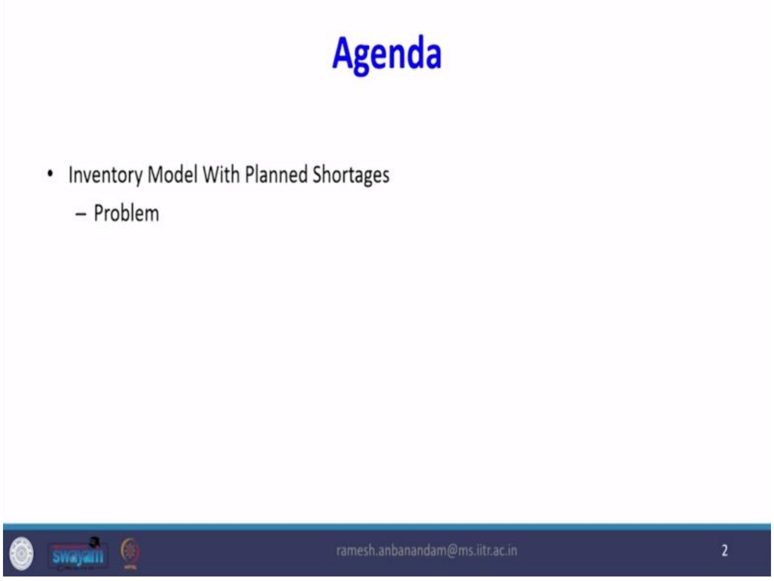
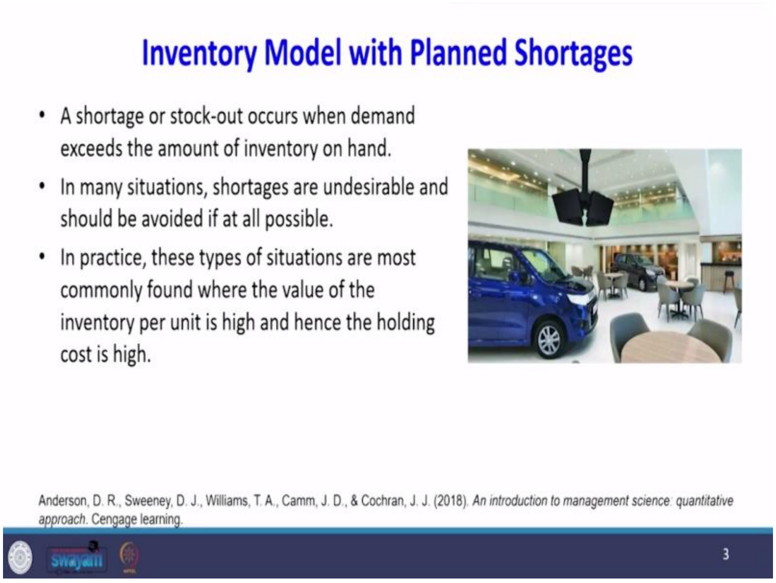
Decision Making with Spreadsheet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology – Roorkee

Lecture – 36
Inventory Model with Planned Shortages

Dear students, today we are going to discuss another inventory model the model name is the inventory model with planned shortages. So far, we have studied economic order quantity and economic lot size quantity there; we did not consider the concept of shortages. So, in this lecture, we are going to consider the concept of shortages, and then we are going to develop a new model for considering this shortage cost.

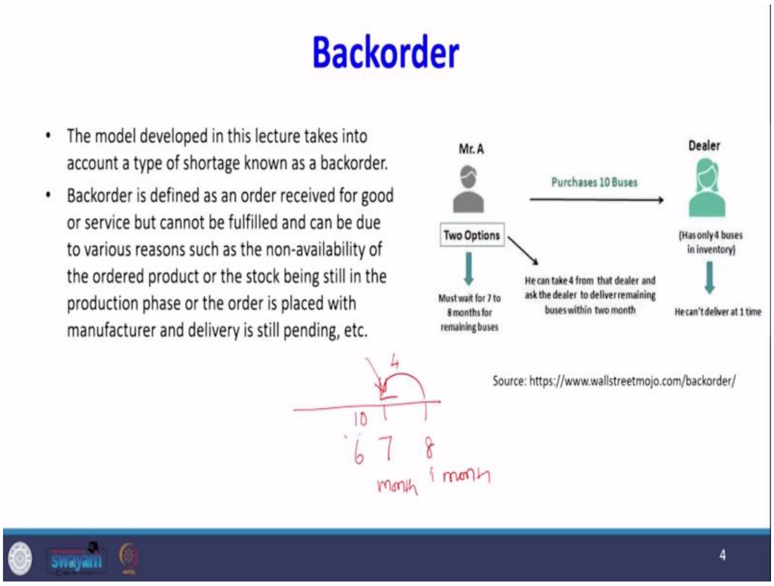


So, the agenda for this lecture is the inventory model with the planned shortage. You remember the shortage is planned in advance. We are planning for shortages because we are going to say that there is an advantage to these shortages. So, that is the agenda for this lecture.



So, inventory model with planned shortages. A shortage or stock-out occurs when the demand exceeds the amount of inventory on hand. Whenever the shortages occur, it happens whenever the demand is more than the inventory on hand. In many situations, shortages are undesirable and should be avoided if at all possible. In practice, these types of situations are most commonly found where the value of the inventory per unit is high, and hence, the holding cost is high.

Look at the example. Look at the picture, which is on the right-hand side. It is a car dealer. Most of the time, if you want to buy a particular car, they will ask you to wait for some days because if they keep that car in inventory, the inventory holding cost is high. So, in these kinds of business scenarios, this kind of inventory model with planned shortages is more suitable where when the inventory holding cost is high.

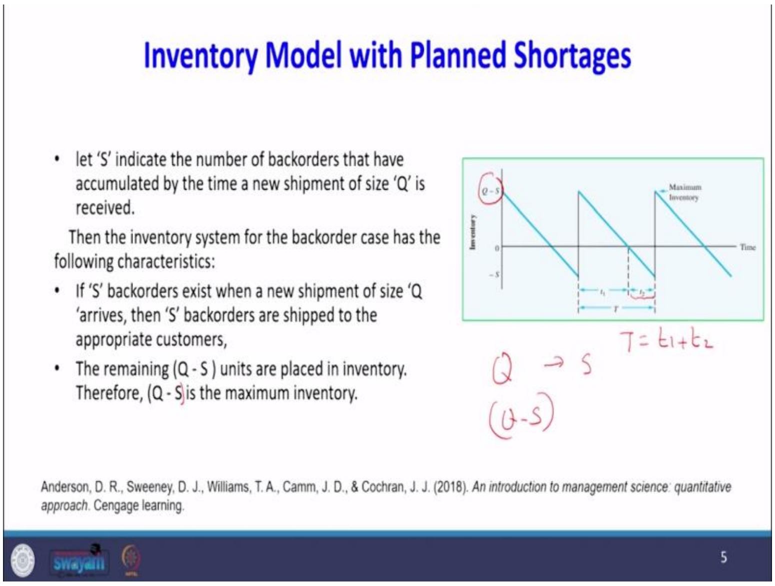


Before going into this topic first we will understand the concept of backorder. What is the backorder? This model developed in this lecture takes into account the type of shortage known as backorder. So, what is a backorder? Backorder is defined as an order received for goods or services but cannot be fulfilled and can be due to various reasons such as the availability of the ordered product, the stock being still in the production phase, or the order being placed with the manufacturer and delivery still pending.

So, this is the definition of backorder. Look at this picture on the right-hand side. There is a person A and a dealer. So, person A wants to buy, say, 10 buses, but the dealer has only 4 buses in inventory. He cannot deliver all 10 buses at a time. So, what can he do? He can take 4 from the dealer and ask the dealer to deliver the remaining buses within 2 months. Person A has to wait for 7 to 8 months for the remaining buses to be received.

This is the concept of back ordering. For example, say this says 7 months. Say this is 8 months, say a month. So, he has ordered something, but this order has not been fulfilled, so what we have done in the next month is first fulfill the previous month's order, which is the meaning of this backorder. In the 7th month, we were not able to fulfill the order; for example, the same problem says that 10 buses are required, but we were able to fulfill only 4 buses.

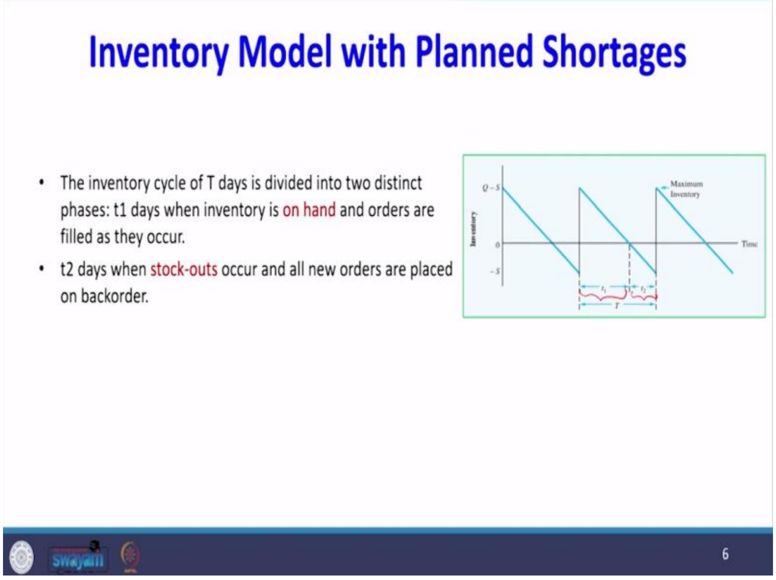
So, the remaining six shortage, for the remaining six we will be manufacturing the 8 months, then we will settle; we will supply the remaining 6 buses to the previous month, so this is the meaning of this back-ordering. So, we are going to consider this concept of back ordering whenever a shortage occurs.



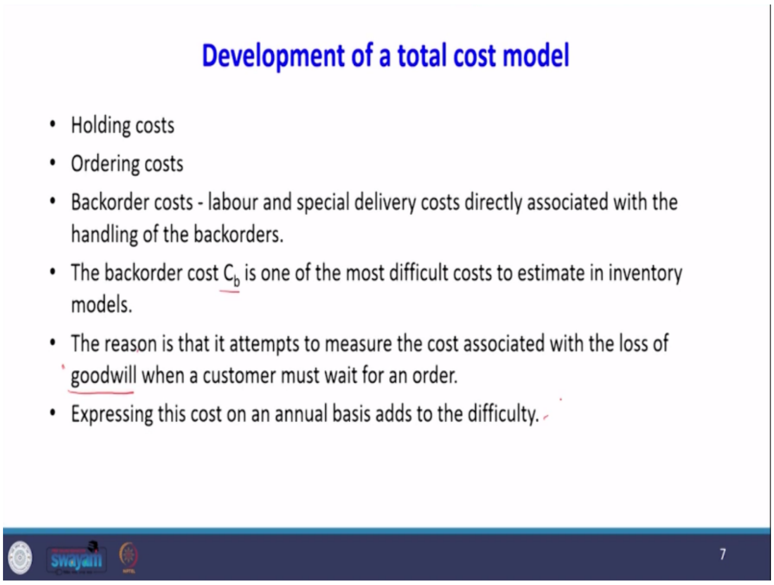
Inventory model with planned shortages let S indicate the number of backorders that have accumulated by the time a new shipment of size Q is received. Here, S is the number of backorder shortages. The inventory system for the backorder case has the following characteristics: If S backorder exists when a new shipment of size Q arrives then S backorders are shipped to the appropriate customers.

So, what is happening? Suppose we are receiving Q quantity already, and there are S shortages, so, as soon as we receive the order, this S quantity will be shipped to the appropriate customers, which means a person who is already waiting. So, the remaining (Q – S) units are placed in inventory. Therefore, the (Q – S) is the maximum inventory. Look at this picture on the right-hand side in the Y axis, the inventory, so the (Q – S) is the maximum inventory.

The negative S represents the shortages there is a t1 and t2. This model is similar to our economic order quantity in that whenever we place the order, the replenishment takes place immediately. See the small t1 during this period there is a shortage's. So, total cycle time is a combination of t1 + t2; t2 is where there are no shortages, and 2 is where there are shortages. So, the maximum inventory is (Q – S).



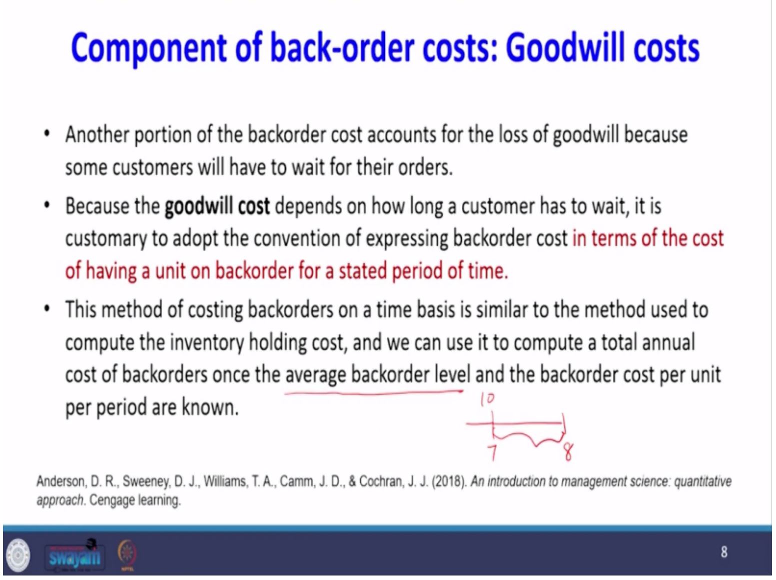
The inventory cycle of T days is divided into two distinct phases t1 days when inventory is on hand and orders are filled as they occur. t2 days when the stock out occurs and all new orders are placed on backorder because due to shortages this will be kept for the backorder.



Unless our previous two techniques are the cost, we have considered previously we have considered the holding cost and ordering cost. So, in this model, apart from these two costs, we are going to consider another cost, which is called backorder cost. What is a backorder cost? Labor and special delivery costs directly associated with the handling of the backorder are called backorder costs.

So, the backorder cost it is denoted as Cb, is one of the most difficult costs to estimate in inventory models. The reason is that it attempts to measure the cost associated with loss of goodwill is perception only goodwill when a customer must wait for an order because we cannot quantify the loss of goodwill. That is why we say that measuring the backorder cost is very difficult.

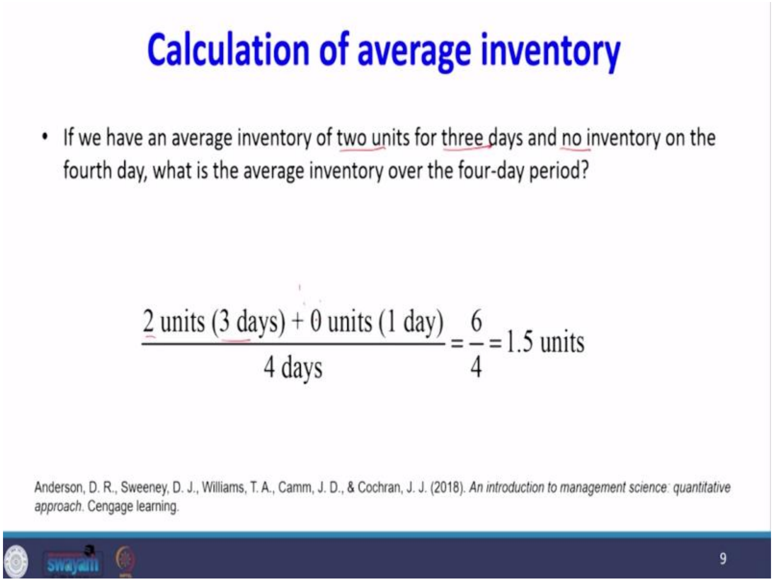
So, expressing this cost on an annual basis adds to the difficulty because this loss of goodwill cannot be measured.



The component of the backorder cost is a goodwill cost. Another portion of the backorder cost accounts for the loss of goodwill because some customers will have to wait for their orders because the goodwill cost depends on how long a customer has to wait; it is customary to adopt the convention of expressing backorder cost in terms of the cost of having a unit on backorder for your stated period of time.

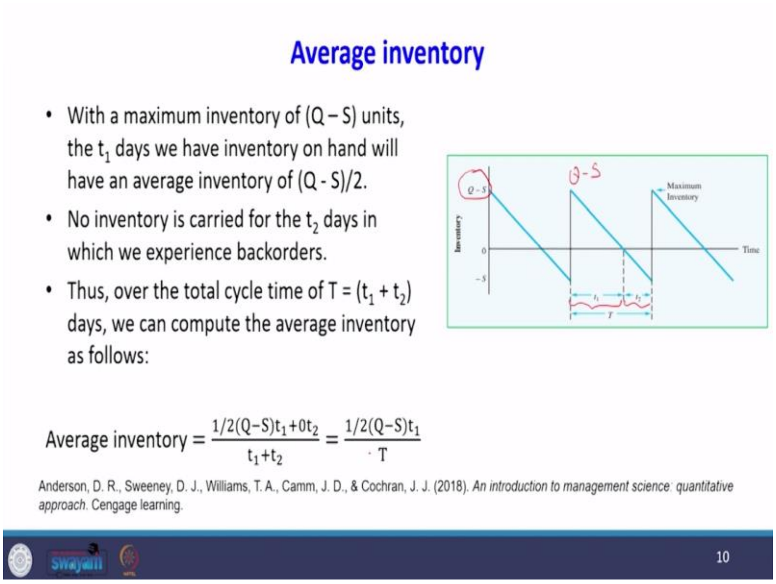
So, what is the meaning of this cost? Suppose, say, the 7th month and this is the 8th month, so the 7th month, there is a shortage. Say the shortage is, let us say, 10 units. So, what will happen in case if you are holding these 10 units for this one period of up to 8 months so that inventory holding cost can be considered as your backorder cost? This method of costing backorder on a time basis is similar to the method used to compute inventory holding costs.

It is similar to inventory holding cost, and we can use it to compute the total annual cost of backorders once the average backorder level and the backorder cost per unit per period are known. So, later we will derive this we will derive this average backorder level then we will multiply that one by the annual backorder cost so that we will get the annual backorder cost.



As usual, we go for the calculation of average inventory. If you have an average inventory of 2 units for 3 days, you have 2 units for 3 days and no inventory on the fourth day; what is the average inventory over the 4-day period? So, for 2 units you are holding for 3 days, and for 0 units, there is a shortage for 1 day. What is the total duration 3 + 1 = 4. So, when you simplify it as 6 upon 4, it is 1.5 units. So, this is the sample calculation for finding the average inventory.

So, this idea here, see that there is a shortage. We are going to use this idea to find the average inventory whenever there is a shortage.



Average inventory with a maximum inventory of (Q – S) units we know that the maximum inventory is (Q – S) and t1 days see this is also (Q – S). On the first day, we have inventory on hand. So, up to this one day, we will have inventory on hand. So, the average inventory is

(Q – S) upon 2. This is the same idea we have used when we are deriving the economic order quantity, as EOQ.

So, we used Q / S maximum inventory divided by 2, but instead of that, we are using (Q – S) upon 2. So, no inventory is carried for the t2 days during this period there is no inventory in which we experience the backorders. Thus, over the total cycle time that is (t1 + t2) days we can compute the average inventory as follows. In the previous slides, I explained how to find the average inventory.

$$\text{Average inventory} = \frac{1/2(Q-S)t_1 + 0t_2}{t_1+t_2} = \frac{1/2(Q-S)t_1}{T}$$

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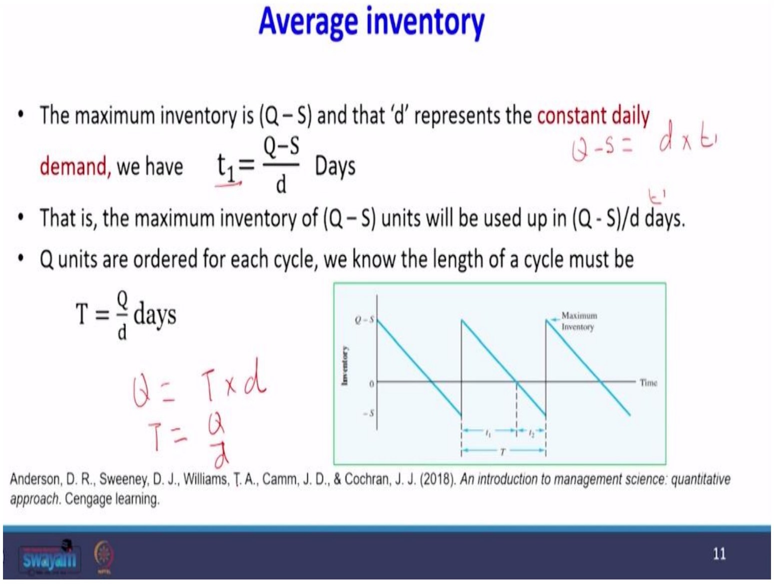
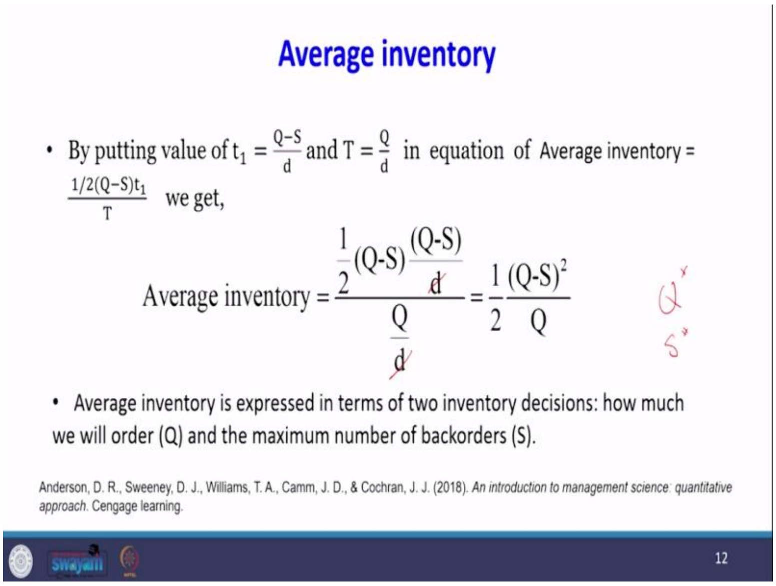


In that previous expression, we are going to replace this small t1 and T in terms of Q, and that is what we are going to do. So, the maximum inventory (Q – S) and that d represents the constant daily demand. So, what will happen when you multiply to get (Q – S); what we have to do is the per day demand multiplied by the t1. So, that will give you the maximum inventory. From this (Q – S) = d multiplied by t, we can get t1. We can get t1 is (Q – S) upon d. So, the t 1 we can express in terms of (Q – S) upon d.

$$t_1 = \frac{Q-S}{d} \text{ Days}$$

What is the small d? This is a constant demand rate that is the maximum inventory (Q – S) units that will be used up to (Q – S) upon d days, that is, your t1 days. Another expression we are going to find out for T is we know that the Q units are ordered for each cycle. We know that the length of the cycle must be seen as Q equal to the length of the cycle and per day demand.

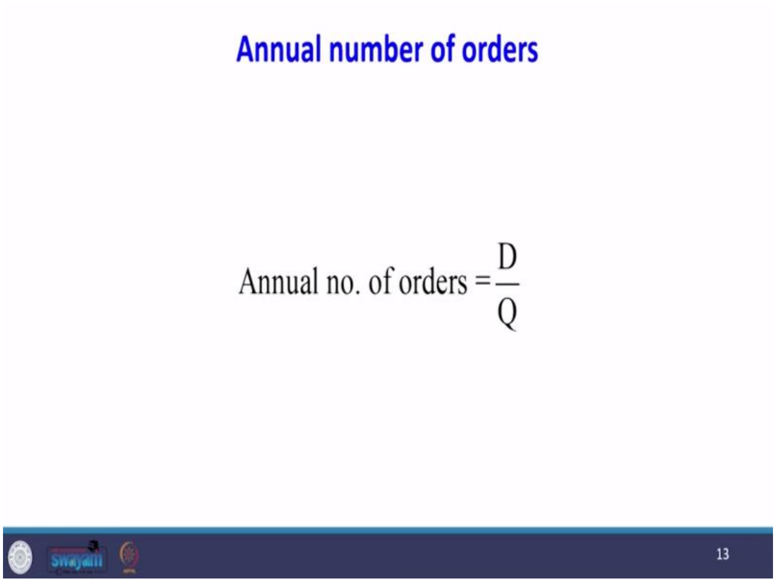
So, from this expression, we can find out the T what is the T it is the Q upon small d. So, we got an expression for small t 1 and capital T. So, we are going to substitute these expressions in our average inventory formula.



So, by putting the value of t1 = (Q – S)upon d and capital T cycle time = Q upon d in the equation of average inventory we are getting one upon two instead of t1. We are going to substitute this term (Q – S)/ d; instead of capital T, we are going to substitute this (Q / d), d is per day demand, and Q is quantity ordered. S is the shortage, so when you simplify this, d gets canceled, so the remaining is 1 / 2 (Q – S) whole square divided by Q.

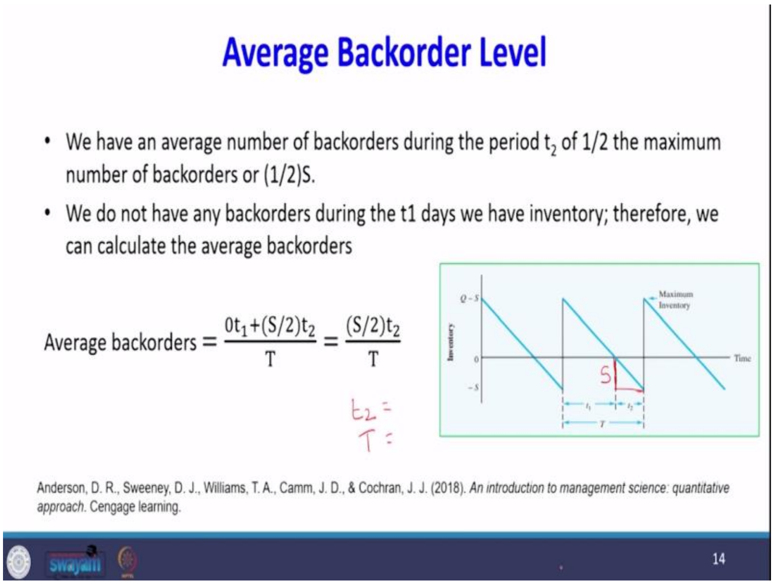
$$\text{Average inventory} = \frac{\frac{1}{2}(Q-S)\frac{(Q-S)}{d}}{\frac{Q}{d}} = \frac{1}{2}\frac{(Q-S)^2}{Q}$$

So, look at this: the average inventory is expressed in terms of two inventory decisions. One is how much we will order Q and the maximum number of backorders that are permitted. In the end, we are going to find out what the optimum order quantity and what the optimum shortages allow. So, this is the average inventory expression for average inventory.



So, from this, we know, as usual, the annual number of orders is D upon Q. D is the annual demand, and Q is the quantity ordered, which is the number of orders.

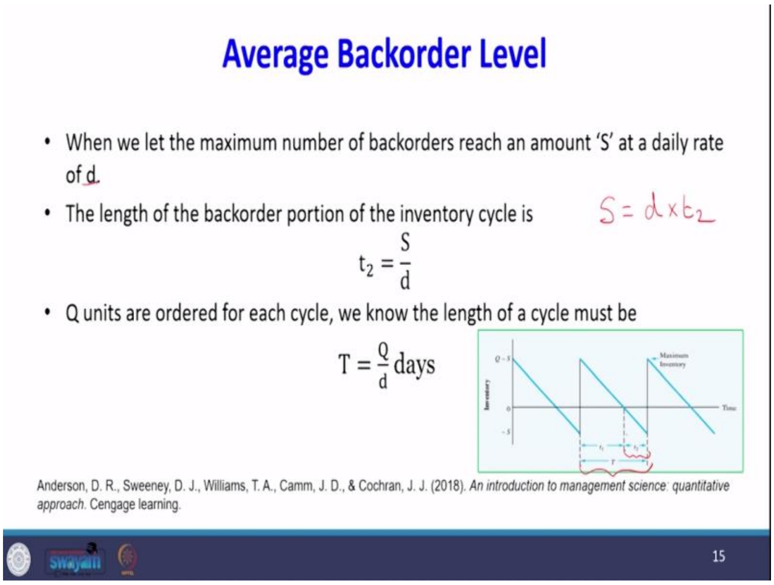
$$\text{Annual no. of orders} = \frac{D}{Q}$$



Now we will derive the expression for the average backorder level. We have an average number of backorders during the period t2 that is half of the maximum number of backorders. Look at this here so this distance is the average number of this is – S average number of backorders this is S when you divide by 2 you will get the average number of backorders.

So, we do not have any backorders during t1. Therefore, we can calculate the average inventory so 0 multiplied by t1 because during t1 there is no shortages plus what is the unit here S / 2 during t2 times. So, when you simplify this (S upon 2) t2 divided by capital T. Here also we are going to find the expression for t2 and T in terms of Q that we are going to substitute here. So, what is t2?

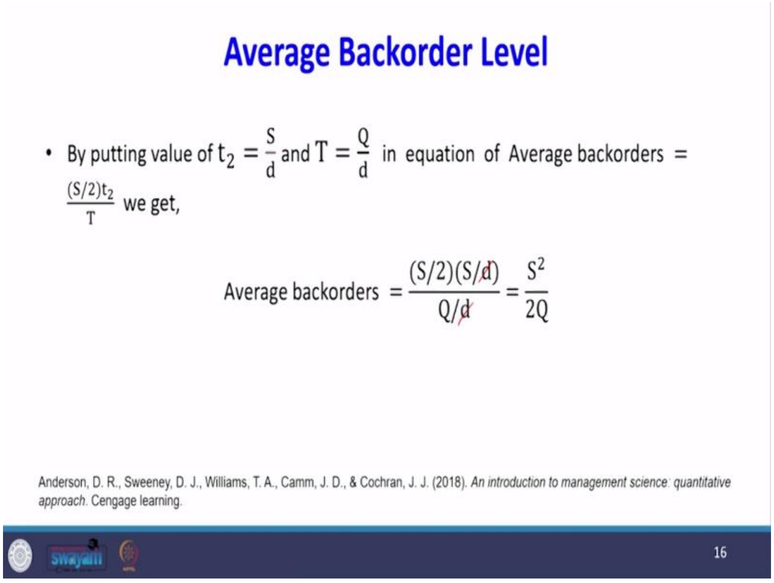
$$\text{Average backorders} = \frac{0t_1 + (S/2)t_2}{T} = \frac{(S/2)t_2}{T}$$



When we let the maximum number of backorders reach an amount S at a daily rate of d. So, d is daily rate of your shortages. So, what will happen? Yes, the total shortage and total number of backorders in terms of the units is the daily rate multiplied by t2. So, from this expression we can get t2 is S upon d. Q units are ordered for each cycle we know the length of the cycle must be T = (Q / d).

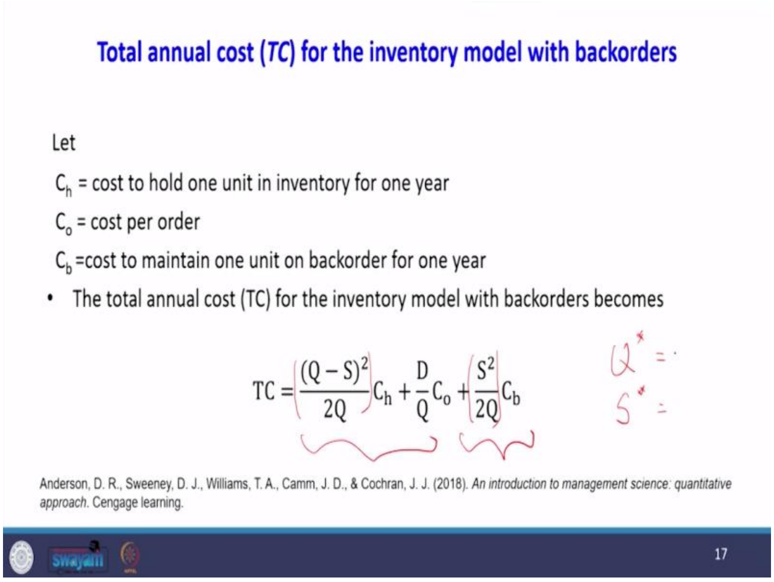
$$T = \frac{Q}{d}\,\text{days}$$

So, we got the expression for this t2, and we already have an expression for T. So, these two expressions we are going to substitute these in our previous equations.



When you substitute there what will happen the average backorder is S upon 2 instead of t2 we can substitute S / d. Instead of capital T we can substitute Q / d. So, when you simplify d d gets cancelled so remaining is S^2 / 2 Q this is average backorder. Previously we have seen the average inventory now we have got the expression for average backorder.

$$\text{Average backorders} = \frac{(S/2)(S/d)}{Q/d} = \frac{S^2}{2Q}$$

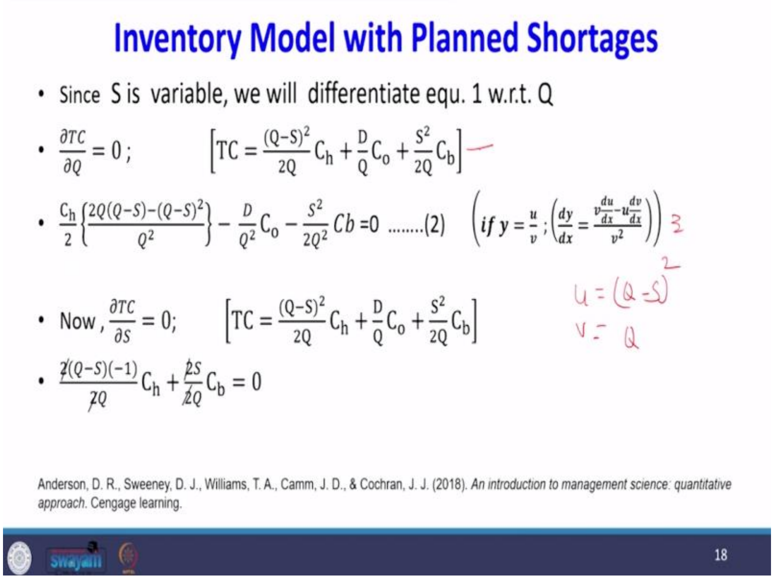


Now, as usual, we find out the total annual cost for the inventory model with the backorders. What are the expressions for different costs? Ch represents the cost of holding one unit in inventory for one year as usual. Co is the cost per order. The new term is introduced that is a Cb cost of maintaining one unit on backorder for one year. That is a backorder cost. We can call it a shortage cost.

So, the total annual cost TC for the inventory model with backorder becomes to see this is the average value of Q; this is the average value of shortages. So, when you multiply by Ch, you will get the holding cost, ordering cost, and backorder cost. In the EOQ definition, we had only these two holding and ordering costs. Now in the shortage inventory model with the shortages, we have one more component, which is the annual backorder cost.

$$TC = \frac{(Q-S)^2}{2Q} C_h + \frac{D}{Q} C_o + \frac{S^2}{2Q} C_b$$

So, this is the equation we are going to here. We have to find out the optimum value of Q, call it Q*, and the optimal value of S. What is the Q ordering quantity? What is the S*? Are there allowable shortages? So, there are two variables here, so I am going to differentiate this equation partially with respect to Q and S and going to equate it to 0. Then, I will get the expression for my Q* and S*.



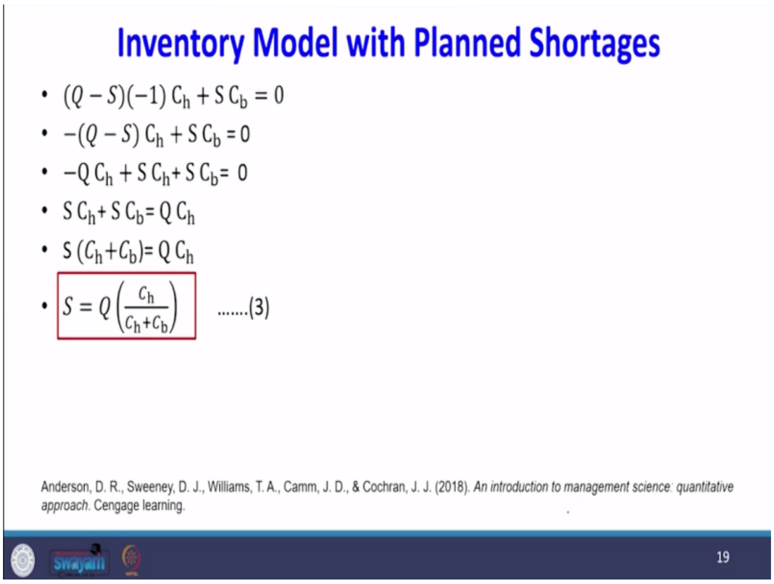
So, this expression is y = u upon v. The formula for differentiation is dy/dx = the denominator v square the numerator v (du/dx) – u (dv/dx). So, in this expression, I am saying about in this expression what the value of u and the value of v. So, here, the value of u is (Q – S) whole square, and the value of v is Q. So, when you substitute when you use this formula for differentiating.

$$y = \frac{u}{v} \; ; \left(\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right)$$

$$\frac{C_h}{2}\left\{\frac{2Q(Q-S)-(Q-S)^2}{Q^2}\right\} - \frac{D}{Q^2}C_o - \frac{S^2}{2Q^2}Cb = 0 \quad \text{........(2)} \quad ($$

Now, $\frac{\partial TC}{\partial S} = 0;$ $\quad \left[TC = \frac{(Q-S)^2}{2Q}C_h + \frac{D}{Q}C_o + \frac{S^2}{2Q}C_b\right]$

$$\frac{\cancel{2}(Q-S)(-1)}{\cancel{2}Q}C_h + \frac{\cancel{2}S}{\cancel{2}Q}C_b = 0$$



When you further simplify

$$(Q-S)(-1)\,C_h + S\,C_b = 0$$

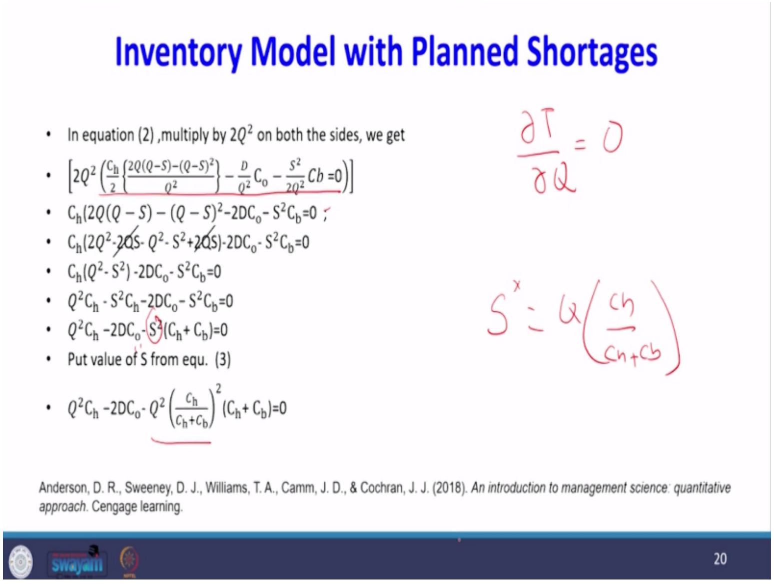
$$-(Q-S)\,C_h + S\,C_b = 0$$

$$-Q\,C_h + S\,C_h + S\,C_b = 0$$

$$S\,C_h + S\,C_b = Q\,C_h$$

$$S\,(C_h + C_b) = Q\,C_h$$

$$\boxed{S = Q\left(\frac{C_h}{C_h + C_b}\right)} \quad \text{.......(3)}$$



Now simplifying further, we get

In equation (2) ,multiply by $2Q^2$ on both the sides, we get

$C_h(2Q(Q-S)-(Q-S)^2$–2DC_o– S^2C_b=0 ; $\left[2Q^2\left(\frac{C_h}{2}\left\{\frac{2Q(Q-S)-(Q-S)^2}{Q^2}\right\}-\frac{D}{Q^2}C_o-\frac{S^2}{2Q^2}Cb=0\right)\right]$

$C_h(2Q^2$-2QS- Q^2- S^2+2QS)-2DC_o- S^2C_b=0

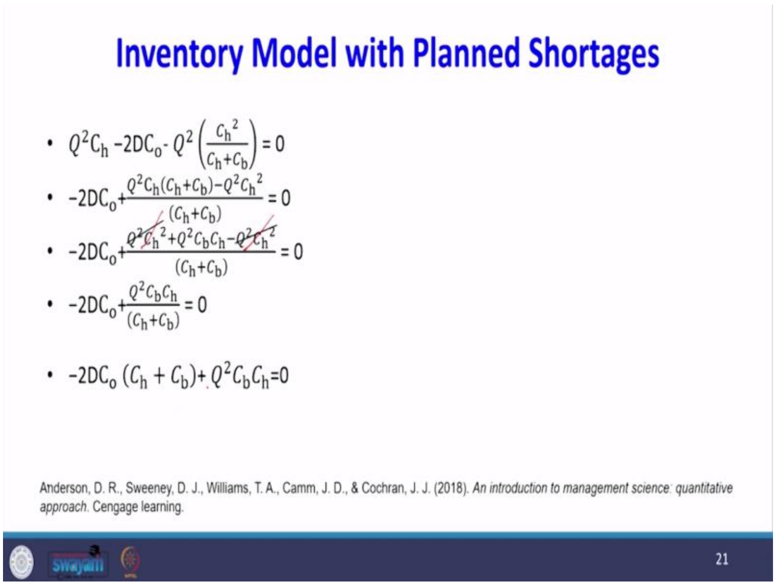
$C_h(Q^2$- S^2) -2DC_o- S^2C_b=0

Q^2C_h - S^2C_h–2DC_o– S^2C_b=0

Q^2C_h –2DC_o- $S^2(C_h+C_b)$=0

Put value of S from equ. (3)

Q^2C_h –2DC_o- $Q^2\left(\frac{C_h}{C_h+C_b}\right)^2(C_h+C_b)$=0



Now, further when you simplify so you are getting

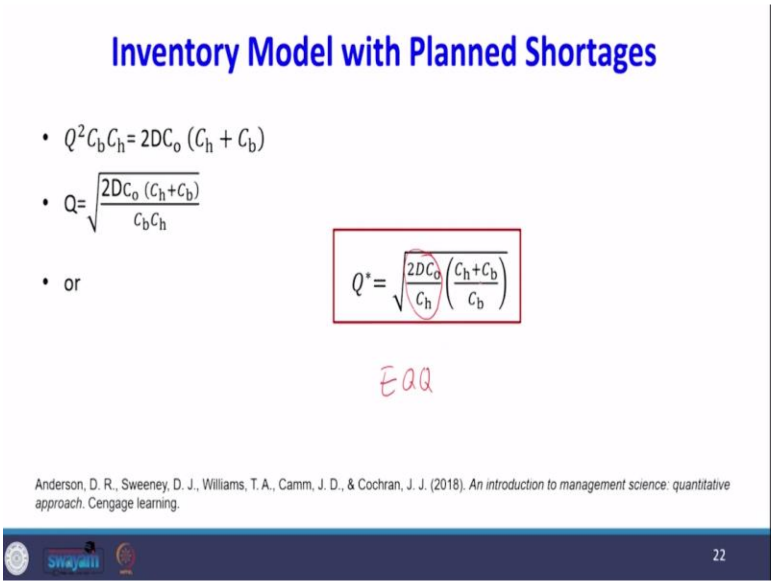
$$Q^2C_h - 2DC_o - Q^2\left(\frac{C_h^2}{C_h + C_b}\right) = 0$$

$$-2DC_o + \frac{Q^2C_h(C_h + C_b) - Q^2C_h^2}{(C_h + C_b)} = 0$$

$$-2DC_o + \frac{Q^2C_h^2 + Q^2C_bC_h - Q^2C_h^2}{(C_h + C_b)} = 0$$

$$-2DC_o + \frac{Q^2C_bC_h}{(C_h + C_b)} = 0$$

$$-2DC_o\,(C_h + C_b) + Q^2C_bC_h = 0$$



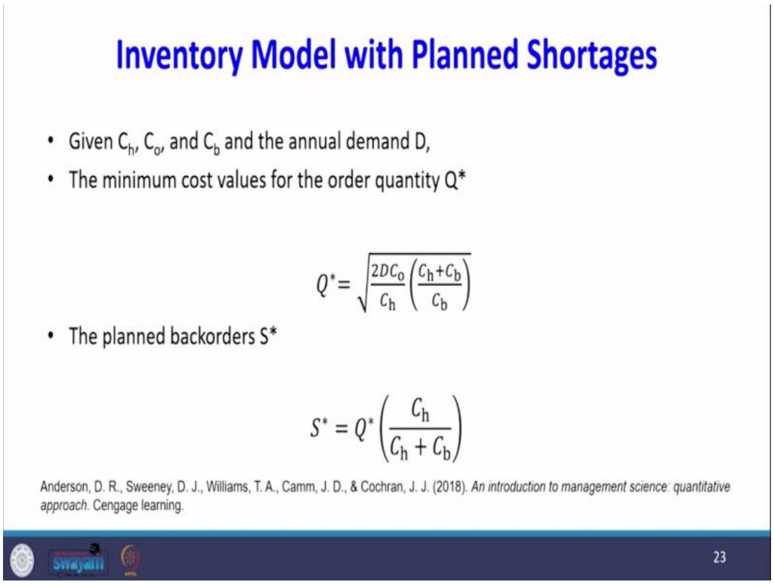
So, when you bring on the right hand side so where we will get the value of Q square. We will get

$$Q^2C_bC_h = 2DC_o\,(C_h + C_b)$$

$$Q = \sqrt{\frac{2DC_o\,(C_h + C_b)}{C_bC_h}}$$

or

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)}$$



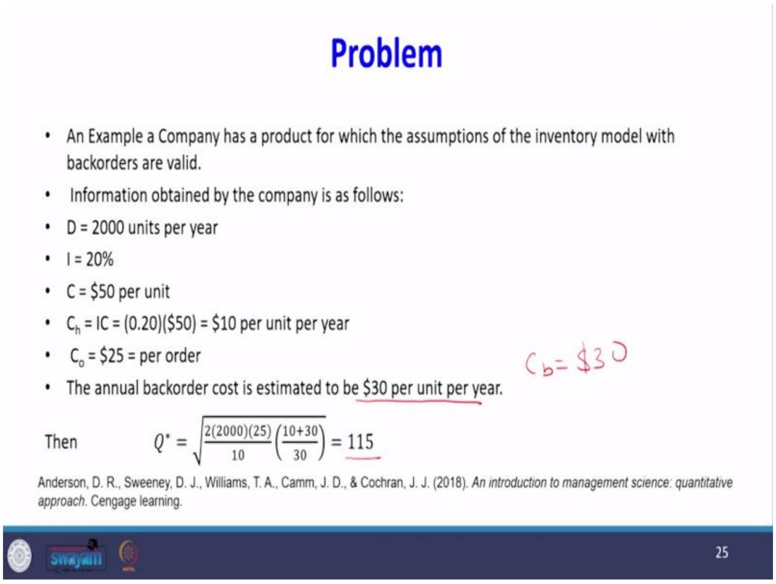
Given Ch holding cost, ordering cost and backorder cost and annual demand D the minimum cost values for the order quantity Q star is this expression root of 2 D Co / Ch multiplied by Ch + Cb / Cb and the planned backorder S star is Q star multiplied by Ch divided by Ch + Cb.

The minimum cost values for the order quantity Q*

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h+C_b}{C_b}\right)}$$

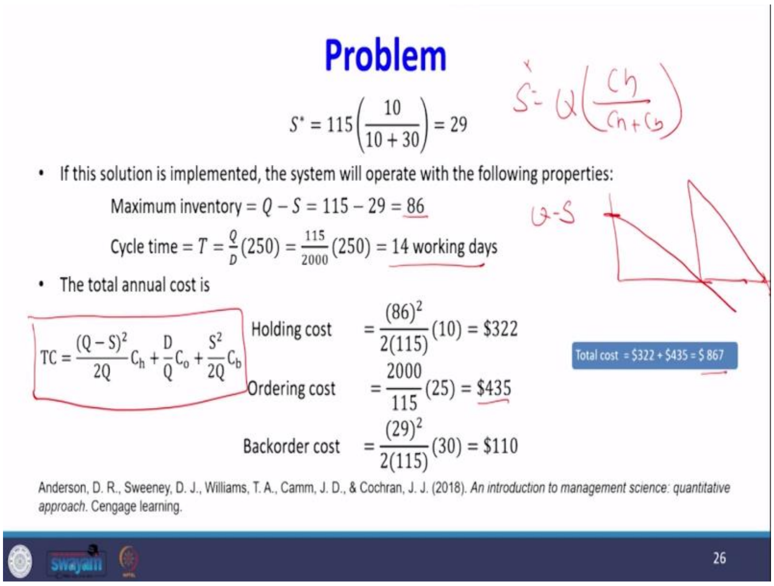
The planned backorders S*

$$S^* = Q^*\left(\frac{C_h}{C_h+C_b}\right)$$



Now, we will take one problem, and then we will apply this planned inventory shortage model. An example is a company that has a product for which the assumptions for the inventory model with backorders are valid. The information obtained by the company is as follows: annual demand is given, interest rate is 20 percent unit cost is 50 dollars per unit, so from this expression, we can find out the unit holding cost per year.

So, 0.20 multiplied by 50, you will get 10 dollars per unit per year. The ordering cost is dollar 25 per order, and the annual backorder cost is estimated to be dollar 30 per unit per year. So, the value of Cb = dollar 30. We already know the expression for Q star, so when you substitute this, we get the optimal order quantity of 115. Next, we will find out the optimal shortages per meter.



We know the formula for shortages S* = Q Ch holding cost divided by holding cost plus shortage cost. So, from the previous expression, we got the value of Q, which is 115. All other values are simply substituted. We are getting the value 29, which. What is the meaning of this 29? Permissible shortages. If this solution is implemented the system will operate with the following properties.

$$S^* = 115\left(\frac{10}{10+30}\right) = 29$$

So, what is the maximum inventory? So, the maximum inventory is (Q – S) what is (Q – S)? (115 – 29) is 86. What is the cycle time? Cycle time is, for example, some periods when there will be shortages. So, what is the cycle time Q / D? Assume that the company works for 250 days per year. So, 250 is divided by the number of orders. When you simplify, it will be Q / D multiplied by 250, so it will be 14 working cycles, and the cycle time is 14 working days.

$$\text{Maximum inventory} = Q - S = 115 - 29 = 86$$

$$\text{Cycle time} = T = \frac{Q}{D}(250) = \frac{115}{2000}(250) = 14 \text{ working days}$$

So, what we can do from this expression is take the annual holding cost and get the expression for the total cost. What are the elements of total cost holding cost, ordering cost, and backorder cost? Individually, we will find out what the total cost holding cost is (Q – S). Just now, we got this 86 divided by 2 Q, what is a Q 115 multiplied by holding cost is dollar 10. So 322 ordering cost D / Q D is 2,000 Q is 115 so ordering cost is 25 435. The backorder cost S square divided by 2 QS is S* is 29 whole square divided by 2 Q is 115 multiplied by 30, 30 is the backorder cost we are getting 110. So, when you add these three costs, we are getting 867, which is the annual inventory cost when there is a shortage. Instead of solving the same problem you consider without shortages.

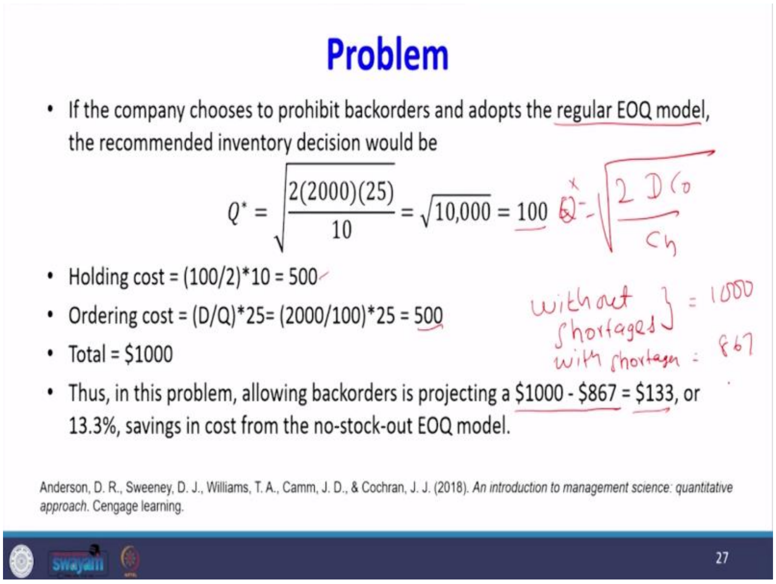
The total annual cost is

$$\text{Holding cost} = \frac{(86)^2}{2(115)}(10) = \$322$$

$$\text{Ordering cost} = \frac{2000}{115}(25) = \$435$$

$$\text{Backorder cost} = \frac{(29)^2}{2(115)}(30) = \$110$$

Total cost = \$322 + \$435 = \$ 867



If the company chooses to prohibit backorders, not allow shortages, and adopt your regular EOQ model, they recommend an inventory decision. We know the formula 2 DCo divided by Ch, which is your Q*. Demand is 2,000, order and cost is dollar 25, and holding cost is dollar 10. So, we got the Q* is 100 units. For the 100 units, we can find out the holding cost. What is the cost? Average inventory Q / 2 multiplied by per unit holding cost.

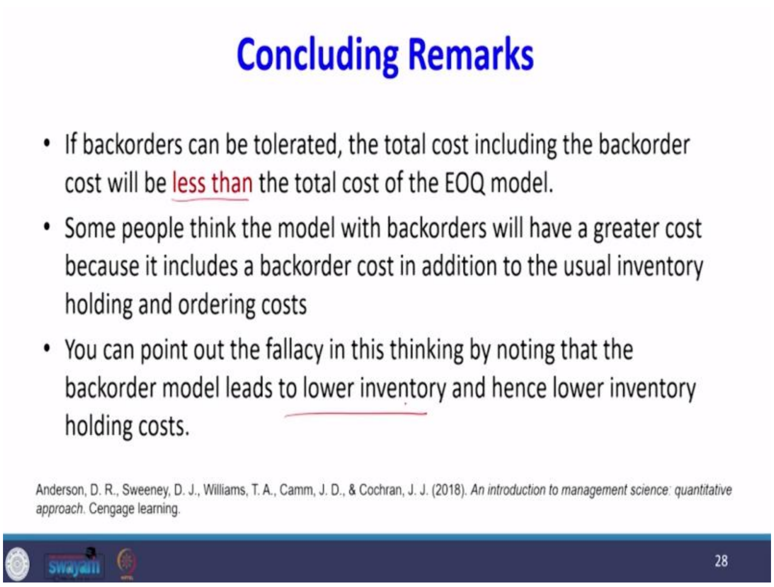
$$Q^* = \sqrt{\frac{2(2000)(25)}{10}} = \sqrt{10{,}000} = 100$$

So, the annual holding cost is 500, and the annual ordering cost is multiplied by the ordering cost. So, the number of orders is 2,000 upon 100 multiplied by 25, which is 5,000. The total cost is 1,000. Now you see in this problem allowing backorder is projecting how much dollar 1000 – 867 = 133 or 13.3 percentage saving in cost from no stock out EOQ model. So, what is the comparison we are making if you go for with shortages and without shortages?

Holding cost = (100/2)*10 = 500
Ordering cost = (D/Q)*25= (2000/100)*25 = 500
Total = $1000
Thus, in this problem, allowing backorders is projecting a $1000 - $867 = $133, o
13.3%, savings in cost from the no-stock-out EOQ model.

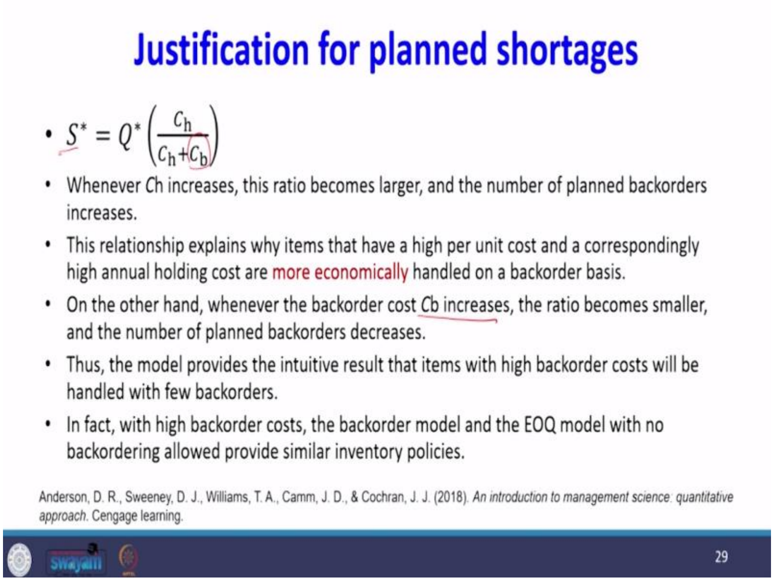
So, without shortages, our cost is dollar 1,000. With shortages, the total annual inventory cost is 867. So, dollar 133 is the saving due to these planned shortages.

*



Concluding remarks, if backorders can be tolerated, the total cost, including backorder cost, will be less than the total cost of the EOQ model, which means that if the customers are willing to wait, then going for shortages is preferable. So, some people think that the model with backorders will have a greater cost because it includes a backorder cost in addition to the usual inventory holding and ordering cost.

You can point out the policy in this thinking by noting that the backorder model leads to lower inventory. Hence the lower inventory holding cost.



I will explain this idea with the help of this expression. So, what is the justification for planned shortages? We have derived the optimal shortage quantity S* = Q* multiplied by Ch upon (Ch + Cb). Here, Ch is the holding cost, and Cb is the backorder cost. Whenever the Ch increases, whenever the holding cost increases, the ratio becomes larger, and the number of planned backorders increases.

$$S^* = Q^*\left(\frac{C_h}{C_h+C_b}\right)$$

So, costly item the value of the items is very high, for example, cars and other items. If the holding cost is high, it is better to go for more shortages. That is this expression phase because this C h is in the numerator. When C h increases, when the holding cost increases, we should go for more shortages. This relationship explains why items that have high per unit cost and a correspondingly high annual holding cost are more economically handled on a backorder basis.

On the other hand, when the back order cost Cb increases, the ratio becomes smaller, and the number of planned backorders decreases. Thus, the model provides an intuitive result that the item with high backorder cost will be handled with few backorders. In fact, high back order costs the backorder model, and the EOQ model with no back ordering allowed provides similar inventory policies.

So, what I mean to say is whenever there is a high backorder cost, whether you solve it by the planned shortages model or over the traditional EOQ model, we will get a similar result. In this lecture, we have seen inventory models with planned shortages. What are the concepts I have covered? I have derived an expression for the total cost. From the total cost expression, we have the value for the optimal order quantity and the optimal shortage quantity.

Then we took a sample problem, then with the help of the sample problem, I explained these concepts. Thank you very much. We will see you in the next class.