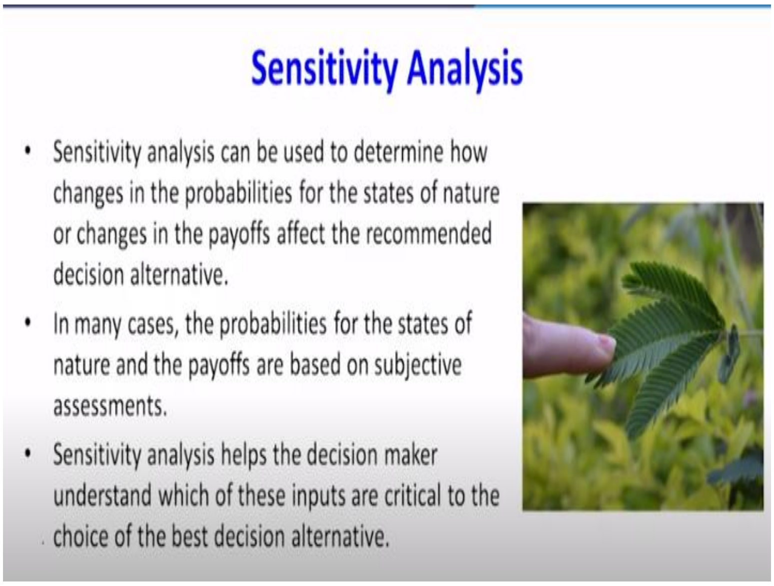
Decision Making with Spreads heet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology-Roorkee

Lecture - 48
Decision Analysis - III

Dear students, in the previous lecture I discussed decision making with probabilities and risk analysis. In this lecture, I am going to discuss sensitivity analysis. So, the agenda for this lecture is how to do sensitivity analysis.

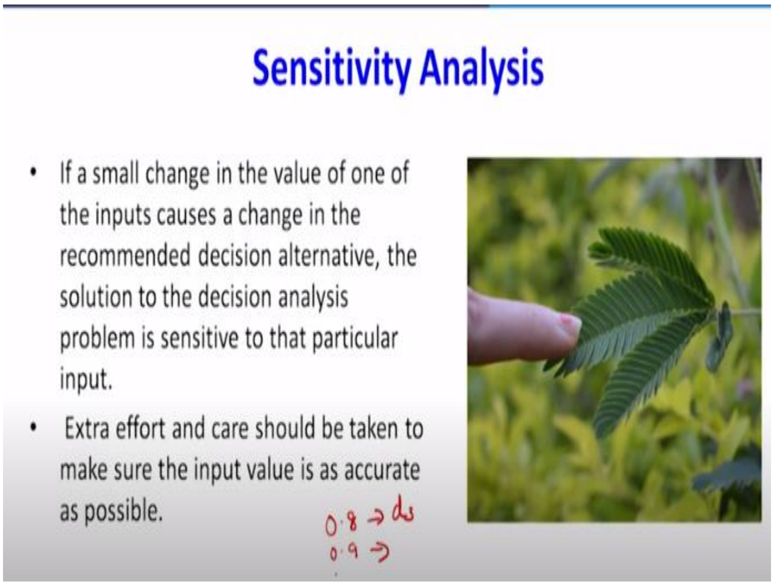


Sensitivity analysis. Sensitivity analysis can be used to determine how changes in the probabilities for the states of nature or changes in the payoff affect the recommended decision alternative. I brought a picture to show what a sensitivity analysis. Suppose there is a small change in probabilities or the payoff; we have to check whether our recommended decision remains the same or that decision also changes.

If there is a small change, our recommended decision also changes, then we can say that the model is a highly sensitive model. In many cases, the probabilities for the states of nature and the payoff are based on subjective assessment. Why are we saying subjective assessment? It is a tentative value. It need not be 100 percent sure.

So sensitivity analysis helps the decision maker understand which of these inputs are critical to the choice of the best decision alternative. So, there are two inputs we have considered. One is the probability, other one is the payoff. Even in the probability for the states of nature also, there are two types of states of nature. One is strong demand and weak demand.

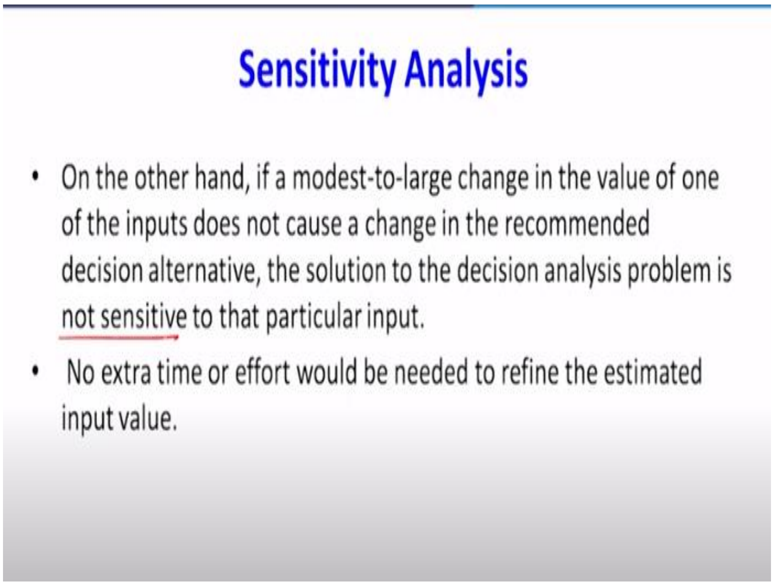
For example, if the probability of strong demand changes, how will our recommended decision change? So, the study of this effect of one variable on our decision is nothing but our sensitivity analysis.



So if a small change in the value of one of the inputs causes a change in the recommended decision alternative, the solution to the decision analysis problem is sensitive to that particular input because there are slight changes affecting our recommended decision. So extra effort and care should be taken to make sure that input value is as accurate as possible.

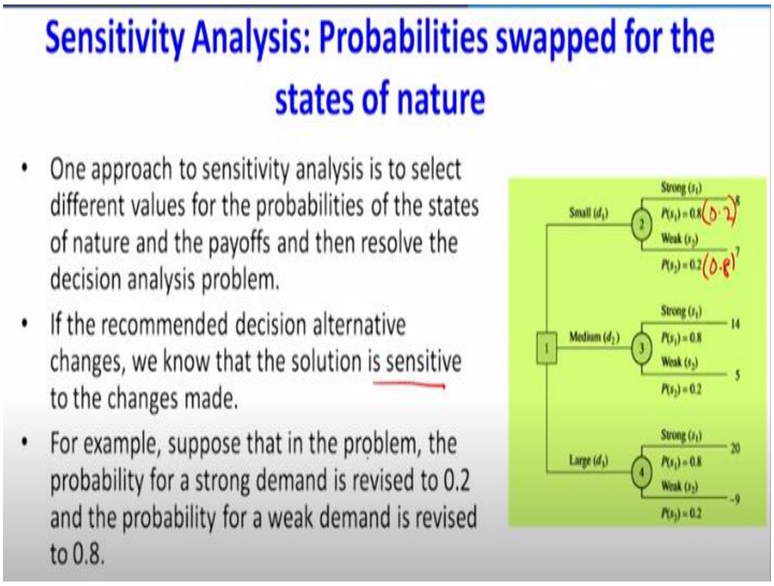
Because it is very sensitive if there are small changes, for example, say, the probability. So probability for strong demand is 0.8. Suppose it is 0.9. Suppose when it is 0.8, we have recommended d_3 is the best decision. Suppose the 0.8 becomes 0.9, whether the d_3 remains our best decision or it changes? If it is changing, then we can say this input is very sensitive input.

Then, we should be very careful in assessing this input. That is assessing and knowing this probability because this is very critical.



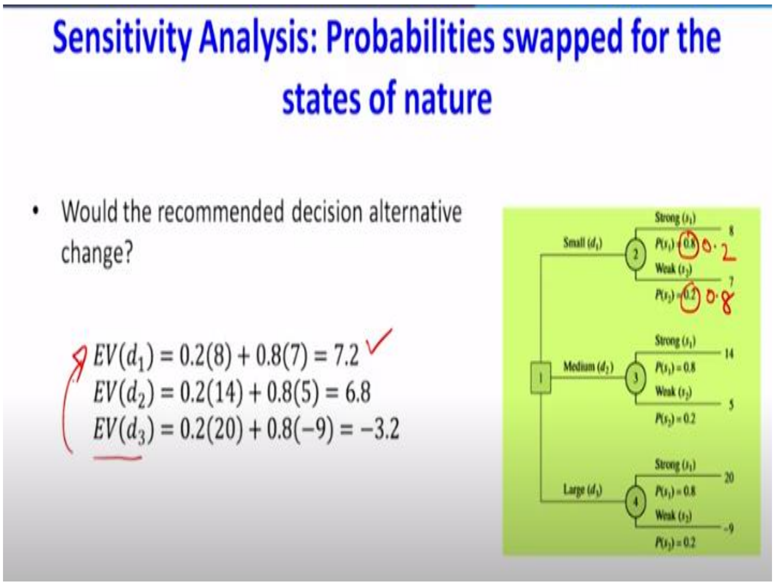
On the other hand, if a modest to a large change in the value of one of the inputs does not cause a change in the recommended decision alternative, the solution to the decision analysis problem is not sensitive to the particular input. Instead of saying not sensitive, we can say it is a very robust model. What do we say robust? Robust in the sense even if there is a small change in the input, that is not affecting the performance of the model.

The context of robust design differs from problem to problem. Sometimes, the problem is that the model needs to be very sensitive. Sometimes, the model should not be sensitive. So no extra time or effort would be needed to refine the estimated input value. When no, extra effort is not required? When the model is not sensitive. That means the model is a robust model, we need not bother about slight changes in the input.



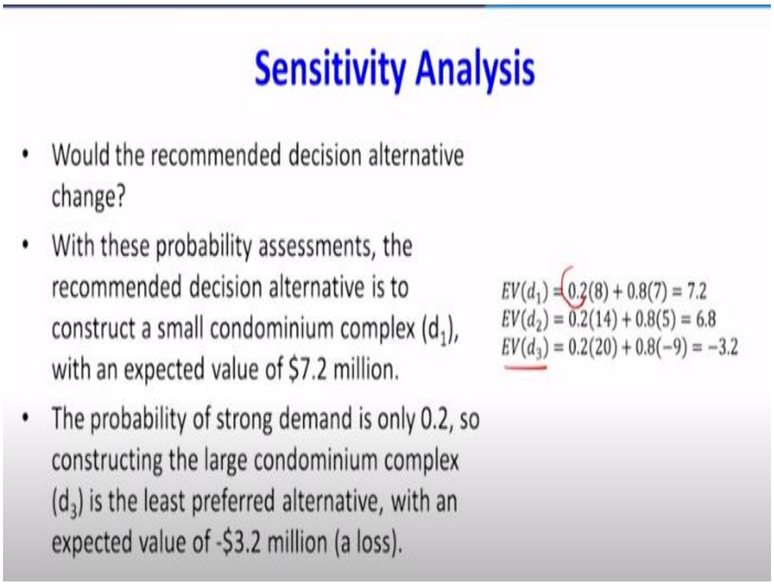
Now, we will see what will happen if the probabilities for strong demand and weak demand are swapped. We know that previously the probability of strong demand is 0.8. Suppose instead of 0.8, suppose if I make 0.2, if I make 0.8, what will happen? That is what we are going to study.

So, one approach to sensitivity analysis is to select different values of probabilities of the states of nature and the payoff and then resolve the decision analysis problem. So when we solve and resolve if the recommended decision is alternative changes, we know that the solution is sensitive to the change made. For example, suppose that in the problem, the probability of strong demand is revised to 0.2, and the probability of weak demand is revised to 0.8. Just I swapped the probability.



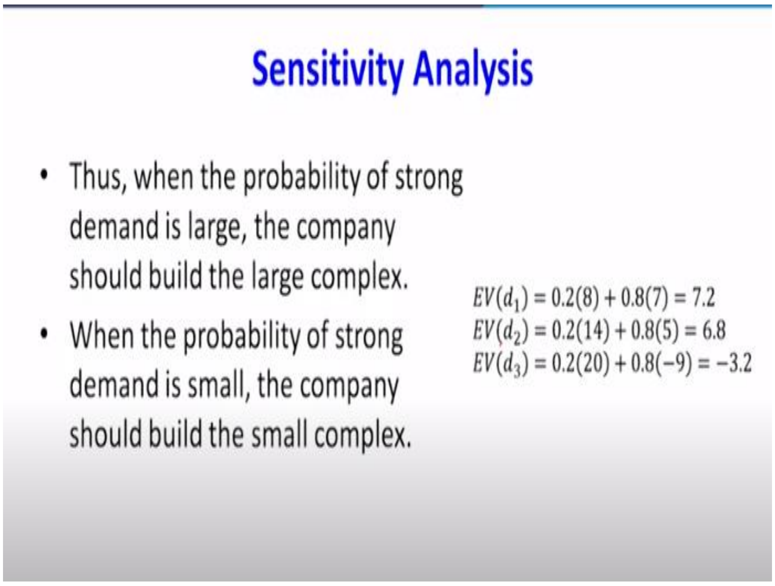
So, when we swap the probability, you see that previously, it was 0.8, now, it has become 0.2. This 0.2 becomes 0.8 everywhere. What will happen here? 0.2 multiplied by 8, 0.8 multiplied by 7. So is getting 7.2 expected value of decision alternative 1. That is a small condominium. And EV(d_2) becomes 6.8, and EV(d_3) has become -3.2. So out of these three, which is the highest one? So, EV(d_1) is the highest one.

So what has happened? We have changed the probability values and our decision to what you have recommended; previously, we recommended d_3 as the best decision. Now we are going to, we are recommending that d_1 is the best decision. Because now the, our recommended alternative has now changed. So, we are saying that now the model is sensitive.

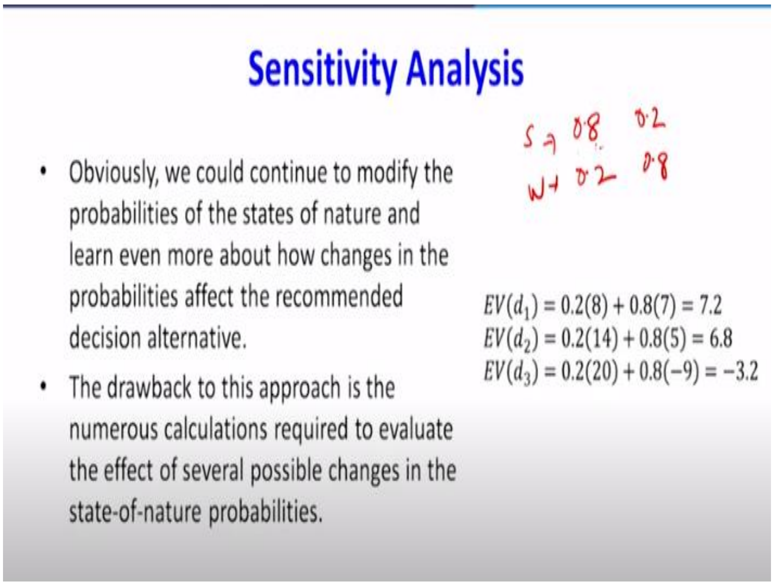


Would the recommended decision alternative change? With this probability assessment, the recommended decision alternative is to construct a small condominium complex. Because the probability of strong demand is only 0.2 and with an expected value of $7.2 million.

The probability of strong demand is only 0.2, so constructing the large condominium complex, which was our previous decision, this one, is the least preferred because that is the lowest value, least preferred alternative with an expected value of -$3.2 million, which is a loss. So, what we are learning here is that the probability of strong demand is changing our decision.

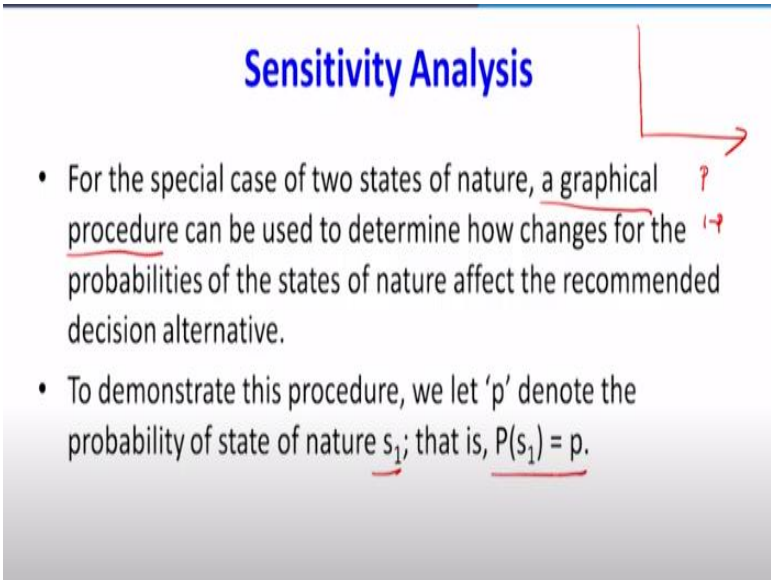


Thus, when the probability of strong demand is large, the company should build a large complex. When the probability of strong demand is small, the company should build a small complex.



Obviously, we could continue to modify the probabilities of the states of nature and learn even more about how changes in the probabilities affect the recommended decision alternative. So, what we have done, previously it was 0.8 and 0.2. This is a strong demand and weak demand. Now we have swapped that. So this is 0.2, and this is 0.8, so we have resolved the problem.

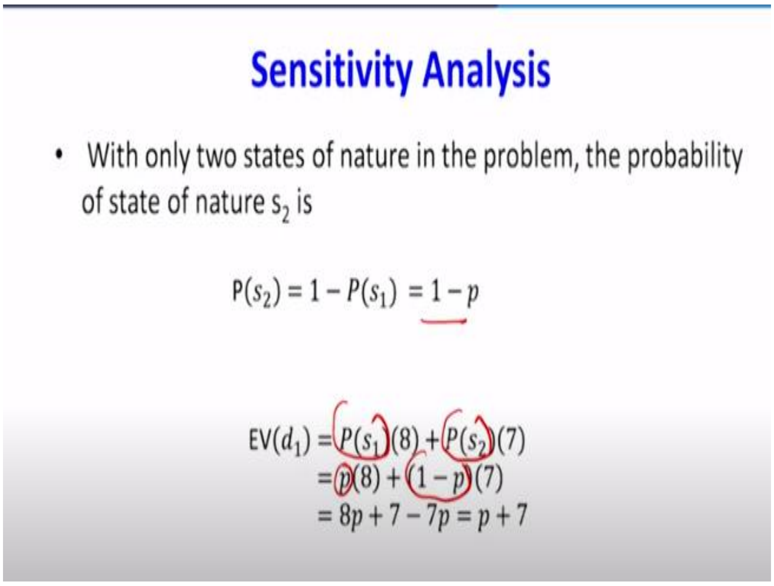
So what we can do instead of this is keep on changing the probability of strong demand, 0.7, 0.9, 0.5, 0.6, and we can resolve the problem. Every time, we can see how it affects our recommended decision. So, the drawback to this approach is the numerous calculations required to evaluate the effect of several possible changes in the states of nature probabilities. So that will become so cumbersome.



For the special case of two states of nature, in our problem, only two states of nature, strong demand, and weak demand, a graphical procedure can be used to determine how changes in the probabilities of the states of nature affect the recommended decision alternatives. If there are only two states of nature okay, we can graphically we can see how these probabilities affect the decision alternative.

When in a graph, we can draw only the two-dimensional figure, and the P and (1 – P) are okay. So, if the probability of strong demand is P, the probability of weak demand can be written as (1 – P). So if there are only two possibilities, we can pictorially represent it. If there are more than three states of nature, we cannot do it pictorially.

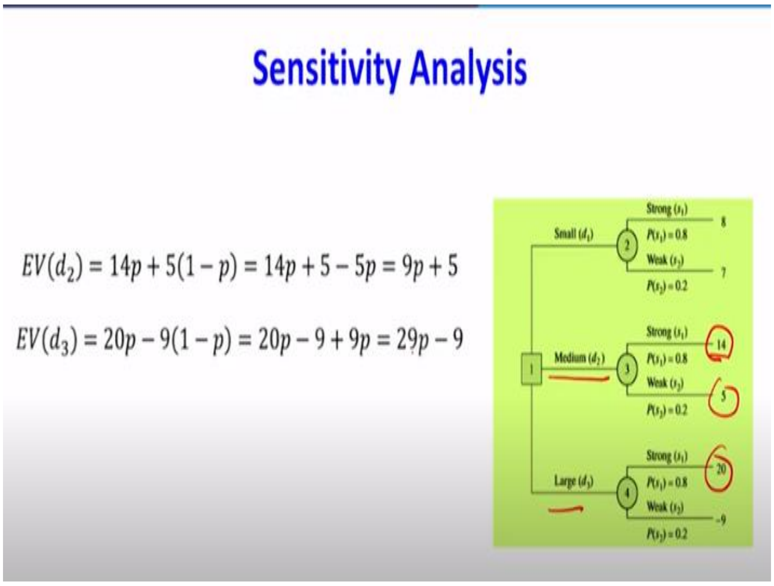
To demonstrate this procedure, we let p denote the probability of states of nature s_1 that is strong demand. So, we are going to consider $P(s_1) = p$.



And p of with only two states of nature in the problem, the probability of states of nature s_2 is

$$P(s_2) = 1 - P(s_1) = 1 - p$$

$$\begin{aligned} EV(d_1) &= P(s_1)(8) + P(s_2)(7) \\ &= p(8) + (1-p)(7) \\ &= 8p + 7 - 7p = p + 7 \end{aligned}$$

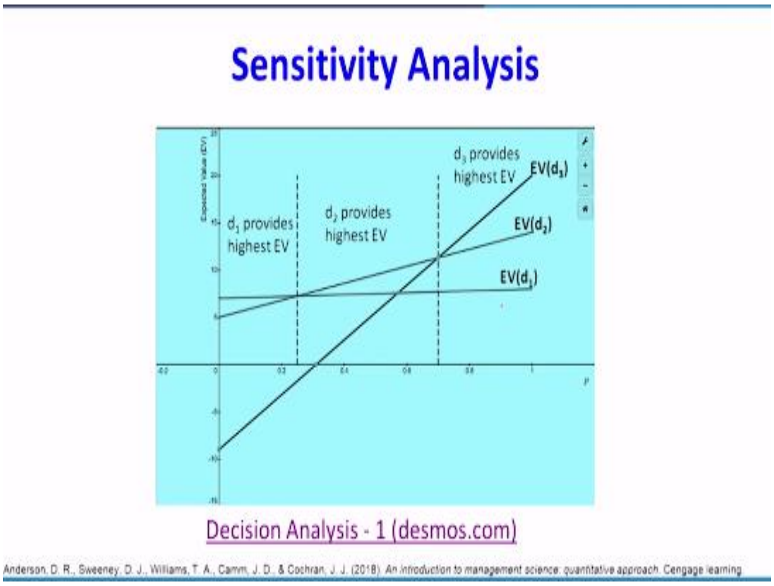


Similarly for d_2 ,

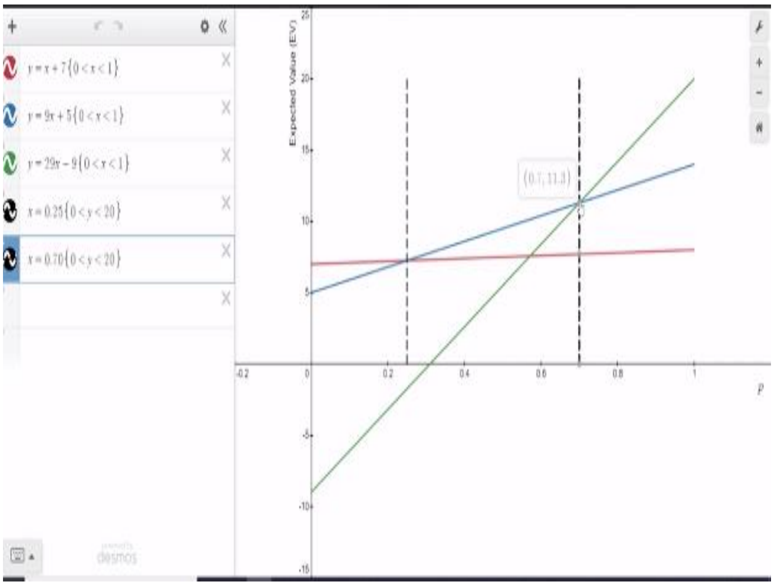
$$EV(d_2) = 14p + 5(1-p) = 14p + 5 - 5p = 9p + 5$$

$$EV(d_3) = 20p - 9(1-p) = 20p - 9 + 9p = 29p - 9$$

.



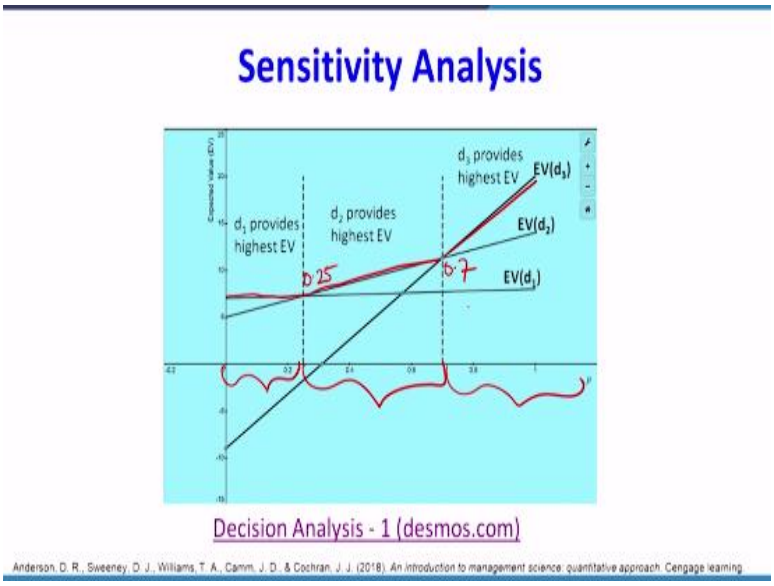
So what am I going to do? With the help of Desmos, I am going to solve these three equations. What are the three equations? This one. $EV(d_1)$, $EV(d_2)$ and $EV(d_3)$. I will plot it in a graph with the help of the Desmos,.



So, see the first equation is $EV(d_1) = p + 7$. But in the Desmos, we have to use only two variables, y and x. So, in $EV(d_1)$, I have written as y, and the pI has taken as x. So, $y = x + 7$. We know x is the probability that should be 0 to 1. So, we have plotted that red color line. Second one $y = 9x + 5$. That is nothing but $EV(d_2) = 9p + 5$, which we have drawn as the blue line.

The third one, $EV(d_3)$, is $y = 29x - 9$, which is our green one. So what will happen? In the x-axis, we have taken the probability; in the y-axis, it is the expected value. So when the red and the blue line intersect, we will get some value of p. What is the value of p? It is 0.25. Similarly, the green and the blue when intersect, we get a value of 0.7, so what we are inferring from this table.

So, this table, I will interpret this; I have taken a screenshot of this picture, and then I have pasted it into the presentation. I will go back to the presentation now. I will interpret from the presentation.

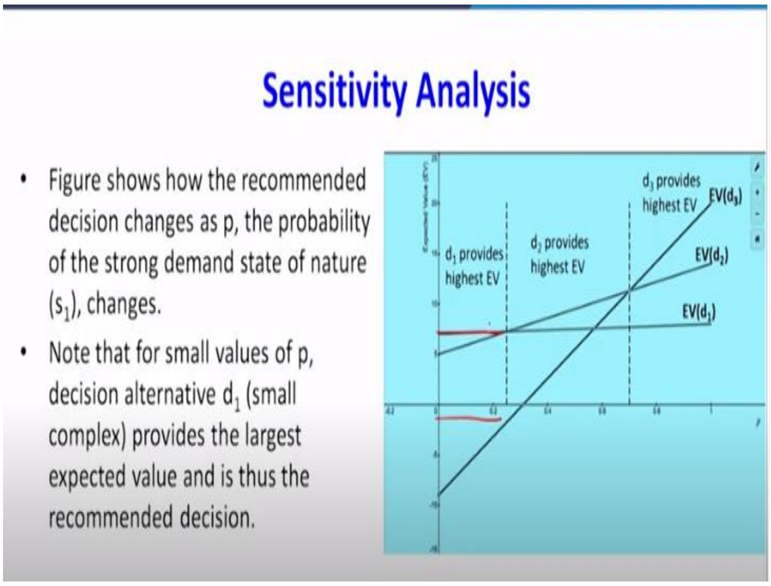


See the Desmos and how the Desmos software works. I have explained in my initial lectures that when we are solving a graphical method, it is okay. So, what we are interpreting, what we are understanding from this figure is whenever the probability of strong demand is less than 0.25, the best alternative is d 1. What is d_1? Going for it because this is the top one. Going to construct a small condominium.

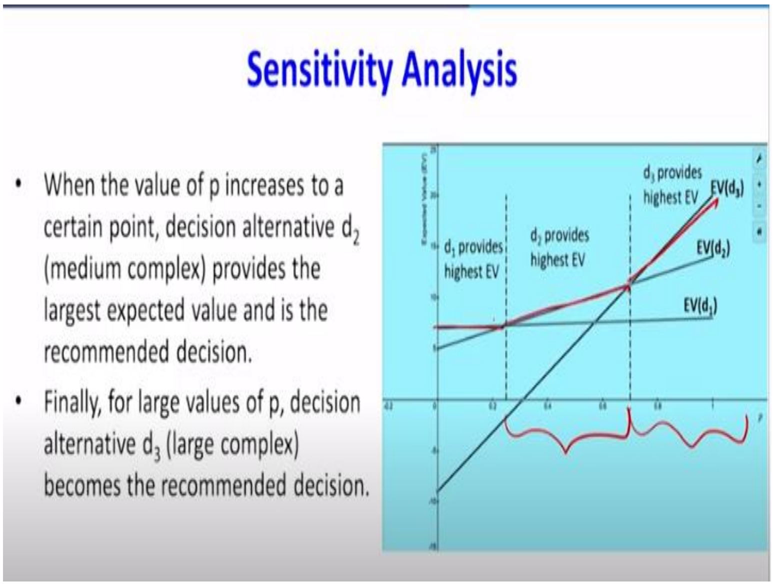
Whenever the probability of strong demand is between 0.25 and 0.7 this line. The best suggestion is to go for d_2, which is constructing a medium-sized condominium. Whenever the probability of strong demand is above 0.7, the best decision is d 3. So, in our given problem, you see the probability of strong demand is 0.8. What was our conclusion? It is d_3; it is going for constructing large-size condominiums.

So, what we understand is that we have a range of probabilities for strong demand. What is the range of probability? Here it is 0 to 0.25 this one. Another one is this one 0.25 to 0.7. So, 0.7 to 1. So up to 0.25 d_1 is the best decision. Between 0.25 and 0.7 d_2

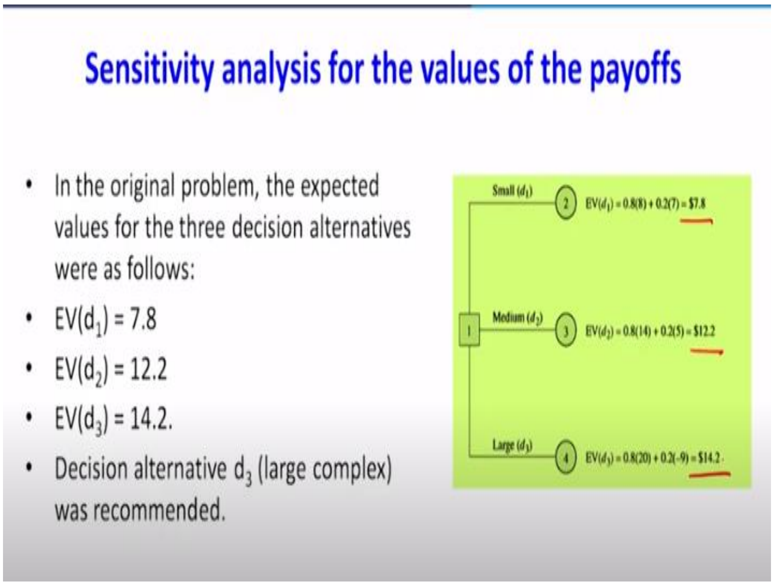
is the best decision. The probability of strong demand goes 0.7 and above d_3 is the best decision. That is our interpretation of this picture.



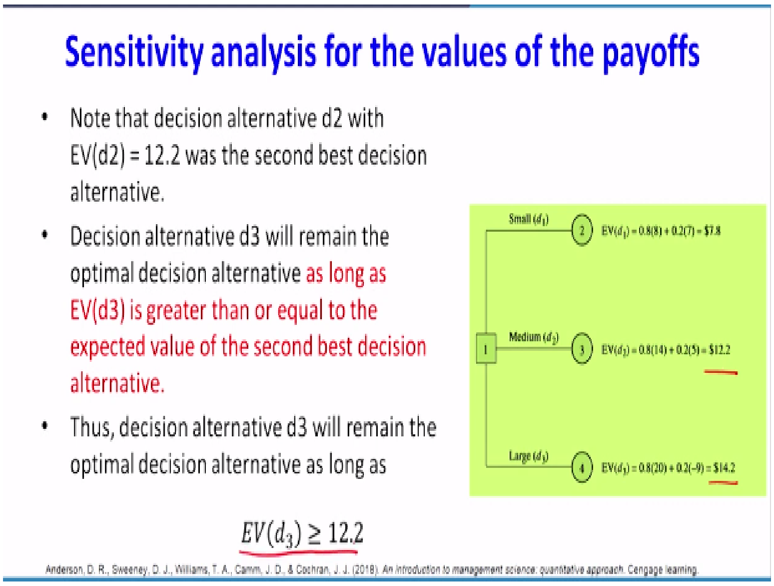
So, the figure shows how the recommended decision changes as p, that is, the probability of strong demand state of nature s_1 changes. Note that for a small value of p here, for a small value of p, the decision alternative d_1 provides, that is, a small complex provides the largest expected value. See, this is the top of these two lines, which one EV (d_1) as and is thus the recommended decision.



When the value of p increases to a certain point, the decision alternative d2, so if the in this range, so 0.25 to 0.7, the d_2 is the best decision because that provides the largest expected value. Finally, for a large value of p in this rage decision, alternative d 3 becomes the recommended decision because this comes on the top. So here this is on the top, here this is on the top.

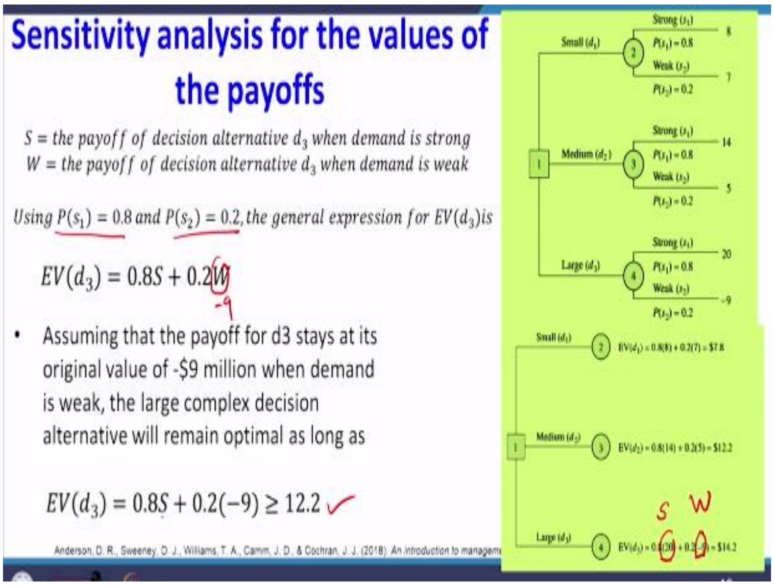


Now, one input is the probability, and the other input is the payoff. Now, we are going to study sensitivity analysis for the values of the payoff. In the original problem, the expected values for the three decision alternatives are 7.8, 12.2, and 14.2. So, we have recommended the decision alternative d_3.



Note that the decision alternative d_2, which is 12.2, was the second-best decision alternative. So, the decision alternative d_3 will remain the optimal decision alternative as long as the expected value of d_3 is greater than or equal to the expected value of the second-best decision alternative. So, to choose d_3, we do not need to have 14.2. If it is greater than the second-best alternative, that is 12.2, the d_3 will be the best decision.

Thus, the decision alternative d_3 will remain the optimal additional alternative as long as the expected value of d_3 is greater than or equal to 12.2.



Now we are going to consider S the payoff decision alternative d_3 when the demand is strong. W is the payoff of the decision alternative d_3 when the demand is weak. We know that the probability of s1 is 0.8, probability of s_2 is 0.2. So the previous is nothing, but instead of 20 and -9, instead of 20, I am going to write s, strong demand. Instead of -9, I am going to write W, which is a weak demand.

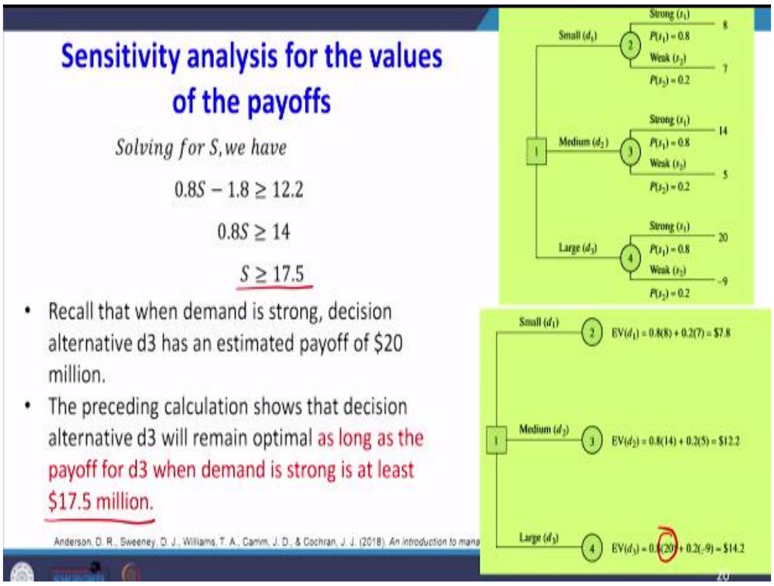
So what will be the expression for this EV(d_3)? 0.8 into S plus 0.2 into W, assuming that the payoff for d 3 stays at its original value of -9.

$$EV(d_3) \geq 12.2$$

Suppose instead of W, we substitute minus $9 million when the demand is weak. So the large complex decision alternative will remain optimal as long as this expression 0.8 into S, 0.2 into -9 is more significant than 12.2.

$$EV(d_3) = 0.8S + 0.2(-9) \geq 12.2$$

So, from this equation, we can find out the value of S. What is the S? The payoff is when there is strong demand.



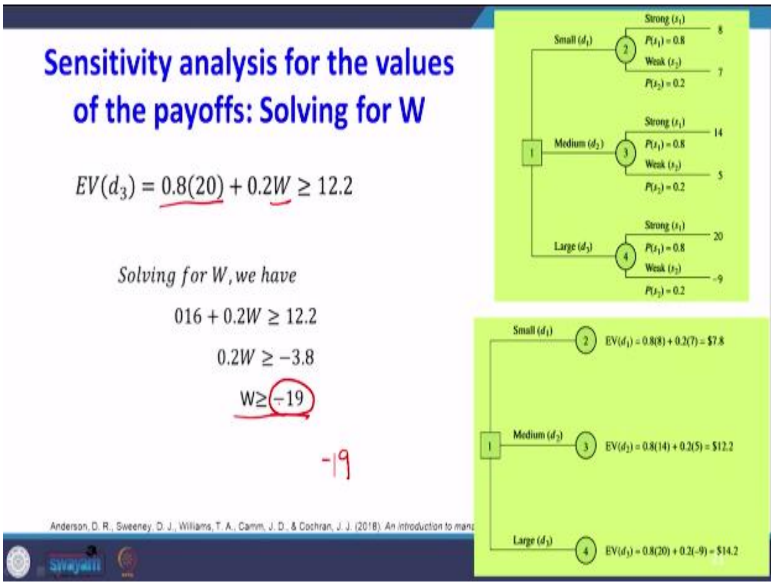
So, solving for S, 0.8 - 1.8 is greater than or equal to 12.2. When you simplify, we get S greater than or equal to 17.5. Recalling that when the demand is strong, the decision alternative d_3 has an estimated payoff of 20 million. You see that this is 20 million here. Now it need not be 20. Looking at the 17.5, even though instead of 20, if it is greater than 17.5, d_3 will still be our best decision.

$$0.8S - 1.8 \geq 12.2$$

$$0.8S \geq 14$$

$$S \geq 17.5$$

So, what we are learning is that the preceding calculation shows that the decision alternative d_3 will remain optimal as long as the payoff for d_3 when the demand is strong is at least \$17.5 million; that is the learning. We have solved the strong demand. Suppose there is weak demand; what will be the range, and what will be the bounding values for the payoff? That we will see that.



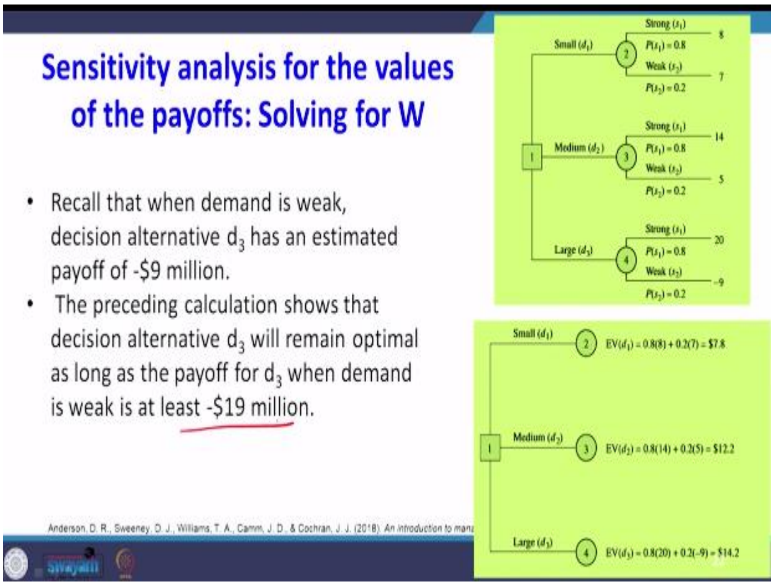
So, what are we going to do now? Now we are going to take this 20 is fixed. What is the 20? When the probability of there is a strong demand, the payoff is 20. That we are going to find out the value for this W. So, when we solve that, we know that this is 12.2. When we solve that, we get a value greater than -19. Now you see that in our problem it is given -19.

$$016 + 0.2W \geq 12.2$$

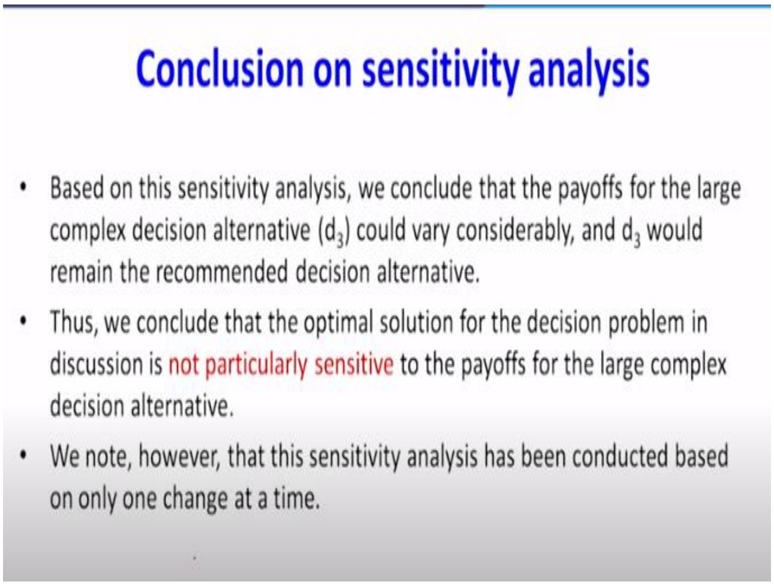
$$0.2W \geq -3.8$$

$$W \geq -19$$

So, what we are learning from this is that even when the demand is weak, the value of payoff, even though it goes up to -19, still our d_3 will be the best decision. What we are learning, we have found the range for our payoff. During that range, our recommended decision d_3 will remain the same.

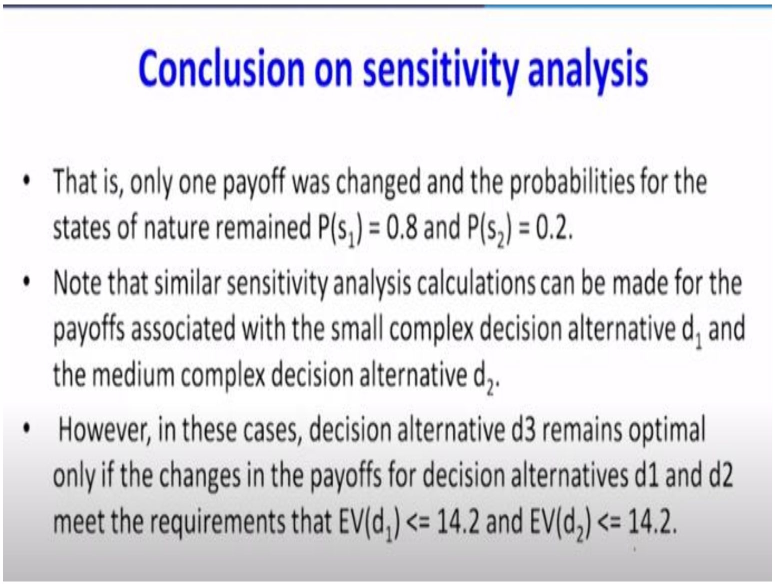


Recall that when the demand is weak, the decision alternative d_3 has an estimated payoff of -$9 million. So, the preceding calculation shows that the decision alternative d_3 will remain optimal as long as the payoff for d_3 when the demand is weak is at least -$19 million.



What is the conclusion of sensitivity analysis? Based on the sensitivity analysis, we conclude that the payoff for the large complex decision alternative d_3 could vary considerably, and d_3 would remain the recommended decision alternative. So, what we are learning even though the payoff changes for d_3, still the d_3 will be the best decision.

Thus, we conclude that the optimal solution for the decision problem in the discussion is not particularly sensitive to the payoff for the large complex decision alternative, which is the d_3. We note, however, that this sensitivity analysis has been conducted based on only one change at a time. In any sensitivity analysis, even at the beginning of the lecture, we have also learned that only one change is permitted at a time.



That is, only one payoff was changed, and the probabilities for the states of nature remained, that is, $P(s_1)$ equal to 0.8 and $P(s_2)$ equal to 0.2. Note that similar sensitivity analysis calculations can be made for the payoff associated with a small complex decision alternative d_1 and d_2. So far, you have done only for d_3. The same thing can be done for d_1 and d_2.

However, in these cases, decision alternative d_3 remains optimal only if the changes in the payoff for decision alternative d_1 and d_2 meet the requirement of the requirement that the $EV(d_1)$ is less than 14.2 and $EV(d_2)$ less than 14.2. So, what we are learning here is that the expected values of d_1 and d_2 are currently less than 14.2. Because it is less than 14.2, the d_3 will always be the best decision.

Dear students, in this lecture, I discussed sensitivity analysis and the concept of robust design. There are two inputs in the given problem. One is the probability of the states of nature, and the other one is the payoff. I have graphically explained how the changes in the probability of states of nature affect our decision alternatives.

So we have seen that even though there is a change in probabilities of states of nature, our decision alternative remained the same. The next input is the payoff. So, I have explained that even though there is a variation in the payoff, that variation in the payoff did not affect our decision alternative. So, we have concluded that our model is robust model, that is, the d_3 is the best decision. Thank you.