Decision Making With Spreadsheet

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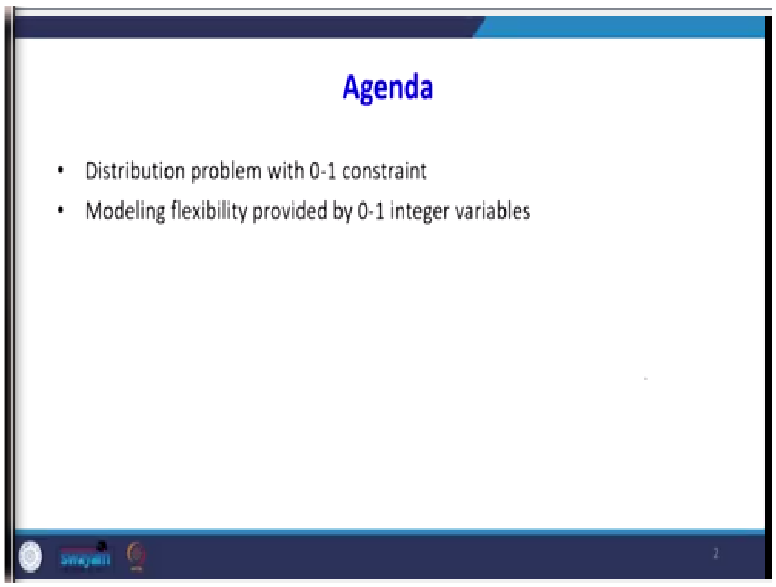
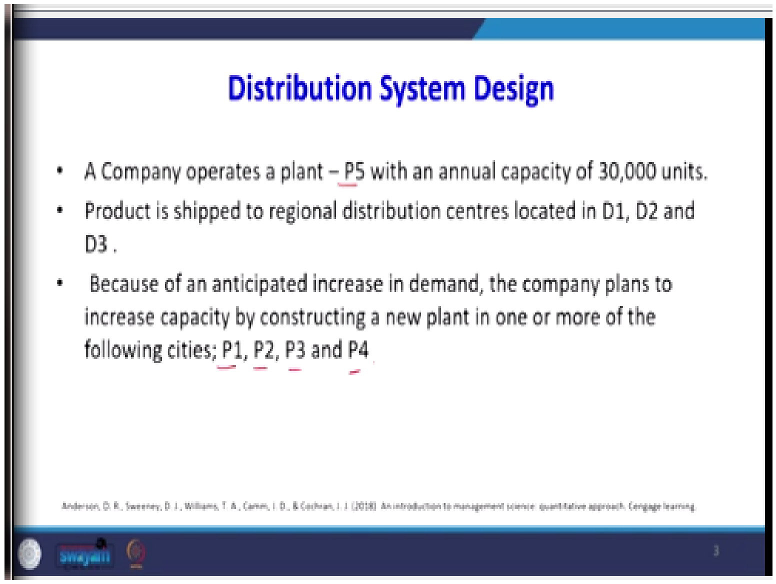
Lecture-23
Integer Programming Distribution Problem

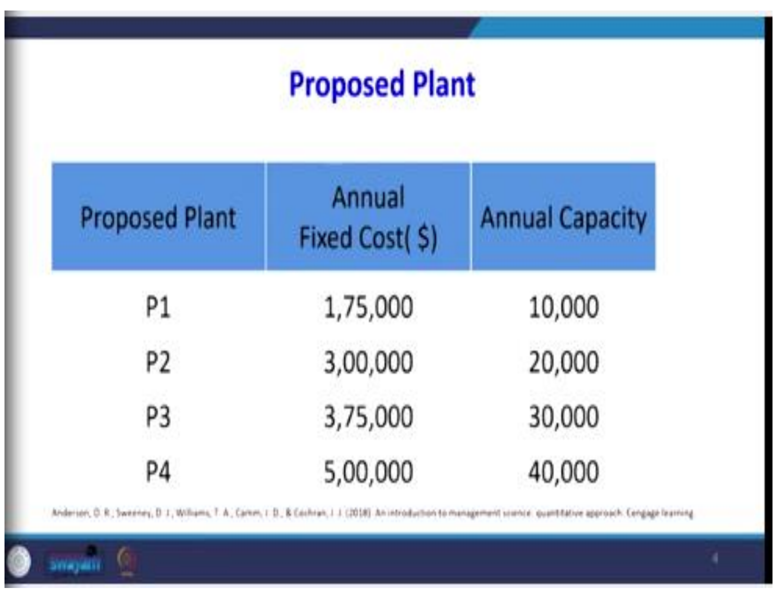
Dear students, in this lecture I will continue with 0-1 integer programming. What have I done in this lecture? I have taken one problem on distribution, which is a standard application supply chain management, where I have used 0-1 constraint for formulating the problem. The second topic that I am going to discuss is some of the modeling flexibility provided by 0-1 integer variables.





Distribution system design: one of the important tasks of supply chain managers is to decide how to design the distribution system. So, I have taken 1 sample problem. This problem is taken from the book by Anderson et al. A company that operates a plant already there is a company they have one plan to say the plant name is P5, with an annual capacity of 30,000 units. The product is shipped to regional distribution centers located in D1, D2, and D3; there are 3 distribution centers and 1 plant.

Because of this anticipated increase in demand the company plans to increase the capacity by constructing a new plant in one or more of the following cities. So, the new cities that they are considering for constructing the new plant are P1, P2, P3, and P4.

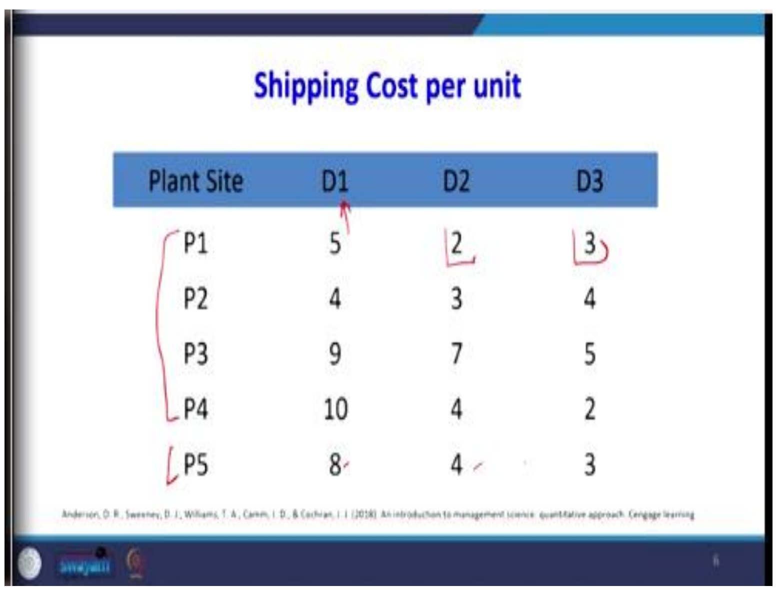


For the proposed plant, there are 4 proposed plants already 1 plant is there, P5, but they are proposing 4 plants, P1, P2, P3, and P4; the annual fixed cost for this proposed plant is given, and the annual capacity is also given.

Distribution Centre

Distribution Centre	Annual Demand
D1	30,000
D2	20,000
D3	20,000

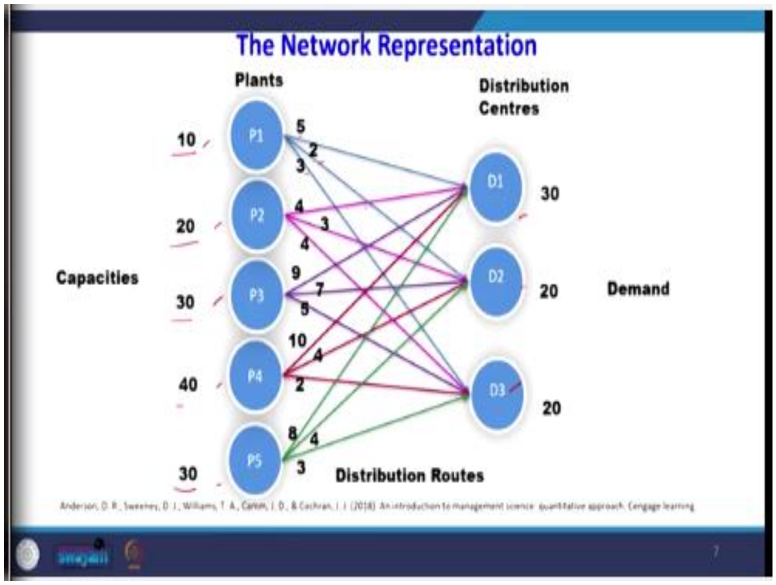
There are 3 distribution centers, D1, D2, and D3, and their annual demand is also given.



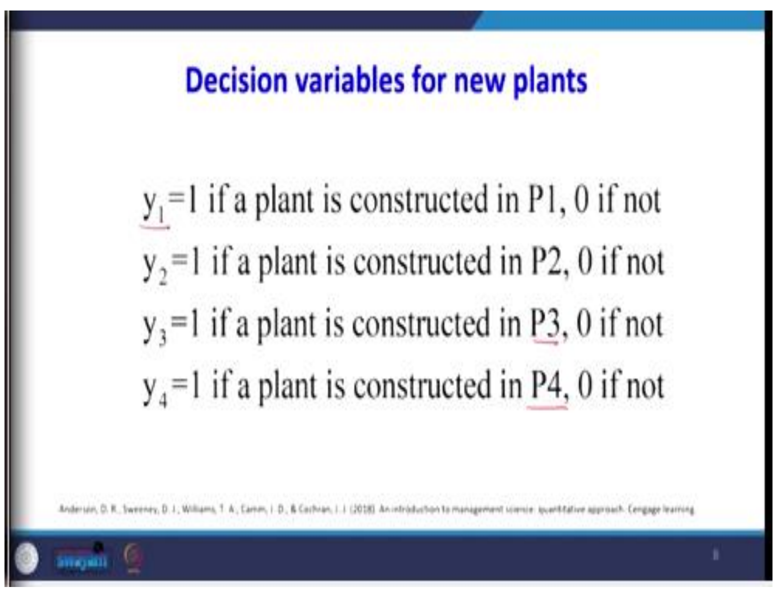
Shipping Cost per unit

Plant Site	D1	D2	D3
P1	5	2	3
P2	4	3	4
P3	9	7	5
P4	10	4	2
P5	8	4	3

Apart from this plant capacity and the demand, the shipping cost from all 4 new plants and the existing plant is also given, so transportation cost is given. So, what is the meaning? In case you are going to construct a plant at P1, if you are shipping the product from P1 to D1, the transportation cost is 5. Similarly, 2, 3, for example, P5 to D1 is 8, P5 to D2 is 4, and P5 to D3 is 3. So, this table represents transportation costs.

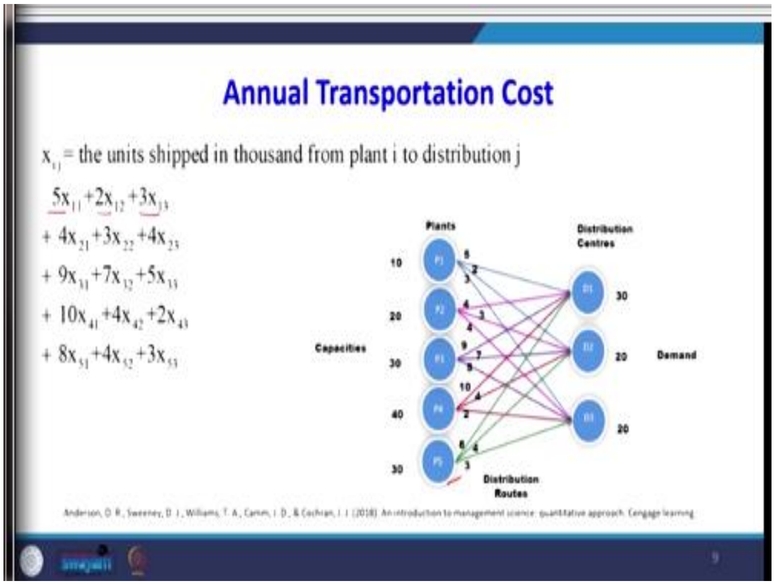


So, now the problem which is there in the previous slide I brought into the network form: what does this network form say? There are 4 proposed plans P1, P2, P3, and P4. This plant already exists; there are 3 demand points D1, D2, and D3. For each proposed plant, there is a capacity, and all the values are in terms of 1000, and capacity is at 10, 20, 30, and 40. This is an existing plant. So, the demand is 30, 20, 21; this number represents, say, P1 to D1, and 5 represents unit transportation cost. P1 to D 2 transport is cost is 2, P1 to D3 transportation cost is 3. So, likewise, I have mentioned the transportation cost of the arc.



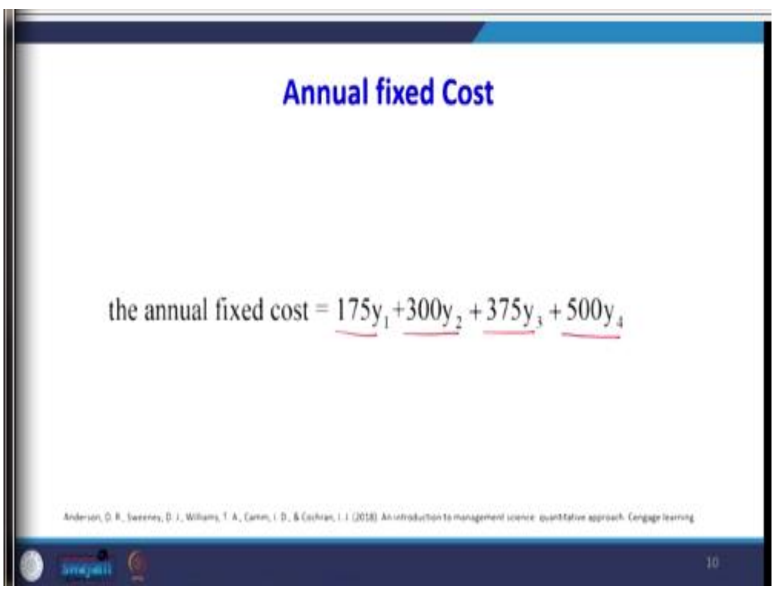
Now I am going to introduce the decision variables for the new plant. We have 4 plants. I am going to consider $y_1 = 1$ if the plant is constructed at P1, $y_2 = 1$ if your plant is constructed at a

P2, $y_3 = 1$ if your plant is constructed to P3, $y_4 = 1$ if the plant is constructed in P4. So, I may suggest 1 plant, 2 plants, 3 plants, or 4 plants, a maximum of 4 plants I can suggest.

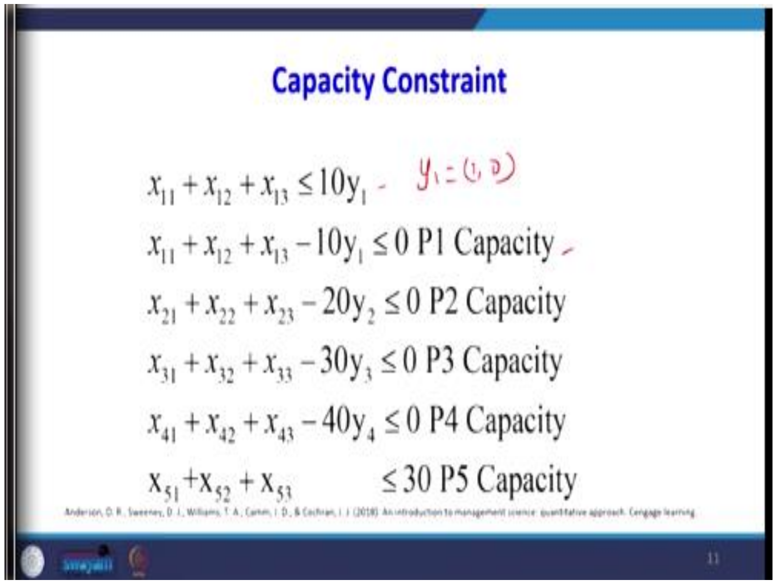


The next element is the annual transportation cost. So, here I am going to introduce the decision variable X ij; i represents the plant, and j represents the demand. So, the unit shipped from in terms of thousands from plant i to distribution j.

So, $5x_{11}$, see this $5x_{11}$, $2x_{12} + 3x_{13}$, similarly $4x_{21}$, $3x_{22}$, $4x_{23}$. Likewise, I have written all the distribution costs along with the decision variables. For example, from P5, P8; $x_{51} + 4x_{52} + 3x_{53}$ represents the total transportation cost.



Now the annual fixed cost is given. So, $175y_1$, $300y_2$, $375y_3$ and $500y_4$. Now there we have already discussed the transportation cost, and now there is a fixed cost also.

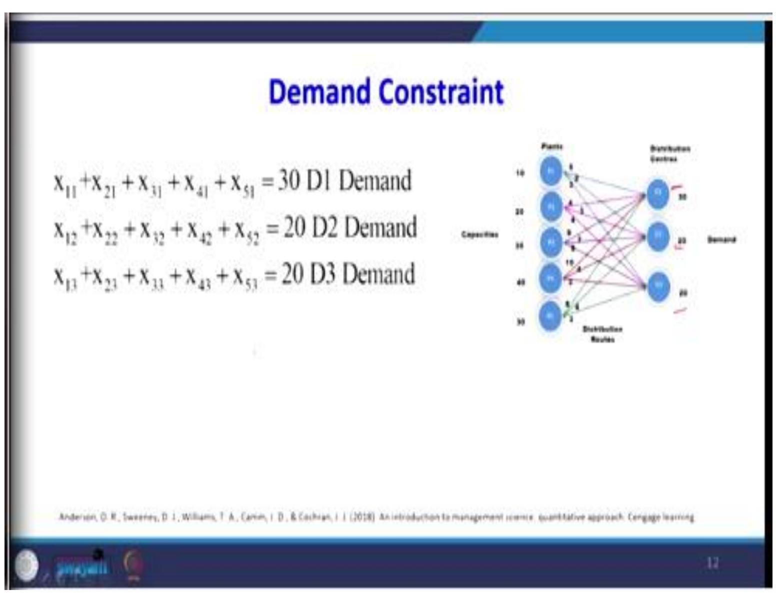


Now we will write about the capacity constraint. For the proposed plant 1, the capacity is 10, so what are the possible items that can be left from plant 1? x_{11}, x_{12}, x_{13} less than or equal to 10y 1? So, the y1 is binary, so this constraint is applicable only if plant 1 is selected, say because we know y1 is 1, 0. So, this constraint is valid only if the plant y1 is selected, so I brought it to the left-hand side, which is -10y1.

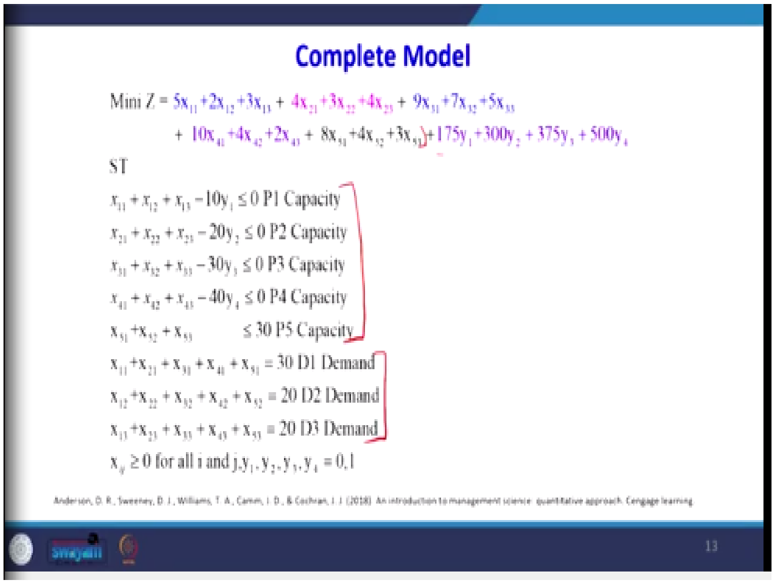
Similarly, the capacity of the proposed plant 2 is 20, so this will be $x_{21} + x_{22} + x_{23} - 20y_2$.

For plant 3 the capacity is 30, so $x_{31} + x_{32} + x_{33}$ - 30y3,

for plant 4, $x_{41} + x_{42} + x_{43} - 40y_4$, and for plant 5 it already exists you need not there will not be any y 5 variables but there will be capacity, that capacity is maximum 30. So, this is our capacity constraint.



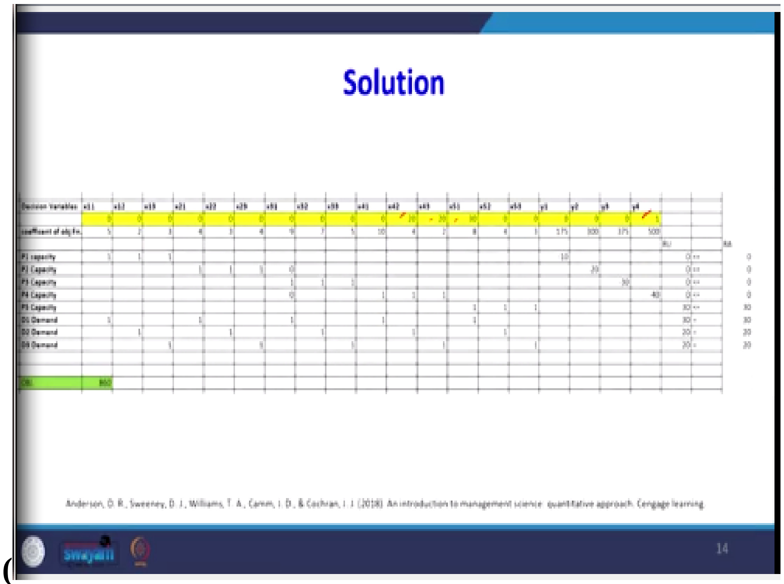
Now, we will be writing on demand constraints; what is the demand constraint? You see, D1, what are the possible ways to reach D1, 11, 21, 31, 41, 51? That should be equal to 30. What are the possible ways to reach D 2? 12, D2, 32, 42, 52, similarly for D3, what are the possible ways? 13, 23, 33, 43, and 53 are the demand constraints.



Now, I have brought the complete model. In the complete model, first, we look at the objective function. Here, the objective function is minimization type; what element is there? There is transportation cost is there, up to this there is a transportation cost, then there is a fixed cost is there. You see that here, fixed cost, and I am subtracting because all the elements are cost, so I can directly minimize the cost.

Then there are our capacity constraints; these are our demand constraints. So, I am going to solve this problem with the help of a solver. Now, I am going to solve this distribution problem with the help of a solver. I have written the decision variables in B3 to U3, then the changing cells from C4 to U4, then the coefficient of objective function I have written, then I have written capacity constraint and demand constraint.

Here, the value of y_1, y_2, y_3, and y_4 is going to be the binary constraint; I will explain what kind of constraint I have considered. So, data, solver, so this is a minimization type. You see, the constraint R4 to U4 is the binary constraint, and all other constraints are as usual. Capacity constraint less than or equal to, demand constraint is equal to type, when I solve it, so the value of the objective function is 860. Now, I will interpret this result in my presentation.



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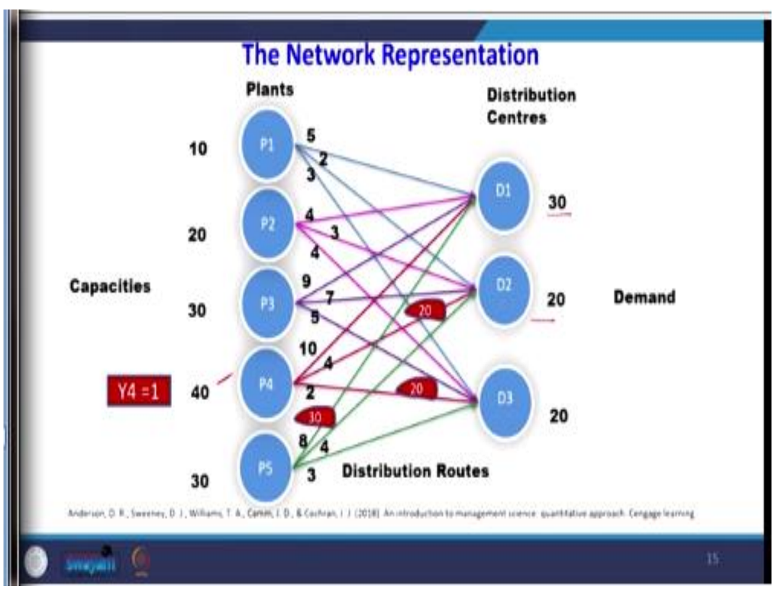
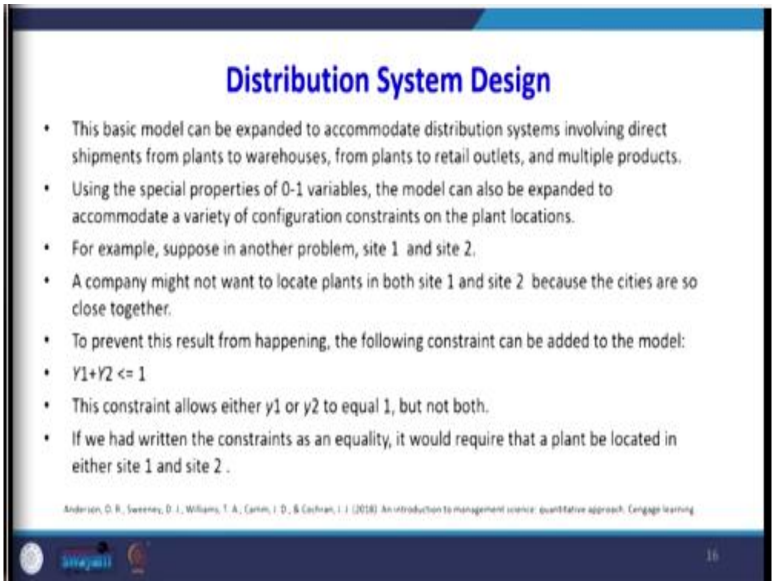
The value of x 42 is 20, x_{43} is 20, and x_{51} is 30, so which plant is opened as per this solution? Only plant 4 is getting selected; we proposed to 4 plants out of 4. We are selecting only one plant to satisfy this current demand.

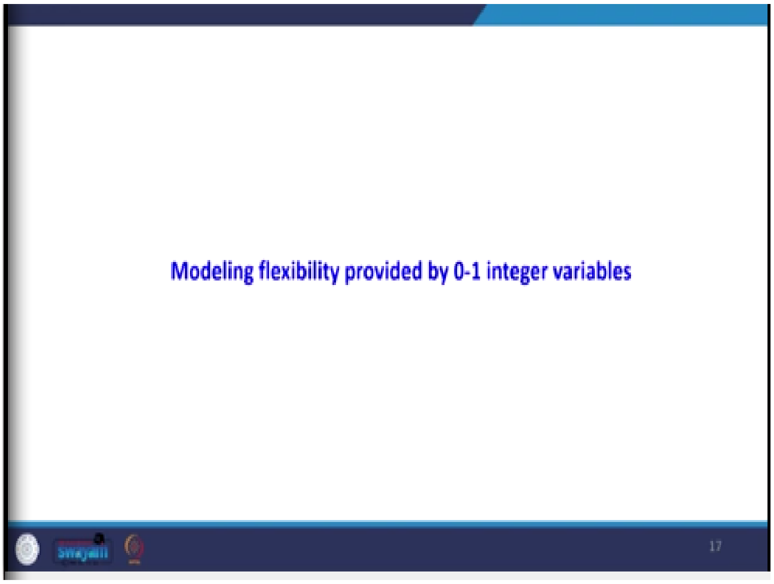


So, on this slide, I have brought your solution. The value of y_4 is 1, which means out of 4, only P4 is getting opened. So, P4 demand is 40, capacity is 40, so from P4, so D1 the demand requirement is 30. So, the P5 30 units are sent to satisfy the D2 demand, which is 20, so we know the P4 plant capacity is 40. So, 20 units are distributed to D2.

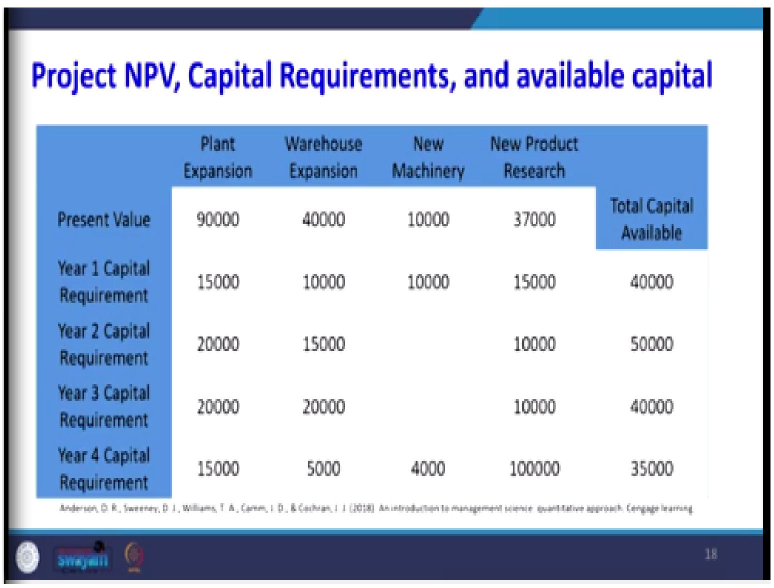


This basic model can be expanded to accommodate distribution systems involving direct shipment from plant to warehouse, from plant to retail outlets, and multiple products. How to consider that? We just have to include a direct arc from the plant to the demand center. Using the special properties of 0-1 variables, the model can also be expanded to accommodate a variety of configuration constraints on the plant location.

For example, suppose in another problem, say site 1 or site 2, a company might not want to locate plants in both site 1 and site 2 because the cities are so close together. To prevent this result from happening, the following constraint can be added, how if you write $y_1 + y_2$ less than equal to 1, what will happen? This constraint allows either y_1 or $y_2 = 1$ but not both, so that is the benefit of a 0-1 binary constraint. If we had written the constraint as an inequality, it would require that a plant be located at either site 1 or site 2.

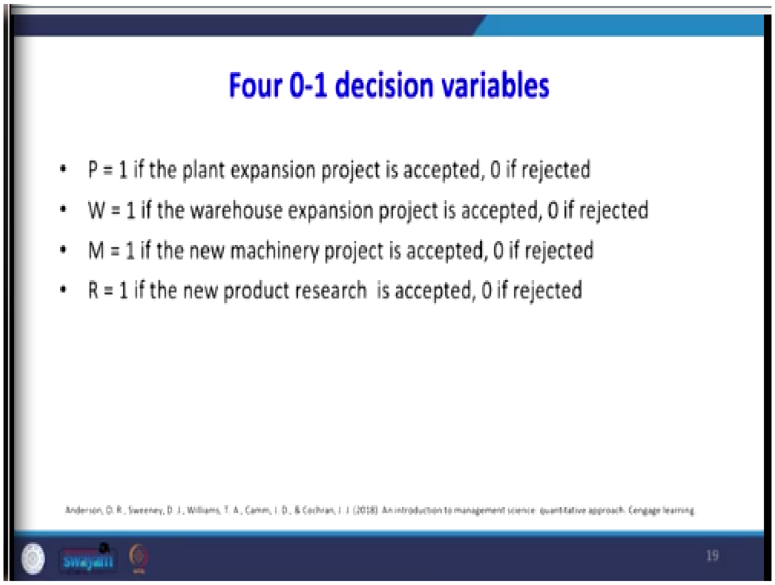


Now, the modeling flexibility is provided by 0-1 integer variables.

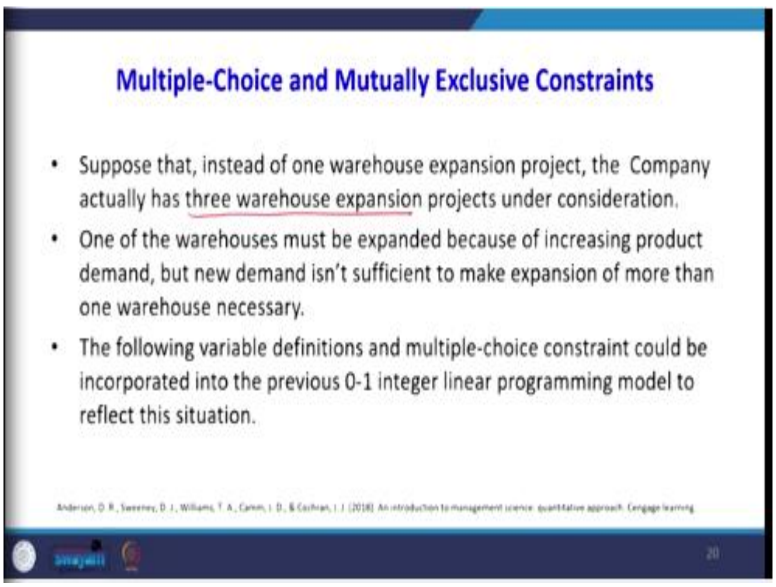


	Plant Expansion	Warehouse Expansion	New Machinery	New Product Research	Total Capital Available
Present Value	90000	40000	10000	37000	
Year 1 Capital Requirement	15000	10000	10000	15000	40000
Year 2 Capital Requirement	20000	15000		10000	50000
Year 3 Capital Requirement	20000	20000		10000	40000
Year 4 Capital Requirement	15000	5000	4000	100000	35000

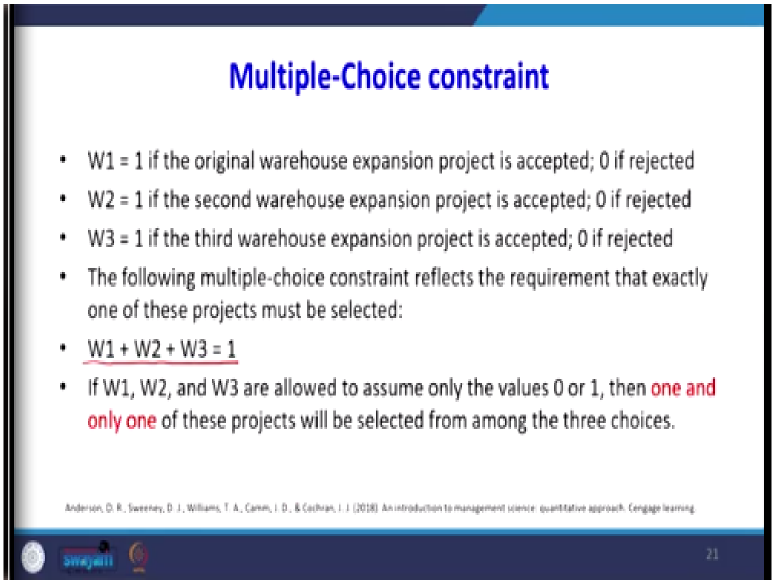
You remember, we have solved this problem, where we must maximize the NPV; what is the data given? Capital requirements and available capital.



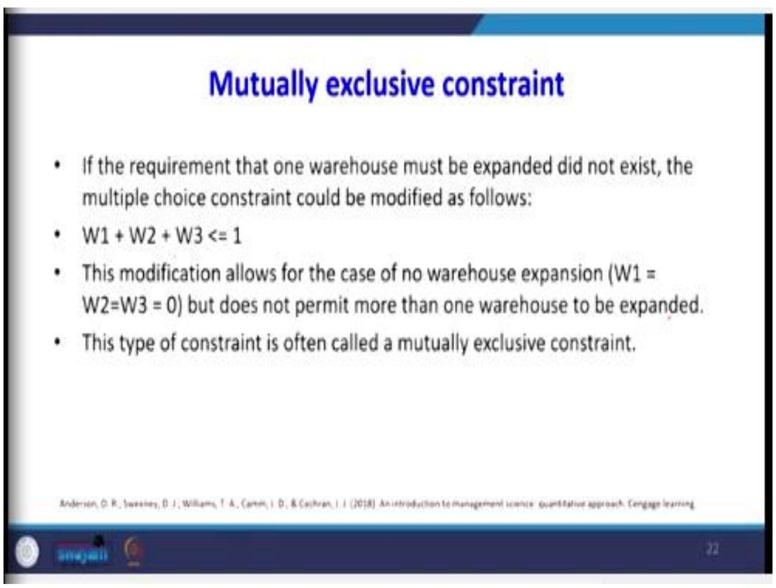
So, P = 1 represents the plant expansion, W = 1 represents the warehouse expansion, M = 1 represents the new machinery, and R = 1 represents new product research.



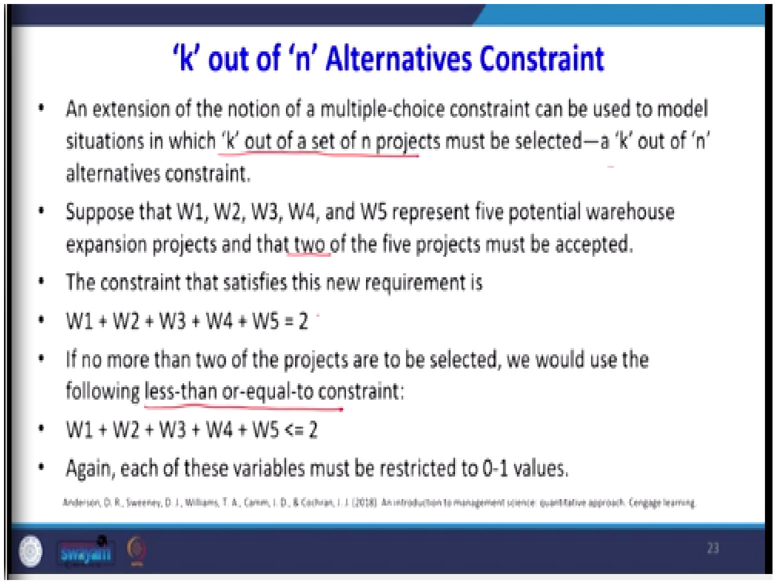
Suppose that instead of one warehouse expansion project, the company has 3 warehouse expansion projects under consideration. So, how to incorporate this kind of situation into our formulation? So, one of the various must be expanded because of increasing product demand but new demand is not sufficient to make the expansion of more than one warehouse necessary. So, the following variable definitions and multiple-choice constraints could be incorporated into the previous 0-1 integer linear program model to reflect this situation.



The first one we are going to consider multiple choice constraints. What is the meaning of multiple choice? Anyone warehouse should be opened; for that purpose, we must introduce W1 + W2 + W3 = 1. In this situation, if W1, W2, and W3 are allowed to assume only the value 0-1, then one and only one of these projects will be selected from among these 3 choices whenever you write equal to sign. This is an example of multiple-choice constraints.

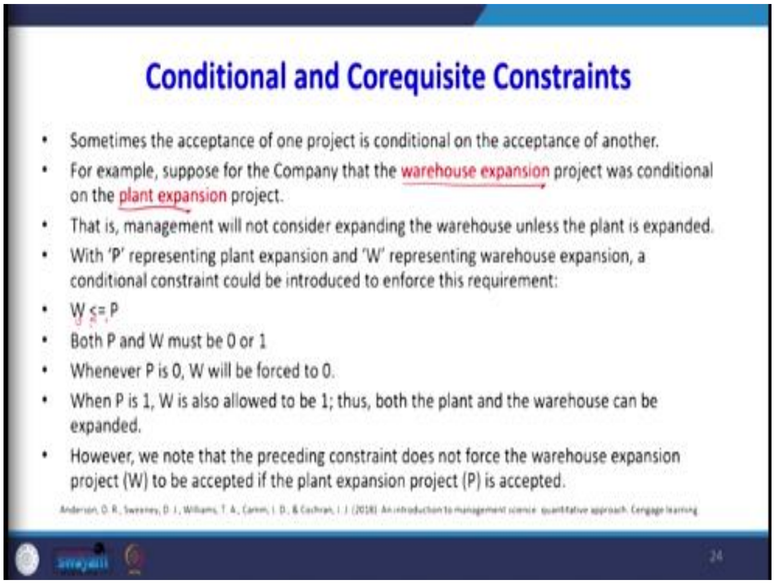


Now, there may be a situation with mutually exclusive constraints; if the requirement that one warehouse must be expanded did not exist, the multiple-choice constraint could be modified as follows. So, if you write W1 + W2 + W3 <= 1, so this modification allows for the case of no warehouse expansion. W1 + W2 + W3 = 0 but does not permit more than 1 warehouse to be expanded. We can have any 1 or it can be all 0, so this is an example of mutually exclusive constraint.



Now there may be a situation k out of an alternative constraint. An extension of this notion of multiple-choice constraints can be used to model situations in which k out of a set of n projects must be selected; simply, we can say a k out of an alternative constraint. Suppose that W1, W2, W3, and W4, and W5 represent 5 potential warehouse expansion projects and that 2 of the 5 projects must be accepted: then how to write this constraint?

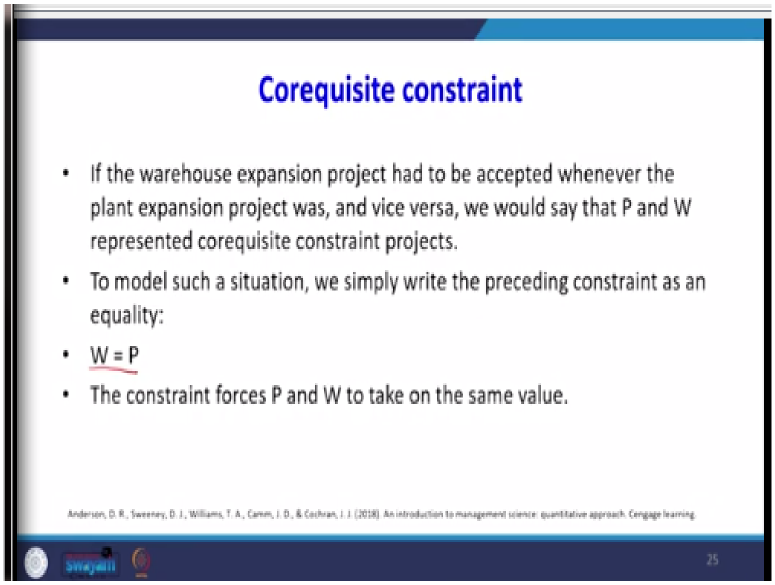
So, this constraint that satisfies this new requirement is that 2 of the 5 can be written W1 + W2 + W3 + W4 + W5 = 2, so that means out of the 5, only 2 will be selected. If no more than 2 of the projects are to be selected, we would use the following less than or equal constraint. So, when you write the same constraint into less than or equal to type, what will happen? Maximum 2 is possible; the first 1 where there is equal to you must have 2 warehouses; when you write less than or equal to, it should be maximum; if 0 is possible, 1 is possible, 2, so maximum we can go for 2 warehouses.



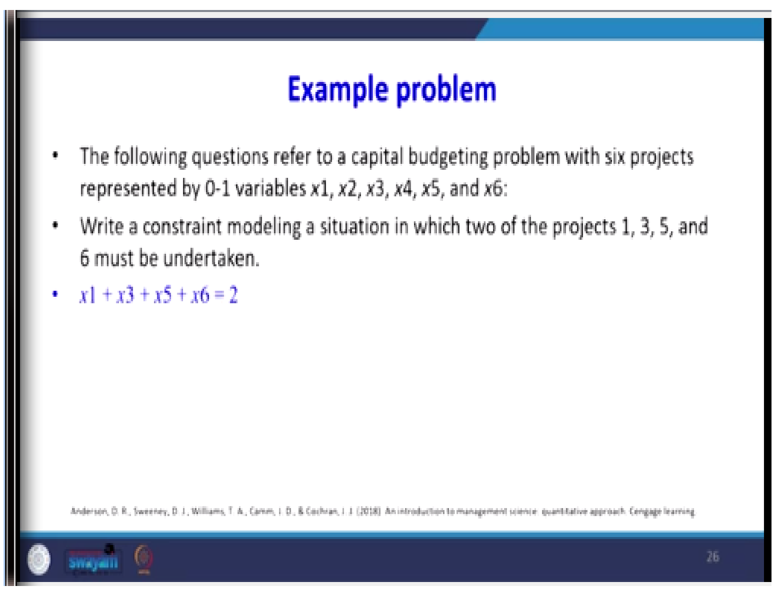
Another way of writing conditional and corequisite constraints. Sometimes the acceptance of one project is conditional on the acceptance of another. For example, the company's warehouse expansion project was conditional on the plant expansion, so what is happening in this type of project is warehouse expansion and plant expansion. So, generally, in a practical context, whenever there is a plant expansion, then only should go for warehouse expansion.

So, how to consider this constraint? That is, the management will not consider expanding the warehouse unless the plant is expanded with P representing the plant expansion and W representing warehouse expansion; a conditional constraint could be introduced to enforce this requirement if W is less than or equal to P. So, what it says is that warehouse expansion is possible only if you go for plant expansion. Here both P and W must be 0 or 1.

So, whenever P is 0, that is, the plant expansion is taking place. If the plant expansion is not taking place, the W will also be forced to 0. If P = 1, W is also allowed to be 1, but it can be 0 also; thus, both the plant and the warehouse can be expanded. However, we know that the preceding constraint does not force the warehouse expansion project to be accepted if the planned expansion project is accepted. So, there is a possibility W can become 0 when there is a plant expansion, but the warehouse expansion is possible only if you go for plant expansion. So, this type of constraint also can be incorporated.

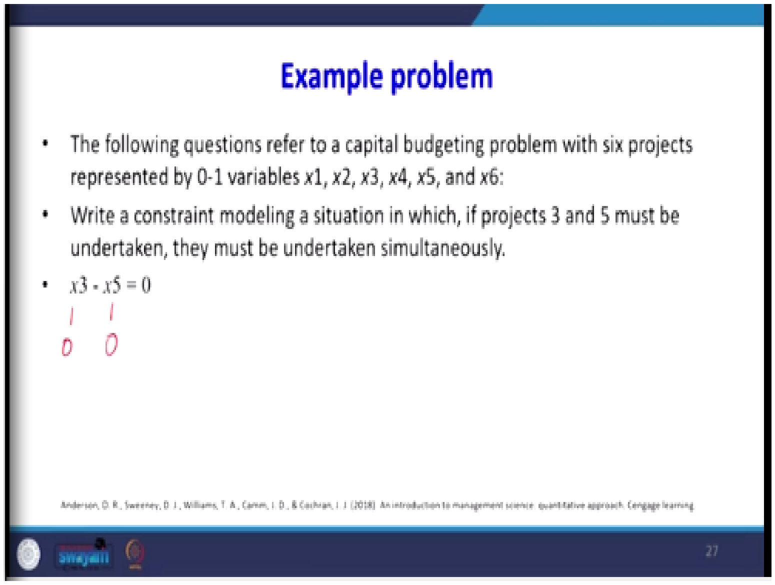


Then let us look at the corequisite constraint. If the warehouse expansion project had to be accepted whenever there was a plant expansion project or vice versa, we would say that P and W represented corequisite constraint projects. How do you write in the form of an equation this corequisite constraint? So, we simply write this corequisite constraint W = P, so this constraint forces P and W to take on the same value whether P = 1 W is also equal to 1. So, this is called the corequisite constraint, P = 0, W also 0.

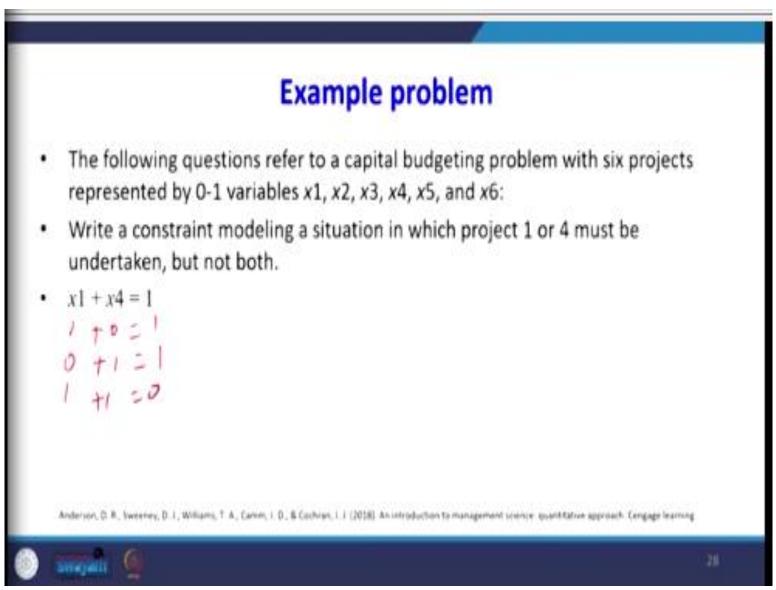


Now we will go for some more flexibility in the binary variables. The following questions refer to the capital budgeting problem with the 6 projects represented by 0-1; what are the 6 projects x_1, x_2, x_3, x_4, x_5, and x_6? Now, write a constraint modeling a situation in which 2 of the projects

1, 3, 5, and 6 must be undertaken. So, how to write that one? When you say x_1, x_3, x_5, x_6 should be equal to 2, this is the first one.

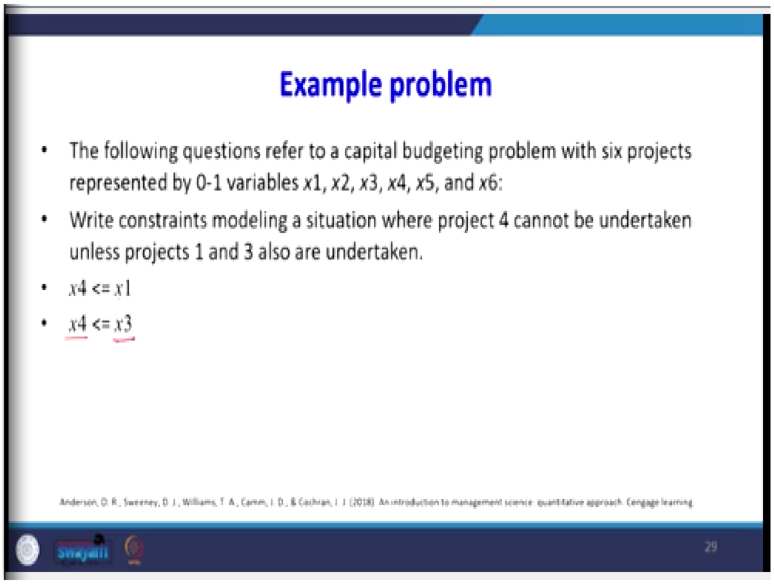


The second question is to write a constraint modeling a situation in which projects 3 and 5 must be undertaken and they must be undertaken simultaneously. So, what would you write? $x_3 = x_5$, so otherwise $x_3 - x_5 = 0$, what is the meaning? Simultaneously, when you go for x_3, then x_5 also should be there; then only we will get 0; if it is 0, this also should be 0; this is the other way.

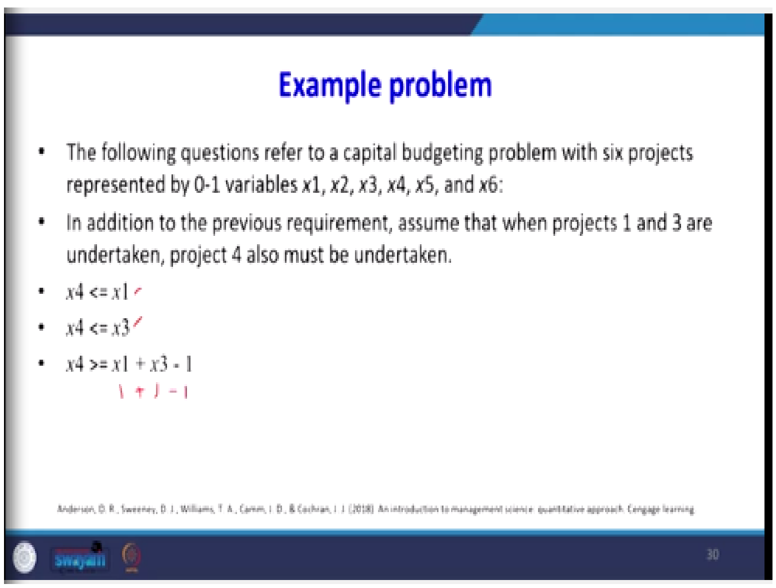


The next question is to write a constraint modeling a situation in which project 1 or 4 must be undertaken but not both.

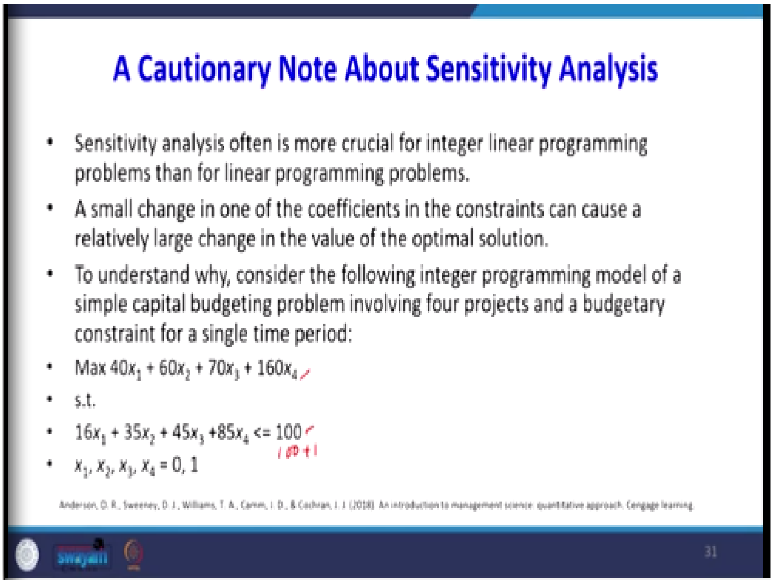
So, when you write $x_1 + x_4 = 1$, either we can go for $1 + 0 = 1$ or $0 + 1 = 1$, but $1 + 1$ is not possible because that will violate your constraint.



The next question is to write a constraint modeling a situation where project 4 cannot be undertaken unless projects 1 and 3 are also undertaken; now, there is a conditional requirement. So, when you write x4 less than or equal to x1, that means that if you are going for x4, x1 must be taken; if you go for the second constraint x4 less than or equal to x3, if you are going for x4, x3 also should be taken.



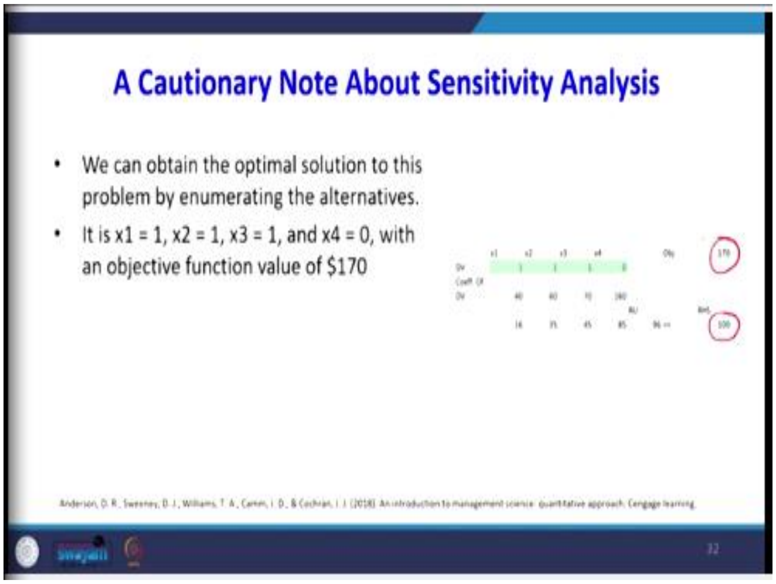
In addition to the previous requirement that we have discussed in the previous slide, assume that when projects 1 and 3 are undertaken, project 4 also must be undertaken. How to represent this one in the form of the equation? So, x_4 is less than or equal to x_1; this is the constraint that we have discussed in the previous slide. Now what is that? Project 4 also must be undertaken, so how to represent this x_4 is greater than or equal to $x_1 + x_3 - 1$. So, when do you represent this way what it says that? When the project 1 and 3 are undertaken, project 4 also must be undertaken. See if you are taking $x_1 = 1$, $x_3 = 1$, so 2 – 1= 1, then x_4 is also undertaken.



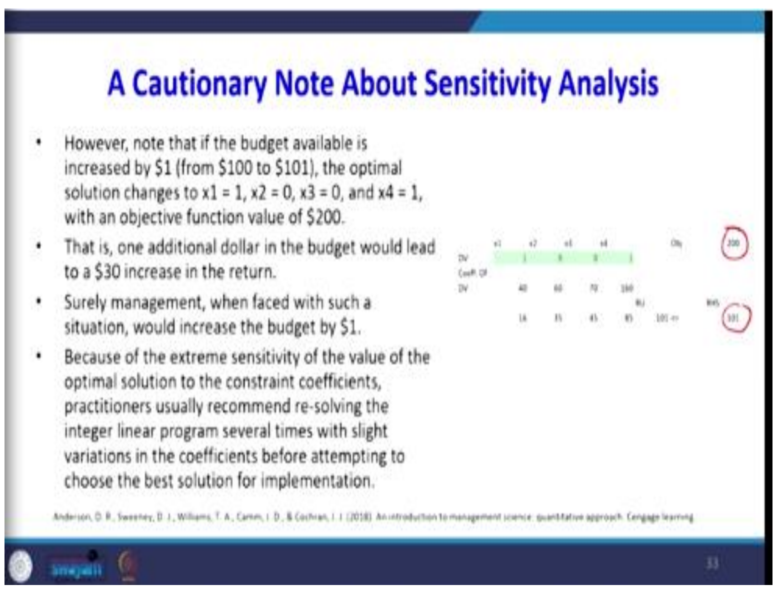
Another important point in integer programming is sensitivity analysis. See, sensitivity analysis is often more crucial for integer linear programming problems than linear programming problems. A small change in one of the coefficients in the constraint can cause a relatively large change in the value of the optimal solution, so there will not be any proportional change. To understand why, consider the following integer programming model of a simple capital budgeting problem involving 4 projects and a budgetary constraint for a single time. So, objective function, the budgetary constraint is 100, so now what am I going to do?

I am going to solve this problem with the help of Excel; then I am going to increase this 1 unit on the right-hand side. Let us see the effect of this 1 unit increase on the right-hand side of the constraint on our objective function; we will go to Excel. Now, I am going to explain the concept of sensitivity analysis in integer programming. So, I have formulated the problem; you see the value of the right-hand side is 100, so go to the solver; look at this: the function is maximization problem C4 to F4 is a binary. When I solve it, the value of my objective function is 170; now, I am going to see the concept of dual value. If I change the right-hand side from 100 to 101, now I resolve it; when I resolve it, you see that the value of the objective function is 200.

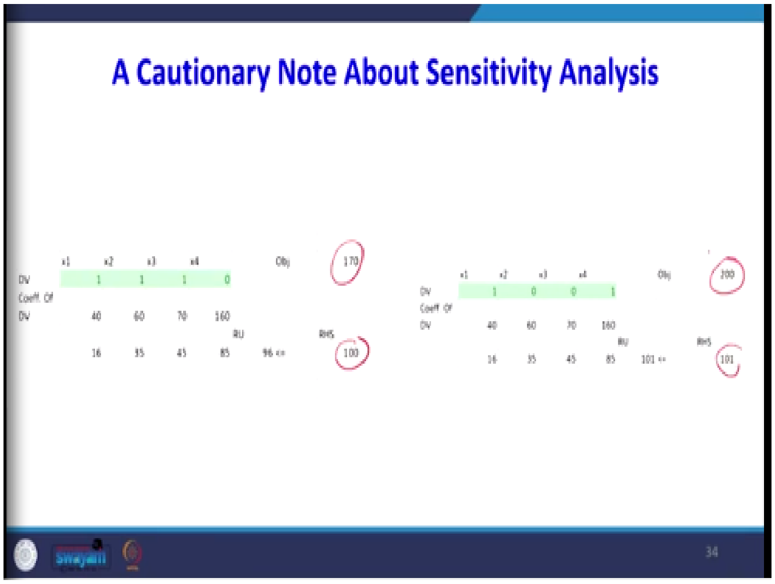
Just for 1 unit increase on the right-hand side, the value of the objective function is drastically increased; that is why you should be very careful while doing the sensitivity analysis. The sensitivity analysis in integer programming means you have to resolve the problem.



When the right-hand side is 100, the value of the objective function is 170.



When I change the right-hand side is 101, the value of the objective function is 200. That is 1 additional dollar in the budget would lead to 30 dollar increase in the return. Surely management when faced with such a situation would increase the budget by 1 dollar because there is a huge increase in the return. Because of the extreme sensitivity of the value of the optimal solution to the constraint coefficients, practitioners usually recommend resolving the integer linear program several times with slight variations in the coefficient of objective functions or the right-hand side value before attempting to choose the best solution for implementation.



This is I brought the same when it was a 100, it was 170 when the right-hand side was 101, there is a 200. In this lecture, I have explained the distribution problem with 0-1 constraint, and after that, I have explained various modeling flexibility provided by 0-1 integer variables. Thank you very much.