Decision Making With Spreadsheet
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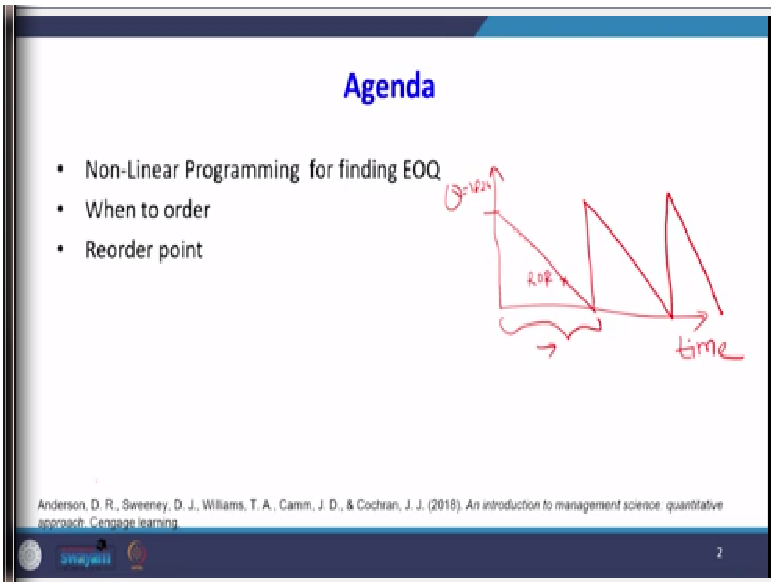
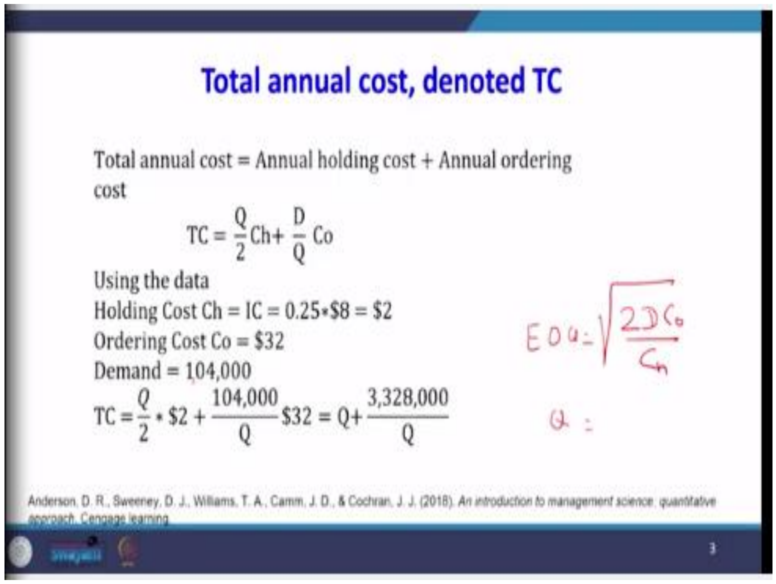
Lecture-34
Inventory Models: Economic Order Quantity.
(EOQ) Model-II

Dear students, I started finding economic order quantity in the previous class, in which I found how much to order. In this lecture I will continue the same problem to answer the following question. One is when to order and knowing what the reorder point is.

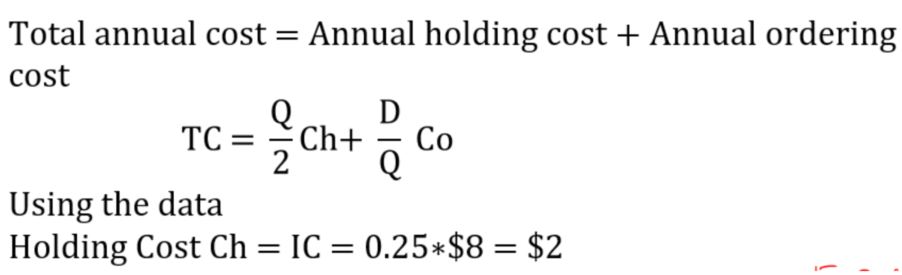


So, the agenda for this lecture is to discuss how to use non-linear programming to solve economic order quantity. To answer these two questions, one is when to order and what is the reorder point. If any inventory management problem was seen in the previous class, we see that it will be in the x-axis the time, and in the y-axis is quantity, so we have found in the previous class what Q, economic order quantity. Now, we should know there may be different cycles, but we may not know that.

We should know when to order, right? We know quantity, so now I will answer in this lecture when to order. Another one is if there is lead time, what is the reorder point? The reorder point is, so suppose I have some quantity says in the previous class we have found the value of Q is 1824. What will happen? When the time increases, we will start consuming these 1824 quantities, so it will keep on consuming. At one point, we have to start the order, so what quantity we have to find out at that point is called your reorder point. That is what we are going to find in this lecture.



First, I will solve this with the EOQ model with the help of non-linear programming. We know that the total annual cost is the sum of the annual holding cost and annual ordering cost. So, what is the annual holding cost? This average inventory Q upon two multiplied by holding cost plus a number of orders multiplied by ordering cost. So, in our problem, we found the interest rate is 25%, and the unit cost is 8 dollars, so 0.25 multiplied by 8, so it is 2 dollars.



The ordering cost is 32 dollars; the demand is 1,04,000; what is the formula for total cost? Q by 2, so I am substituting the ordering cost, so 2 demand divided by Q multiplied by ordering cost. So, when you simplify you will get Q + 3,328,000 upon Q, so we must find out this Q value.

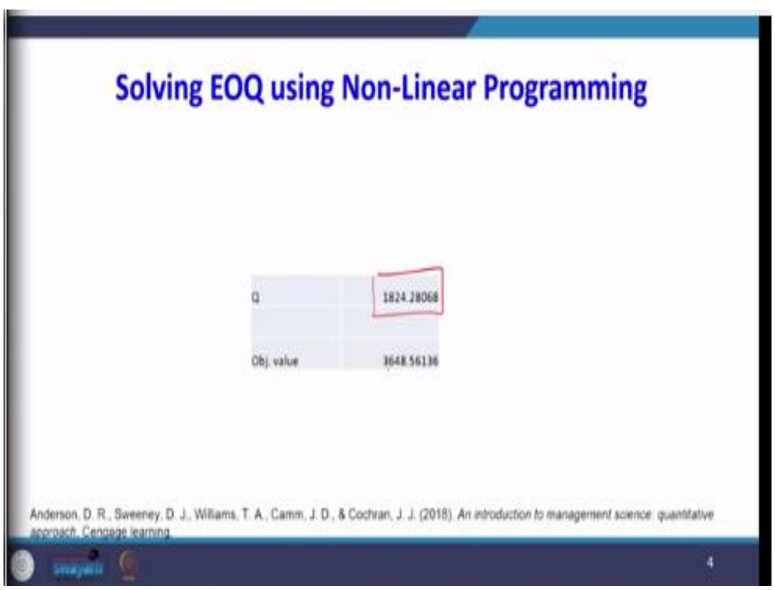
So, instead of deriving this, we can use the idea of non-linear programming to find out the optimal value of this Q; that is what we are going to do. So, I am going to open an Excel, I am going to write this equation, then I am going to find out the value of Q. Now I am going to find out the optimal value of Q. We know the function for total cost Q + 3,328,000 upon Q.

The total cost expression is:

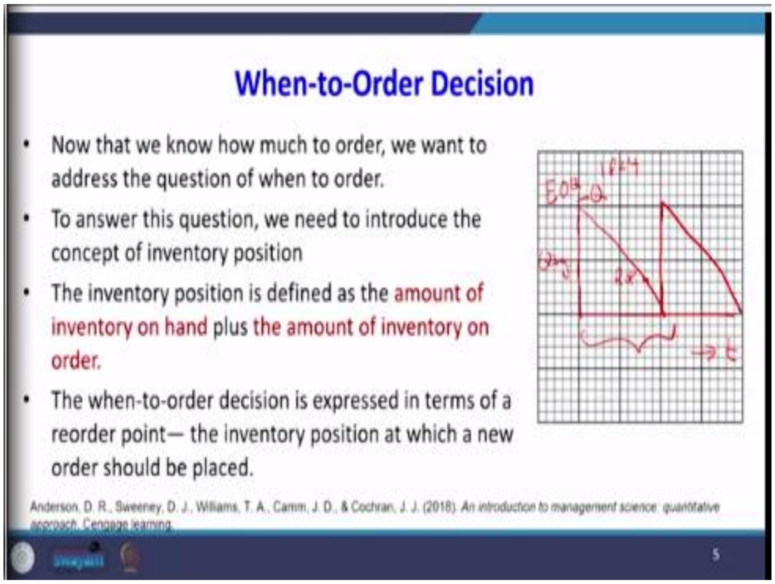
$$\mathrm{TC} = \frac{Q}{2} * \$2 + \frac{104{,}000}{\mathrm{Q}} \$32 = \mathrm{Q} + \frac{3{,}328{,}000}{\mathrm{Q}}$$

So, Q is one of my decision variables; look at the objective function F7; see in the F7, I have written this expression; what is that? So, the value of the objective function is the Q that is F5 + demand 3,328,000 upon F5. Now, this expression is non-linear, so if I go to the solver, go to data, solver, now F7 is my objective function, it is a minimization function, and F5 is my changing cell.

So, we know that there should be a value of F5 should be greater than or equal to 0, here you must select GRG non-linear. So, when I click Solver, and you see the result, the value of Q is 1824; this is the exact value that we got using our EOQ formula; now, I will go back to my presentation.



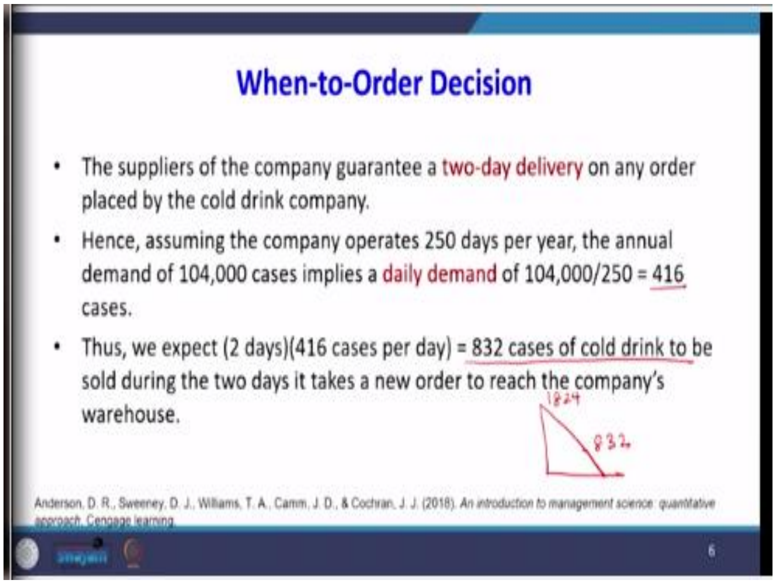
This is the exact value of Q; the objective function value is 3648. So, what point we are taking from here is that any EOQ formula can be solved with the help of a solver by using the concept of non-linear programming, which you have studied in detail in our previous lectures.



Now, we are going to find out when to make a decision. As I told you in the previous class assume that this inventory cycle. So, we have got a value of EOQ that is a Q value; now we should know when to order, so this is the answer we are going to get now. Now that we know how much to order, which is the value of Q, we want to address the question of when to order. We know the x-axis is the time, y-axis is quantity. To answer this question, we need to introduce the concept of inventory position.

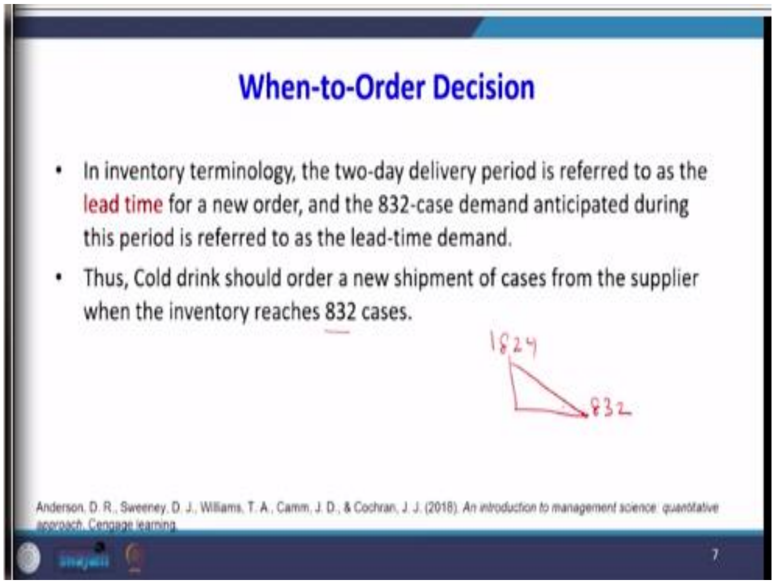
So, the inventory position is defined as the amount of inventory on hand plus the amount of inventory on order, which is the meaning of inventory position. So, the when-to-order decision is expressed in terms of a reorder point, that is, the inventory position at which a new order should be placed; what is the meaning? Suppose we have 1824 will keep on consuming, so it will reach a particular position. So, when 1824 reaches a particular position, we have to start ordering another cycle, so that position is called your reorder point.

You may ask if we can order once it reaches inventory 0, but what will happen? There will be a lead time for receiving any order, so in advance, we have to order so that when the quantity reaches 0, we may get the order that we have ordered previously so that there will not be any shortage. Here, the assumption is that we are not allowing any shortage, so in advance, we have to order. So, the point at which the inventory has to be ordered is called the reorder point.



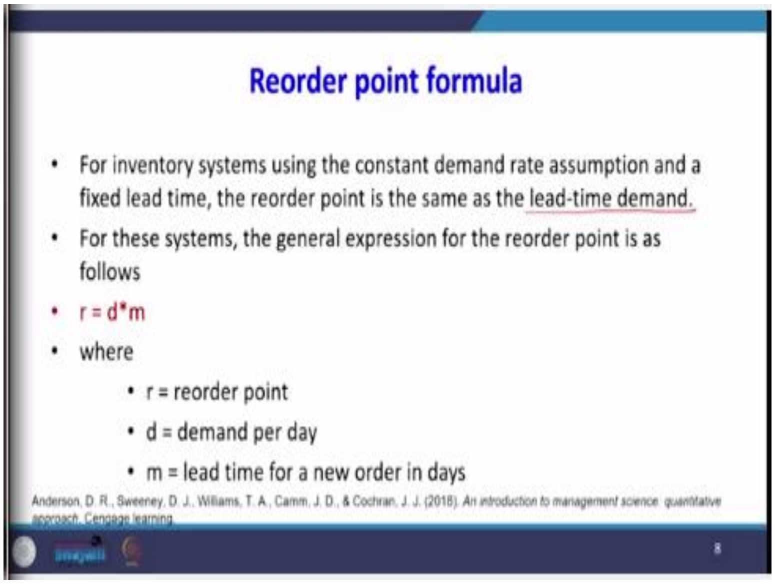
Now there is some more information about this problem we are going to study. The suppliers of this company guarantee a 2-day delivery on any order placed by the cold drink company. So, here is the inventory; the lead time is 2 days. Hence, assuming the company operates 250 days per year, the annual demand we know of 1,04,000 cases implies daily demand; what is the daily demand? The annual demand is divided by the number of working days, so the daily demand is 416 cases. We know that the lead time is 2 days, so what do you have to do?

We expect 2 days and multiplied by these 416 cases per day; this is 832 cases of cold drinks to be sold during the 2 days. So, it takes a new order to reach the company's warehouse, so it is like this. So, when the point reaches 18242, when the point reaches 832, then we have to make another order. How did we get this 832? Because lead time is 2 days, per day demand is 416, so when you multiply lead time and per day demand, that will give you the reorder point. That is what the answer is: 832.



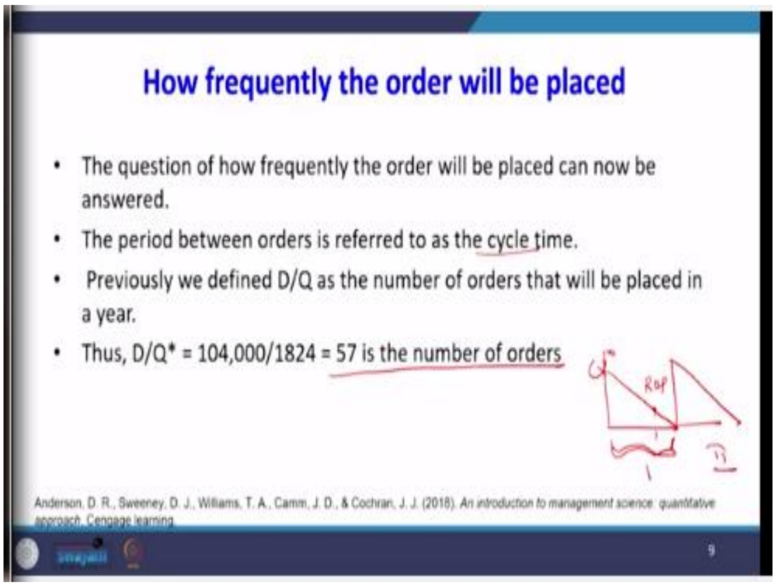
So, in inventory terminology, the 2-day delivery period is referred to as the lead time for a new order, and 832-case demand is anticipated during this period, which means the lead time is referred to as the lead time demand. Thus, the cold drink company should order a new shipment of cases from the supplier when the inventory reaches 832 cases. So, this 832 is nothing, but your reorder point. Still, we did not answer when to order.

The when to order we are saying in terms of quantity; when the quantity reaches 18242, when it reaches 832, we have to make another order. Here in terms of quantity but I want to say in terms of time when should I make the next order? So, we are going to answer that.



So, for example this problem can be solved by using a formula reorder point formula. For an inventory system using constant demand rate assumptions and a fixed lead time, here also remember the lead time is also fixed, and the demand rate is also fixed. So, the reorder point is the same as the lead time demand, so the reorder point is nothing but your demand during the lead time.

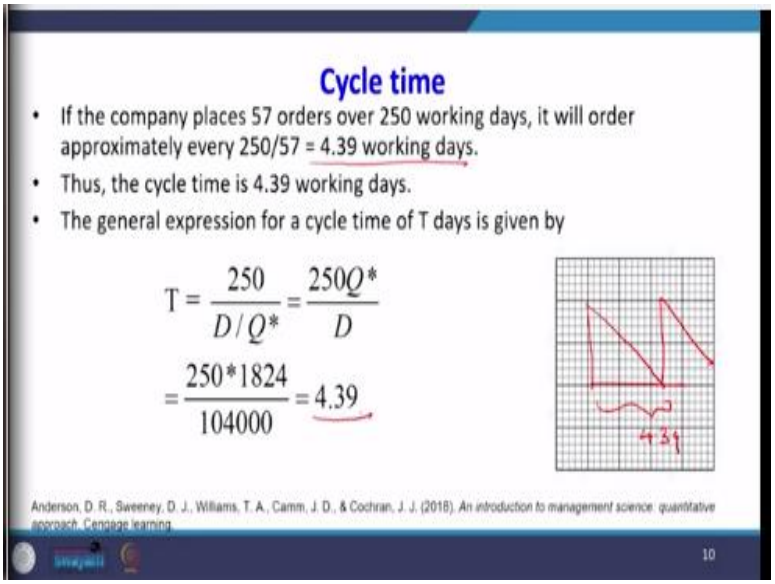
So, for this system, the general expression for the reorder point is reorder point r = d, which is the demand per day, and m is the lead time for a new order in days. We can use this as a formula to determine the reorder point.



Now, the next question is, how frequently will the order be placed? We have answered when to order, but we did not mention the time. The time we are going to know by answering this question: what is that? How frequently will the order be placed? That means we are asking what should be the timing for making the following order. The question of how frequently the order will be placed can now be answered.

The period between orders is referred to as the cycle time, so this is cycle time 1; suppose you order another thing. This is cycle time 2. So, here we got a reorder point, so this duration is our cycle time. Previously, we defined D by Q as the number of orders that will be placed in the year. So, how we got this number of orders? D upon Q star, Q star is our value, and we can Q star, economic order quantity. So, 1,04,000 upon 184 there will be 57 times you have to make the order, so 57 is the number of orders.

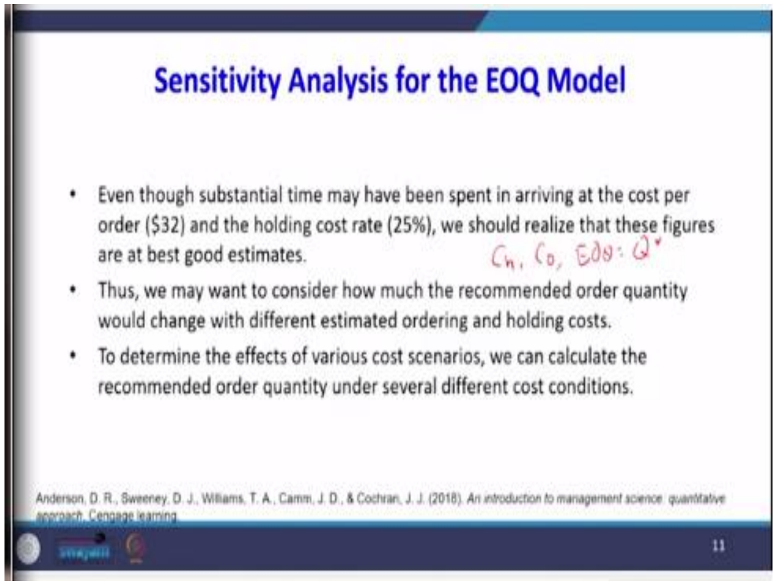
D/Q* = 104,000/1824 = 57



Now we can find out the cycle time, if the company places 57 orders over 250 working days it will order approximately every 250 upon 57, 4.39 working days. So, this distance now, we got this distance, so this distance is 4.39. So, every 4.39 times the company should make the order, so the cycle time is 4.39 working days. So, the general expression for cycle time of T days is given by this formula T = number of working days divided by number of orders. So, 250 upon 57 we are getting this 4.39.

$$T = \frac{250}{D/Q^*} = \frac{250Q^*}{D}$$

$$= \frac{250*1824}{104000} = 4.39$$



Now the sensitivity analysis for the EOQ model. What is the meaning of sensitivity analysis? In the total cost expression, we have the annual holding cost and annual ordering cost; if the value of the annual holding cost and the value of the annual ordering cost change, how will that affect our economic order quantity? So, we want to know how robust our model is, which is the EOQ model; that is what we are going to do with the help of Excel.

So, even though substantial time may have been spent in arriving at the cost per order, we know that it is 32 dollars, the ordering cost, and the holding cost rate is 25% of the unit cost. Now, we should realize that these figures are, at best, good estimates that mean there is a chance for variability. Thus, we may want to consider how much the recommended order quantity EOQ would change with the different estimated ordering and holding costs. So, if the value of C_h and Co is changing, what is the range of C_h and the range of Co, and how does it affect our value of EOQ?

That is nothing but Q*. So, to determine the effect of various cost scenarios, we can calculate the recommended order quantity under several different cost conditions. Now, I am going to explain how to use Excel for doing this sensitivity analysis. The demand I have taken is 1,04,000, the interest rate is 25%, the unit cost is 8 dollars, and the ordering cost is 32. So, in H7 to H10, I am going to change this interest rate. I am considering only 2 scenarios.

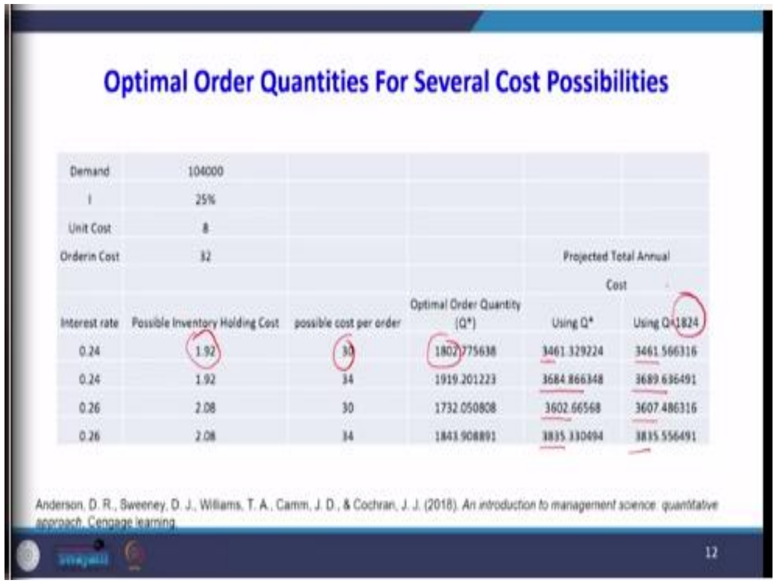
Instead of 25, if it goes to 24, what will happen? If it goes to 24, you see the value of I7, so if I keep the cursor on I7, you see the ordering cost is 192; previously, it was 2 dollars, and it is decreasing; the inventory holding cost is now is 1.92 dollars. Instead of ordering a cost of 32, if I change it to 30, I am going to substitute that formula. When I click K7, I use our traditional formula, the square root of 2DCo divided by C_h.

So, now the ordering quantity for 1.92 holding cost and 30 ordering cost, the EOQ is 1802. So, for this 1802, what is the total cost? So, I am going to substitute this 1802 into my total cost function, which is what I have done in K7. So, you see the formula for L7. I am adding the total expression, the expression for the total cost, so the average cost is K7 upon 2 multiplied by I7 plus the number of orders; what is the number of orders?

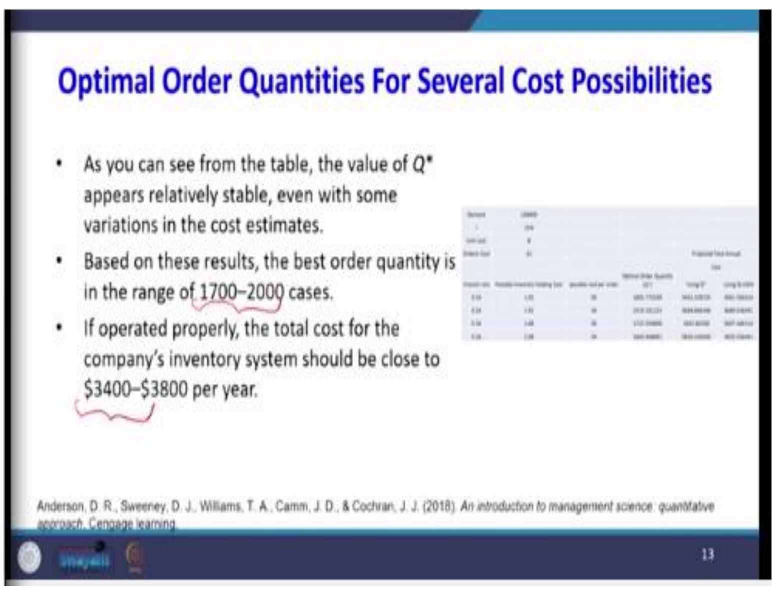
The demand upon EOQ multiplied by J7, which is the ordering cost, so it is 3461. But initially, we know the value of Q is how much? 1824. For this set of inventory holding costs and ordering costs, if you are ordering 1824 units, the total cost is 3461. You see that there are not many changes, whether you order 1802 or 1824. Now look at the next scenario; the holding cost is the same. You see I8, but I changed the ordering cost.

So, when I changed the ordering cost, now EOQ is changing, EOQ is 1919. Now the corresponding cost is 3684. For this set of costs for our initial value EOQ, which we got 1824, now the cost is 3689., you see the cost wise only a 5 rupees difference is there. Similarly, when the interest rate goes to 26%, and the order cost is 30, we are getting another set of Q*; Q* is 3602, no 1732. For that, the cost is again only 3602.

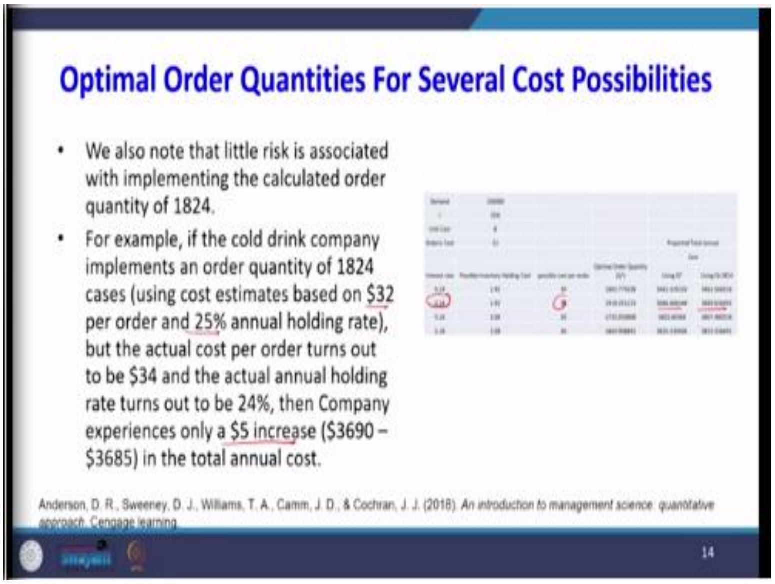
But instead of 1732, if you order our initial value of 1824 again, the cost is only how much? There is a 5 rupees difference. So, what are we learning? Even though there is a variation in the holding cost and ordering cost, there is not much variation in the total cost. So, what we are concluding is that the EOQ model that is 1824, the answer we got, is robust; it will not affected by small changes in our ordering cost and inventory holding cost. I will go back to the presentation now.



So, I have brought the output. Look at this. There are changes 192 and 30. This is EOQ but look at the cost. This is 3461, and it is also 3461. In the second scenario, 3684 and 3689, only a 5 rupee difference; in the third scenario, 3602 and 3000, there also only a 602 rupee is difference; and in the fourth scenario, 3835, it remains the same. So, what are we concluding about that? Our EOQ model, which we got in 1824, is the robust one.

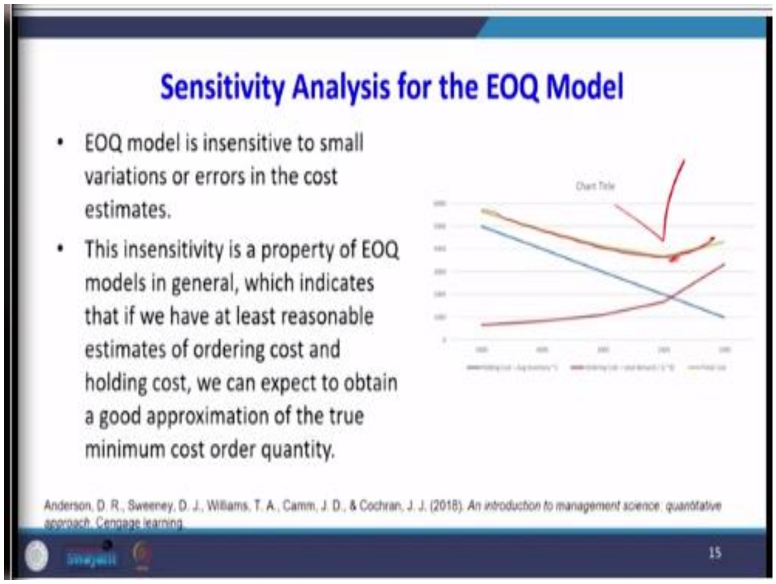


As you can see from the table, the value of Q* appears relatively stable even with some variations in the cost estimates. So, based on this result the best order quantity is in the range of 1700 to 2000; if operated properly the total cost for the company's inventory system should be close to 3400 to 3800 per year instead of saying yes to a single unit, so we can give the range of inventory, the range for the EOQ that can be given. The range for the total cost so maybe the more robust this is, takes care if there is any fluctuation in the ordering cost.



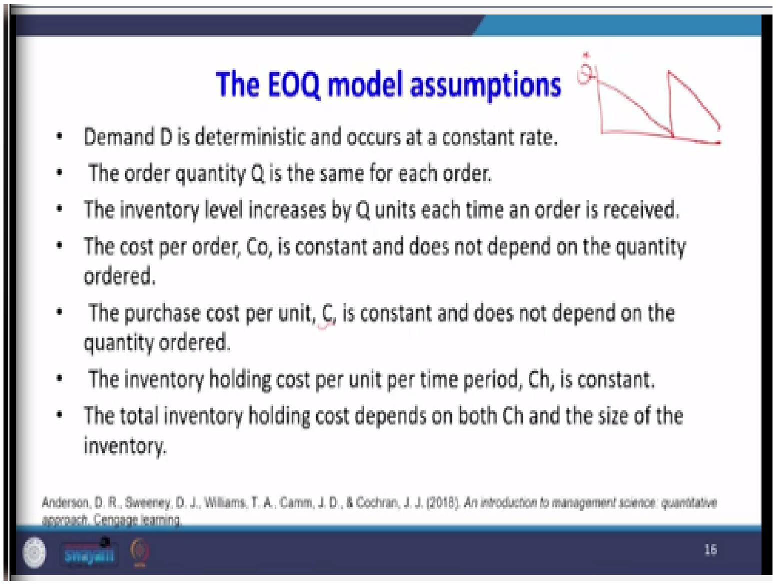
We also note that there is little risk associated with implementing the calculated order quantity of 1824. What is that risk? For example, if the cold drink company implements an order quantity of 1824 cases by using the cost estimate of 32 dollars per order cost and 25% for the holding cost. But if the actual cost for an order turns out to be 34, and the actual annual holding cost turns out to be 24, then the company experiences only a 5% increase.

I am saying 24, 24, and 34; the difference is seen between these two here: 84, it is 89, and there is only a 5-dollar increase in the total annual cost. So, it will not affect the model that we proposed, which was the robust model.



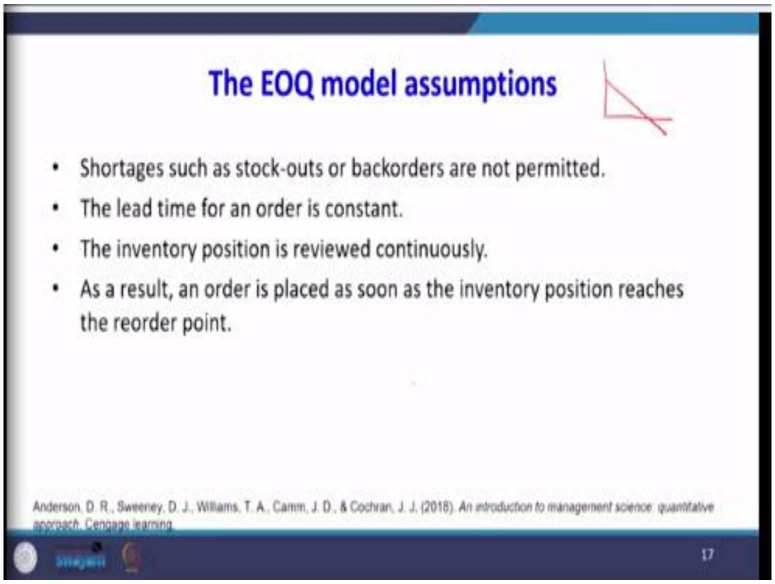
The same thing can be understood with the help of this figure; when you look at this figure, see this total holding cost is somewhat flat here because the increase in the total cost is very, very small; it is not like this. In case the total cost curve goes like this, there is a sudden increase in total cost; now, this is flat, so this also indicates that the model is more robust; that is what we are saying.

The EOQ model is insensitive to small variations or errors in the cost estimate. This insensitivity is a property of EOQ models in general, which indicates that if you have at least reasonable estimates of ordering cost and holding cost, we can expect to obtain a good approximation of the true minimum cost order quantity. So, we need not bother about whether there is any fluctuation in the ordering and holding cost.



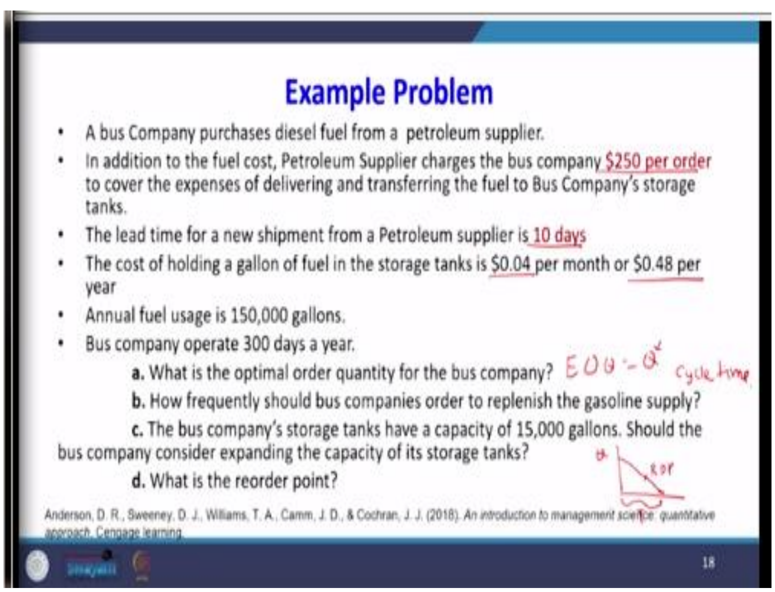
Now we can see the assumptions for economic order quantity. What are the assumptions? The demand is deterministic and occurs at a constant rate. The order quantity Q is the same for each order, what I mean is like this. The demand is constantly decreasing; the Q* says the economic order quantity is the same for each cycle. The cost per order is constant and does not depend on the quantity ordered.

The purchase cost per unit C is constant and does not depend on the quantity ordered. Because many times in practice, the demand also cannot be deterministic, and the unit cost also will not be the same because when you order more cash, there is the possibility of a discount. So, if there is a discount, we can cover that kind of problem in the coming lectures. So, the inventory holding cost per unit per time period is constant. So, the total inventory holding cost depends on both the cost of carrying inventory and the inventory size. You see that the ordering cost is not like this, the ordering cost is independent of the size of the inventory, but the holding cost is directly proportional to the size of the inventory.



Shortages such as stock-outs or backorders are not permitted. What is the stock-out if the demand goes on the negative side that is not permitted? What is the backorder? For example, the January order cannot be fulfilled in the month of February, which is the meaning of a backorder. The lead time for an order is constant; the inventory position is reviewed continuously. What is the meaning of this inventory position being reviewed continuously?

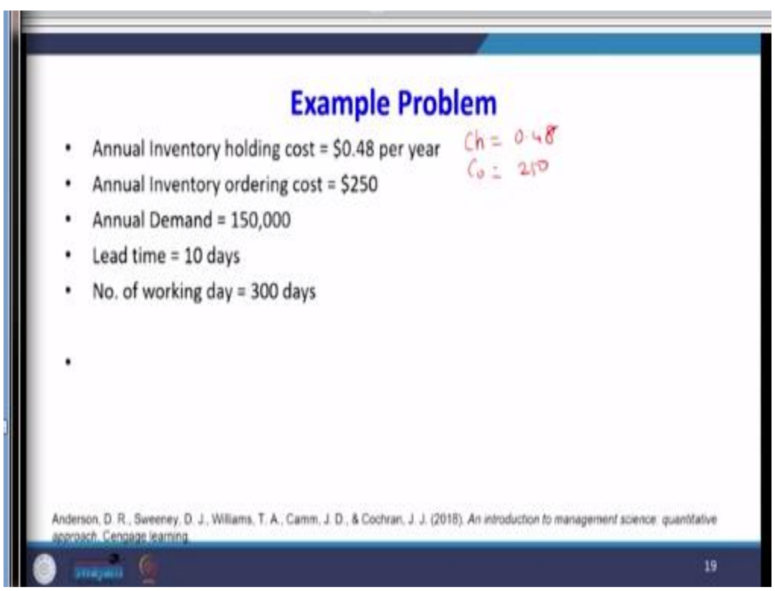
We are closely monitoring the inventory level as soon as it reaches the reorder point. Immediately, we are placing another order, which is the meaning of the inventory position, which is reviewed continuously. As a result, an order is placed as soon as the inventory position reaches the reorder point.



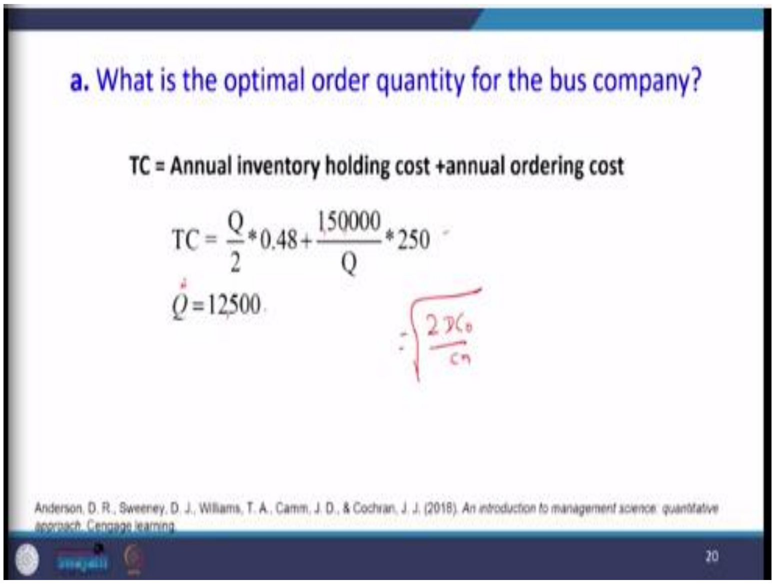
So far, we have seen about EOQ assumptions; now, I will take another problem. For that problem, I will find out what the EOQ is. How frequently should we order, and what is the reorder point? So, the problem is like this: a bus company purchases diesel fuel from a petroleum supplier. In addition to the fuel cost, the petroleum supplier charges the bus company 250 dollars per order, so we got the ordering cost to cover the expenses of delivering and transferring the fuel to the bus company’s storage tank.

The lead time for a new shipment from that petroleum supplier is 10 days; the lead time is 10 days. The cost of holding a gallon of fuel in the storage tank is 0.04 dollars per month; when you multiply by 12, it is 0.48 per year. The annual fuel usage is 150,000 gallons; the annual demand is this much. However, the company operates 300 days a year, so what questions need to be answered? What is the optimum order quantity EOQ? Otherwise, you can call it Q*.

How frequently should bus companies order to replenish the gasoline supply? So, we have to know the cycle time. The bus company storage tanks have a capacity of 15,000 gallons. Should bus companies consider expanding the capacity of storage? What is the reorder point for the next one? So, we should find out the following things: we should know what the Q* is, we should know what the reorder point is, and we should know what the cycle time T is. Now, what data is given?



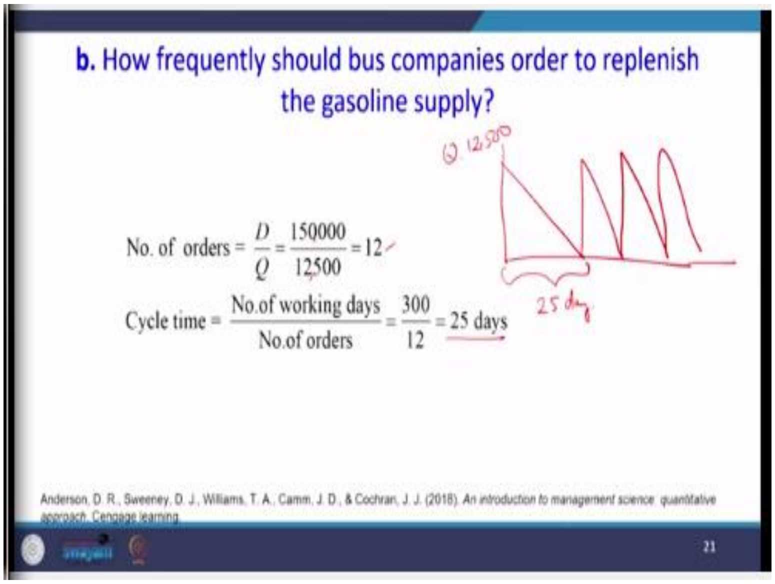
So, we know the annual inventory holding cost C_h is 0.48, the annual inventory ordering cost Co is 250 dollars, the annual demand is 150,000, the lead time is 10 days, and the number of working days is 300 days. First, we will solve this question.



What is the first question? What is the optimal order quantity for the bus company, EOQ? We know that the total cost is annual inventory holding cost plus annual ordering cost, so Q upon 2 0.48 + 150,000 upon Q multiplied by 250, so the value of Q is 12500. We can solve this with the help of Excel, so I will go back to Excel. To know the Q*, there are 2 ways: one, we can use our traditional formula; otherwise, we can solve this equation by using the concept of non-linear programming; you will get a Q*.

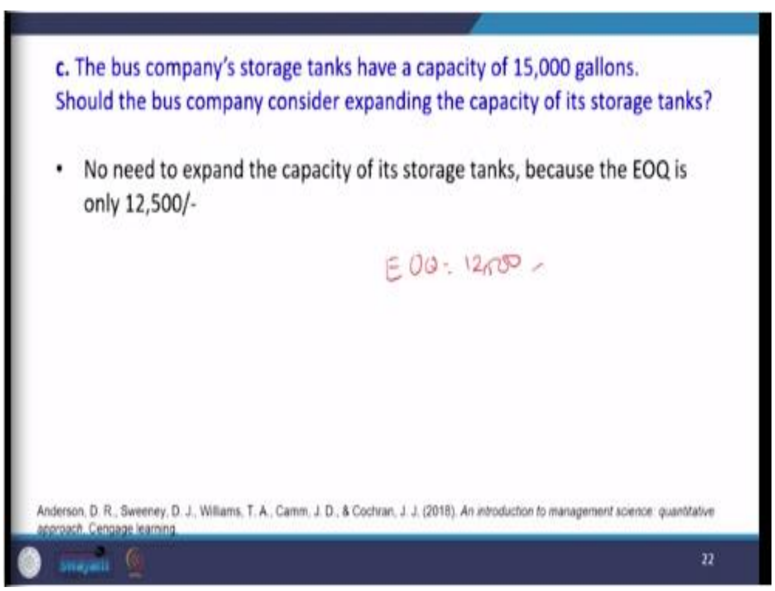
$$\text{TC} = \frac{\text{Q}}{2} * 0.48 + \frac{150000}{\text{Q}} * 250$$

$$Q = 12500$$

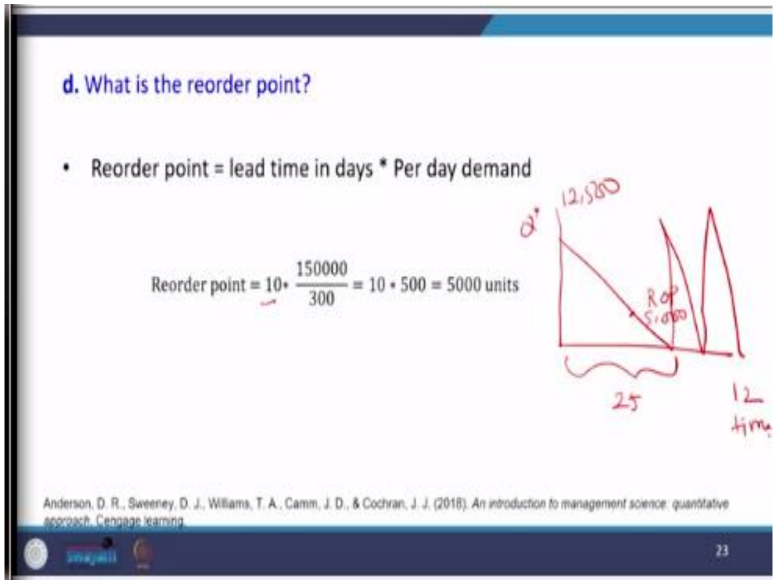


Now, how frequently should bus companies order to replenish the gasoline supply? To answer this, first, you should know the number of orders; what is the number of orders? 150,000s upon 12500, it is 12. So, the number of orders per year is 12. So, what is the cycle time? Number of working days upon a number of orders. So, 300 is the number of working days per year for 12 orders, so every 25 days, that company should place the order. What we have found is like this.

So, we know what the Q* is. What is the Q*? Q* is 12500. So, what is the cycle time? The cycle term is 25 days. What is the number of orders? 12. So, like 2, 3, 4, so up to 12, the number of cycles is 12, so up to 12 cycles, we have to do it. So, every 25 days, you must place the order.



The next question is whether the bus company storage tanks have a capacity of 15,000 gallons. Should the bus company consider expanding the capacity of a storage tank? It is not required because our EOQ itself only 12,500, and the tank capacity is 15,000, so every time we are going to buy, we are going to store only 12,500 gallons, so there is no requirement for expanding the capacity of the storage tank.



So, what is the reorder point? The reorder point is lead time in days multiplied by per day demand. We know the lead time is 10 days, and per day demand is annual demand driven by the number of working days, which is 500, so 10 multiplied by 500 is 5000. So, what have we found? We found the Q*, the cycle time, and the reorder point.

So, our Q* is 12500; when the demand reaches, say, 5000, you place another order; how many orders? The number of orders is 12, so this is 25 days; like this, there are up to 12 times. Dear students, in this lecture, I explained how to use the concept of non-linear programming to find economic order quantity. Then I answered what is a when to order?

Then we found the reorder point, so when the inventory reaches that point, we have to place another order; that is the answer for our when to order. Then, we have found what the cycle time is. What is the meaning of cycle time? How frequently do we have to make the order? That we have found in this lecture, thank you very much.