Decision Making with Spreadsheet
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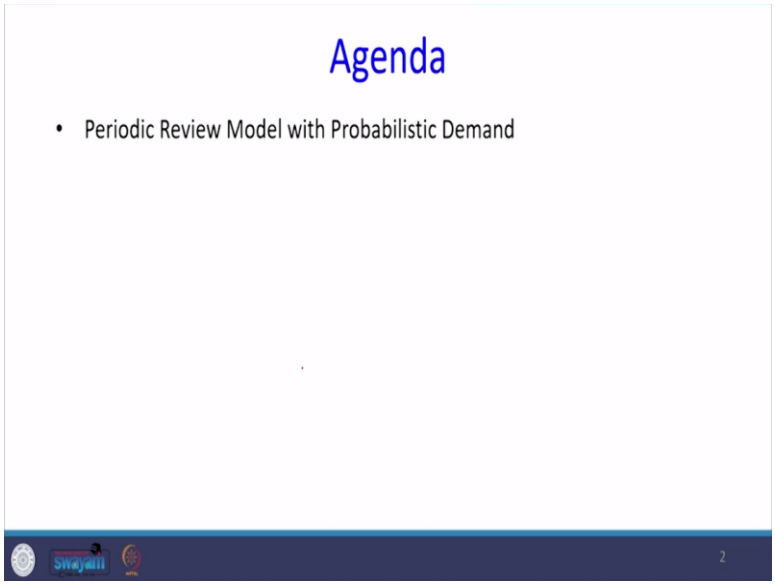
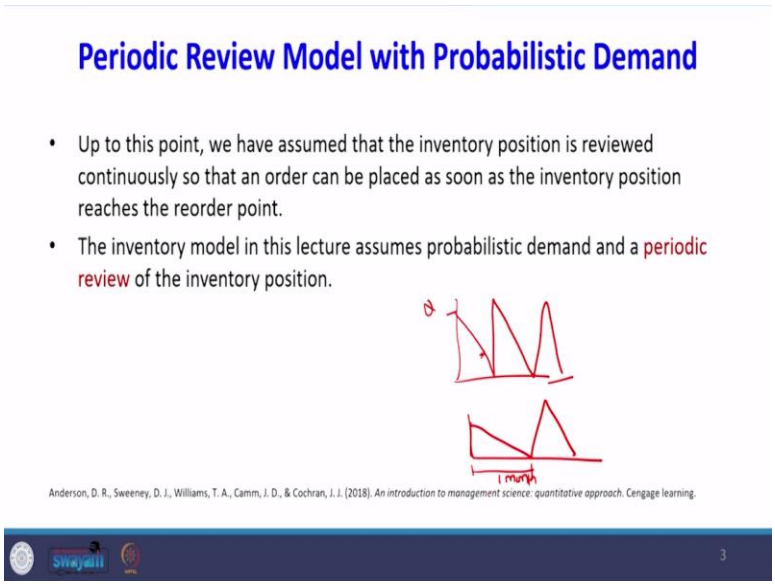
Lecture – 40
Periodic Review Model P Type with Probabilistic Demand

Dear students. Today, I am going to discuss a new inventory model called the periodic review model. Another name called which is a P-type inventory model. So far, what we have studied is a continuous review model that is called a Q-type inventory system model. So, in this lecture, I am going to discuss the P-type. I will explain in detail what this P type is and how it differs from the Q type.



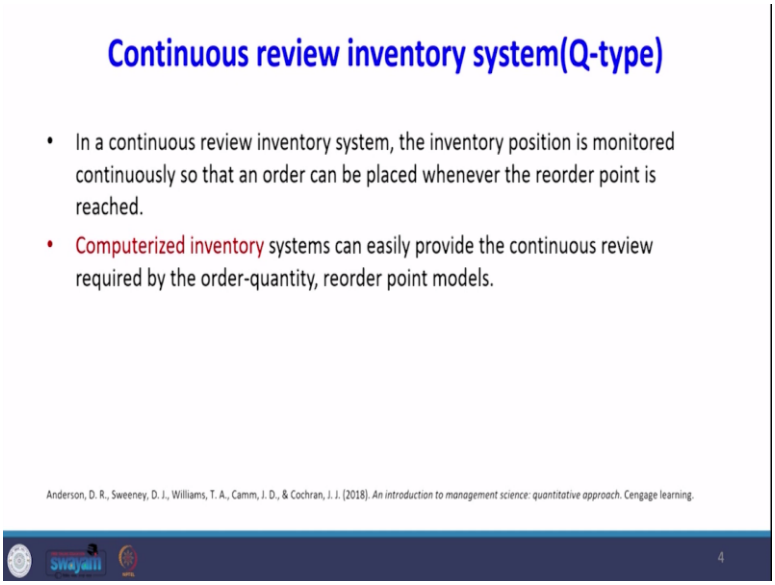
So, the agenda for this lecture is a periodic review model with probabilistic demand.



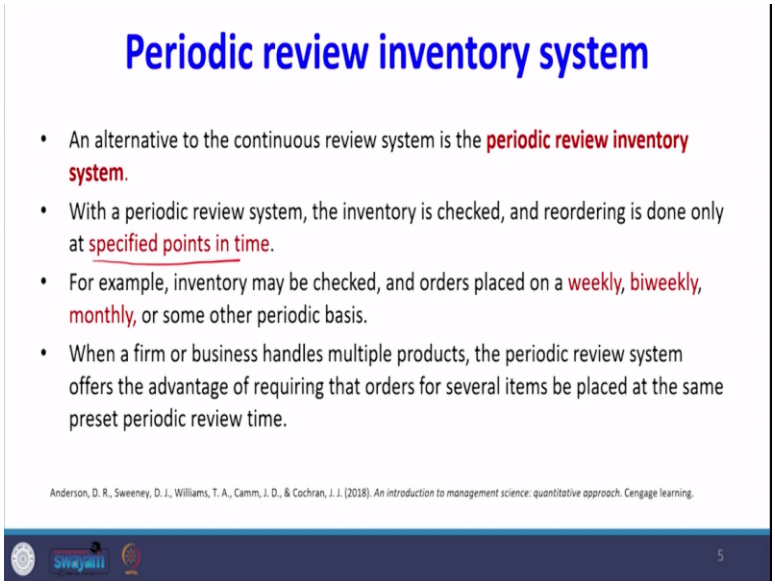
What is this periodic review model? Up to this point, we have assumed that the inventory position is reviewed continuously so that an order can be placed as soon as the inventory position reaches the reorder point. So, what we have done so far, suppose we are ordering it goes like this, it goes like this, it goes like this, this is a different Q. As soon as the point reaches here we are making the order.

So this type of model is called the continuous review model. Inventory model in this lecture, we are going to assume the probabilistic demand and the periodic review of the inventory position. What is the periodic review we may order? Now, the order cycle may be like this. Here the period means maybe this may be one month, or it may be one week like this.

So, here, the period is fixed, then we have to see if the period is fixed. What should be the ordering quantity? Previously, the Q was fixed. What is Q? Are the order and quantity fixed? We are ordering at different points in time, but here, we are not going to do it that way. In the fixed interval we are going to make the order.



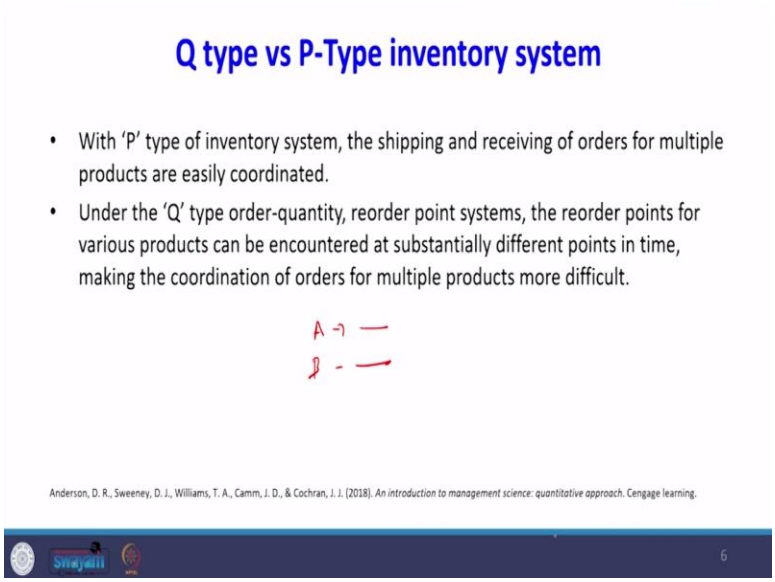
So, in a continuous review inventory system that is a Q-type inventory system, the inventory position is monitored continuously so that an order can be placed whenever the reorder point is reached. To have this continuous inventory system, we need a computerized inventory system that can easily provide the continuous review required by the order quantity reorder point so what point I am trying to say here is that we need a computerized inventory system to know what should be the ordering quantity, to know what should be the reorder point and the whole inventory system has to be monitored closely. So, this Q-type inventory system is a little more complicated.



Now, we will go over what this periodic review inventory system is. An alternative to the continuous review system is the periodic review inventory system. With a periodic review system, inventory is checked, and recording reordering is done only at a specified point in time. So, the order and quantity and reordering are done at a specified point in time. For example, inventory may be checked, and the order may be placed on a weekly, biweekly, monthly, or some other periodic basis.

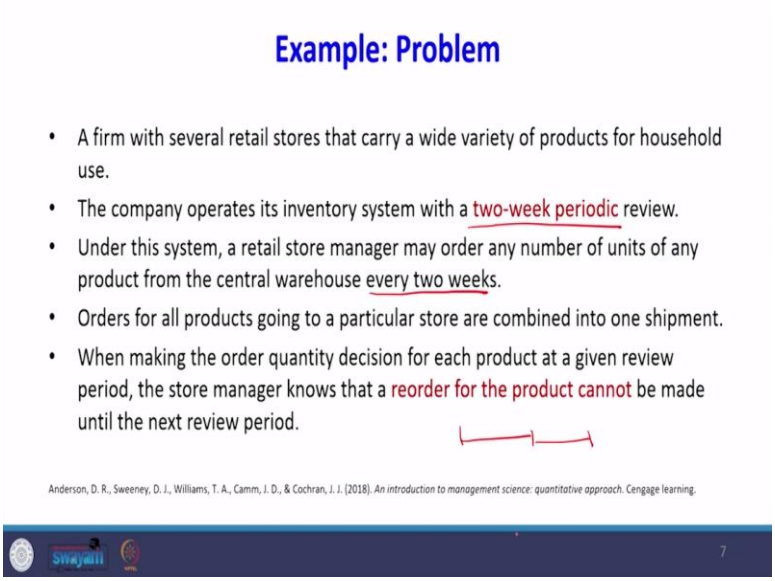
You see that the ordering interval is fixed when a firm or business handles multiple products. In reality, it is so because a company may monitor, for example, a small retail store that can have multiple products. So, when your firm handles multiple products, the periodic review system offers the advantage of requiring that orders for several items be placed at the same present periodic review time.

So, it is better if the company is handling multiple products, ordering, and taking advantage of transportation because you can order all the products at a time. Taking the economies of scale is more beneficial and more useful for us.



With the P type of inventory system, the shipping and receiving of orders for multiple products are easily coordinated and under the Q-type order quantity, which was our initial traditional method, not the traditional what we say inventory system that was studied in the previous lectures. Reorder point systems, reorder points, and various products can be encountered at substantially different points in time.

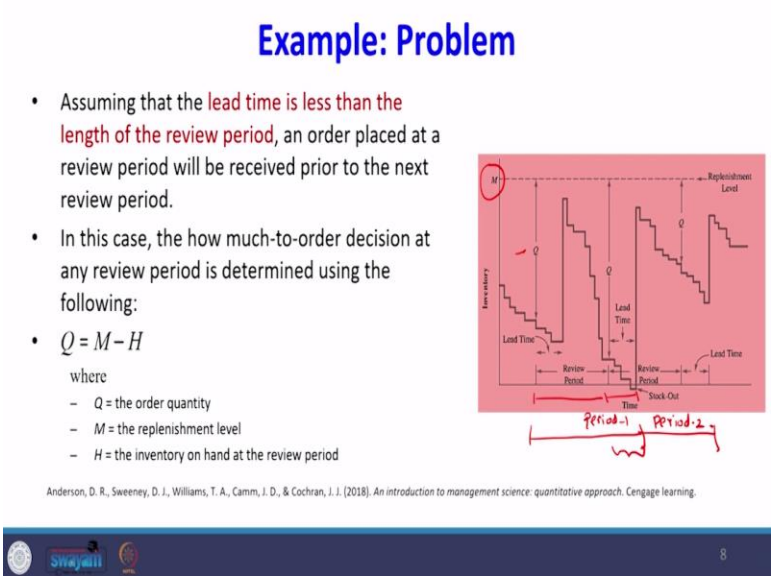
Making the coordination of orders for multiple products more difficult. See product A. There may be some reorder points. Product B and there may be some other reorder points. So, different products have different reorder points. So, managing inventory is more difficult under Q type when the company has multiple products, but the P-type inventory system is so convenient because all the orders can be clubbed because we are making an order on a fixed interval.



Now, I will explain this periodic review inventory model with the help of an example. This example is taken as a reference for this book by Anderson et al., A firm with several retail stores that carry a wide variety of products for household use. The company operates its inventory system with a two-week periodic review. What is the meaning of these two weeks? Every two weeks, they will check the inventory level.

Under this system, a retail store manager may order any number of units of any product from the central warehouse every two weeks. So, this person will order any quantity in any number, but every two weeks, from the central warehouse. Orders of all products going to a particular store are combined into one shipment. Here, we can take advantage of economies of scale because we can club the order.

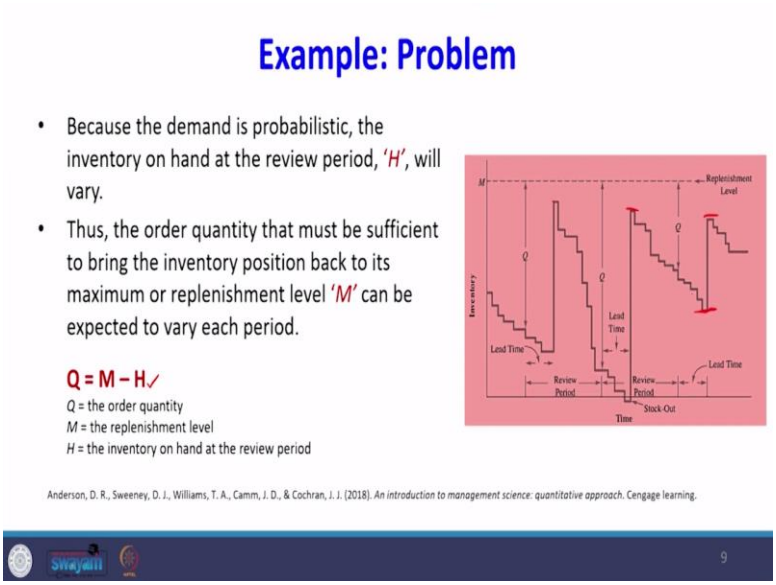
Club the product, and then it will be easy for our transportation. When making the order quantity decision for each product at a given review period, the store manager knows that a reorder point for a product cannot be made until the next review period. So, suppose if I am making an order every month, the next order can happen only in the next month. So, the reorder point cannot be made until the next review period.



Assuming that the lead time is less than the length of the review period, an order placed during a review period will be received prior to the next review period. Look at this picture on the right-hand side of the x-axis. Time on the y-axis is the inventory level. The M represents the replenishment levels, and Q represents your order quantity. You see, the periodic review period is this long, and the lead time is this long.

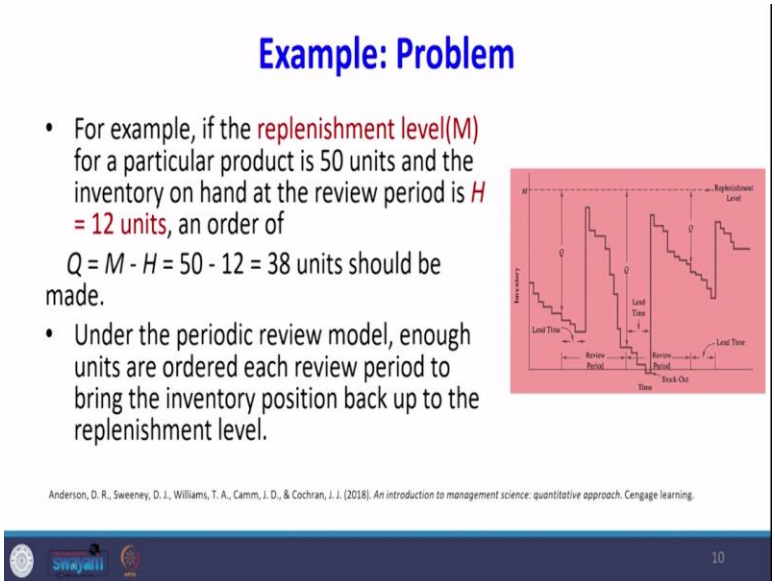
And there is a chance because we have assumed the demand is probabilistic, there is a chance there may be a stock out. So, this is one cycle, including the lead time. This is another cycle, including the lead time cycle 1 and cycle 2; otherwise, we can say period 1 and period 2. What we assume here is that the lead time, for example, this lead time, is less than the review period, and orders placed at a review period will be received prior to the next review period.

Suppose if I am making this order before the next review period you will receive the order. In this case, how much to order a decision at any review period is determined using the following equation. Q: How much do you order? Q = M – H. What is an M? M is the replenishment level, and H is the inventory on hand during the review period. We will explain this equation with the help of an example in the next slide.



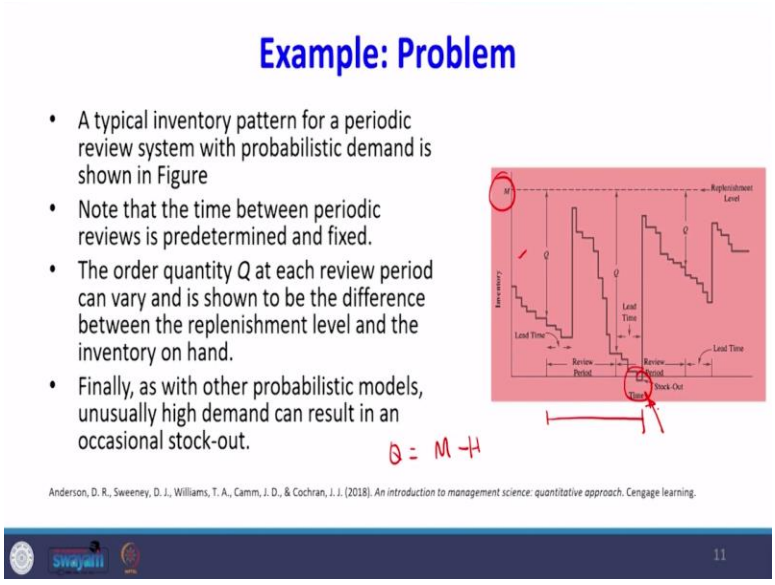
Because the demand is probabilistic, the inventory on hand at the review period, which is called H, will vary. So, this H also will vary; thus, the order quantity that must be sufficient to bring the inventory position back to its maximum or replenishment level M can be expected to vary each period. So, what will happen is we have to order the Q quantity so that we reach the M, M is called the replenishment level.

So, M can be expected to vary. You see that sometimes it is here, sometimes here, sometimes here you see that now, sometimes the M is here, so this is expected to vary.



For example, if the replenishment level for a product is, say, 50 units and the inventory on hand at the review period is, say, 12 units, then what will be the order quantity? So, the order quantity Q = M – H, what is M? 50 that is the replenishment level – H is 12, so 50 – 12, so 38 units should be ordered so that you can reach your replenishment level. So, under the periodic review model, enough units are ordered during each review period to bring the inventory position back up to the replenishment level.

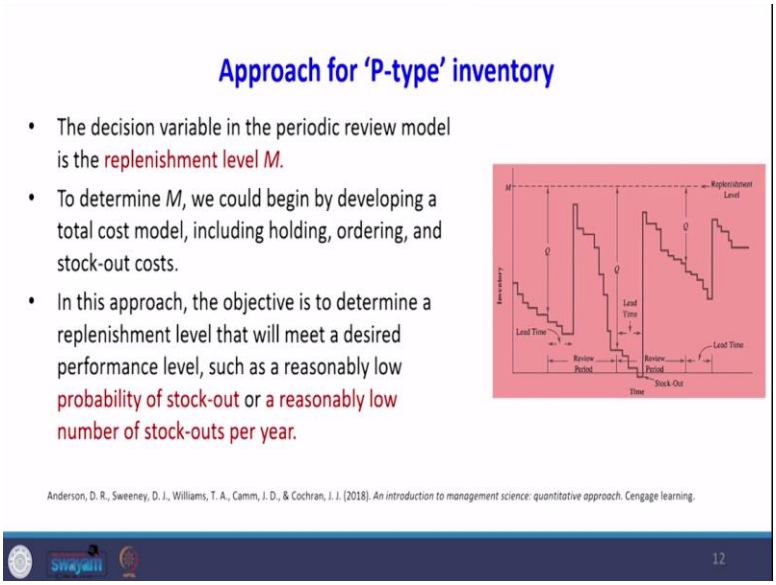
So, we have to order enough quantity in Q so that the M reaches its required level. Here M is how much 50.



A typical inventory pattern for a periodic review system with the probabilistic demand is shown in this figure. Please look at the figure which is given on the right-hand side. Note that the time between periodic reviews is predetermined and fixed. So, this duration is fixed. So,

the order quantity Q, Q at each review period can vary and be shown to be the difference between the replenishment level M.

So, this M and the inventory on hand is your H. So, Q = M – H. finally, as with another probabilistic model, unusually high demand can result in an occasional stock out. You see that there is a possibility of stock out.

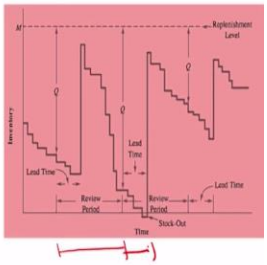


So how are you going to solve the P-type inventory problems? What approach is needed? So, here are the decision variables in the periodic review model at replenishment level M. To determine M, we could begin by developing a total cost model, including holding cost, ordering cost, and stock-out cost. So, instead of doing that, we are going to follow another approach. What approach are we going to follow?

The objective is to determine the replenishment level M that will meet a desired performance level, such as a reasonably low probability of stock out or a reasonably low number of stock out per year instead of finding the total holding cost, ordering cost, and stock out cost we are going to measure the performance inventory system based on how many number of stock-outs are permitted.

Approach for 'P-type' inventory

- In the example problem, we assume that management's objective is to determine the replenishment level with only a 1% chance of a stock-out.
- In the periodic review model, the order quantity at each review period must be sufficient to cover *demand for the review period plus the demand for the following lead time.*



Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

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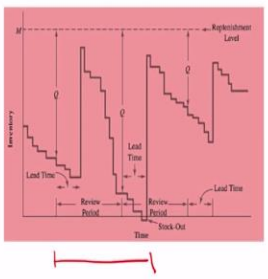
So, in the example problem, we assume that the management objective is to determine the replenishment level with only a 1 percent chance of stock out. So, the management says that 1 percent of stock out is permitted, or they can tolerate it. In the periodic review model, the order quantity at each review period must be sufficient to cover the demand for the review period plus the demand for following the lead time.

So, when we order, that should satisfy the requirement during the review period and the lead time.

Approach for 'P-type' inventory

- Suppose that an order is to be placed at time '*t*'
- To determine this order quantity, we must realize that the quantity ordered at time *t* must last until the next time inventory is replenished, which will be time (review period + lead time)
- Thus, the total length of time that the order quantity at time 't' must last is equal to the review period plus the lead time.



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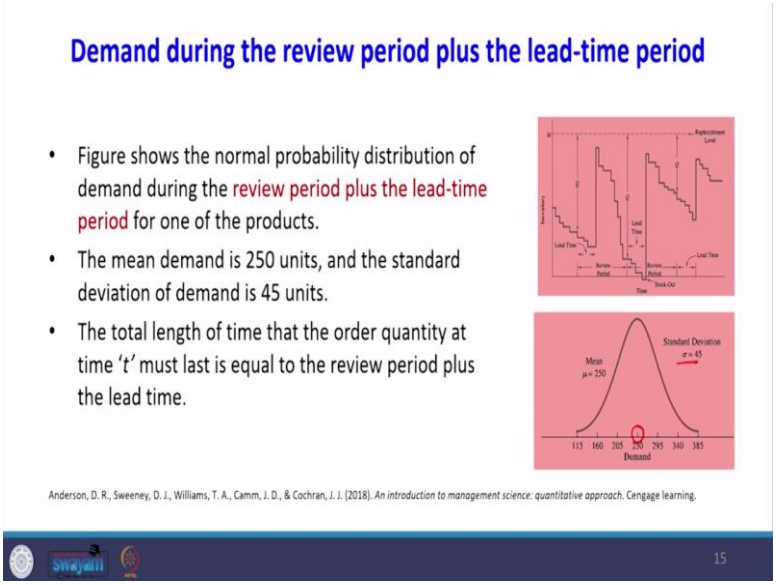
Suppose that an order is to be placed at time t. Suppose we are making an ordering a time t. To determine this order quantity, we must realize that the quantity ordered at time 't' must last until the next time inventory is replenished, which will be the time that is review period time plus lead time. So, what I mean here is that once we make the order, that order will

satisfy the requirements during the review period and the same time lead time period as this one.

Thus, the total length of the time that the order quantity at time t must last is equal to the review period plus lead time. So, once we make the order, it will satisfy the requirements of the review period demand and the lead time demand.

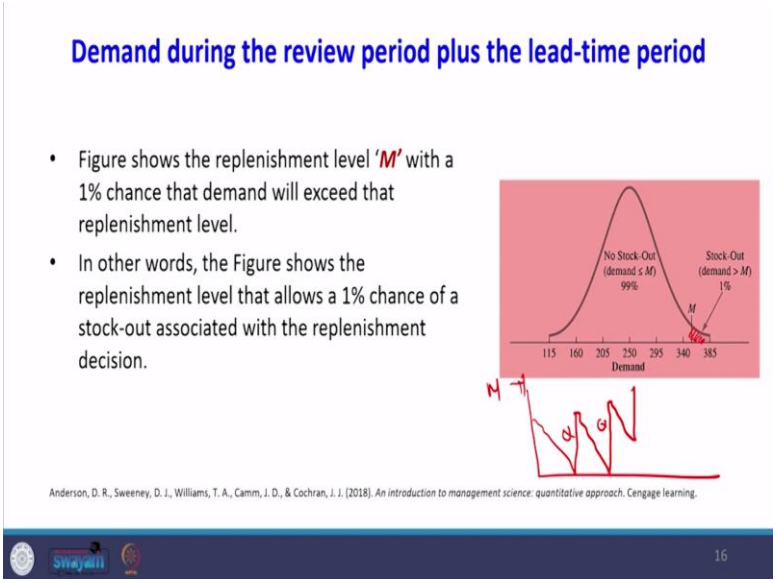


Now, the demand during the review period plus the lead time period since both are probabilistic in nature, so what information is provided here is the demand information during the review period, and the lead time period is given. So, the figure shows the normal probability distribution of demand during the review period plus the lead time period for one of the products.

Look at this figure the mean of the demand at this point mean of the demand is 250, and the standard deviation of the demand is 45. So, demand follows normal distribution. Remember this pattern: this demand is both for the review period and the lead time period. Remember, when you recollect the previous class, the previous class only the lead time demand was given because the other period time, we assume that it is following normal distribution that is in good approximation.

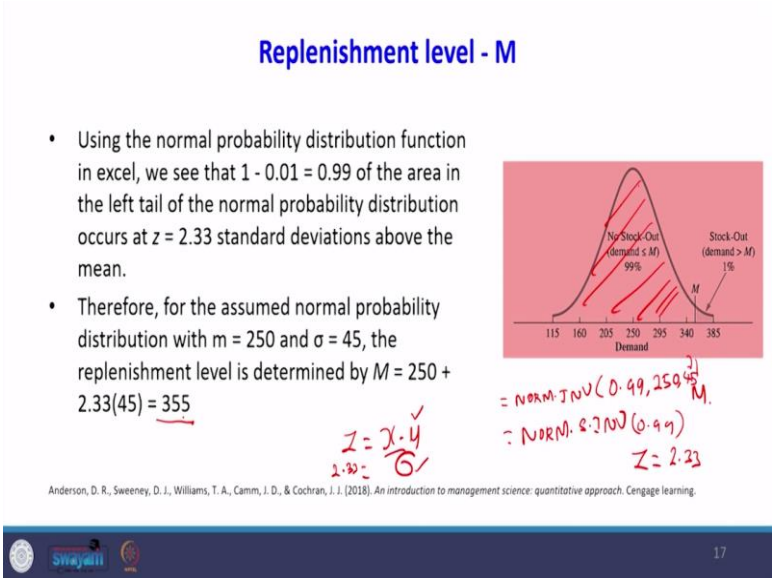
But here we are considering the demand during the review period plus lead time, which is a difference between the problem that we discussed in the previous lecture and this lecture. So, the mean demand is 250 units, and the standard deviation of the demand is 45 units. The total

length of the time that the order quantity at time t must last is equal to the review period plus the lead time period.



So, in the previous slide, we have seen the demand. Now, the figure shows that the replenishment level M has a 1 percent chance that the demand will exceed the replenishment level. So, M is given here, so the right-hand side area area shows that stock out, then the stock out will come when the demand exceeds the value of M. You remember M was like this.

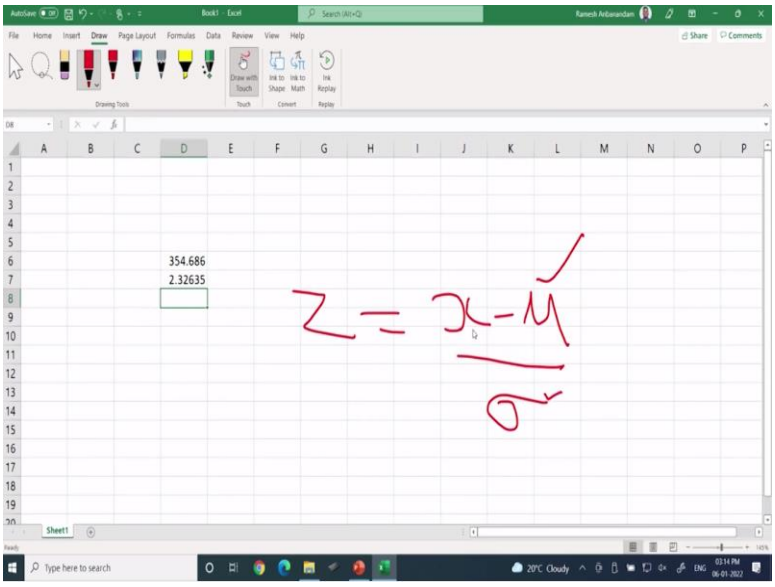
So, M was here, there was a Q, this is an M, there is an ordering quantity, here, it is a Q, here it is a Q. So when this stock out will come when the demand is exceeding the replenishment level. In other words, the figure shows the replenishment level that allows a 1 percent chance of a stock out associated with replenishment time associated with the replenishment decision.



Now, we will calculate the replenishment level M. So, using the normal probability distribution function in Excel, we know that the right-hand side area is one percentage, so what will be the left-side area? This area is 0.99. So, if the left-hand side area is 0.99, we have to see what the value of M? Now this, I am going to explain with the help of Excel. So, what am I going to do if the area of the left-hand side is 0.99? What is the M value?

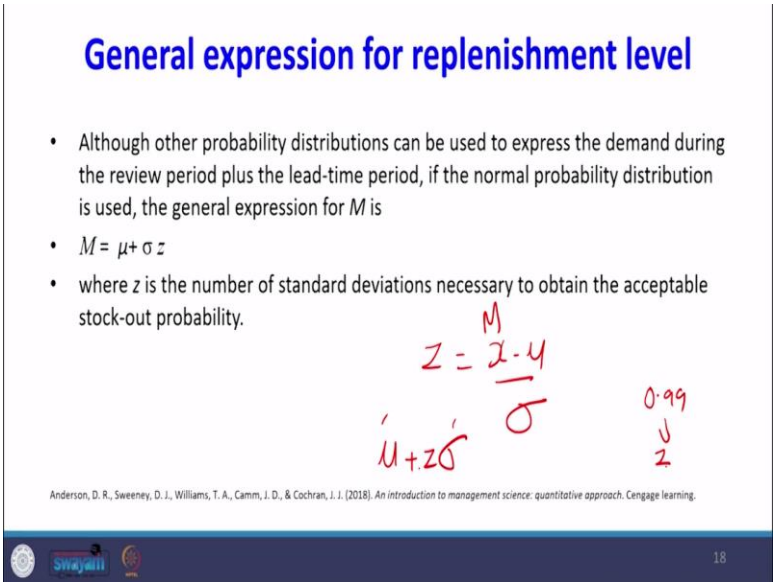
You remember there are two ways which I told you. What are the two ways? One is there are two functions in excel. One is NORM.INV another one is NORM.S.INV. Suppose I use this function NORM.INV, we have to provide a supply to the left side area, which is the mean of the distribution. What if the left side area is 0.99, the mean of the distribution is 250, and the standard deviation of the distribution is 45? Then you will get directly the M value.

Suppose we are using this function standardized normal distribution function, so you have to provide only 0.99 in that value of Z. That Z value is 2.33. So, you know the Z value is 2.33, nu is given 250 sigma is given 45, then you can find out the x value. So, when you will do that you will get the value of M is 355. So, this I am going to do with the help of excel.



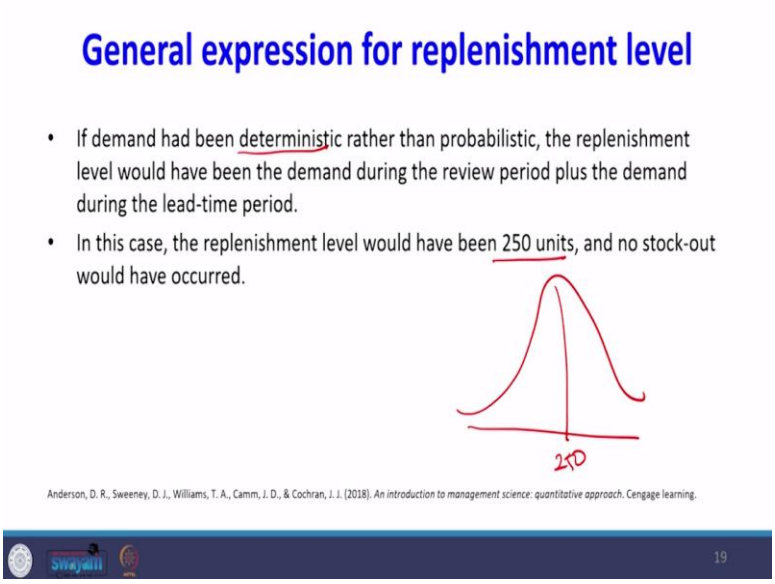
Now, I will explain how to find the value of M, which is the replenishment level we know that I told you about NORM. So, the left side area is 0.99, the mean is 250, and the standard deviation is 45, See that we are getting the value of f is 354. You see that we are also getting these values of 355. Suppose we use the other formula, and we have to use this one: NORM.S.INV 0.99. So, we are getting the Z value is 2.32.

So, if the Z is 2.32, we know what the sigma value is from that; we can find out the value of x is 355. So, we got the value of M is 355.



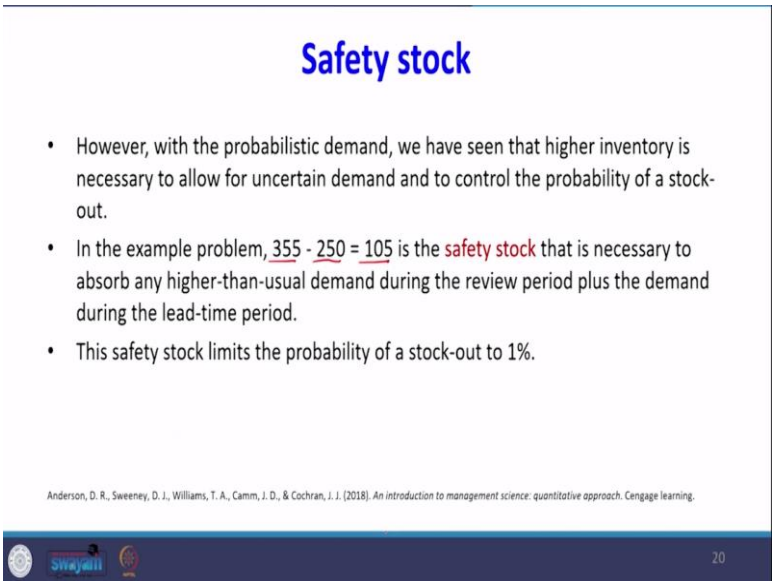
So what is the general expression for knowing this replenishment level? Although other probability distributions can be used to express the demand during the review period plus lead time period, if the normal probability distribution is used, the general expression for M is u + z sigma where z is the number of standard deviations necessary to obtain the acceptable stock out probability.

So, here, instead of x, we are going to call it M, so suppose we use M. So, if you want to know the value of M, this will be the mean of the demand sigma is the standard deviation of the demand. If the area is 0.999, what does the corresponding Z value that value is here on this side?



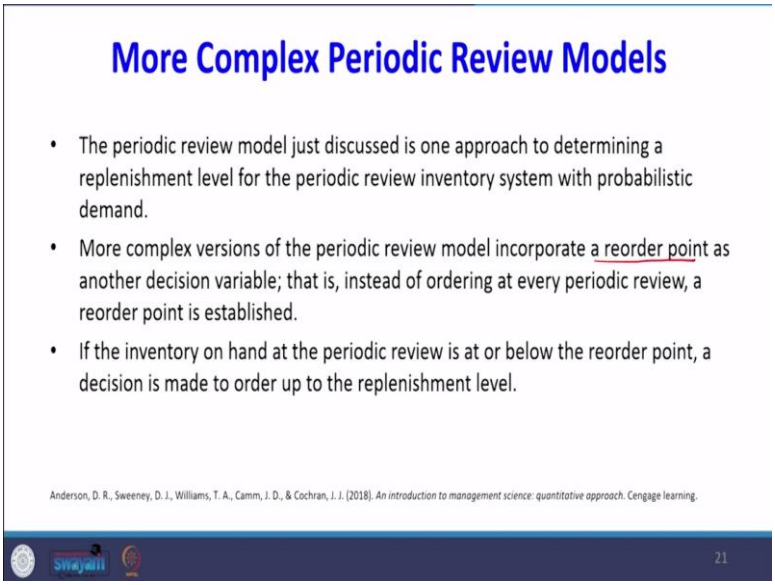
Now, we will go for a general expression of the replenishment level. If the demand had been deterministic rather than probabilistic, the replenishment level would have been the demand during the review period plus the demand during the lead time. So, in this case, the replenishment level would have been 250 units because the demand remembered initially it was given.

So, the demand is 250, and no stock out would occur because we know in advance what the demand is. So, if the demand is deterministic, the order and quantity will be 250, and the replenishment level will be 250.



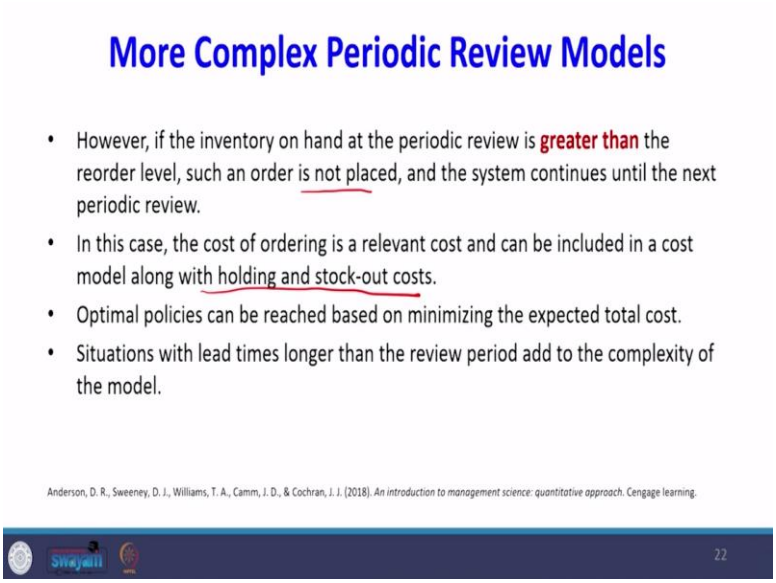
However, with the probabilistic demand, we have seen that a higher inventory is necessary to allow for uncertain demand and to control the probability of stock out. So, in our example, when we assume that the demand is probabilistic, our order and quantity were 355. When we assume the demand is deterministic, the ordering quantity is 250, so this difference of 105 is the safety stock.

What is safety stock that is necessary to absorb any higher than unusual demand during the review period plus the demand during the lead time? So, these 105 units of safety stock limit the probability of stock out to 1 percent. So, the chances for stock out is only one percentage that can be achieved by having 105 additional units apart from our 250 units.



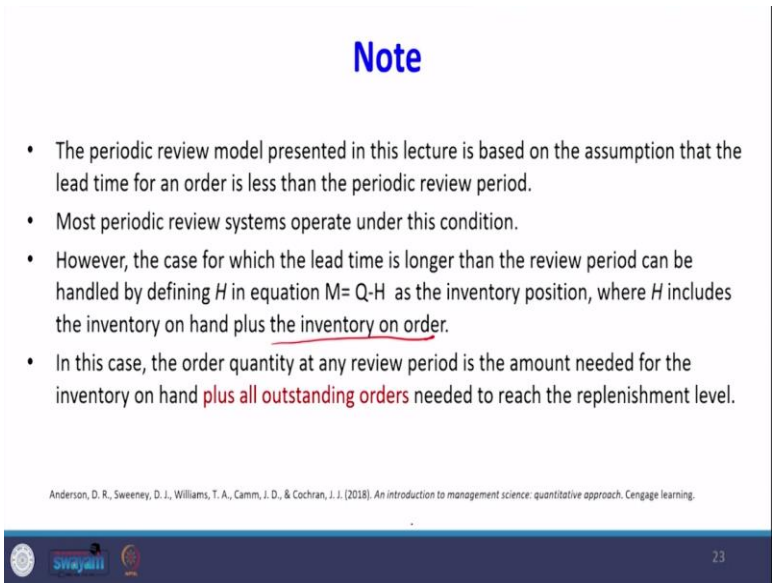
Now, we can discuss some more complex periodic review models. The periodic review model just discussed is one approach to determining the replenishment level M for the periodic review inventory system with the probabilistic demand. A more complex version of the periodic review model incorporates the reorder point as another decisional variable. Here, our decision variable is only M, which is the replenishment level.

If you consider more complex versions, even the reorder point can also be considered as another decision variable. That is, instead of ordering at every periodic review, the order point is established. So, whenever the demand reaches that point, we will make an order. If the inventory on hand at the periodic review is at or below the reorder point, a decision is made to order up to the replenishment level. This is what we have seen if the inventory on hand on periodic review is below the reorder point, a decision is made to order up to the replenishment level.



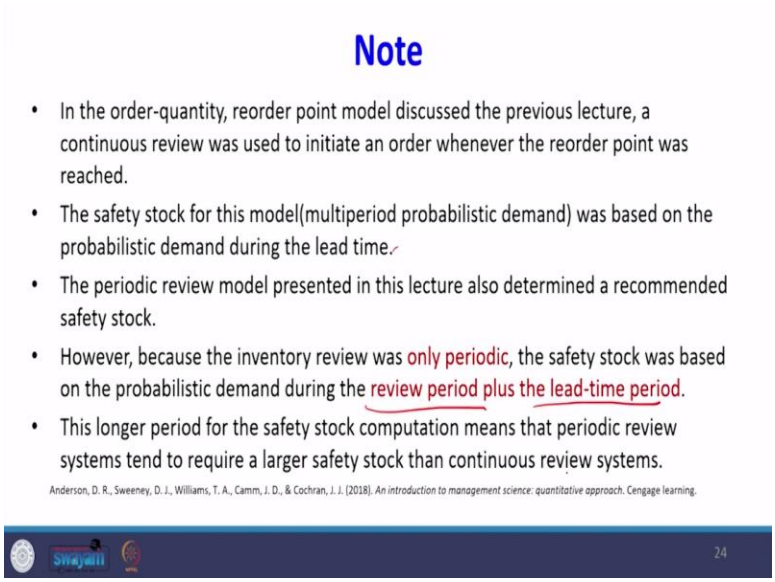
However, if the inventory on hand at the periodic review is greater than the reorder level, we are making more stocks than the reorder level. If such an order is not placed, we will not go for a new order, and the system continues until the next periodic review. In this case, the cost of ordering is a relevant cost and can be included in a cost model along with holding and stock-out costs.

So, in this situation, because we are ordering only for the next period, we can consider the ordering cost along with holding and stock-out costs. So, optimal policies can be reached based on minimizing the expected total cost. Situations with a lead time longer than the review period add to the complexity of the problem. So, we have seen now the lead time is smaller than the review period; sometimes, the lead time is longer than the review period, and the complication increases.



Some important note you should remember the periodic review model presented in this lecture is based on the assumption that the lead time for an order is less than the periodic period. If you remember my first slide there this was my assumption that the lead time is lesser than the periodic review period. Most periodic review systems operate under this condition, but however, the case for which the lead time is longer than the review period can be handled by defining H in an equation M = Q – H as inventory position where the H includes inventory on hand plus inventory on order.

So, the meaning of H differs when the lead time is longer than our review period, so here, H is inventory on hand plus inventory on order that is some order to be received. So, in this case, the order quantity at any review period is the amount needed for inventory on hand plus all outstanding orders needed to reach the replenishment level. At this point, you should remember. This kind of problem we have discussed in our previous lecture, also where the reorder point is greater than the Q. Q is the order quantity.

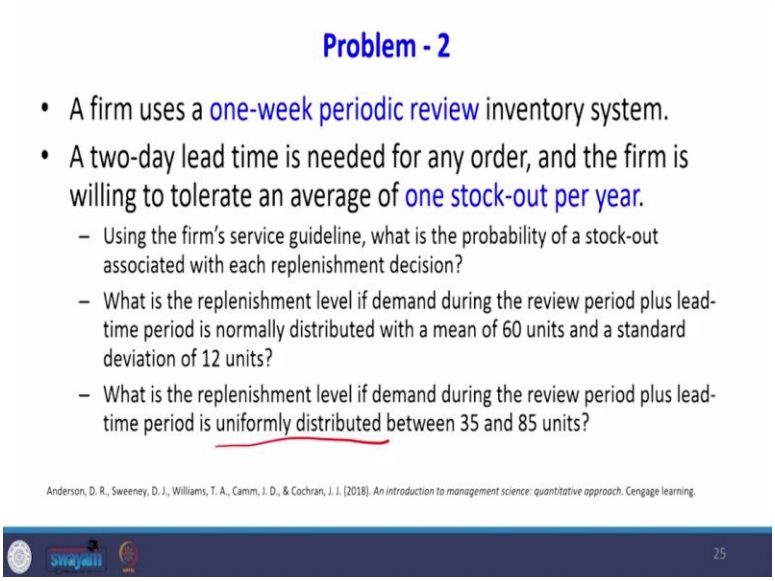


In the order quantity reorder point model discussed in the previous lecture, a continuous review was used to initiate an order whenever the reorder point was reached. In the previous lecture, we were discussing about the continuous review system. The safety stock for this model What is this model? The model you discussed in the previous class, multi-period probabilistic demand, was based on the probabilistic demand during the lead time.

You remember, in the previous lecture, we considered only the demand during the lead time. With the help of demand during lead time, we found the average that is annual demand, I think, 8008 units, but here it is different. The periodic review model presented in this lecture

was also determined as a recommended safety stock; however, because the inventory review was only periodic, the safety stock was based on probabilistic demand during the review period.

The lead time period is the point where we differ from the previous lecture. In the previous lecture, we discussed the demand during lead time, but now we are considering demand during the review period plus the lead time period. So, the longer period of safety stock computation means that the periodic review system tends to require a larger safety stock than a continuous review system.

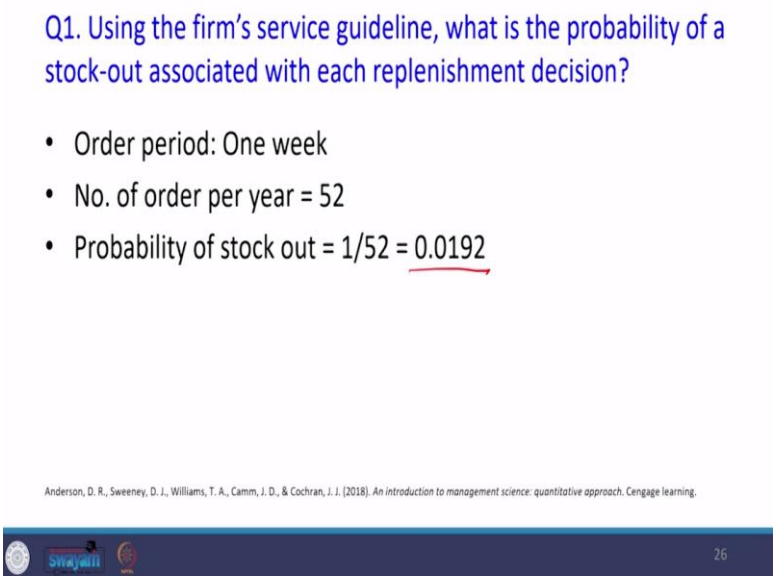


So, previously we have seen the demand follow normal distribution. So, I am going to take another example here the demand is going to follow uniform distribution. What is this problem? A firm uses a one-week periodic review inventory system, so that means every week, they are going for ordering. A two-day lead time is two days. You see that lead time is shorter than the review period.

A two-day lead time is needed for any order and the firm is willing to tolerate an average of one stock out per year. So, using the firm service guideline, what is the probability of stock out associated with each replenishment decision? The second question is, what is the replenishment level if the demand during the review period plus the lead time period is normally distributed with a mean of 60 and a standard deviation of 12?

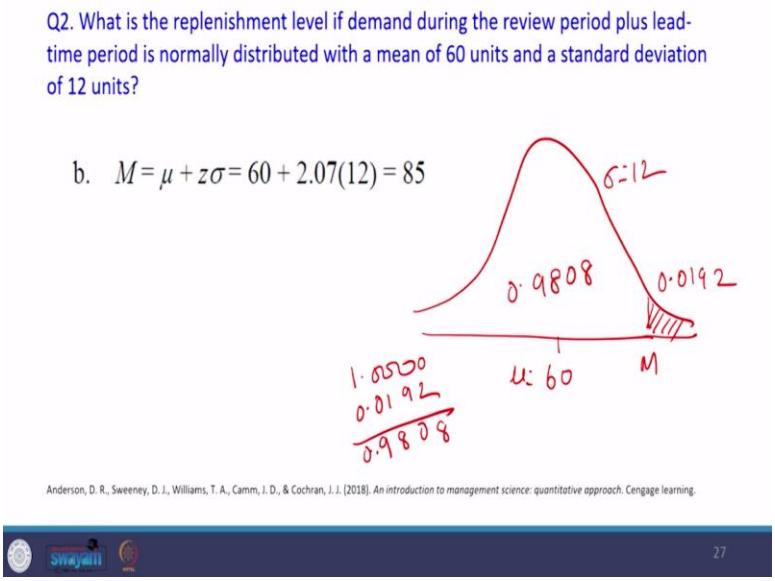
Then, what is the replenishment level? The demand during the review period plus the lead time period is uniformly distributed. Now, we are changing the demand. If it is uniformly

distributed, what is the replenishment level? If the demand follows normal distribution, what is the replenishment level?



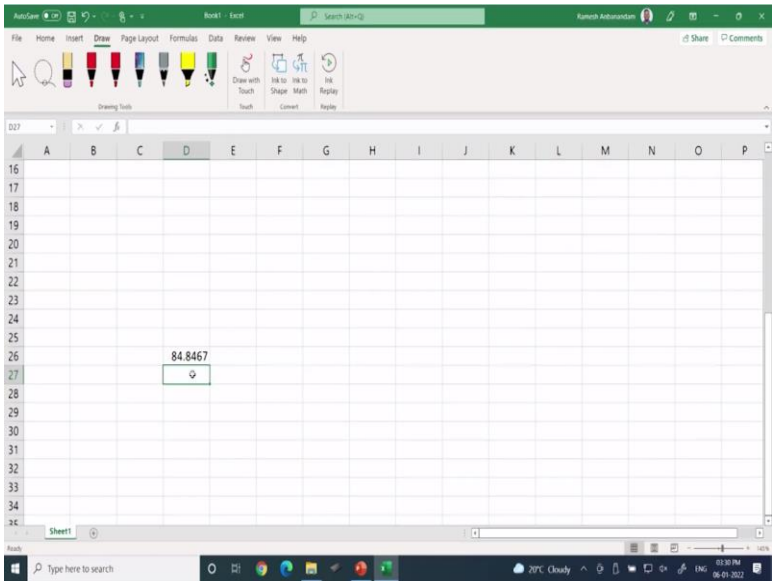
So, the first question is what is the probability of stock out associated with each replenishment level? We have seen in the previous lecture also the probability of stock out is we should know what is the total number of orders in that we have to see how many orders we can go for stock out. So, here the order period is one week, which is given in the problem. So, if it is one week, what will the number of orders per year be? It was 52 weeks, so there will be 52 orders.

Then, the management is willing to tolerate one stock out, and if that is the case, what is the probability? So, the probability of stock out is 1 upon 52, which is 0.0192.

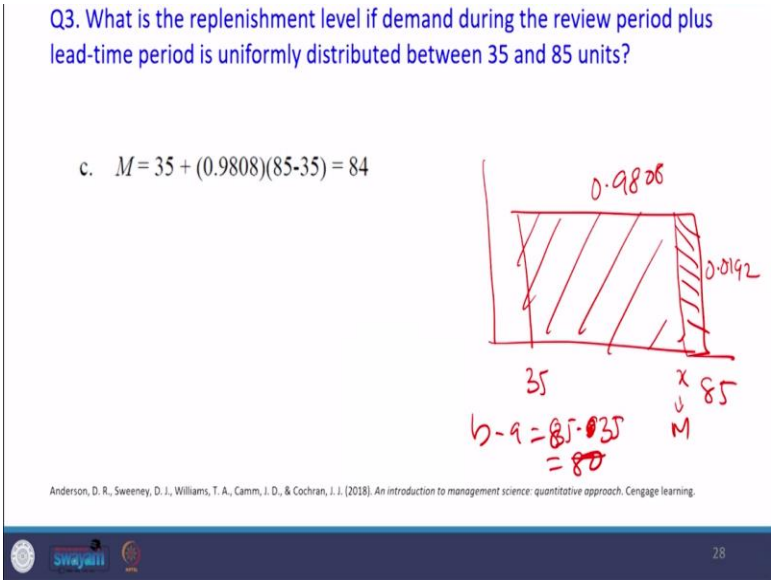


What is the replenishment level if the demand during the review period plus the lead time period is normally distributed with a mean of 60 and the standard deviation is 12? So, what we have to know is that from the previous question, we have seen the probability of stock out is 0.0192. The mean is given, the mean is 60, and the standard deviation is 12. Now, we have to find out what is the value of M.

So, what we have to do is know that Excel is providing the minus infinity to plus x area, so what is the left side area? So, the left side area is 1 – 0.0192. What is the left side area? This one is 0.9808. So, if the left side area is 0.9808, we should know what the M value is.



So, I will explain this with the help of Excel, so equal to NORM.INV 0.9808, mean 60 standard deviation is 12. So, we are getting the replenishment level is 85 and the replenishment level is 85.



The next question is, what is the replenishment level if the demand during the review period plus the lead time period is uniformly distributed between 35 and 85? So, what is the shape of the uniform distribution? The shape of the uniform distribution is 85, and this is 35, so we should know if the right-side area is this much. What is this area? This is 0.9808. We should know what the value of this value M is instead of x, which we can call M.

So, first we must see 0.980 percentage this 85 – 35. So, when you subtract 85 -35, it is 50. So, when you multiply 0. 9808 by 50, then you add with this 35, that will be 84. So, how did we get 0.98? Because this is the right-side area we have seen previously, the stock out is 0.0192, so the side area. So, the left side area is 0.9808, so this percentage of your (b – a) is 85 -35 = 50, so 50 multiplied by 0.908 plus 35 will get 84.

So, dear student, in this lecture, I have discussed the periodic review model, which is called the P-type inventory model with probabilistic demand. I have had two problems. I have explained the concept of a P-type inventory system with the help of two models. In two problems, in one problem, the demand follows normal distribution. In the other problem, the demand follows both normal and uniform distribution. So, thank you very much, students. I will see you in the next class.