Decision Making with Spreadsheet
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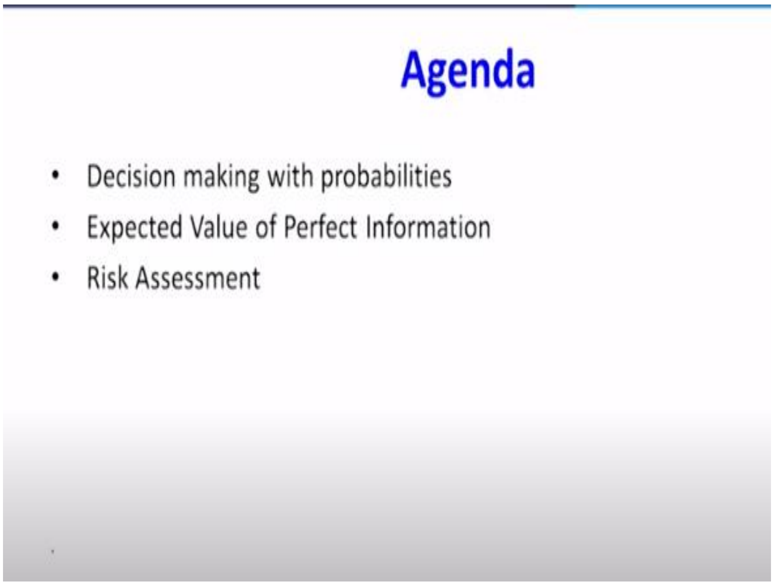
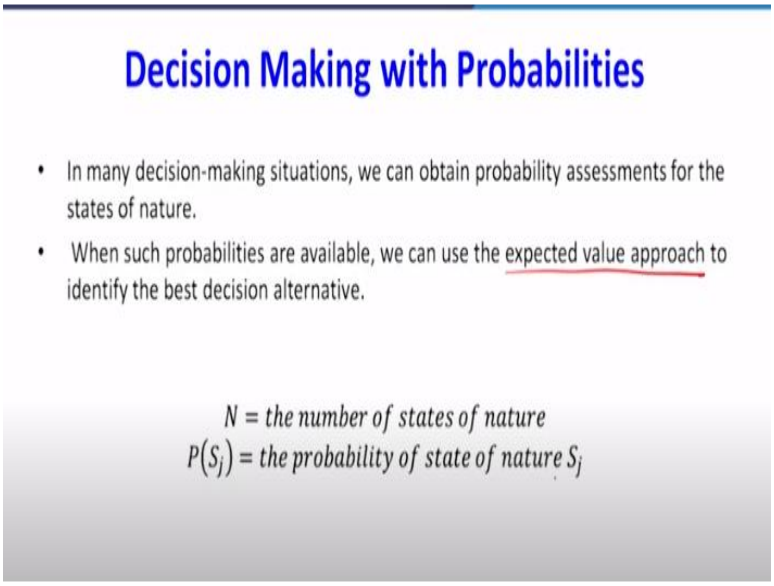
Lecture - 47
Decision Analysis - II

Dear students, in the previous lecture, I discussed decision-making without probabilities. In this lecture, I am going to discuss how to make a decision when the probability is given. What is the meaning of probability? Here is the probability for our state of nature. After that, I am going to discuss the expected value of perfect information. What is the meaning of perfect information here?

If somebody is going to give you the states of nature, what is the value of that information? After that, I am going to discuss risk analysis for decision-making.

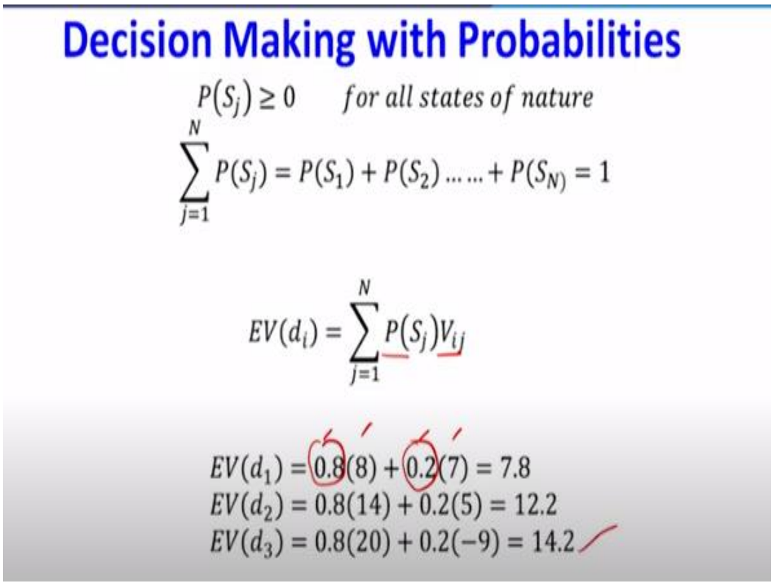


So, the agenda for this lecture is to make decisions with probabilities and find the expected value of perfect information—finally, the risk assessment.



In many decision-making situations, we can obtain a probability assessment of the states of nature. When the probabilities are available, we can use the expected value approach. You might have known; you might have studied this expected value approach. Nothing but the mean value, weighted mean value approach to identify the best decision alternative.

So when we say capital N is the number of states of nature in our problem, too, the PS j represents the probability of states of nature. Here, j, there is a possibility of strong demand and weak demand.

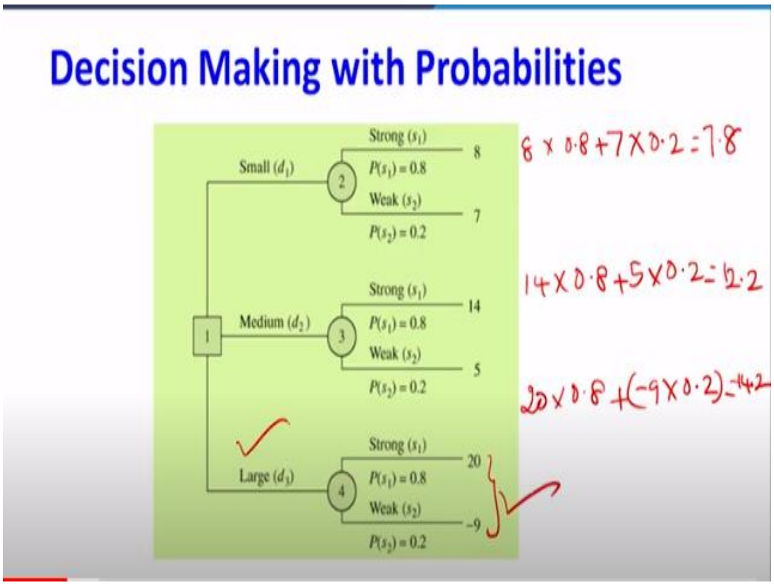


So, what are the properties? The value of probability should be greater than or equal to 0 for all states of nature. The other property is the sum of probability should be 1. So, what is the meaning of expected value for different decision alternatives? The expected value is nothing, but the probability multiplied by corresponding payoff for all states of nature. For example, the probability of strong demand is 0.8, and the payoff is 8.

So multiply 0.8 multiply by 8. The probability of weak demand is 0.2; the payoff is 7. So when you multiply this, we are getting 7.8. So, the expected value of decision 1 is 7.8. The expected value of d_2 for medium complex size is 12.2. The expected value of d3, going for a large complex, is 14.2.

$$EV(d_1) = 0.8(8) + 0.2(7) = 7.8$$
$$EV(d_2) = 0.8(14) + 0.2(5) = 12.2$$
$$EV(d_3) = 0.8(20) + 0.2(-9) = 14.2$$

So, what is the difference between decision-making without and with probability?
In the previous lecture, this 0.8 and 0.2 is not given. But now, the probability is given. So, we are looking for the expected value. So out of these three expected values, which is the highest one? The highest one is d_3. So, we are suggesting going to a large complex.

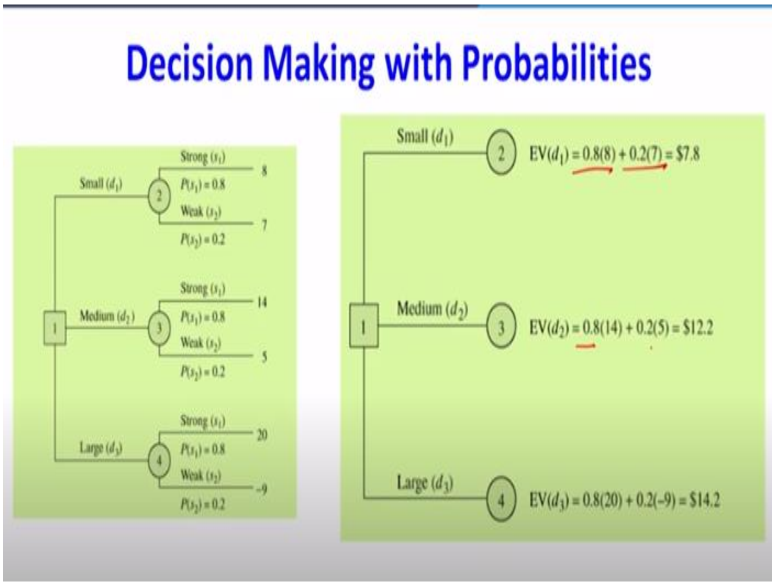


So, how are we doing this? So, this is 8 multiplied by 0.8 plus 7 multiplied by 0.2. Here 14 multiplied by 0.8 plus 5 multiplied by 0.2. Here 20 multiplied by 0.8 plus (-9) multiplied by 0.2. So, we got the values. So, what are the values? For the first one, it is 7.8. For the second one, it is 12.2. For the third one, it is 14.2. So out of these three, this value is the highest one. So, we go for this one d_3.

$$8 \times 0.8 + 7 \times 0.2 = 7.8$$

$14 \times 0.8 + 5 \times 0.2 = 12.2$

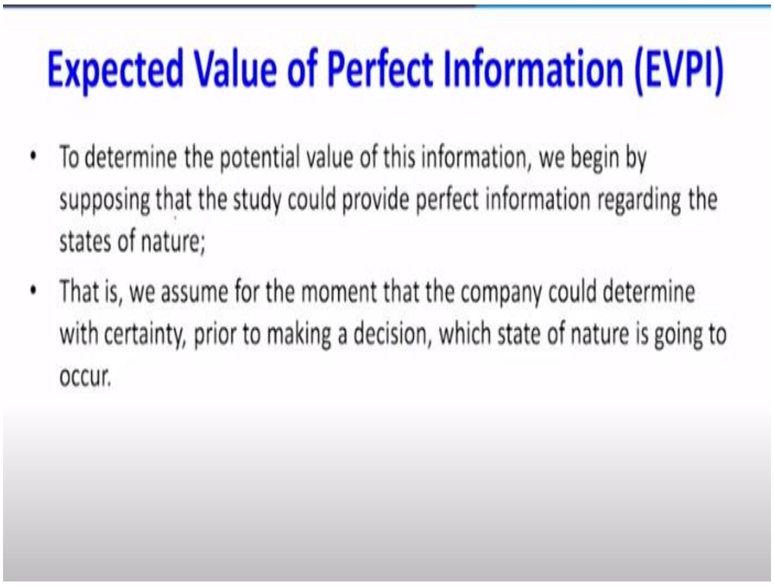
$20 \times 0.8 + (-9 \times 0.2) = 14.2$



Here, I have shown how we got 7.8, nothing but this 0.8 into 8, 0.2 into 7, 0.8 into, 14, 0.2 into 5. So, we got 7.8, 12.2, and 14.2.

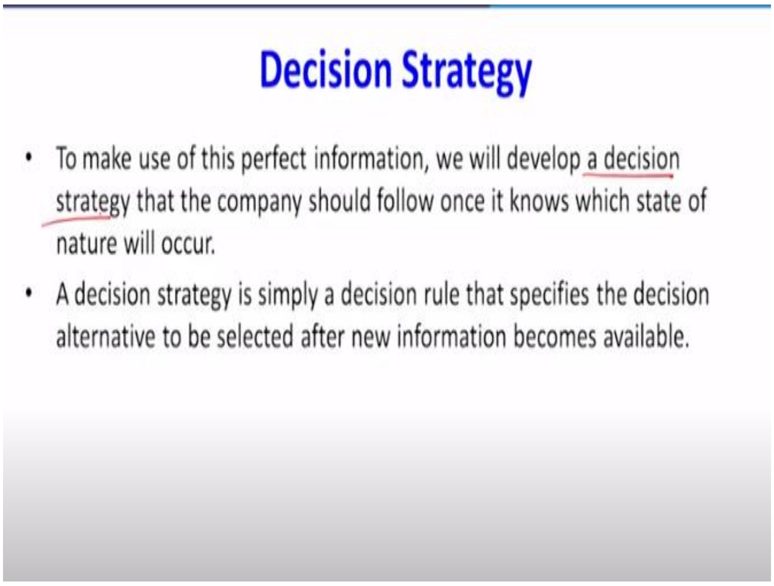
Our suggestion is to go for a large-size complex. Now we are going to find another term called the expected value of perfect information. What is the meaning of this perfect information? Suppose that the company has the opportunity to conduct a market research study. So, the company is going for a market research study that would help evaluate buyers' interest in condominium projects and provide information that the management could use to improve the probability assessment for the states of nature.

The company has assigned a marketing company, so they are doing some market research studies. So, they are going to provide information about the states of nature.

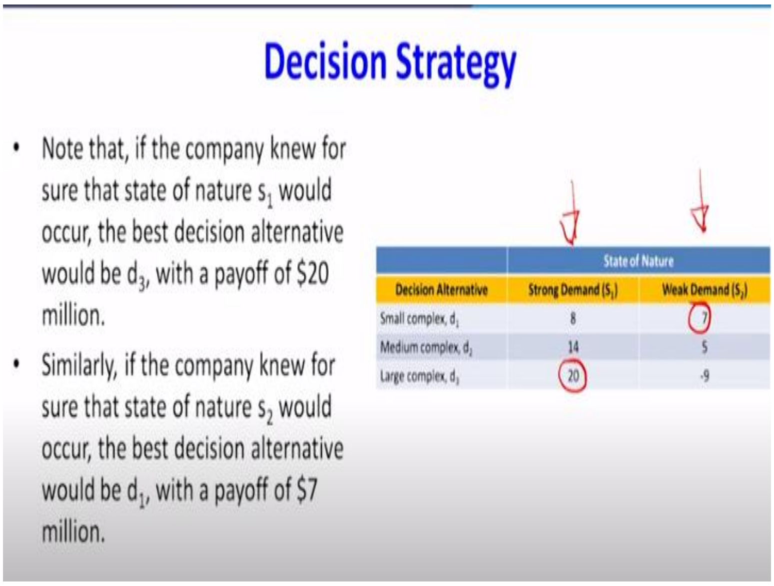


To determine the potential value of this information, what is that information, and what are going to be the states of nature? So, we begin by supposing that the study could provide perfect information regarding states of nature. So, we are going to find out what will be the value of this perfect information, perfect information about the states of nature. So, the marketing research company will determine whether the demand is going to be strong or weak demand.

That is, we assume for the moment that the company could determine with certainty, that is, with the help of this marketing research company, that is, prior to making a decision, which states of nature are going to occur.

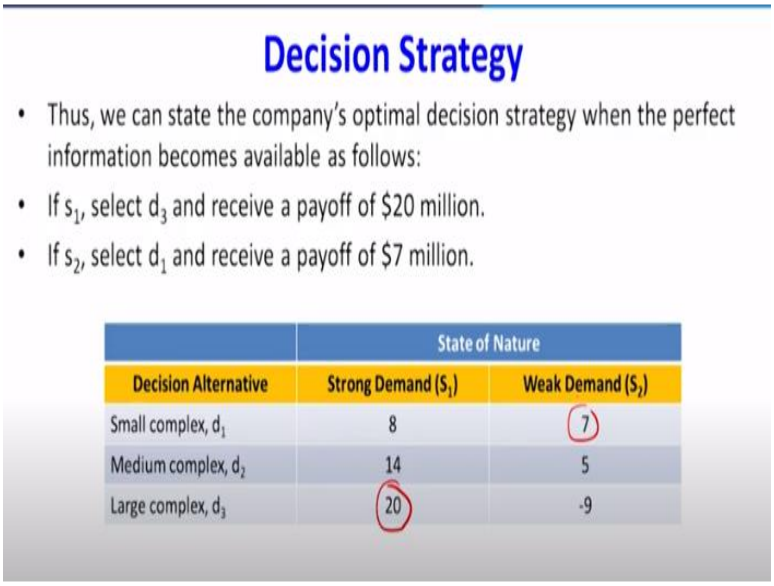


To use this perfect information, we will develop a decision strategy that the company should follow once it knows which states of nature will occur. What is a decision strategy? A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available. Here is new information about the states of nature.

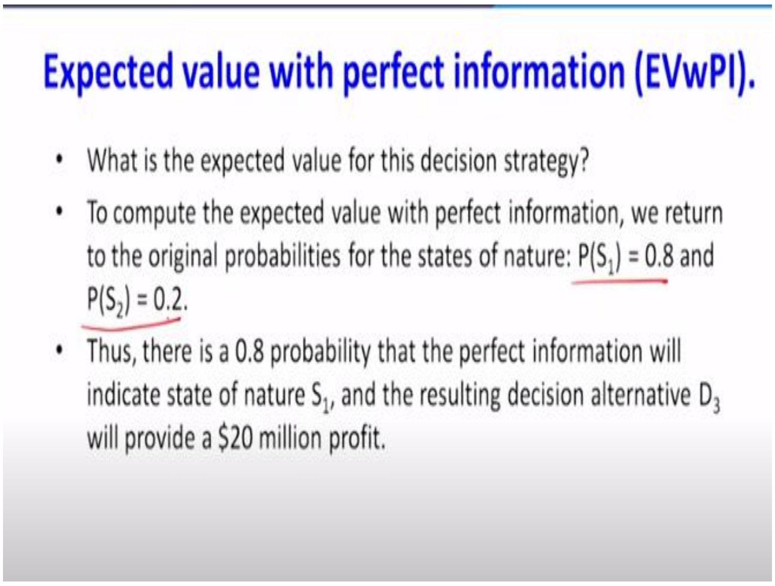


Note that if the company knew for sure that state of nature S_1 would occur. The company says that with the help of the research marketing in research company, the state of nature is going to be in strong demand. So, what will be the best decision alternative? So, if the company says this is going to be states of nature, our best decision alternative is d_3, which is $20 million.

Similarly, if the company knew for sure that the state of nature S_2 would occur, the best decision alternative would be d_1. So, if the company knows in advance states of nature are going to be weak demand, we will be going for this one, which is a pay of $7 million.

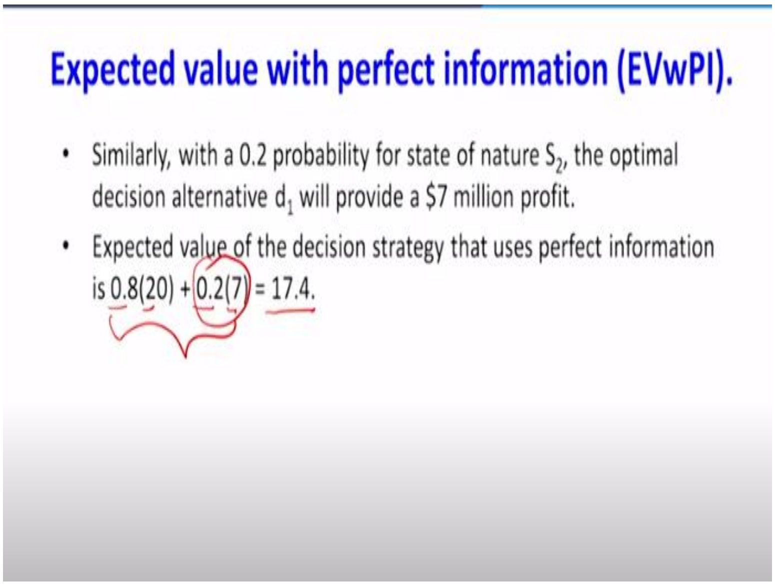


Thus, we can state the company's optimal decision strategy with perfect information, which becomes available as follows: If S_1 is going to occur, select d 3 and receive a payoff of $20 million. If S_2 occurs, select d_1 because this is the highest d_1 and receive a payoff of $7 million.



So, what is the expected value of this decision strategy? As we know, the expected value is multiplying with the corresponding probability. So, to compute the expected value with perfect information, we return to the original probabilities for the states of nature. We know that the probability of strong demand is 0.8 and weak demand is 0.2.

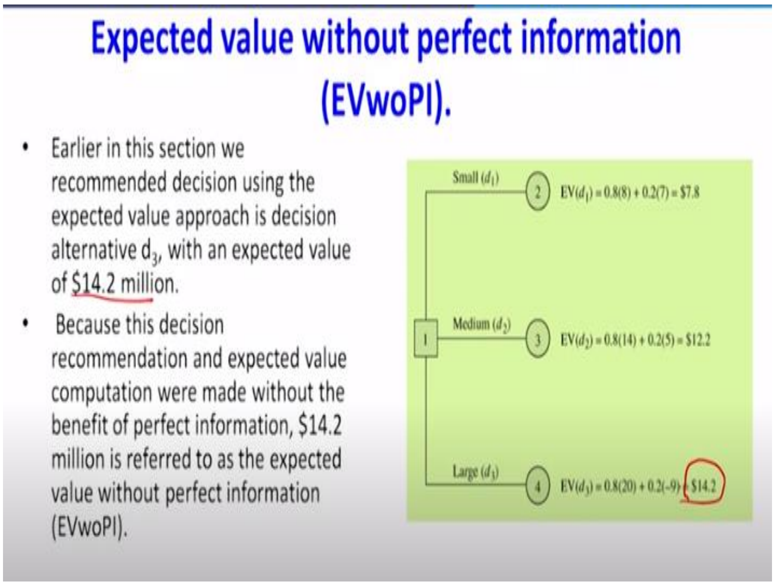
Thus, there is a 0.8 probability that the perfect information will indicate states of nature S_1 and the resulting decision alternative d_3 will provide a 20 million profit.



Similarly, with a 0.2 probability for states of nature S_2, the optimal decision alternative d_1 will provide a 7 million profit. So, the expected value of the digital strategy that uses the perfect information is multiplying this probability into 20 and 0.2 into 7 for 17.4. Sometimes, the student asks when we know the perfect information, what is that?

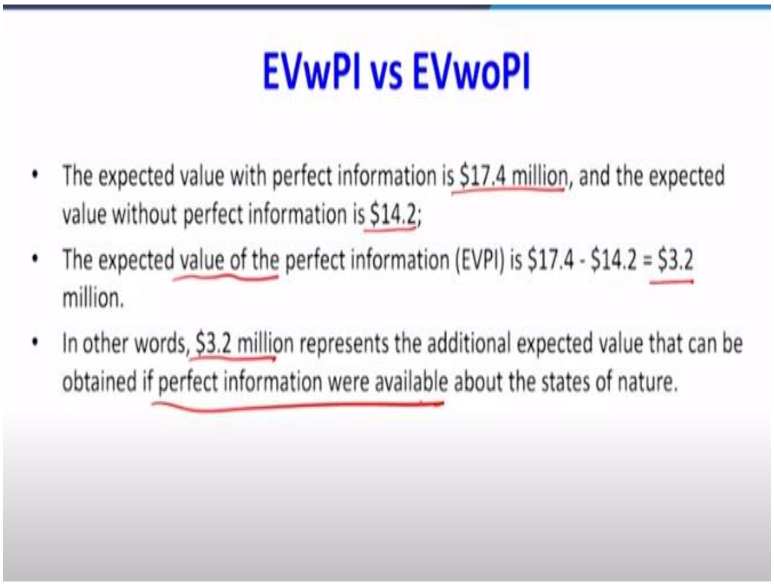
The perfect information is, say, it is going to be a strong demand, why do we have to multiply this by 0.2? Because there are two possibilities. The marketing research company can provide two possibilities. They may say it is going to be strong demand, but at the same time, they may say it is going to be weak demand also. So, in advance, you may not know what the suggestion of this marketing research company is.

So, you have to consider both possibilities but with the highest payoff because we know that we know in advance this is going to happen. So, what you have to do is the probability of the highest payoff for one state of nature. For another state of nature probability with the highest payoff when you multiply, this will be $17.4 million.



Earlier in this section, we recommended a decision using the expected value approach, which is decision alternative d_3 with an expected value of $14.2 million. Remember, we were suggesting when we go for the expected value, our expected payoff was 14.2. This decision recommendation and the expected computation were made without the benefit of perfect information.

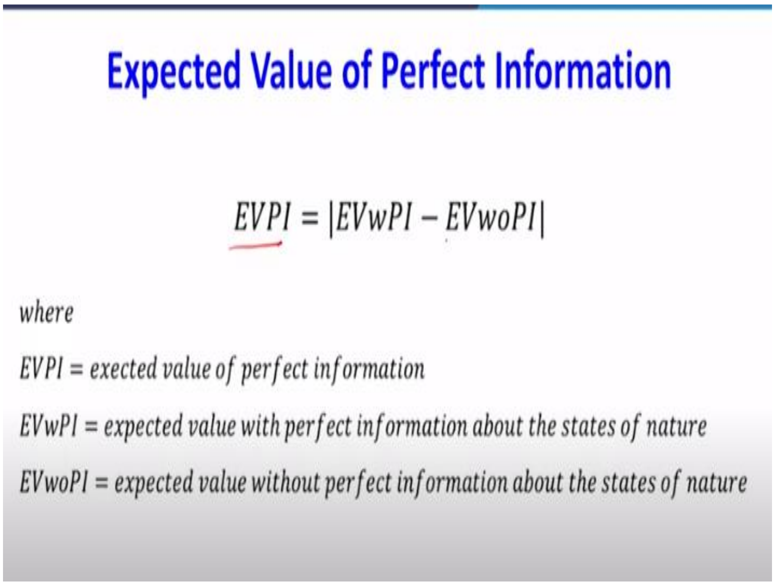
So, we did not consider the perfect information. So, $14.2 million is referred to as the expected value without perfect information because we do not know. So, this $14.2 million is the *expected value without perfect information* (EVwoPI).



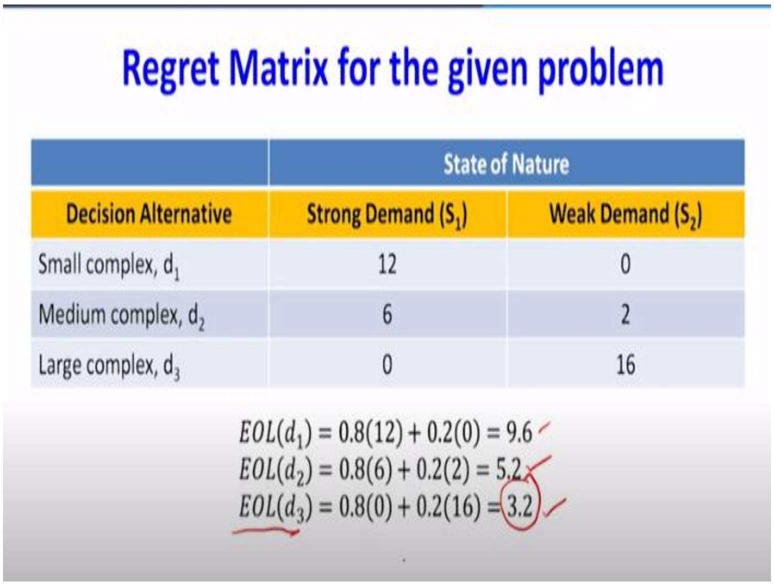
Now you should remember what is the expected value of with the perfect information versus the expected value of without perfect information. Just now, you have seen the expected value with perfect information is 17.4 million, and the expected value without perfect information is 14.2 million. The expected value of perfect information: now you see that we are finding the value of perfect information. EVwPI is the difference between these two.

What is that? The expected value with perfect information is 17.4 minus the expected value without perfect information, which is your $3.2. In other words, $3.2 million represents the additional expected value that can be obtained if perfect information was available about the states of nature.

In another way, we can say the amount of the consultation fee that you can provide to the marketing research company should not exceed $3.2 million dollar because by knowing perfect information, by not knowing perfect information, the difference is only $3.2 million.

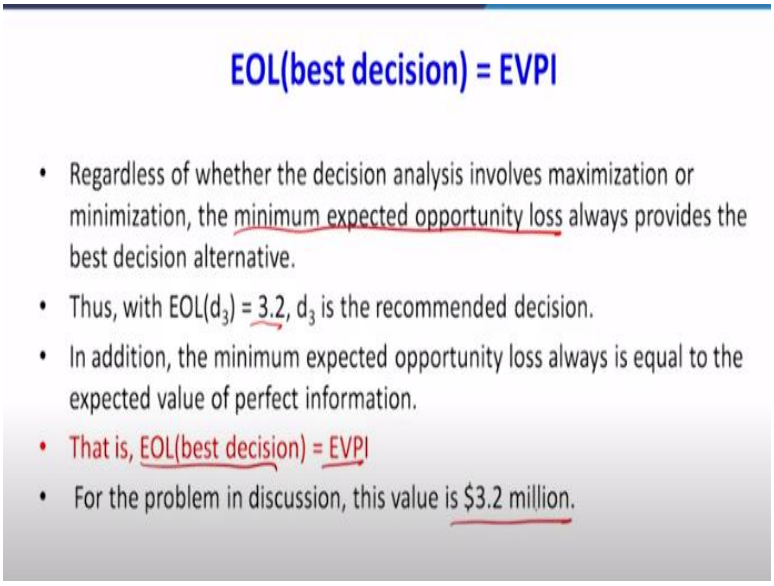


So, what is the expected value of perfect information? Is the difference between the expected value with perfect information minus the expected value without perfect information?



Now, I am going to connect this expected value of perfect information with the help of this regret matrix. You remember we have found the regret matrix. But we are going to multiply this regret matrix with corresponding probability. Regret is the loss. So, what will happen? The probability of strong demand is 0.8. So, 0.8 multiplied by 12 plus 0.2 into 0 9.6. Expected loss: if you go for d_2, it is 5.2. Expected loss: if you go for the d_3 decision, it is 3.2. Now, out of these three, which is the minimum? The expected loss when you go for d_3 is 3.2. You see that our expected value for perfect information equals our expected loss of regret matrix.

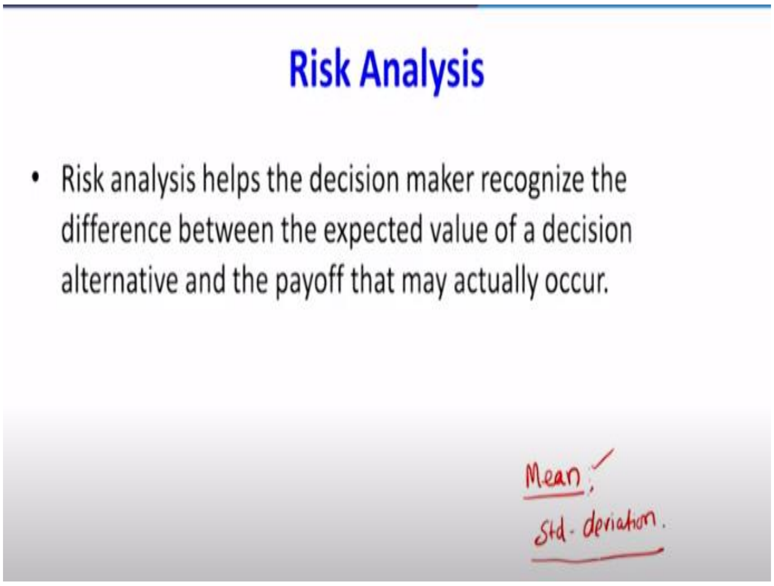
$$EOL(d_1) = 0.8(12) + 0.2(0) = 9.6$$
$$EOL(d_2) = 0.8(6) + 0.2(2) = 5.2$$
$$EOL(d_3) = 0.8(0) + 0.2(16) = 3.2$$



So, we are saying that the expected loss of the best decision is equal to our expected value of perfect information. So, regardless of whether the decision analysis involves maximization or minimization, the minimum expected opportunity loss always provides the best decision alternative.

So, what I am saying is that the minimum expected opportunity loss, whether the problem is a maximization problem or minimization problem, if you go for minimum expected opportunity loss, will always provide the best decision alternative. So here, the expected loss for the d_3 decision is 3.2. So d_3 is the recommended decision.

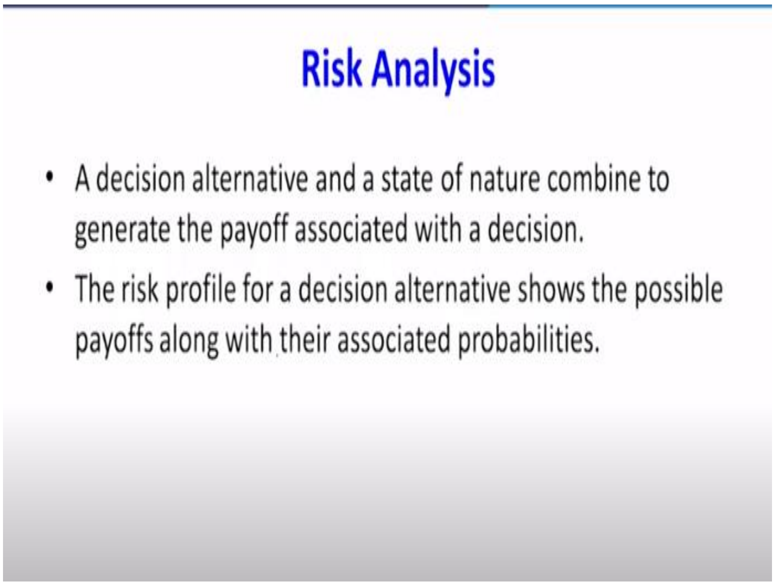
In addition, the minimum expected opportunity loss always is equal to the expected value of perfect information. So, the expected loss of the best decision is equal to the expected value of perfect information. So, in our problem, this value is $3.2 million.



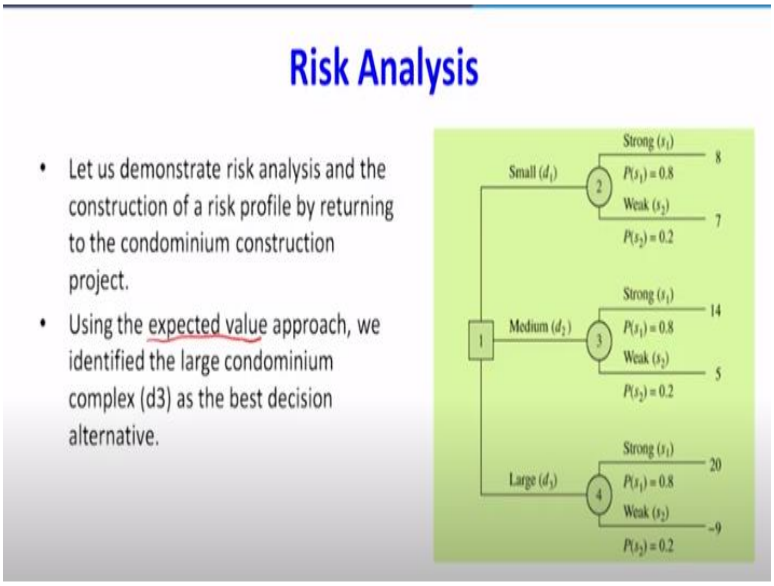
Dear students, so far, we have made the decision based on the expected value. So, what is the meaning of this expected value? The expected value is nothing but your mean. We know that the mean many times, right, mislead the decision maker unless you are not considering the standard deviation. So, what is the meaning of risk analysis?

Risk analysis is when you make a decision, you should not look only for the mean. You have to consider the standard deviation also. So, the risk is measured by its standard deviation. So, risk analysis helps the decision maker recognize the difference between the expected value of a decision alternative and the payoff that may actually occur, the deviations, the deviation from what you are expecting, and what you are actually getting.

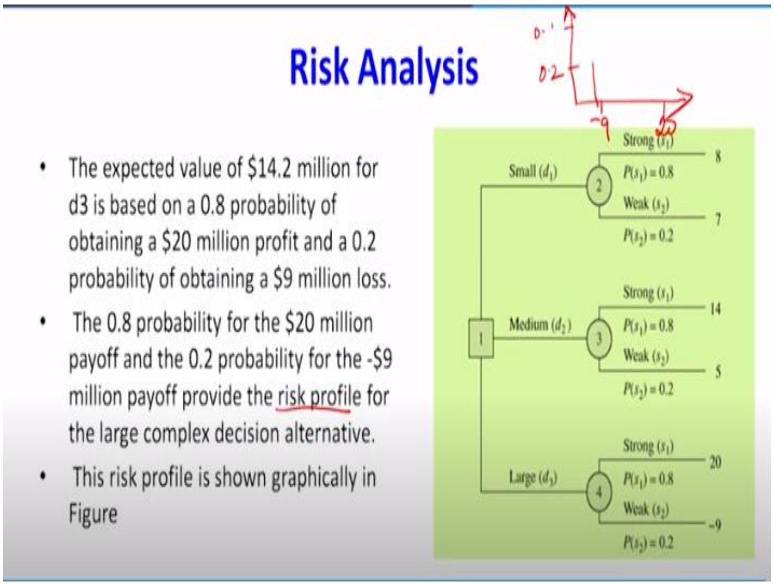
So that deviation is measured with the help of the standard deviation. What are we going to do for each additional alternative? You see that we have recommended d 3, right? So, for d3 decisions, we are going to analyze the risk of that decision.



So, a decision alternative and the states of nature combined to generate a payoff associated with a decision. So the risk profile for a decision alternative shows the possible payoff along with their associated probabilities.

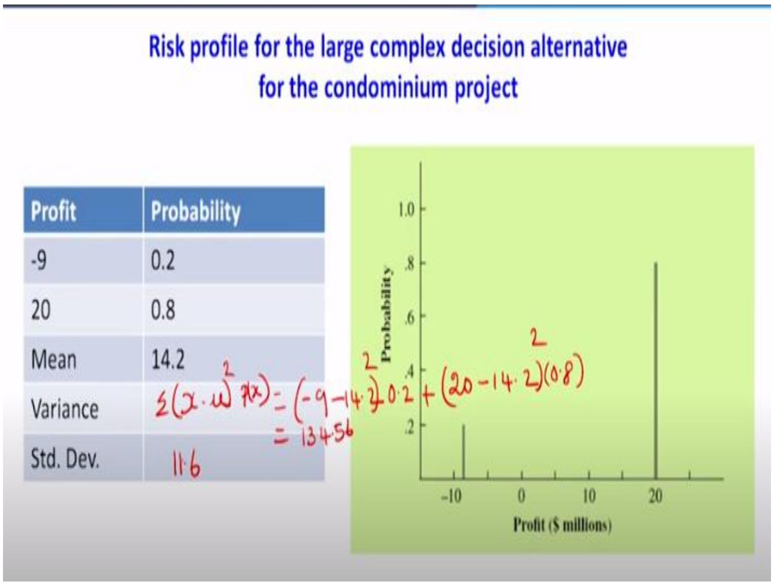


Risk Analysis. Let us demonstrate risk analysis and the construction of a risk profile by returning to our initial problem. Using the expected value approach, we identified the largest condominium complex, d3, as the best decision alternative. So we consider only the expected value. We did not consider the risk, the standard deviation, the deviation from what you expect, and what actually is going to occur.



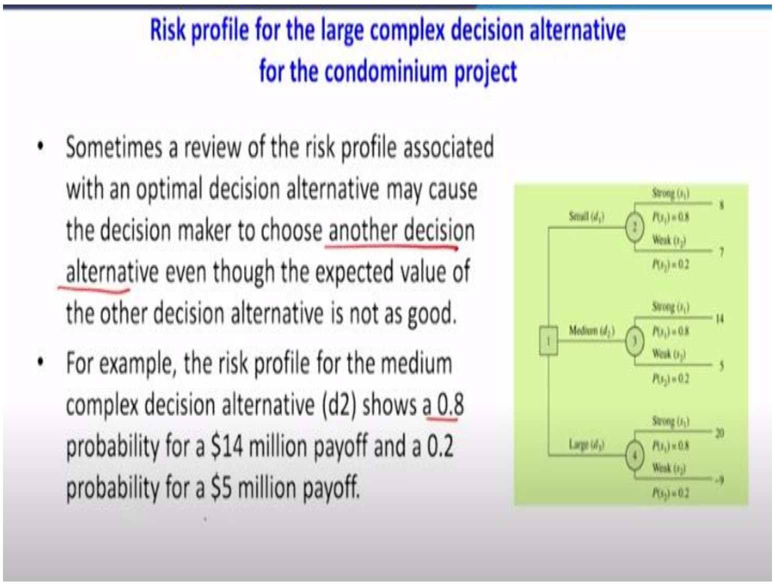
The expected value of $14.2 million for d 3 is based on a 0.8 probability of obtaining a $20 million profit and a 0.2 probability of obtaining a $9 million loss. The 0.8 probability for the 20 million payoff and 0.2 probability for the - $9 million payoff provide the risk profile for the large complex decision alternative. So, what is the risk profile? What is the risk profile on the x-axis? I am going to write the payoff.

In the y-axis, I am going to write the probability. So – 9, 0.2, and 20.8. So, this is nothing but my risk profile.

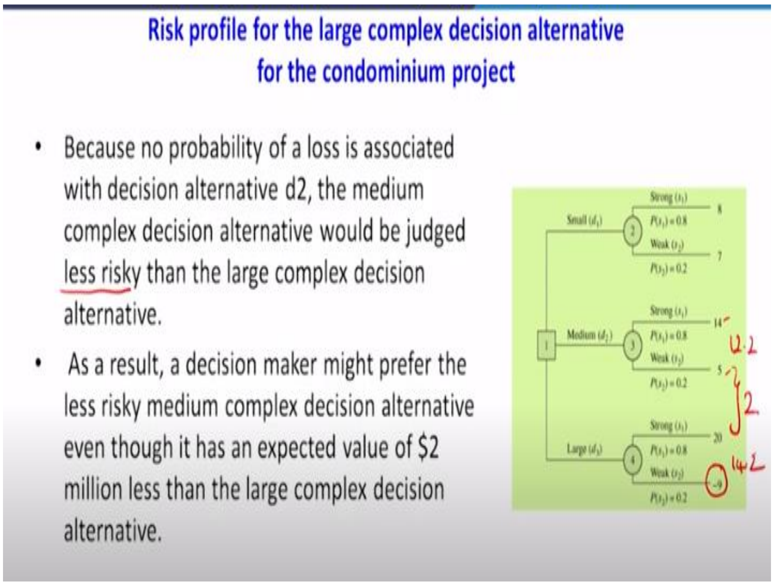


Now look at the risk profile, which is on the right-hand side. So now, we know that for the d_3 decision, the mean is 14.2. So, if I want to measure the risk, what do I have to do to find out the standard deviation? So, for the standard deviation, I have to know the value for variance. So, what is the formula for variance

The x is our payoff. P(x) is the probability. So, for the first one, if I expand this -9 minus 14.2 whole square 0.2 plus 20 minus 14.2 whole square 0.8. So, the students, if we quantify the risk of taking this decision, that is 11.6.

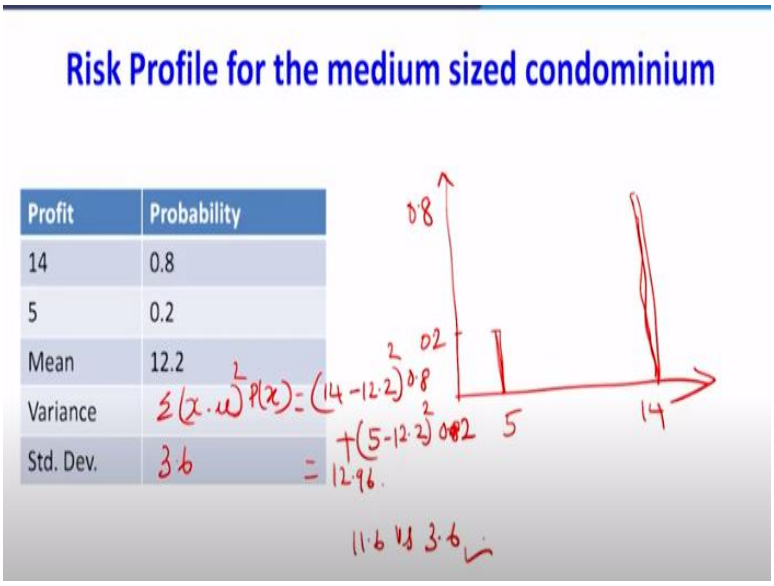


Sometimes, a review of the risk profile associated with an optimal decision alternative may cause the decision maker to choose another decision alternative even though the expected value of the decision alternative is not as good. For example, the risk profile for the medium complex decision alternative d_2 shows a 0.8 probability for a $14 million payoff and a probability of 0.2 for a $5 million payoff, so if we find the risk profile for the d2, that is our second best decision.



Because no probability of loss is associated with the decision alternative d_2, the medium complex decision alternative would be judged less risky than the large complex decision alternative. Look at the d_2 and d_3. In d_3, there is a chance for loss, but in d_2, there is no chance for loss. So we can say the d_2 decision is less risky than, lesser risky than the large-size complex.

So, as a result, the decision maker might prefer a less risky medium complex additional alternative, that is, d_2, even though it has an expected value of 2 million less than the large complex decision alternative. So here we know that it is 12.2, this is 14.2. Even though the difference is $2 million still, some people may prefer medium size condominiums.



Now, we can quantify this one. So first we have to draw the risk profile. As we know, what is the risk profile? So, it is 5, it is 14. So, 5, the probability is 0.2. For 14, the probability is 0.8. So, if you quantify this risk, we know the mean is 12.2. As we know, the formula for variance is x minus mu whole square P(x) equal to 14 minus 12.2 whole square 0.8 plus 5 minus 12.2 whole square 0.2.

The formula for variance (σ^2) is:

$$\sigma^2 = \sum (x - \mu)^2 P(x)$$

Now, applying the given values:

$$\sigma^2 = (14 - 12.2)^2 \cdot 0.8 + (5 - 12.2)^2 \cdot 0.2$$

So first, we have to find out the variance. So, the risk profile for the medium-sized condominium, when we quantify this risk profile, we got a variance of 12.96 and a standard deviation of 3.6. So, this is your d_2 decision. You see the d_3 decision previously. The previous standard deviation was 11.6.

$$\sigma^2 = 2.592 + 10.368 = 12.96$$

$$\sigma = \sqrt{12.96}$$

$$\sigma = 3.6$$

So, when we compare 11.6 versus 3.6, so 3.6 is the second-best alternative. That is going for the d_2 decision, which is less risky than the d_3. Dear students, in this lecture, I have discussed decision-making with probability. Otherwise, we can say decision-making with risk. After that, I discussed the expected value of perfect information. If somebody is going to be willing to give information about the states of nature, what will be the value of that information?

We have discussed that. After that we discussed the risk of each decision alternative. So, we have studied that the importance of considering risk while making the decision. Thank you very much.