Decision Making With Spreadsheet
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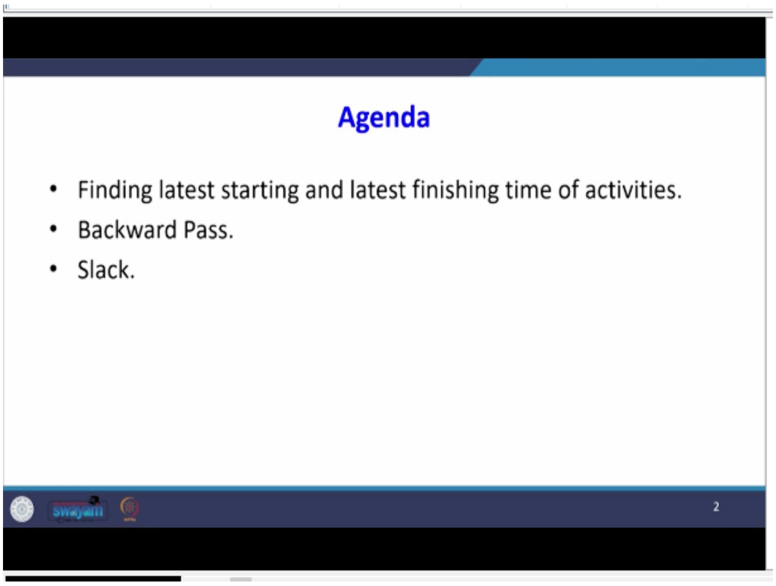
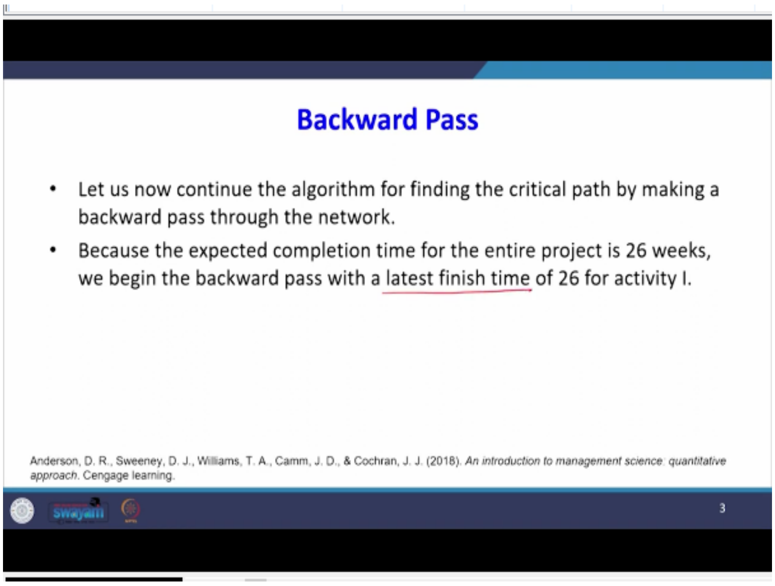
Lecture-30
Project Scheduling: PERT/CPM-II

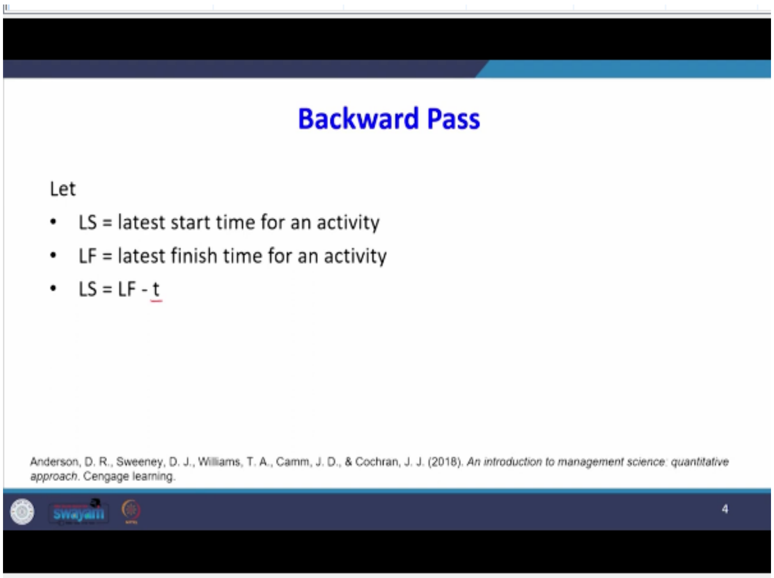
Dear students in the previous class, I have explained about forward passes for the purpose of forward passes to find out the earliest starting time and earliest finishing time. In this lecture, I am going to explain how to find out the latest finishing time and latest starting time. For that the algorithm which I am going to use is called backward pass.



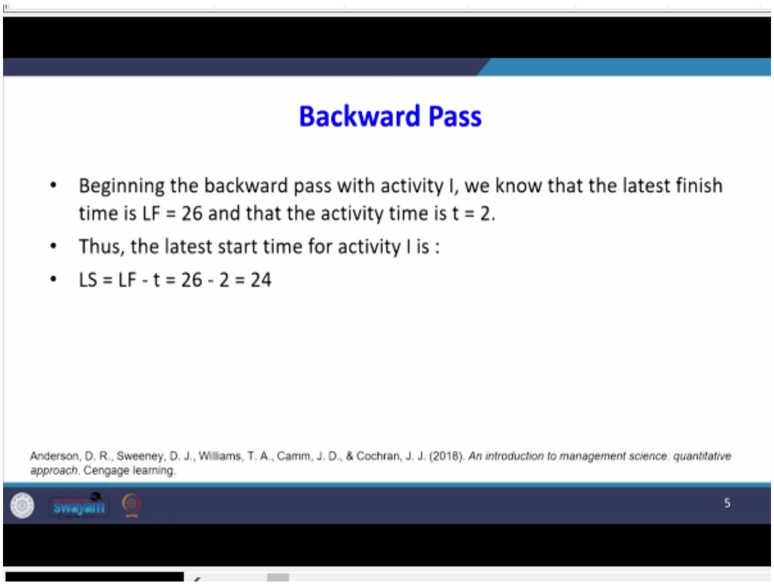
So, the agenda for this lecture is finding latest starting and latest finishing time of activities by using backward pass. Then, I am going to explain a concept called Slack. The meaning of Slack is that many days a project can be delayed, which will not affect our total project completion time. That is the meaning of slack.



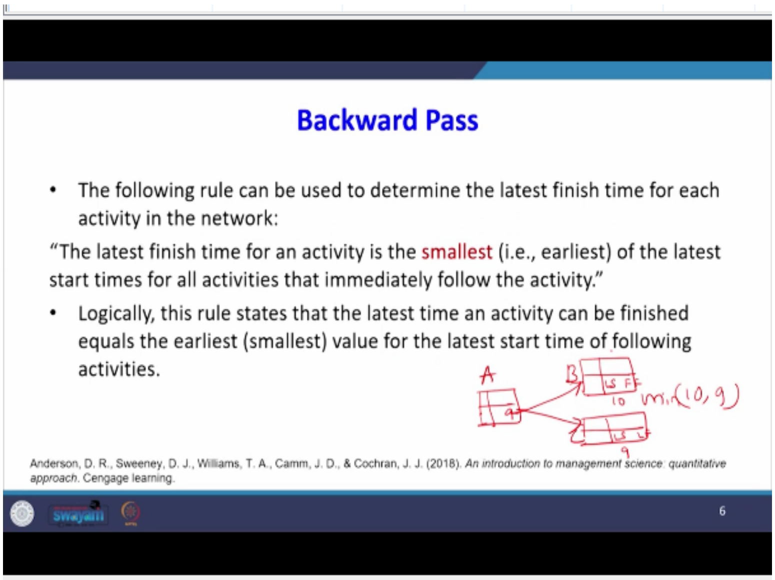
So, what is the backward pass? Let us now continue the algorithm for finding the critical path by making a backward pass through the network. Because the expected completion time for the entire project is 26 weeks, we begin the backward pass with the latest finishing time 26 for the activity I. I is the last activity, so the latest finishing time is 26.



So, LS, let LS be the latest starting time for an activity, and LF is the latest finishing time for an activity. So, then the latest starting time is the latest finishing time minus the duration of that activity. That is the t; t is the duration of that activity.



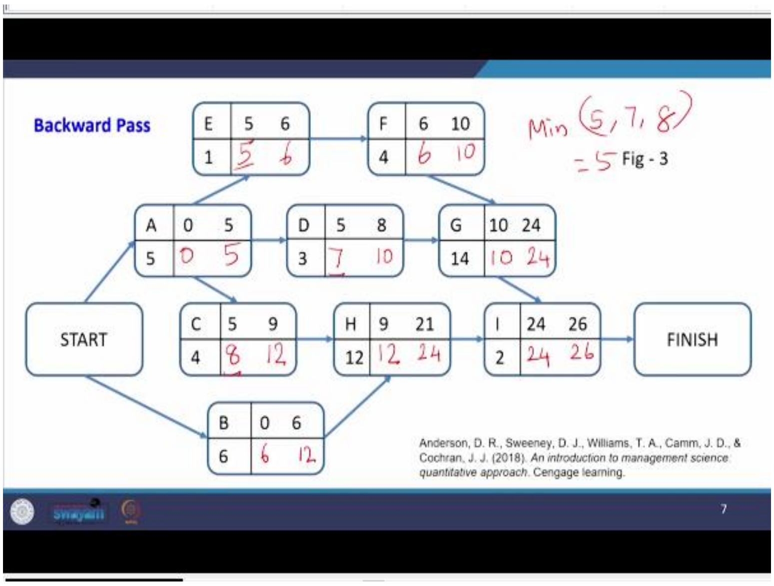
Beginning the backward pass with activity I, we know that the latest finish time is 26, LF is 26, and that activity time is the duration of that activity is two units. So, the latest start time for activity I is LS equal to the latest finishing time, how much 26 minus the duration of that activity is 2. So, 24 is our latest starting time.



The following rule can be used to determine the latest finish time for each activity on the network. What is that? The latest finish time for an activity is the smallest of that is the earliest time earliest of the latest start time for all activities that immediately follow the activity. What is the meaning of this? Suppose there are 2 activities that say like this; suppose there is one preceding activity. So, what we have to do here is know this is the latest starting time and finishing time. This is the latest starting time and latest finishing time.

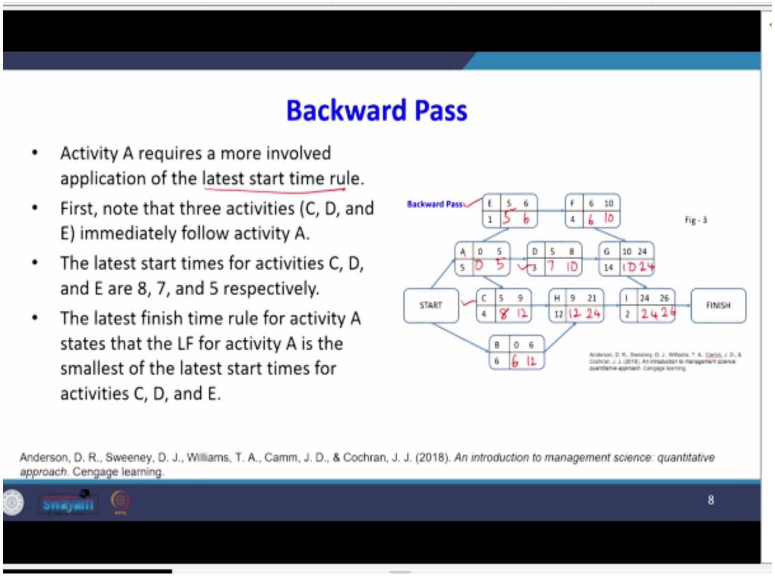
So, for this activity, the latest finishing time of, say, assume that this is A, this is B, this is C, it goes like this, the arrow is like this. You see, for activity B, for example, the latest start time is, say, 10; for activity C, the latest start time is, say, 9; and then the latest finish time for activity A is a minimum of 10, 9, and a minimum of 10. 9. What is that? So, 9. So, 9 will be the latest finish time of the preceding activity.

That is the meaning of this sentence, what is that? The latest finish time for an activity is the smallest time for the latest start time for activities that immediately follow the activity. So, what are the 2 activities which are following B and C? So, if you want to know the latest finish time that will be the smallest of the latest start time of the succeeding activities. Logically this rule states that the latest time in activity can be finished equal to the earliest value of the latest start time of following activities.



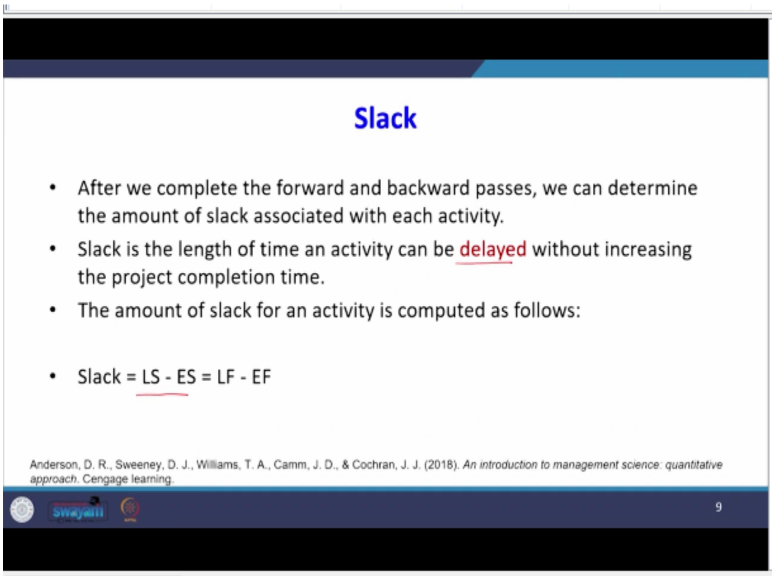
Now I am going to apply for this backward pass. So, what is this 26? So, this is my latest finishing time for I. So, 26 - 2 is 24. Then it is 24 here, so 24 - 12 is 12. So, when you come here for activity G, the latest finish time is 24, 24 - 14, say 10. So, for activity F, the latest finishing time is 10, 10 – 4, it is 6. Now for activity D the latest finish time is 10, so 10 - 3 7. So, for activity F, the latest finishing time is 6, and 6 – 1 is 5. That is the latest start time.

For activity B, the latest finishing time is 12; 12 - 6, it is 6; for activity C, the latest finishing time is 12; 12 - 4 is 8, your earliest starting time. Now you come to activity A. In activity A, there are 3 succeeding activities. What are they? A, D, and C. Now you see here; this is the earliest to start time 5, this is 7, this is 8. So, I am writing what is the minimum of 5, 7, 8? What is the minimum? So, 5 is minimum. So, the 5 will be the latest finishing time for activity A. So, the duration is 5. So, 5 - 5 is 0. So, now we have the earliest start time, earliest finish time the latest start time, and the latest finish time.

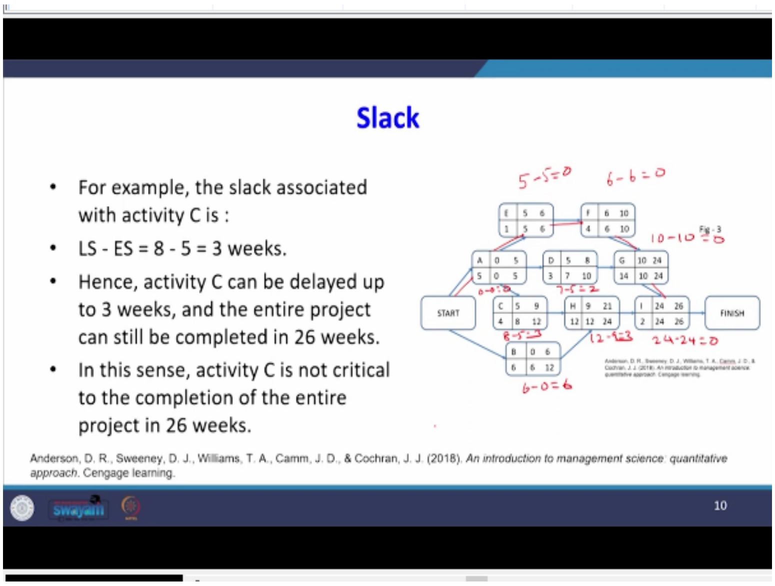


So, now I will explain step by step. So, activity A requires a more involved application of the latest start time rule that I have already explained to you. First, note that the 3 activities are C, D, and E. So, I will reproduce the value here, so 26, 24, and 24, and I want to write again. Please check this one; just reproduce the cell. Now, see that activity A requires a more involved application of the latest start time rule.

First, note that 3 activities are C, D, E. So, the C, D, E, and their latest start times are 8, 7, and 5, respectively. So, the latest finish time rule for activity A states that the latest finish time for activity A is the smallest of the latest start times for activities C, D, and E. So, what is the smallest among 5, 7, and 8? So, 5 is the smallest, so we are writing 5. So, 5 - 5 it is 0.



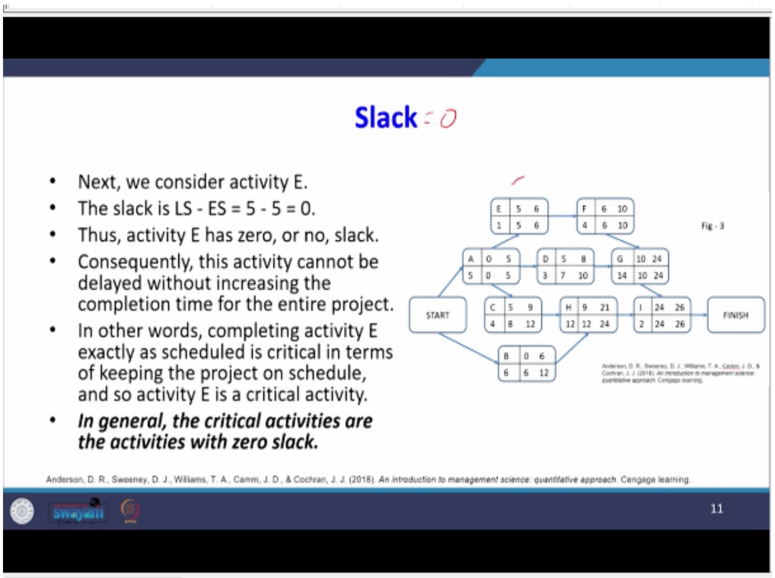
Now, I am going to explain an important concept called Slack. After we complete the forward and backward process, we can determine the amount of slack associated with each activity. What is the slack? Slack is the length of time an activity can be delayed without increasing the project completion time. So, the amount of slack for an activity is computed as the difference between the latest start time minus the earliest start time or the latest finish time minus the earliest finish time.



Now, for each activity, for example, activity E, the latest start time is five minus the earliest start time of 5, which is 0. For activity F, it is 6 - 6 = 0. Activity G, 10 - 10 = 0. Activity I 24 – 24 = 0. So, activity H 12 – 9 = 3. For activity D 7 - 5 = 2. So, activity A 0 - 0 = 0. Here 8 - 5 = 3. Here 6 - 0 = 6. Now I have found the slack for all activities. For example, in activity A, there is a 0 slack. That means this activity cannot be delayed.

Similarly, in activity E, I am connecting all the activities which are having 0 slack. So, here is 0 slack. So, activity F also has 0 slack, activity G also has 0 slack, and activity I also has 0 slack. So, if you connect all the activities which are having 0 slack, that path is nothing but a critical path. Now, for example, say activity C. So, in activity C, the latest start time is 8, and the earliest start time is 5.

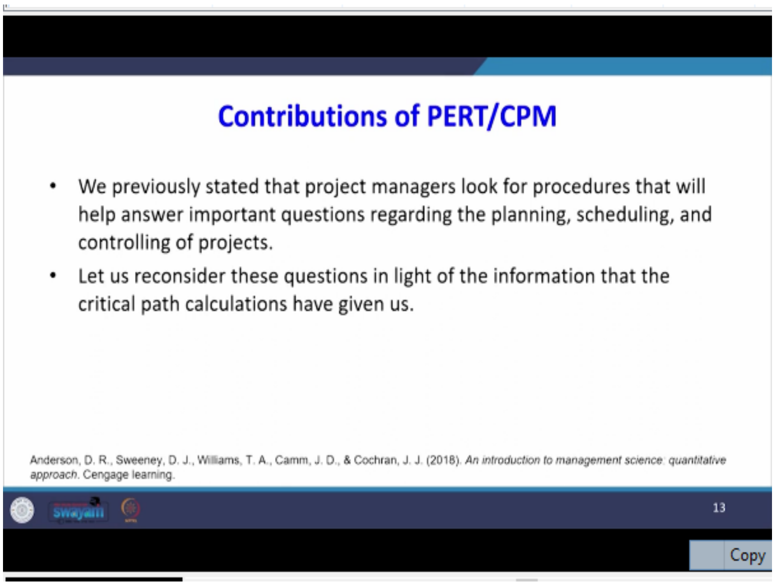
So, the difference is 3 weeks. What is the meaning of the 3 weeks? So, the activity C can be delayed up to 3 weeks, and the entire project can still be completed in 26 weeks. That is the meaning of the slack. You can delay that much amount but that will not affect our total project completion time. So, in this case, activity C is not critical to the completion of the entire project in 26 weeks. So, wherever there is a positive slack, those activities are called noncritical activities.



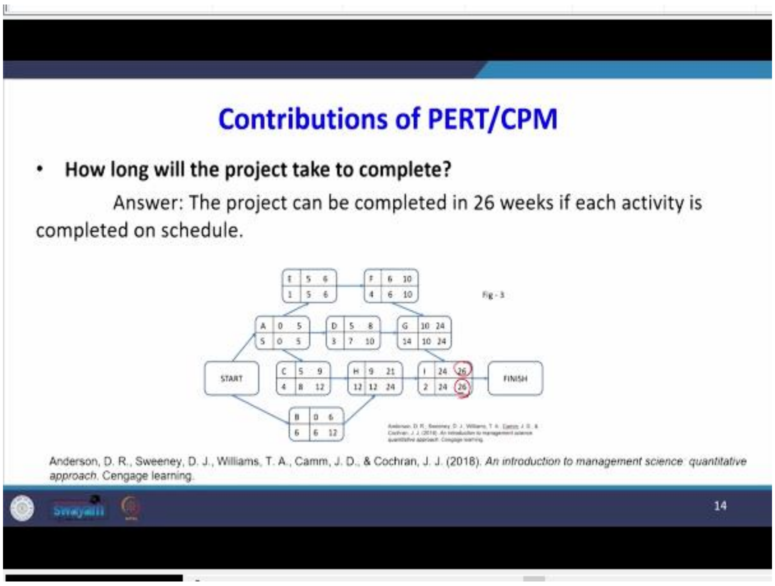
So, next, we consider activity E; here, the latest start time is 5, and the earliest start time is also 5. So, the slack is 0. So, what is the meaning of this 0? So, activity E has 0 or no slack. Consequently, this activity cannot be delayed without increasing the completion time for the entire project. In other words, completing activity E exactly as scheduled is critical in terms of keeping the project on schedule so activity E is a critical activity wherever the slack is 0 those corresponding activities critical activities. In general, the critical activities are the activities with 0 slack.

Activity	ES	LS	EF	LF	Slack (LS – ES)	Critical Path?
A	0	0	5	5	0	YES
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	YES
F	6	6	10	10	0	YES
G	10	10	24	24	0	YES
H	9	12	21	24	3	
I	24	24	26	26	0	YES

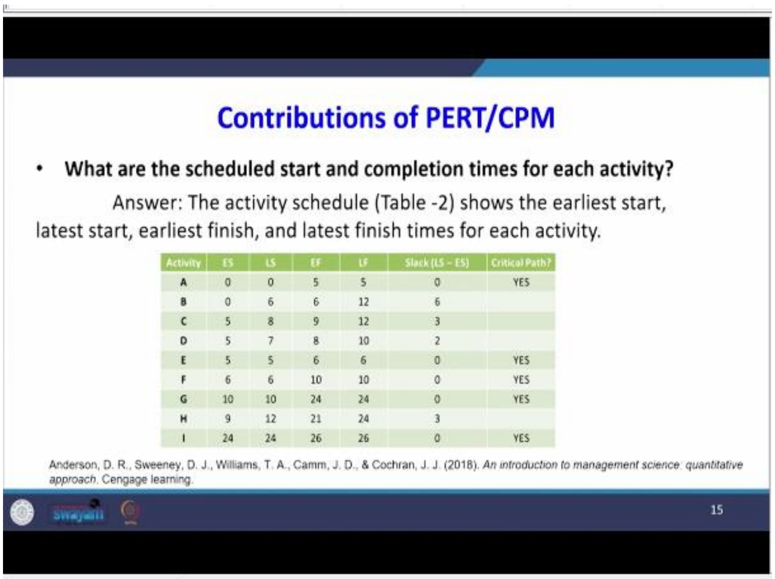
So, with the help of the previous network problem and by using forward and backward pass, I found the earliest start time and latest start time, earliest finish time, and latest finish time. So, what is the slack? Latest start time and earliest start time, 0, 6, 3, 2, 0, 0, 0, 0, 3, 0. So, wherever the slack is 0. So, I am writing that path is the critical path. What is the critical path? A is there, the next one is E, the next one is F, the next one is G, and the next one is I. So, A, E, F, G, I is our critical path. So, see this A E F G I, this is our critical path.



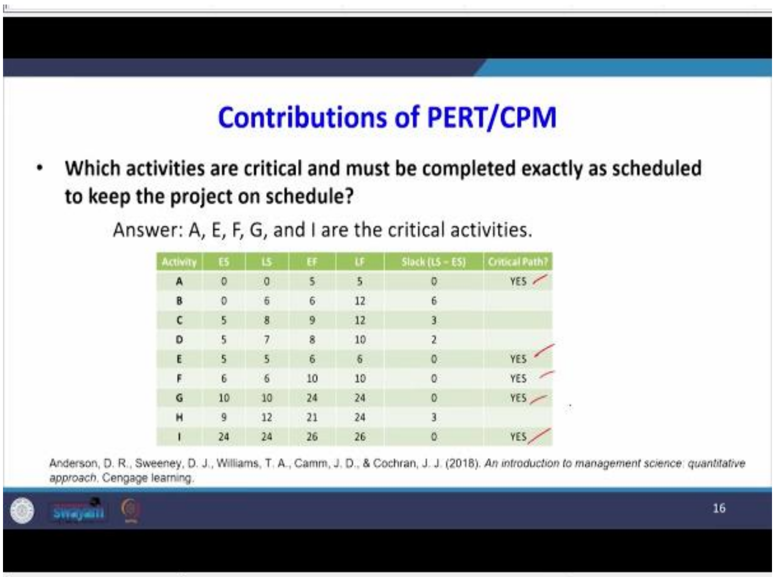
So, what is the contribution of this PERT and CPM for the managers? We previously stated that the project managers look for procedures that will help answer important questions regarding the planning, scheduling, and controlling of the projects. Let us consider these questions in light of the information the critical path calculations gave us.



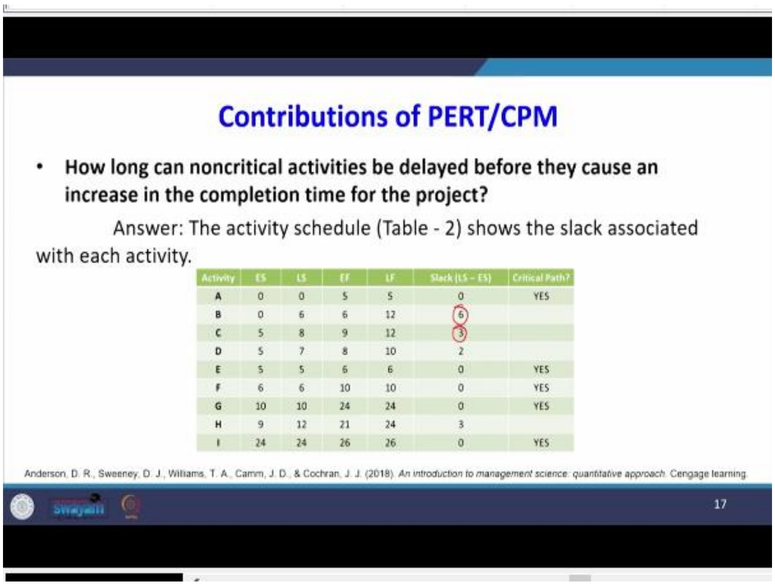
The first question that the manager is wanted to know how long will the project take to complete. It is an obvious question. So, the project can be completed in 26 weeks because you see that this 26 week, here also 26 weeks. If each activity is completed on schedule that is the answer to this question.



The next question manager wants to know what are the scheduled start and completion times for each activity. So, this table shows the earliest start, latest start, earliest to finish, and latest finish time for each activity.

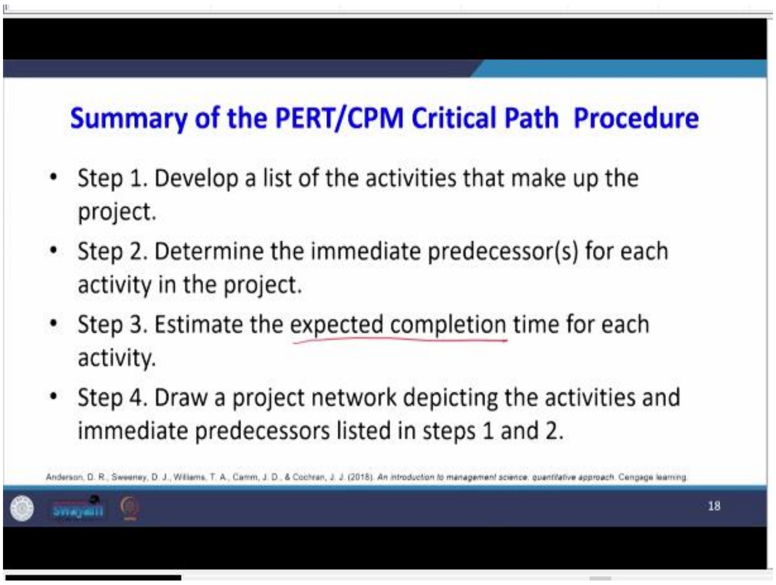


The next question the manager wants to know is which activities are critical and must be completed exactly as scheduled to keep the project on schedule, so, what we have to look at from this table. We have to look at wherever there is a 0 slack. So, the corresponding activity activities are called critical activities that should not be delayed at all. So, what answer do we have A, E, F, G, I?



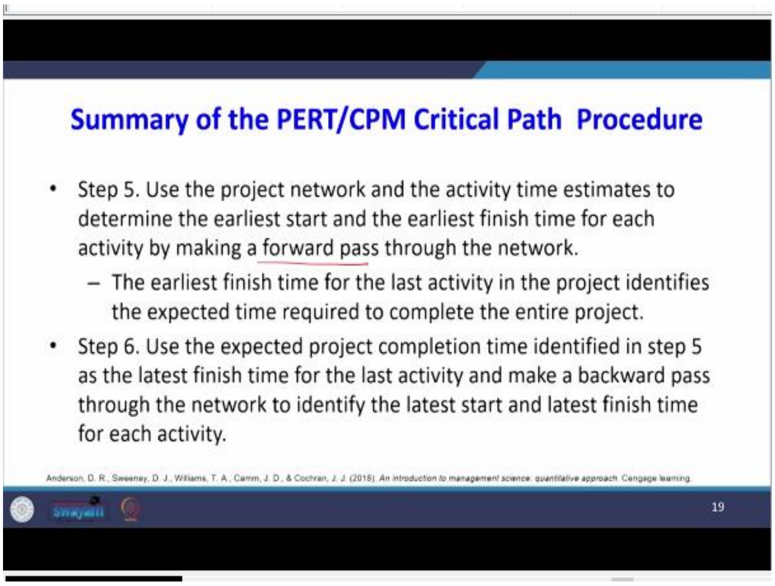
The other question the manager may want to know is how long noncritical activities can be delayed before they cause an increase in the completion time for the project. So, here we have to look at the column where the slack is there. So, for example, activity B can be delayed 6 days, and activity C can be delayed 3 days; you should be very careful at a time when you can delay only one activity; you cannot simultaneously delay activities B and C.

So, the mean meaning of slack is by keeping all other duration of activities, the schedule of the activity is the same, and you can delay only the activities of, say, B; what is the meaning of that? The other activities are not delayed; only one activity, delayed at a time is a meaning.



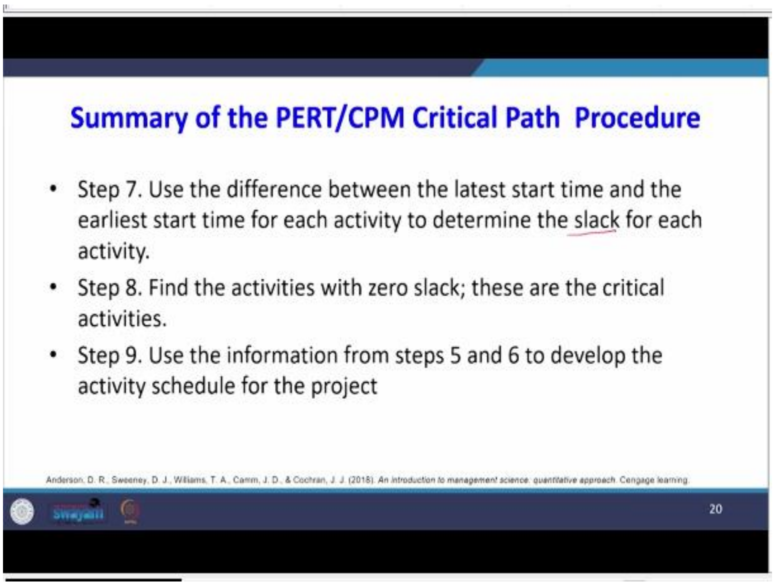
Now, what is the summary of the PERT/CPM? Critical path procedures: step 1: develop a list of activities that make up the project. So, we have to identify what are the activities that form our project network. Step 2: determine the immediate predecessors for each activity in the project. So, we have to find out what their predecessors are. Step 3: estimate the expected completion time for each activity.

Sometimes, this expected completion time may be deterministic. That means only one value may be given; sometimes, there may be 3 values. What are the 3 values? Optimistic, pessimistic, and normal time. Step 4: draw the project network, debating the activities and immediate predecessors listed in steps 1 and 2. Next, you have to draw the activity.

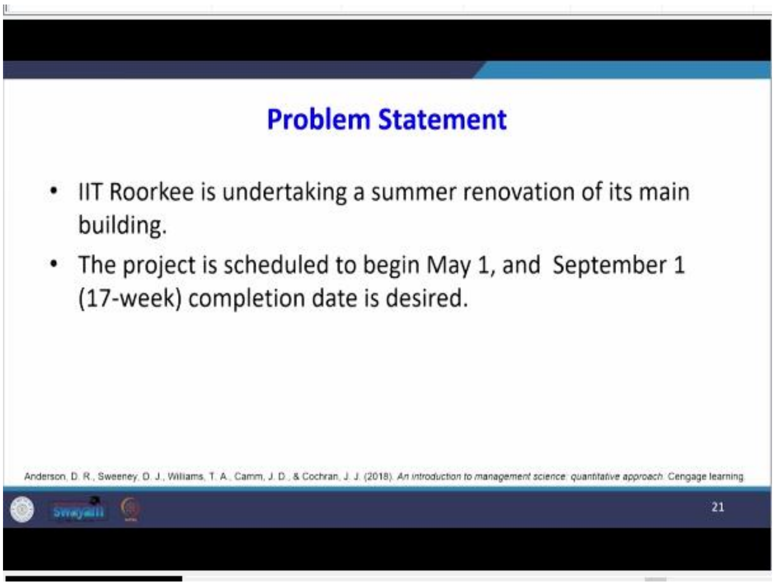


Step 5: use the project network and the activity time estimates to determine the earliest start and the earliest finish time for each activity by making a forward pass through the network. The earliest finish time for the last activity in the project identifies the expected time required to complete the entire project. We have seen that values are 26, for example. Step 6: use the expected project completion time identified in Step 5 as the latest finish time for the last activity.

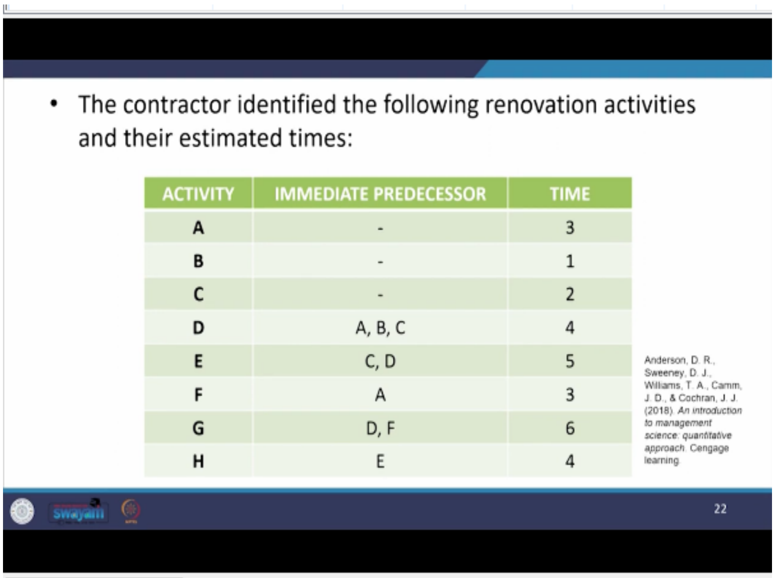
Then make a backward pass through the network to identify each activity's latest start and finish time. So, by fixing the latest finish time for the last activity from there we have to use the backward pass to find out the latest start time and latest finish time for all the activities.



Step 7: use the difference between the latest start time and the earliest start time for each activity to determine the slack for each activity. What is step 8? Find the activities with 0 slack. So, these are the critical activities. Step 9: use the information from steps 5 and 6 to develop an activity schedule for the project. So, we might have received the value by using forward pass and backward pass. By using that information, we can prepare the schedule for each activity. What we can prepare is the earliest start time, earliest finish time, latest start time, and latest finish time.

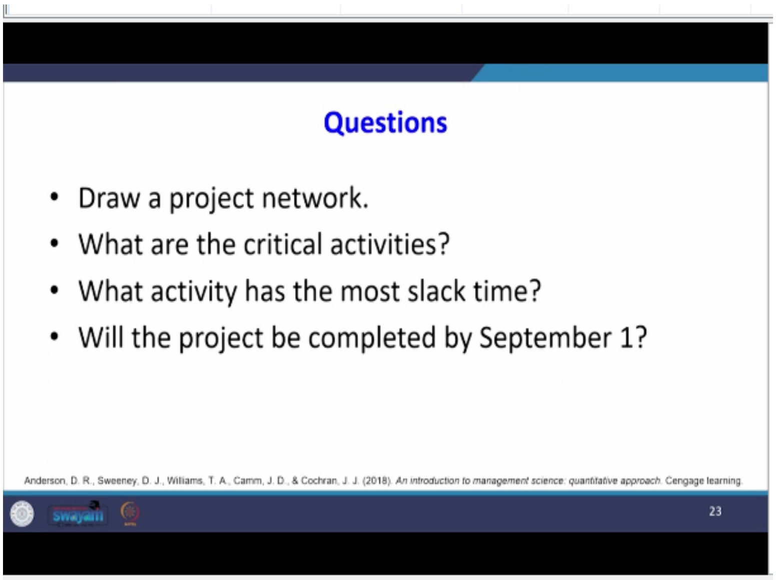


Now I have taken another problem; I am going to follow these steps in this problem, and then I am going to explain the forward and backward pass. So, IIT Roorkee is undertaking a summer renovation for its main building. The project is scheduled to begin May 1, and the September 1 completion date is desired. So, how many weeks are there between May 1, May, August, and September? It is 17 weeks. So, the renovation has to be finished within 17 weeks.

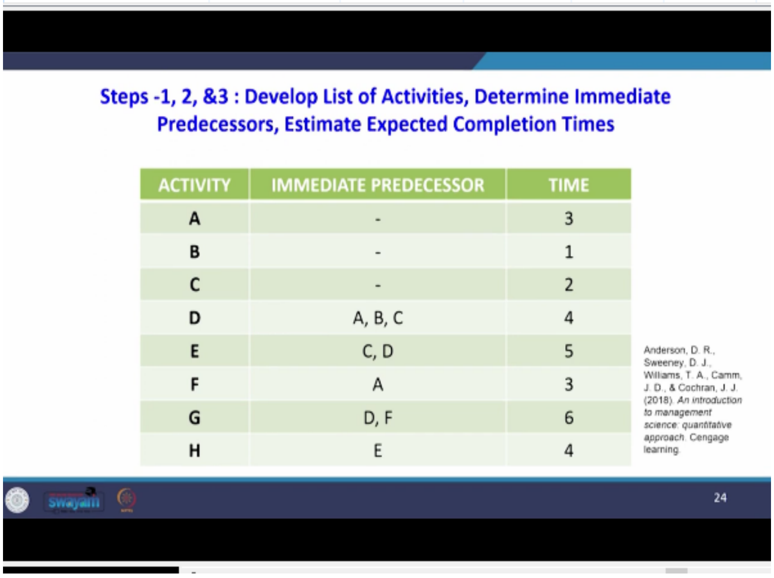


ACTIVITY	IMMEDIATE PREDECESSOR	TIME
A	-	3
B	-	1
C	-	2
D	A, B, C	4
E	C, D	5
F	A	3
G	D, F	6
H	E	4

The contractor identified the following renovation activities and their estimated times. So, there are 3 activities, A and B, C. There are no predecessors. For D, there is a predecessor A, B, and C; for E, there is a predecessor C and D. For F, there is a predecessor A; for G, there is a predecessor D and F; for H, there is a predecessor E. And the time for each activity is given.



So, as per the steps, the first step is the questions which you have to answer draw a project network, what are the critical activities, what activity has the most slack time, and will the project be completed by September 1? These are the questions.



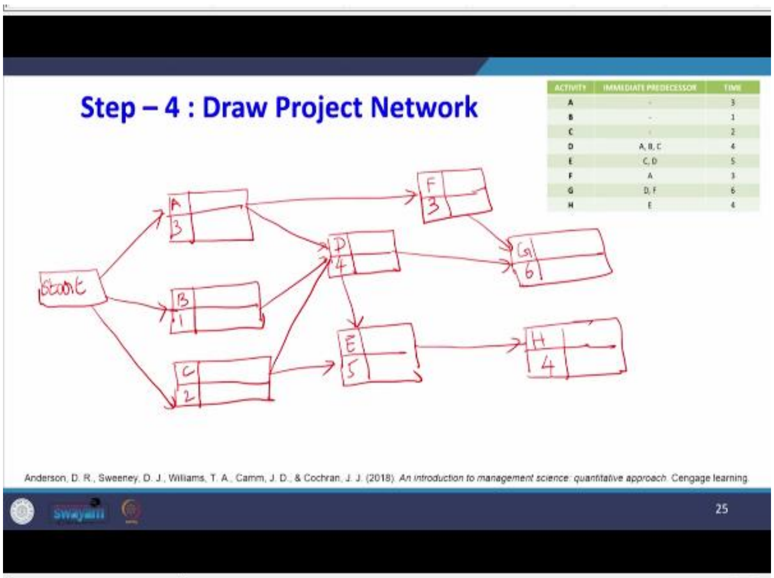
Steps -1, 2, &3 : Develop List of Activities, Determine Immediate Predecessors, Estimate Expected Completion Times

ACTIVITY	IMMEDIATE PREDECESSOR	TIME
A	-	3
B	-	1
C	-	2
D	A, B, C	4
E	C, D	5
F	A	3
G	D, F	6
H	E	4

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

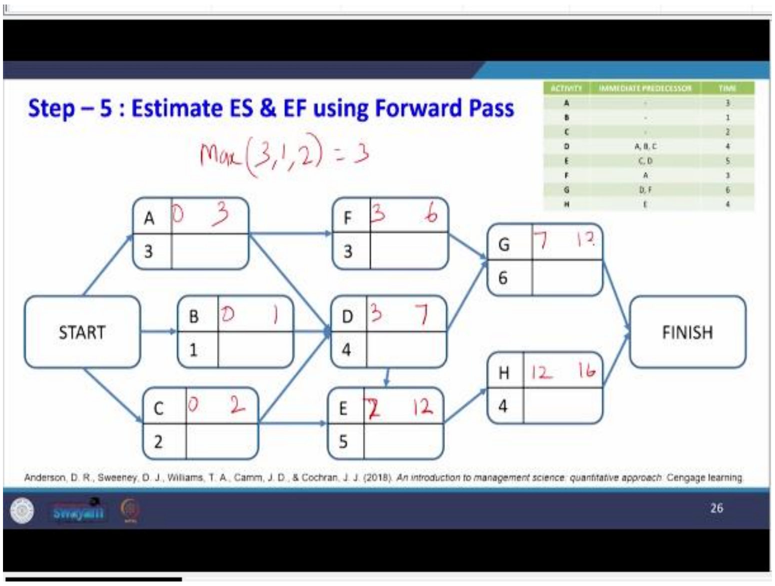
24

First we have completed steps 1, step 2, and 3. They develop a list of activities, determine immediate predecessors, and estimate the expected completion time that we have completed. Now, we will go to step 4.



Step 4 is drawing the project network. So, we start from here; there are 3 activities, and there are no predecessors. So, A, B, and C, duration 3, 1, 2, this is a start. Then for activity D, there are 3 predecessors. A, B, C. So, I am going to draw the activity D here. So. the predecessor for activity D is A, B, C. So, A, B, and C. Activity E, C, and D are predecessors.

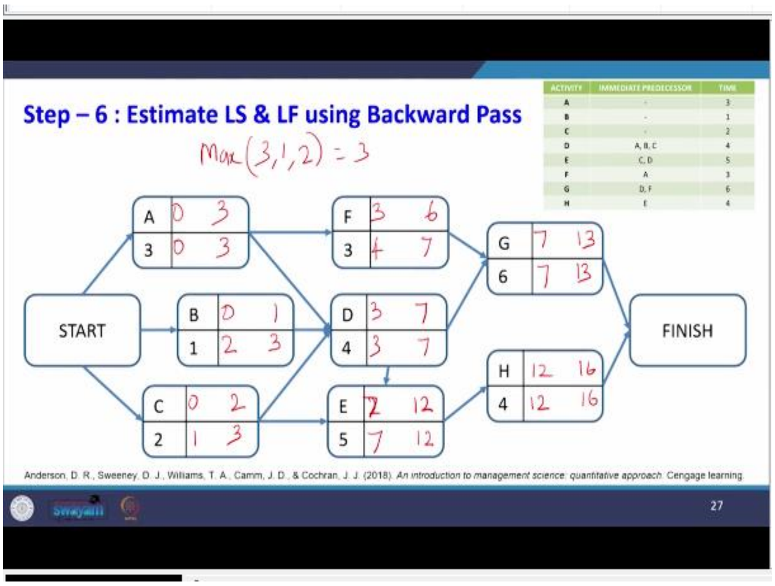
So, here I am writing activity E. The predecessors are C and D. Then, for activity F, I am writing that the predecessor is A. So, here I am writing, so the predecessor is A, then for activity G, the predecessor is D and F. So, B and F, for activity H, the predecessor is E. So, we have drawn the project network. Now, we will go to step 5.



Step 5 estimates the earliest start time and earliest finish time using a forward pass. I am going to use the forward pass. So, wherever there are no predecessors, the earliest start time is 0; here, it is 0; here, it is 0; here, it is 0. So, the earliest finish time is the earliest start time plus the duration of that activity 0 + 3 = 3, 0 + 1 = 0 + 2 = 0. Now, for activity F, the earliest start time is equal to the earliest finish time of the preceding activity, which is A. So, it is 3, so 3 is the earliest start time.

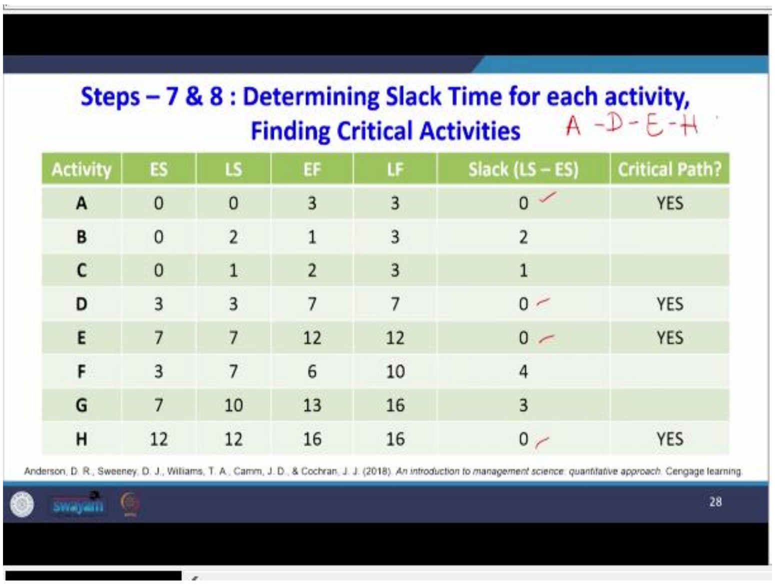
So, this 3, this 3, so 3 + 3 is 6. Now you see for activity D. For activity D, there are 3 earliest finish times. What are they? For activity A, it is 3; for activity B, it is 1; for activity C, this is 2. So, we have to find out the maximum of 3, 1, and 2 in the forward pass; the maximum is 3. So, 3 will be the earliest start time for activity D. So, 3 + 4 is 7. So. here it is a 2 + 5. So, now for activity E, there are 2 predecessors. What are they?

D and C, which is the largest value of their earliest finish time. So, between 2 and 7, 7 is the largest one, so 7 will be the earliest start time. So, this will be 12. Now 12, it is 16, now it is 7. Now you see why it is 7: for activity G, there are 2 predecessors, F and D. The earliest finish time for F is 6, and for D, it is 7. So, the highest one is 7. So, it is 13.



Now, I am going to estimate the latest start time and the latest finish time using the backward pass. So, what do I have to do? I have to start at 16. So, 16 is my latest finish time. So, 16 - 4 is 12, here it is 13, 13 - 6 is 7. So, here it is 12, 12 - 5 is 7. So, for activity F it is 7, so 7 - 3 is 4. For activity D, it is 7, 7 – 4, it is 3. Now look at the activity A. There are two latest start times. What are they? 4 and 3, the largest value is 4, the smallest value is between 4 and 3, and the smallest value is 3. So, 3 should be written, so 3 - 0 0.

Now this is 3; for activity B 3 - 1, it is 2. Now, for activity C, there are 2 succeeding activities, D and E. There are two latest start times, 3 and 7, so which is a minimum? 3 is a minimum, so 3 – 2 it is 1. So, now I have completed the latest start time and the latest finish time.



Now, steps 7 and 8 are determining the slack time for each activity and finding critical activity. So, we know what is slack; slack is the latest start time minus the earliest start time. So, here, activity A has 0 slack, and activity D has 0 slack. So, activity E has 0 slack, and activity H has 0 slack. Now, wherever the slack is 0, those activities are called critical activities. So, what is a critical path we can take?

The critical path is A, D, E, and H. So when you go back to this route A, D, E, H. So, this E then H. So, this route A, D, E, H, this route is the critical path. So, the duration is 16. So, in this class, I explained how to find the latest start time and finish time using the backward pass. Then I explained the meaning of slack for each activity, I developed a project schedule, and then I found the critical path. So, I have taken one sample problem, and I have explained all the concepts with the help of that sample problem; thank you.