal programming approach was developed precisely for this kind of problem, like 2 goals that are conflicting. Goal programming can be used to identify a portfolio that comes closest to achieving both goals.

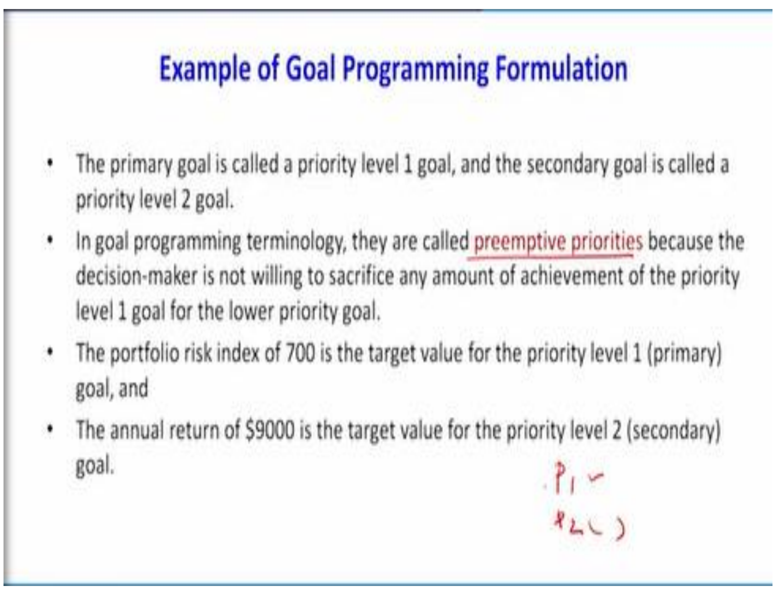


Before applying the methodology, the client must determine which, if either, the goal is more important. So, we have discussed 2 goals P1 and P2. So, the client should say which goal is more important for him. Suppose that the client's top priority goal is to restrict the risk he says the P1 is his first priority. That is keeping the portfolio risk index at 700 or less is so important that the client is not willing to trade.

The achievement of this goal is for any amount of increase in annual return. As long as the portfolio risk index does not exceed 700, the client seeks the best possible return.

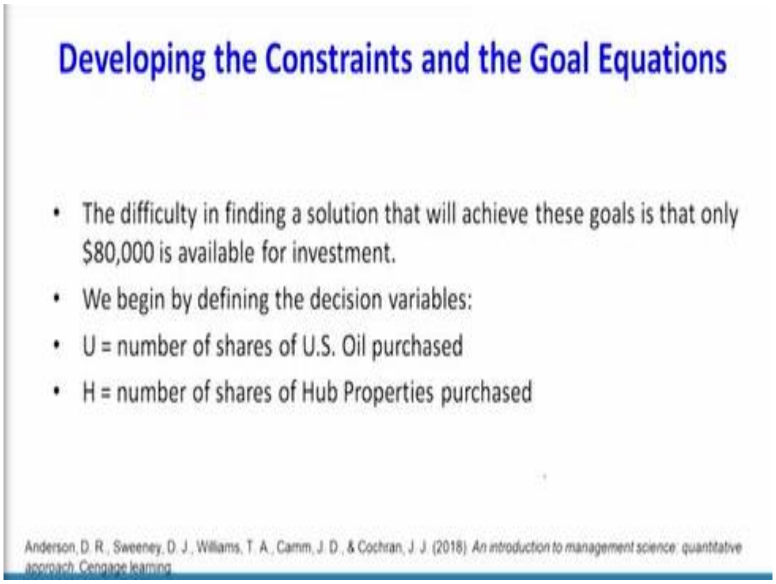


Now, I am going to write the priority levels of goals. Based on this statement of priorities, the goals for the problem are as follows. What is the primary goal? Priority level 1 goal 1: find a portfolio that has a risk index of 700 or less. Secondary goal: goal 2: find a portfolio that will provide an annual return of at least 9000 dollars.

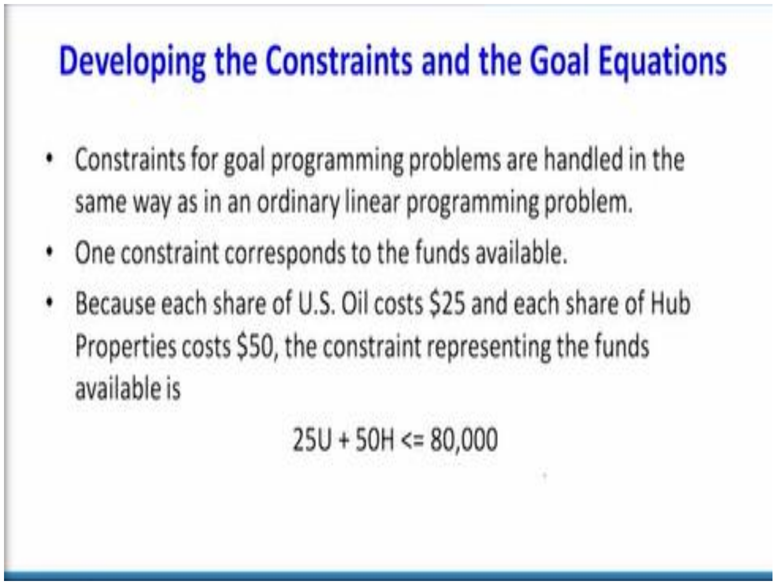


The primary goal is called a priority level 1 goal, and the secondary goal is called a priority level 2 goal. In goal programming terminology, they are called preemptive priorities because the decision maker is unwilling to sacrifice any amount of achievement of the priority level 1 goal for the lower priority goal. The portfolio risk index of 700 is the target value for the priority level 1 primary goal.

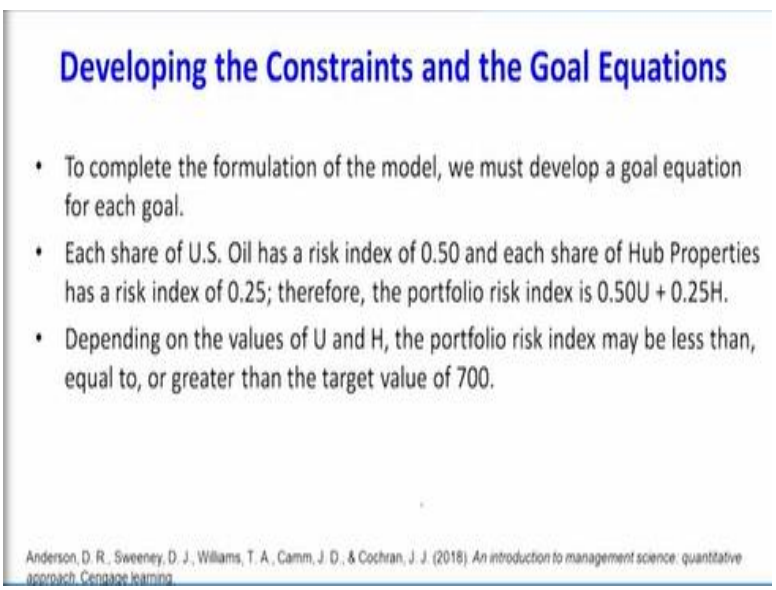
The annual return of 9,000 dollars is the target value for priority level 2. That is the secondary goal. So, there is a P1 and P2, and P1 is the priority. So, in P2, when he is achieving P2, he is not willing to sacrifice P1. So, P1 has to be completed, P2 need not be achieved, and he is willing to sacrifice for P2. But the P1 should not be sacrificed.



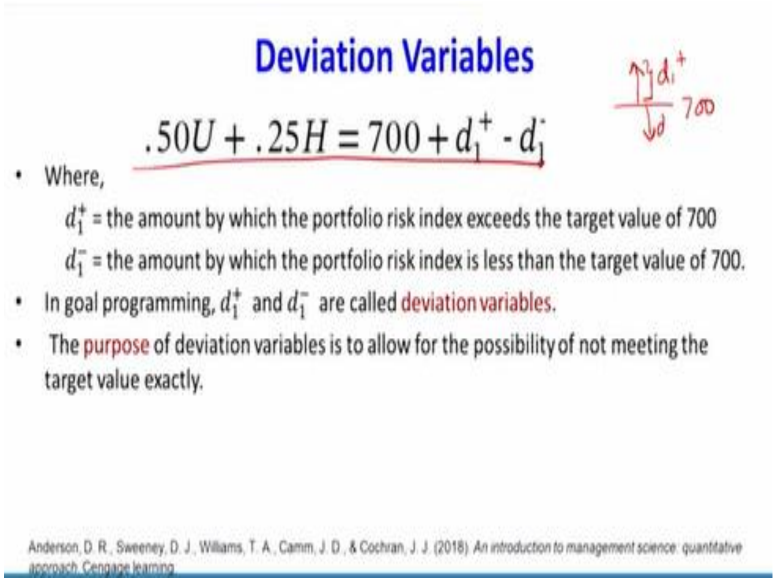
The difficulty in finding a solution that will achieve these goals is that only 80,000 dollars is available for the investment. As usual, we begin by defining the decision variables. So, decision variable U is the number of shares of U.S. Oil purchased. H is the number of shares of Hub properties purchased. So, we have to recommend using this 80,000 to the client to determine how many shares he should purchase from you and how many shares he should buy from H.



Now, developing the constraint and goal equations. Constraints for goal programming problems are handled in the same way as in an ordinary linear programming problem. Here, the one constraint corresponds to the funds available. Because each share of U.S. Oil costs 25 dollars and each share of Hub properties costs 50 dollars, the constraint representing the funds available is 25U + 50H <= 80,000.

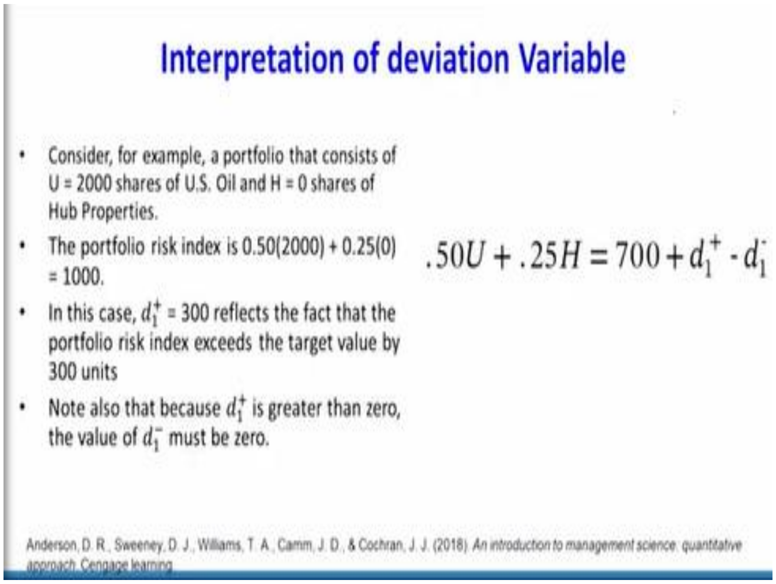


To complete the formulation of the model, we must develop an equation for each goal. Each share of U.S. Oil has a risk index of 0.5, and each share of Hub properties has a risk index of 0.25; therefore, the portfolio risk index is 0.5U + 0.25H. Depending on the value of U and H the portfolio risk index may be less than or equal to or greater than the target value of 700.



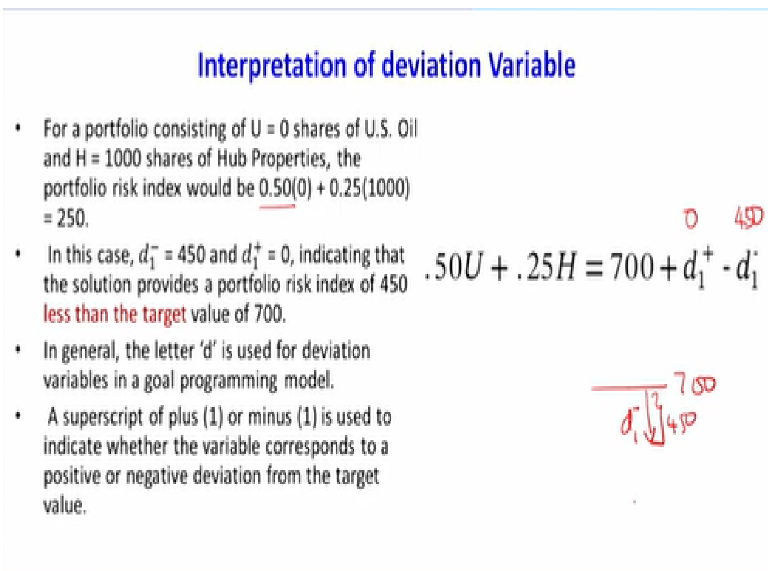
Now, I am going to introduce a new variable called the deviation variable. So, what will happen? Look at this equation $0.5U + 0.25H = 700 + d_1^+ - d_1^-$. Here, the d_1^+ is the amount by which the portfolio risk index exceeds the target value. For example, suppose I write, say this is 700. So, this is the upper side. Say this is the lower side. So, I can call this distance d_1^+. So, d_1^+ is the amount by which the portfolio risk index exceeds the target value of 700.

The bottom one is d_1^- the amount by which the portfolio risk index is less than the target value of 700. In goal programming d_1^+ and d_1^- are called deviation variables. The purpose of deviation variables is to allow for the possibility of not meeting the target value exactly. That means in case the d_1^- has some positive value, we may exceed 700. If the d_1^- has some positive value, our value may be less than 700.



Consider a portfolio that consists of U = 2000 shares of U.S. Oil and H = 0 shares of Hub properties. What will happen? The portfolio risk index will be 0.50(2000) + 0.25(0) = 1000, which is 1000 shares. So, 1000 shares in these shares have a $d1^+$ value of 300. That means here it is the value of this one is 300. That reflects that the portfolio risk index exceeds the target value by 300 units.

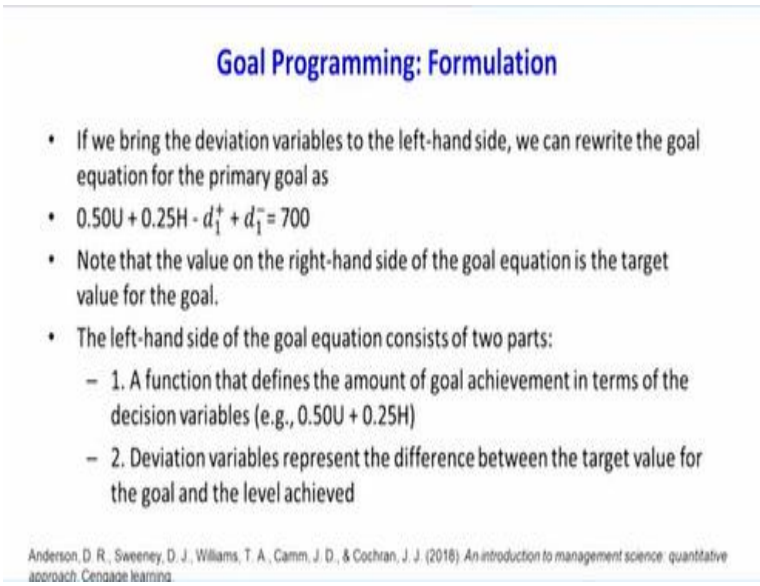
Note that because d1+ is greater than 0, the value of d_1^- must be 0. So, this value will be 0. It can have only one value, whether d_1^+ or d_1^-.



For a portfolio consisting of U = 0 shares of U.S. Oil and H = 1000 shares of Hub properties, the portfolio risk index would be 0.50(0) + 0.25(1000) = 250. In this case, the value of d_1^- = 450. So,

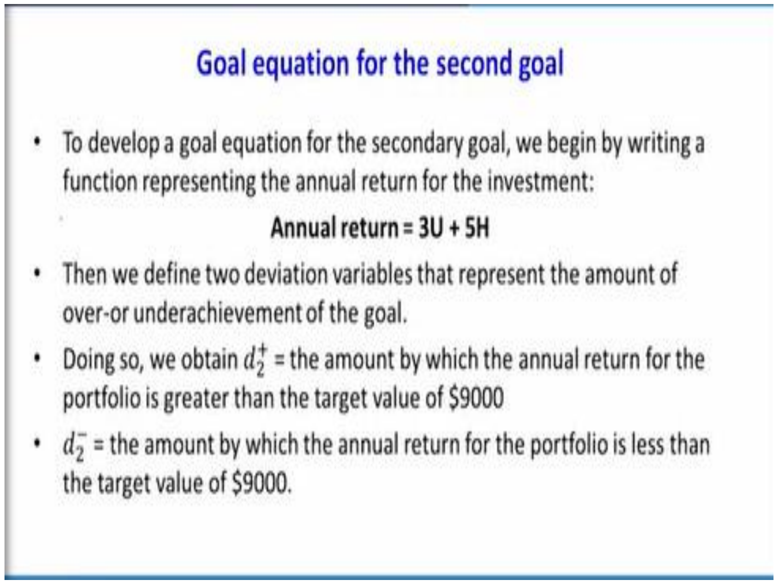
d_1^- value = 450 and $d_1^+ = 0$, indicating that the solution provides a portfolio risk index of 450 less than the target value of 700. As I told you, suppose this is 700, so here the value of, say, this is d_1^- so, this value is here 450. Here it is called under achievement.

In general, the letter d is used for deviation variables in a goal programming model. A superscript of plus 1 or minus 1 is used to indicate whether the variable corresponds to a positive or negative deviation from the target value. So, if I say d_1^{+}, it is called over-achievement positive deviation. If I write d_1^- it is called under achievement.



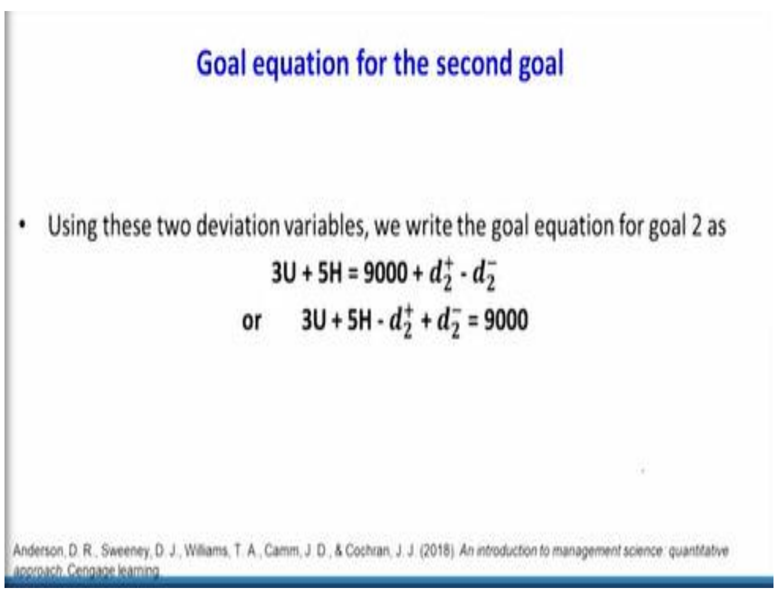
If we bring the deviation variables to the left-hand side, we can rewrite the goal equation for the primary goal as $0.5U + 0.25H - d_1^+ + d_1^- = 700$. Note that the value on the right-hand side of the goal equation is a target value of the goal. The left-hand side of the goal equation consists of two parts. One part is a function that defines the amount of goal achievement in terms of decision variables. This part is this part.

Another part is the deviation variables, which represent the difference between the target value for the goal and the level achieved.

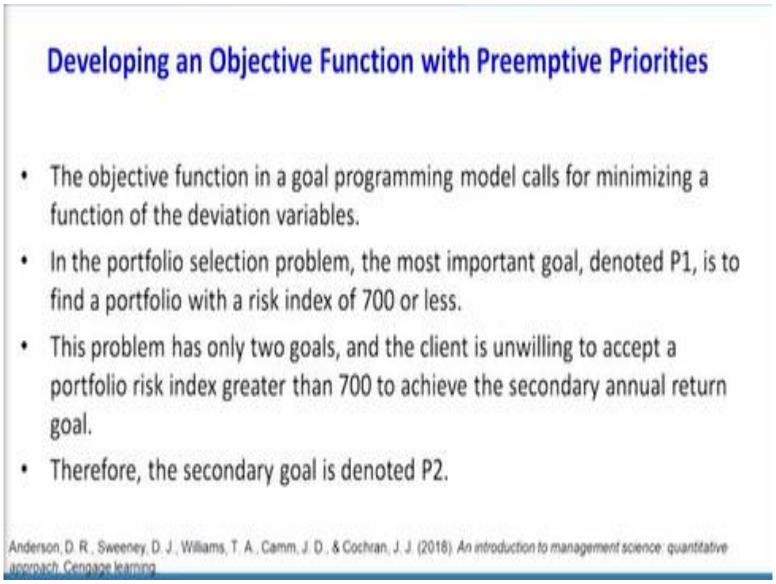


Now, we can go for the goal equation for the second goal. To develop a goal equation for the secondary goal, we begin by writing a function representing the annual return for the investment. We have seen previously that the annual return for stock 1 is 3, and for stock 2, it is 5, so it is a 3U + 5H. Then, we define two deviation variables that represent the amount of over or under-achievement of the goal. In doing so, we obtained d_2^+, which is why we wrote d_2^+ for the second goal.

If it is plus, we know the amount by which the annual return for the portfolio is greater than the target value of 9000 d_2^- the amount by which the annual return for the portfolio is less than the target value of 9000 dollars.

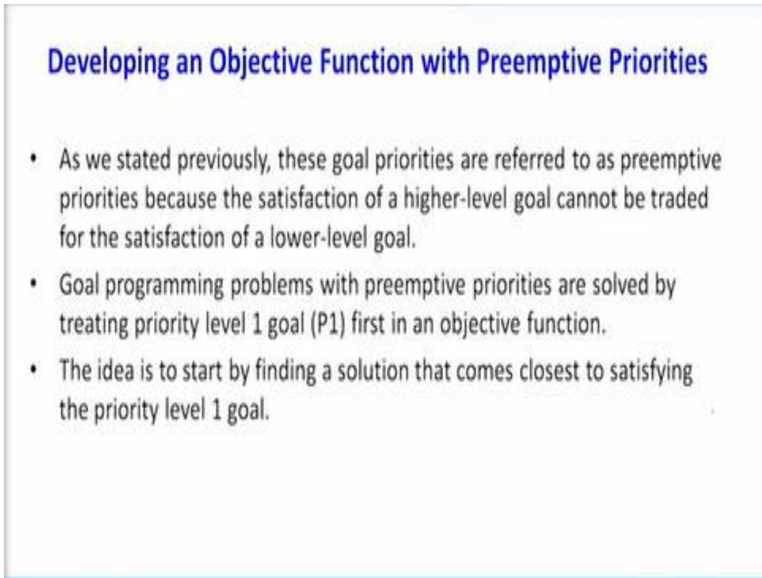


So, using these 2 deviation variables, we write the goal equation for goal 2 as $3U + 5H = 9000 + d_2^+ - d_2^-$. When you bring on the left-hand side it will become $3U + 5H - d_2^+ + d_2^- = 9000$.

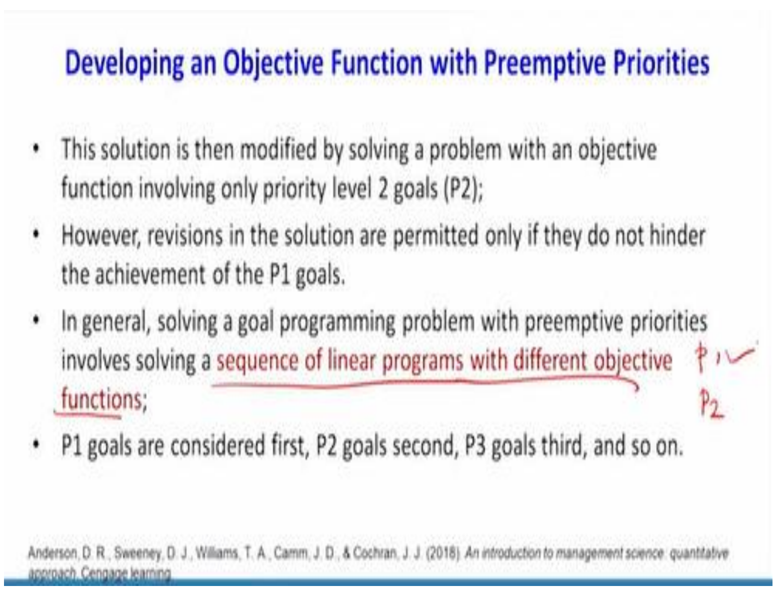


The objective function in a goal programming model calls for minimizing a function of deviation variables. So, in the goal programming context, the objective function will always be minimization, minimizing the deviation variables. In the portfolio selection problem, the most important goal denoted by P1 is to find a portfolio with a risk index of 700 or less. This problem has only two goals, and the client is unwilling to accept the portfolio risk index greater than 700 to achieve the secondary annual return goal.

That means his priority is that the risk index should be less than equal to 700. For the secondary annual return goal, he may be accepting less than his expected target. Therefore, the secondary goal is denoted by P2.

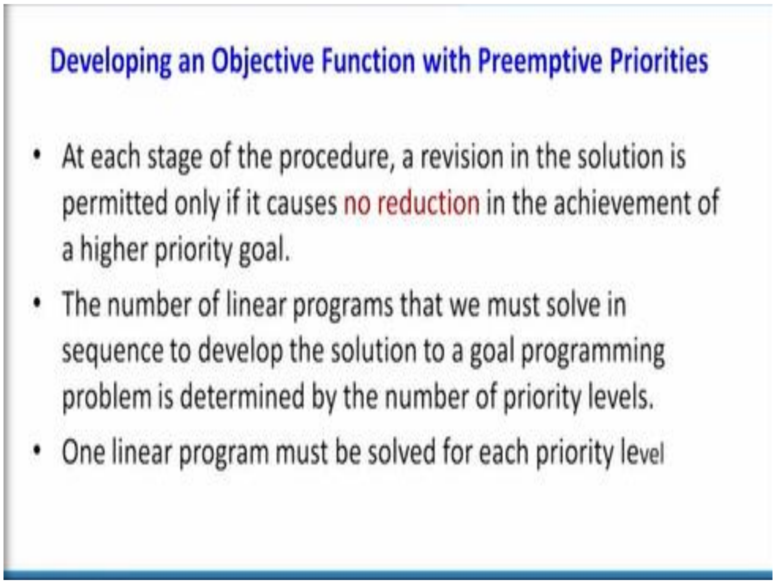


As we stated previously, these goal priorities are referred to as preemptive priorities because the satisfaction of higher-level goal P 1 cannot be traded for the satisfaction of a lower-level goal. Goal programming problems with preemptive priorities are solved by treating the level 1 goal that is a P1 first in the objective function. The idea is to start by finding a solution closest to satisfying the priority level 1 goal.



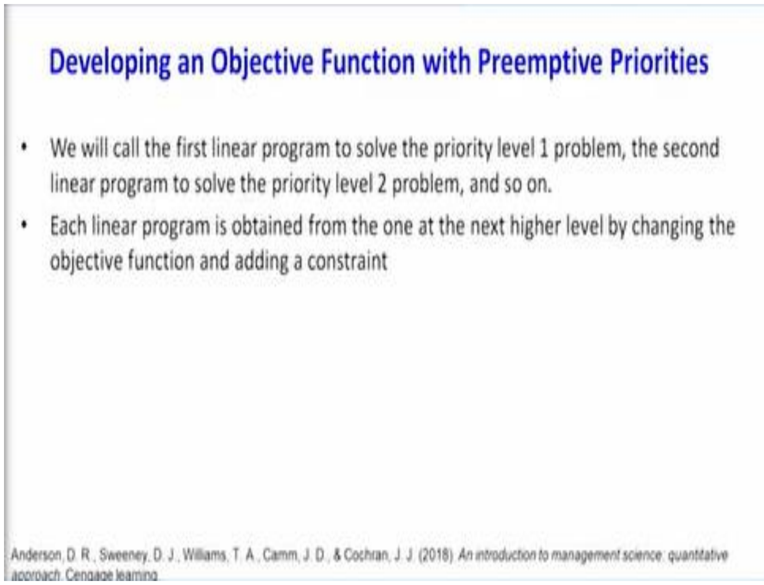
After solving goal 1 then, we are going to solve goal 2. So, this solution is then modified. Which one? Goal 1 solution by solving the problem with an objective function involving only priority level 2 goals. So, we will be solving for goal 1 after achieving goal 1 then we will go for solving goal 2. However, the revisions in the solution are permitted only if they do not hinder the achievement of P1 goals.

That means in goal 2, we can achieve goal 2 without disturbing the solutions of goal 1. In general, solving a goal programming problem with preemptive priorities involves solving a sequence of linear programs with different objective functions. For example, first, we have to solve P1, and then we should solve P2. While solving P2, the solution that is obtained for P1 should not be sacrificed. So, P1 goals are considered first, P2 goals are second, and P3 goals are third, and so on.

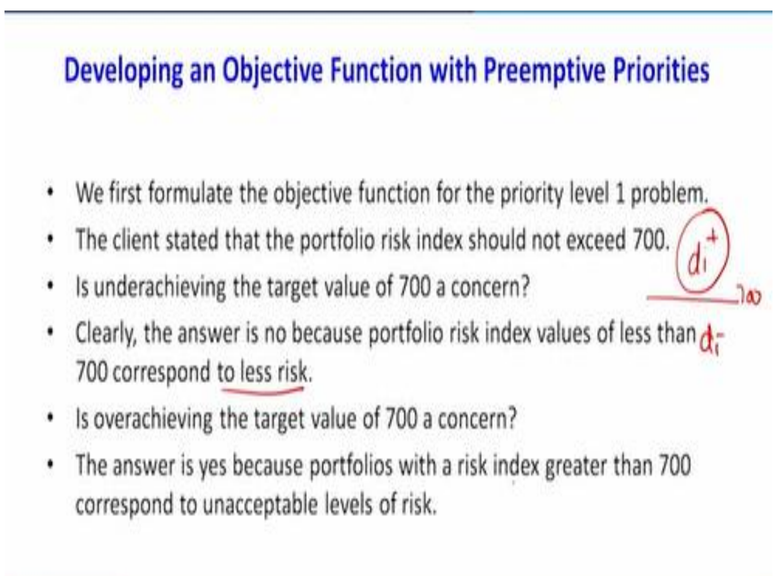


At each stage of the procedure, a revision in the solution is permitted only if it causes no reduction in the achievement of a higher priority level. That means we have got the solution for P1. We are going to get the solution for P2. However, while getting the solution for P2, the solutions for P1 should not be sacrificed. The number of linear programs that we must solve in sequence to develop the solution to a goal programming problem is determined by the number of priority levels.

In our problem, we have two priorities: P1 and P2. So, we have to solve it two times. So, one linear program must be solved for each priority level.



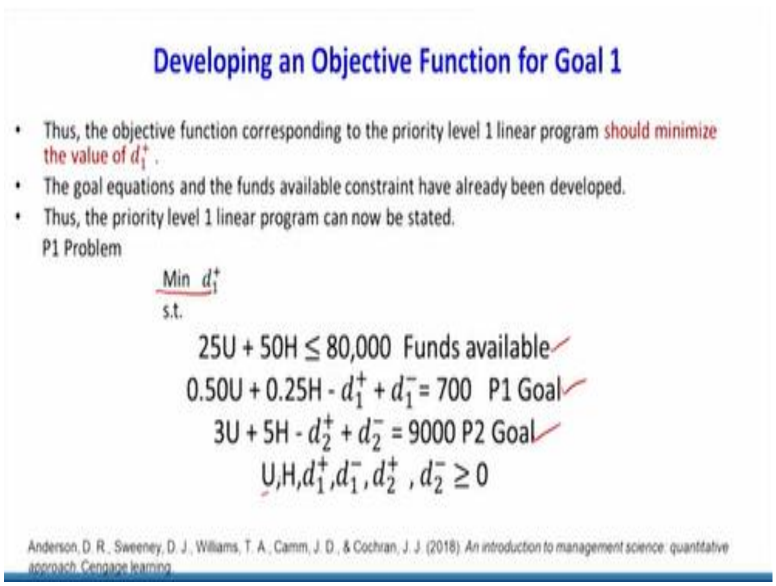
We will call the first linear program to solve the priority level 1 and the second linear program to solve the priority level 2 and so on. Each linear program is obtained from the one at the next higher level by changing the objective function and adding a constraint. So, what we will be doing? After solving goal 1, when we are going to solve for goal 2, we will change the objective function, and then we will add a constraint for goal 1 that I will explain.



We first formulate the objective function for the priority level 1 problem. The client stated that the portfolio risk index should not exceed 700. Now we have to see there are two things. One is overachieving. One is d_1^+, another one is d_1^-. For example, this is 700. Assume that this is a 700 linear scale. Now d_1^+ and d_1^-; we must choose which is more important for the client because this is a risk. Suppose if the value of d_1^+ is positive, we exceed the 700.

So, that is not preferable. Suppose the value of d_1^- is positive. That means we are underachieving our risk is below 700. So, that is preferable. So, what is more important here is that overachievement is not desirable. So, our objective function is to minimize the d_1^+ that is overachieved. Now I will come back to the point. Is underachieving the target value of 700 a concern?

Clearly, the answer is no, because the portfolio risk index value of less than 700 corresponds to less risk. Now, is over-achieving the target value of 700 a concern? The answer is yes because the portfolios with higher indexes greater than 700 correspond to an unacceptable level of risk. So, which is more serious? The d_1^+ is serious. If we exceed 700, that is more serious.



Thus, the objective function corresponding to the priority level 1 linear program should minimize the value of $d1^+$ because we cannot exceed more than 700. So, our objective function is minimizing d_1^+. Thus, the priority level 1 linear program can now be stated as a P1 problem. What is that? We have to minimize the overachievement, which means the risk index should not exceed 700. The next constraint is the funds available constraint, followed by Goal 1 and Goal 2.

Then, the decision variable U, H, d_1^+, d_1^-, d_2^+, and d_2^-. So, for this addition problem, we should find the value of U and H. Dear students, in this lecture, I have discussed examples and the need for a multi-criteria model. Also, I have explained how to formulate the constraint and the objective function. In the next lecture, I will explain how to solve the goal programming problem graphically using the software Desmos. Thank you.