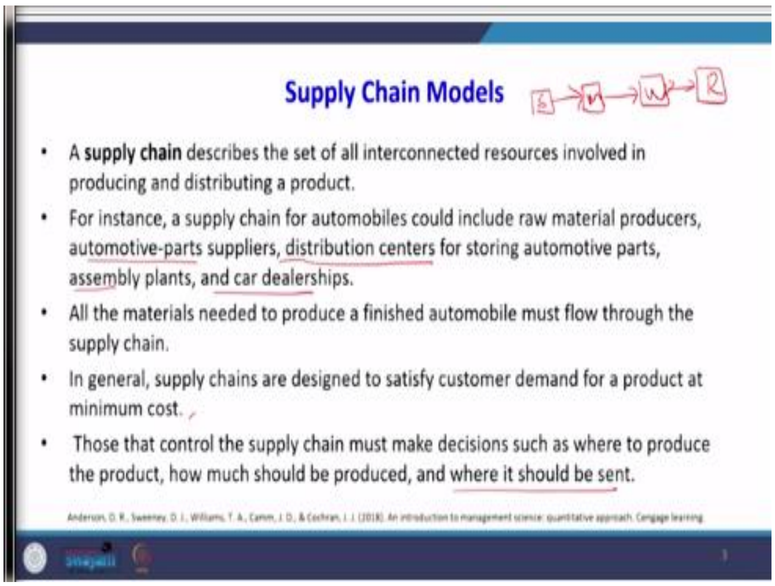
Decision Making With Spreadsheet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology-Roorkee

Lecture-19
Distribution and Network Models: Transportation Problem

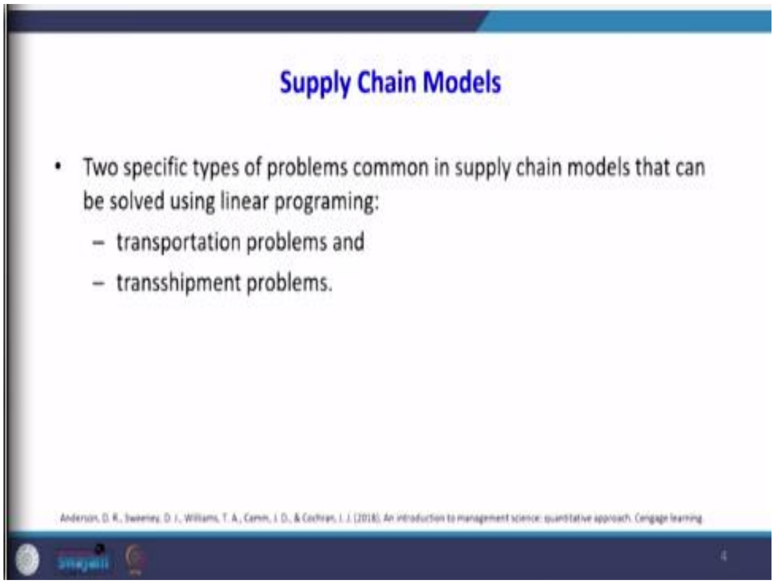
So far, I have discussed some advanced linear programming applications. In this lecture, I am going to explain distribution and network models; I am going to discuss 2 important problems: one is transportation problems, and one is transshipment problems.



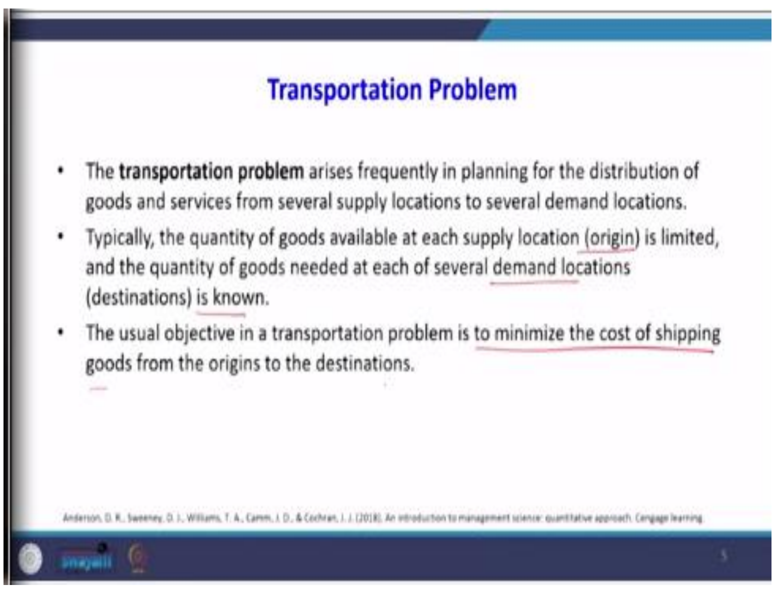
These 2 problems are very useful in modeling the supply chain. What is the supply chain? A supply chain describes the set of interconnected resources involved in producing and distributing a product. For instance, a supply chain for automobiles could include raw material producers, automotive part suppliers, distribution centers for storing automotive parts, assembly plants, and car dealers.

So, what is the supply chain? There may be a supplier, there is a manufacturer, there is a warehouse, there is a retailer, so supplier, manufacturer, warehouse, and retailer, this is the supply chain. So, each entity is connected by a network, so modeling this network is an important application of linear programming problems. Here, in the supply chain network model, all materials needed to produce a finished automobile must flow through the supply chain.

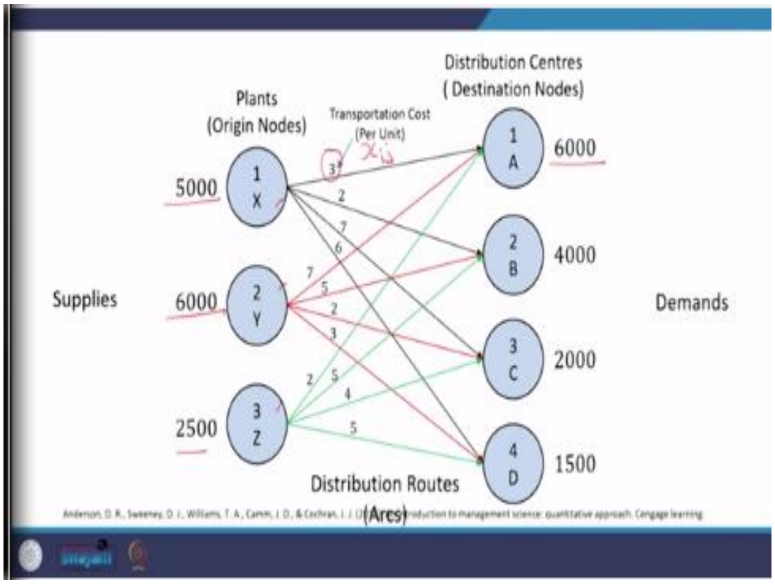
In general supply chains are designed to satisfy customer demand for a product at a minimum cost. So, the distribution network must reduce overall supply chain costs. Those who control the supply chain must make decisions such as where to produce the product, how much should be produced, and where it should be sent. Here is where an important decision of supply chain network models.



Two specific types of problems common in supply chain models that can be solved using linear programming are transportation and transshipment problems.

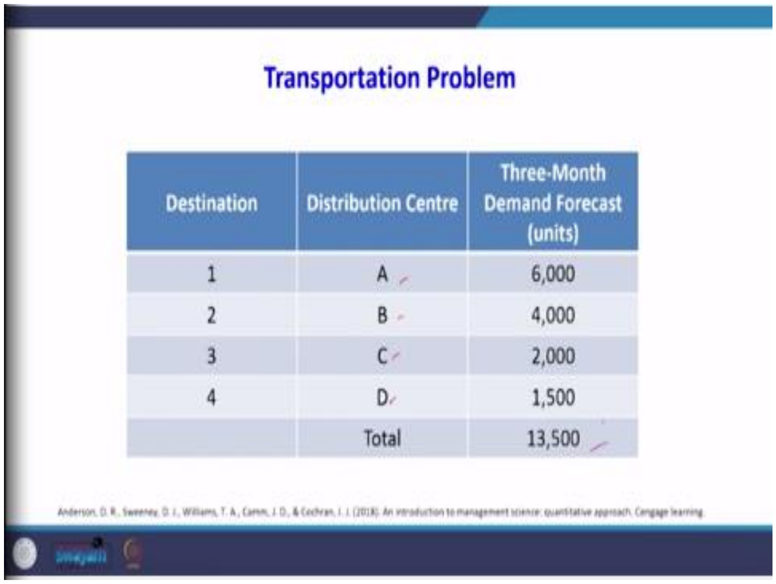


Now we will discuss the transportation problem. The transportation problem arises frequently in planning for the distribution of goods and services from several supply chain locations to several demand locations. Typically, the quantity of goods available at each origin supply chain location is limited. The quantity of goods needed at each of several demand locations or destinations is known. The usual objective in transportation problems is to minimize the cost of shipping goods from their origins to their destinations.

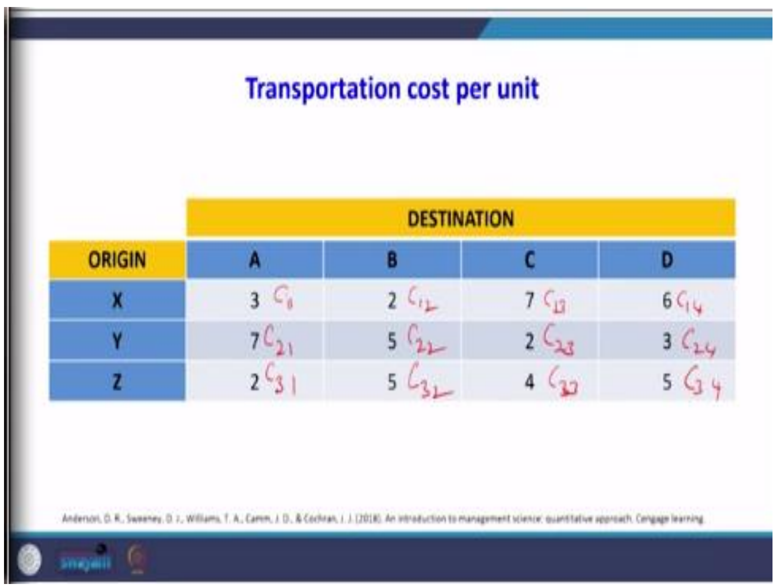


Now, I have taken a sample of a network problem; this example is taken from the Anderson et al. book. There are three supply nodes X, Y, and Z plant for example, there are four demand nodes distribution centers A, B, C, and D. So, plant 1 has a capacity of 5000 units, plant 2 6000, plant 3 2500, the distribution center is having the demand of 6000, for node B it is a 4000, 2000, 1500. So, what is the objective of the transportation problem is how many units must be picked from each plant and distributed to satisfy the demand.

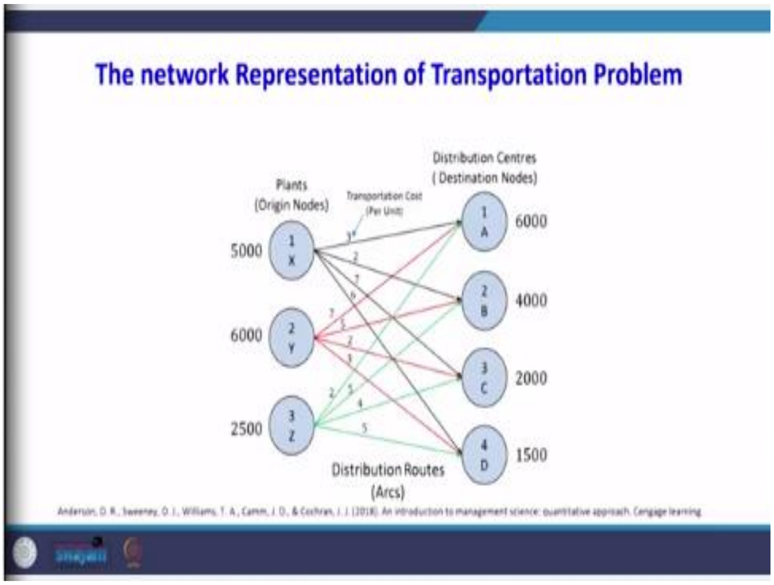
Here, the unit transportation cost is given; here, the decision variable is when I say xij, how many units have to be picked from each plant? I represent the origin, and j represents the destinations. So, xij is the decision variable, and one more thing, the distance we are not considering only the transportation cost is given; it indirectly captures the distances.



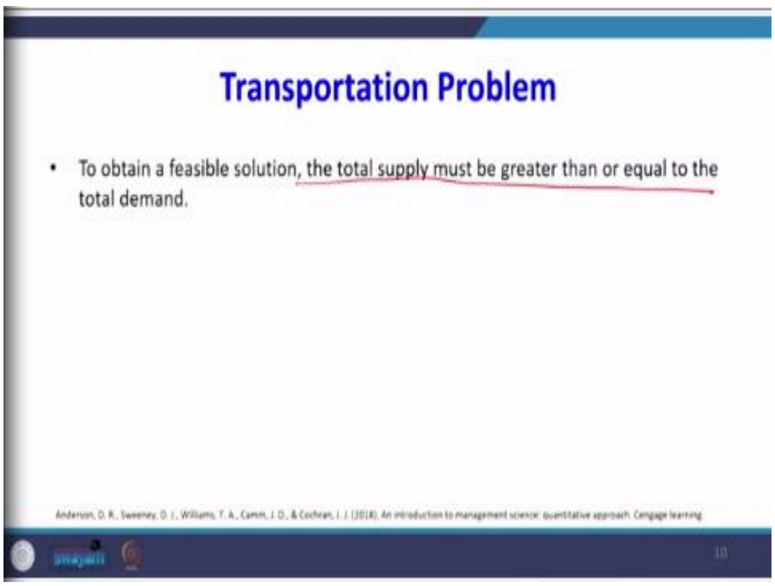
Now, there are 4 destinations: A, B, C, and D; there is a demand for each destination. So, the total demand is 13500.



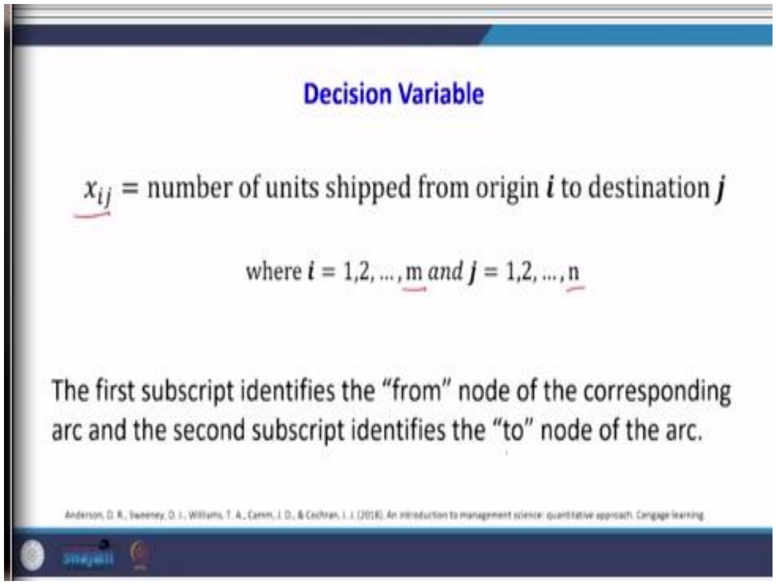
The transportation cost per unit is given. For example, from origin X to destination A, the unit transport cost is 3, so we can say C11, C12, C13, C14, so we can call it C21, C22, C23, C24, so C31, C32, C33, C34 like this, this is the unit transportation cost.



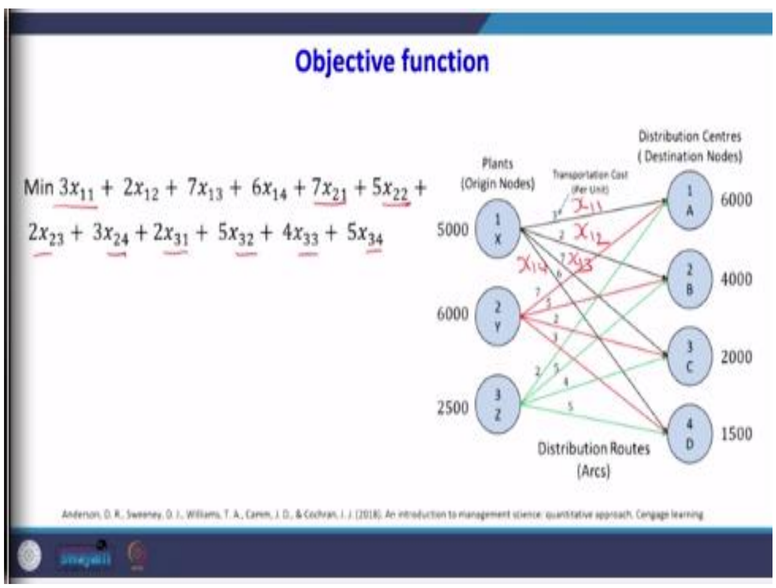
The transportation problem can be represented in the form of a network.



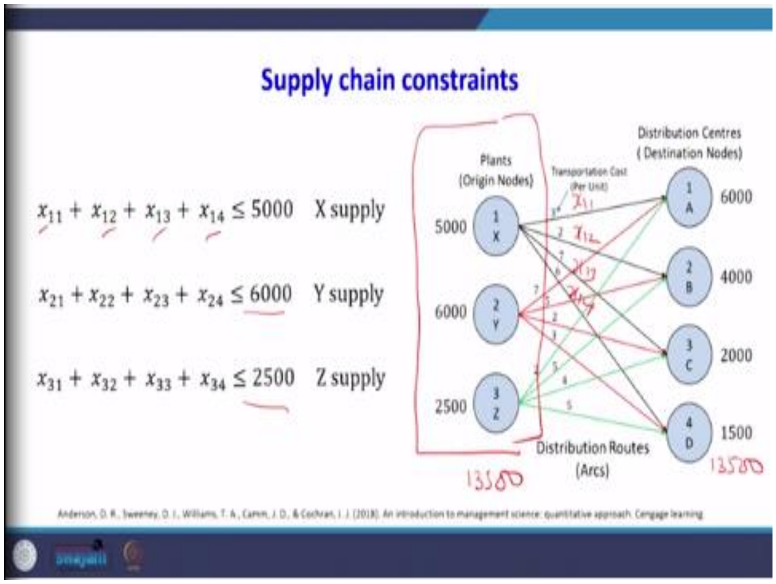
To obtain a feasible solution, the total supply must be greater than or equal to the total demand; the supply should be greater than the demand. In case this condition is not satisfied, you will get an infeasible solution.



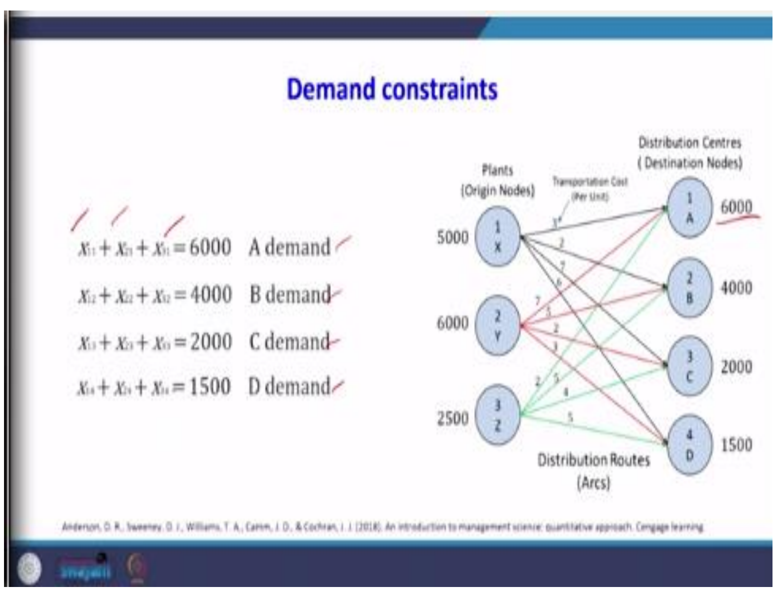
Now, what kind of decision variables xij? As I told you, the number of units shipped from origin i to destination j, where i represents m origin, n destination. The first subscript is identified from the node of the corresponding arc. The second subscript identifies the 2 nodes of the arc.



So, first, we will write an objective function, so I can write this is x11, this is x12, this is x 13, this is x 14. Similarly, I can write for other nodes, so here, 3x11, the unit cast is 3, and the quantity shipped is x 11, so 3x11 + 2x12 + 7x13 + 6x14. From node 2, it is x21, x22, x23, x24 multiplied by the corresponding unit transportation cost. Then 31, 32, 33, and 34 are multiplied by the corresponding unit transportation cost. So, the objective function is the minimization of the cost that has to be minimized.



The next constraint that we have to write is the supply constraint; what is the supply? This portion is the supply. So, the number of units going away from each node, so x11, x12, x13, x14, this is x11, x12, x13, x14, so that cannot exceed 5000 units because it is capacity is only 5000. Similarly for second x21, 22, 23, 24, 6000, then 31, 32, 33, 34, 2500. So, when you add a total supply of 13500, the total demand is 10000, 12000, 13500. Now, the supply is equal to demand, so we can proceed with the problem. Sometimes there may be a mismatch, which I will explain to you, that is a special case of transportation problem.



The next is the demand constraint. So, here demand is 6000, what are the different possibilities x11, x21, x31 = 6000, 4000, 2000, 1500. Here, it should be equal to the sign because demand must be satisfied. Now, I am going to solve this transportation problem with the help of a solver.

So, you can see here I have entered the origin and the destination; origins are X, Y, and Z, and destinations are A, B, C, and D. On the supply side, I have written total supply, and the demand side, I have written the total demand.

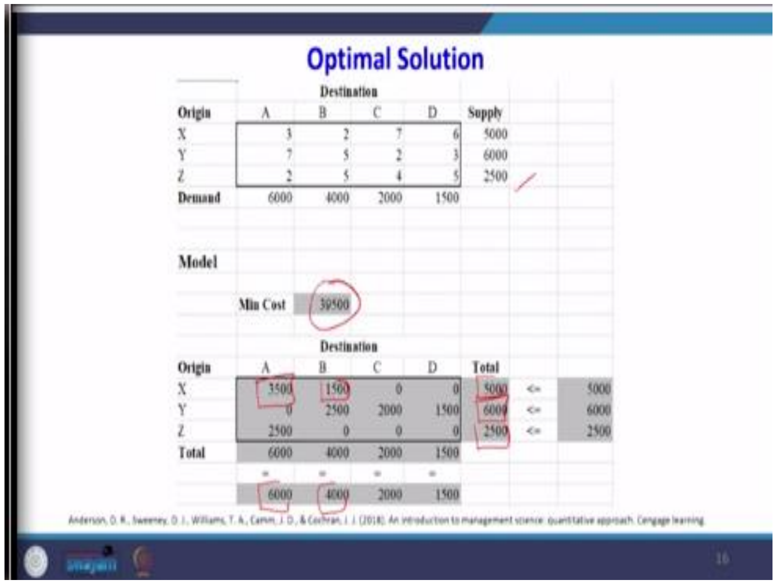
Now, at the bottom, where I am going to get the answer, that is the origin I have written. Testing what I have written, you see the formula of F17, so the sum of that row. The bottom one, F18, is the sum of this row; similarly, it looks at this B20, so the sum of this column is the C20 sum of this column. At the bottom, I have written the B18; suppose this value should be B8; what is the B8? That is the total demand.

So, the column sum should be equal to the total demand. Similarly, the row sum should be equal to F5, which is your total supply. So, this is the model for the transportation problems; now, I will go to data and then the solver. Look at this; this is a minimization problem, so objective function. Look at the changing variable cell, which is B17 to E19, so you have to select B17 to E19 at a time. Then I wrote the demand constraint and the supply constraint, and when I solved it, now I got the value.

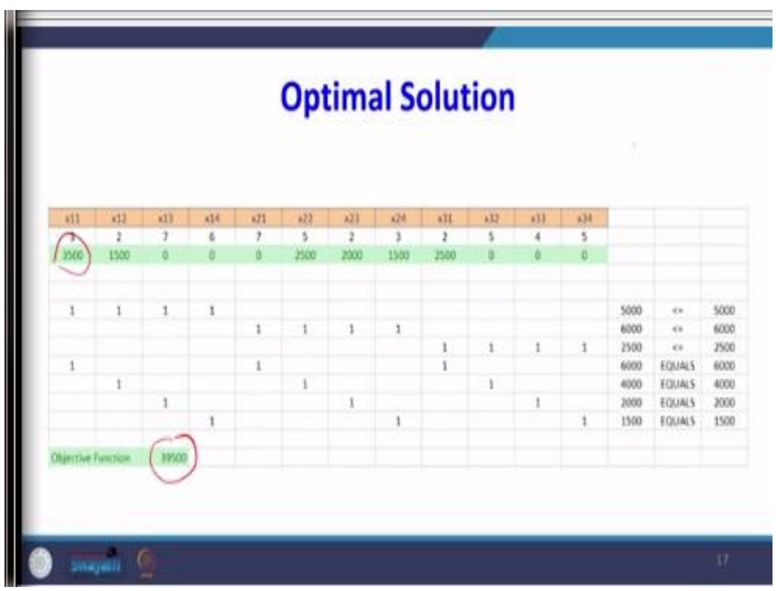
You see that B17, 3500, so what is the meaning? From origin X to destination A, I should transport 3500 units; from origin X to destination B, I have to transfer 1500. So, again, if I press the control tilde again, I can see the actual value; you see that, you see F17 to F19, my supply constraint F17 to F19, so my supply constraints are satisfied. Similarly, when you look at B 22 to E 22, my demand constraints are also satisfied.

This is the one way to solve the LP model with the help of Excel; this is the easiest way to interpret. But there is another way you can solve the same problem: just write all the equations, like LP formulations, and then you can solve it, which I am going to show you. Now, I have formulated from our formulations, such as our simple linear programming problems. So, I have written the decision variables, coefficient objective function, the value of the objective function on the right-hand side, and the constraints and resources utilized.

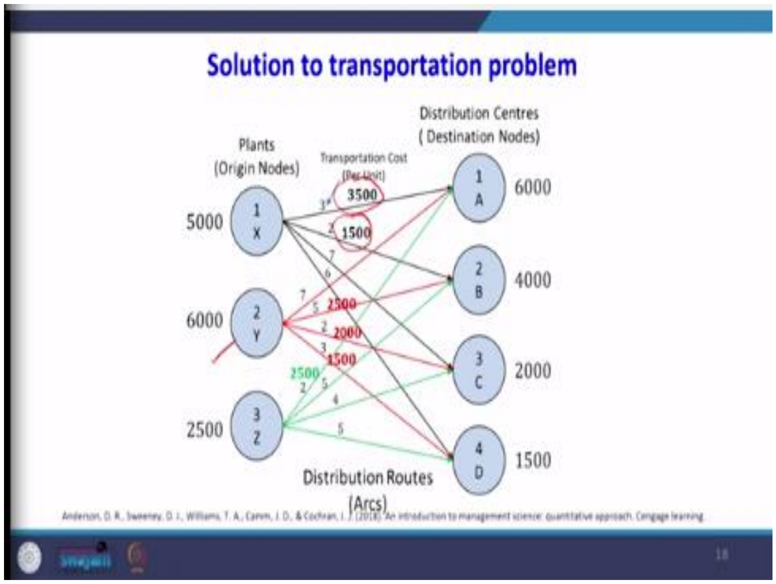
Now, when I go to data solver, it is a minimization problem, so when I solve it, the total transportation cost is 39500. So, how many units do you have to transport from X1 to 1 3500, 12 to 1500, and so on? So, here is what we are learning here. For the same problem, we can make an Excel model, or you can solve it with simple formulations and solving equations.



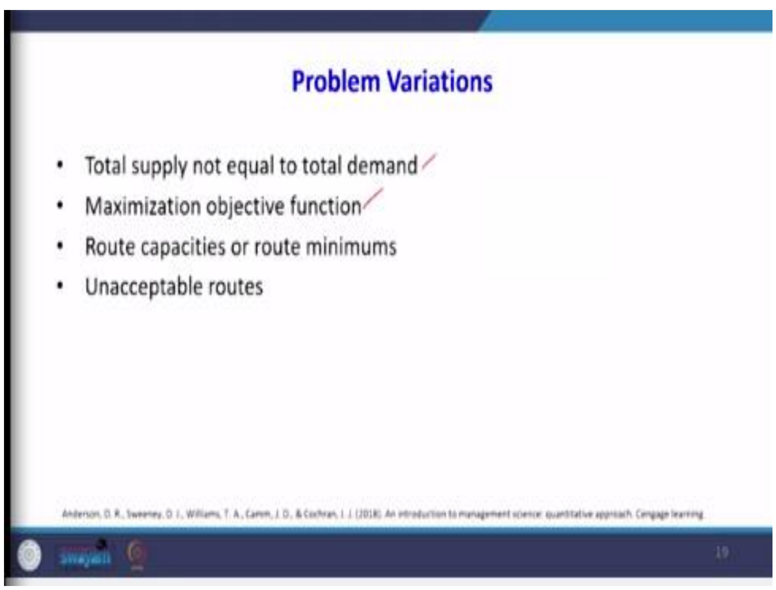
Now, I have brought the screenshot of our Excel model. So, here we can see this is our given problem, the bottom where you can get the answer here 3500, 1500. So, this is our total supply from X, total supply from Y, and total supply from Z; this is our demand, so this is satisfied. What is the value of our optimal objective function? 39500.



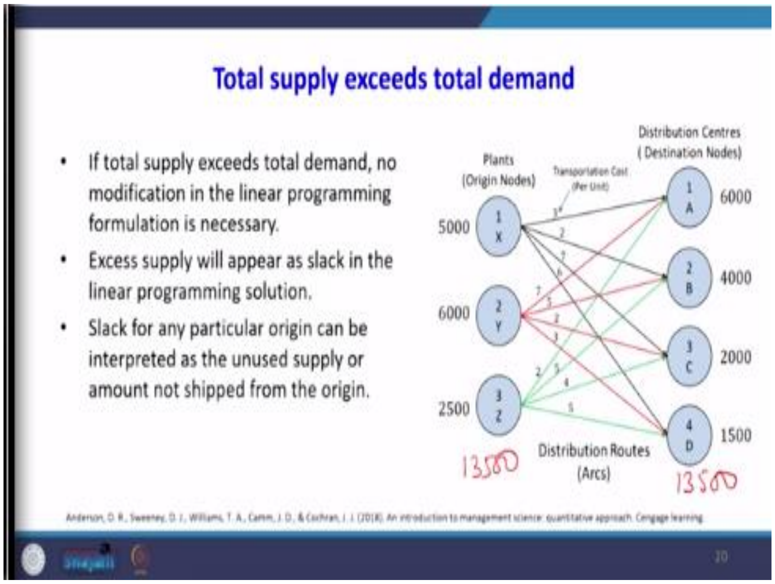
When we solve this problem in the form of equations, we also get 39500, and we get various cell values, which are nothing but from origin 1 to destination 1; these many units have to be transported. So, solving the second method, which is taking the equation solving with the help of Excel, is somewhat more straightforward than the previous method; those who do not have any knowledge of Excel can simply solve the model instead of going with the previous method, which I told you.



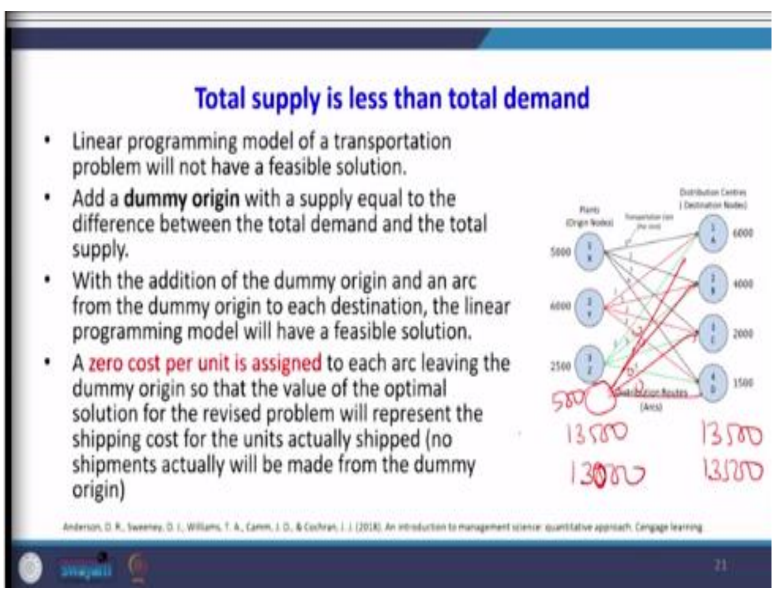
Now I have brought the solution to the transportation problem. So, we are getting from see X1 to 1, 3500 units, X1 to 2, 1500, so now 5000 is satisfied. So, 2500, 4500, again 6000, yes from destination 2 to 2 we must transport 2500, 2 to 3, 2000, 2 to 4, 1500, from 3 to 1, 2500, that is all. So, this is our solution to our transportation problem.



Now we will see the different variances and variations in our transportation problem. There are 4 types of variation that are possible: one is total supply, not equal to total demand. In our problem, which we have discussed, here supply is equal to demand. The next one is sometimes the objective function may be maximization type; that time, what to do? In the next one, there may be a root capacity or root minimum, and there may be unacceptable routes, so how can we handle these problems? That we will see now.



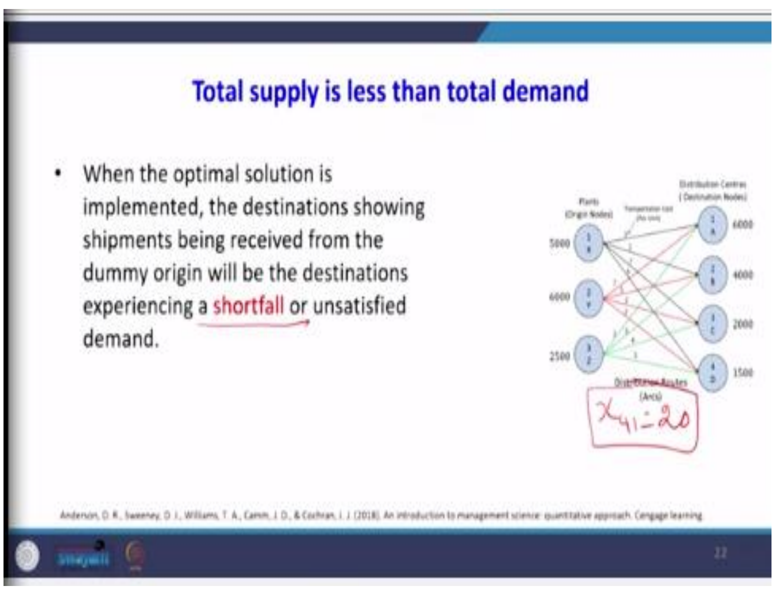
The first one is when the total supply exceeds total demand; here, we are getting 13500, and here, also 3500. Sometimes, what will happen? Here, the supply is higher; demand is less, and then there is no problem. If the total supply exceeds the total demand, no modification in LP formulation is necessary, so what will happen? If it is an excess supply, that will appear as slack in our LP model, that is the materials that are not transported. So, slack of any origin can be interpreted as an unused supply or amount not shipped from the origin.



Another situation may come when the total supply is less than the total demand. So, we see this is 13500, and sometimes we see this is 13000, for example. In this case, the right-hand side is 13500, but on the left, this is only 13000, not 13500. Here, if the total supply is less than the total demand, the LP model of a transportation problem will not have a feasible solution; if the supply is less than the demand, there will not be any feasible solution; how can we handle this?

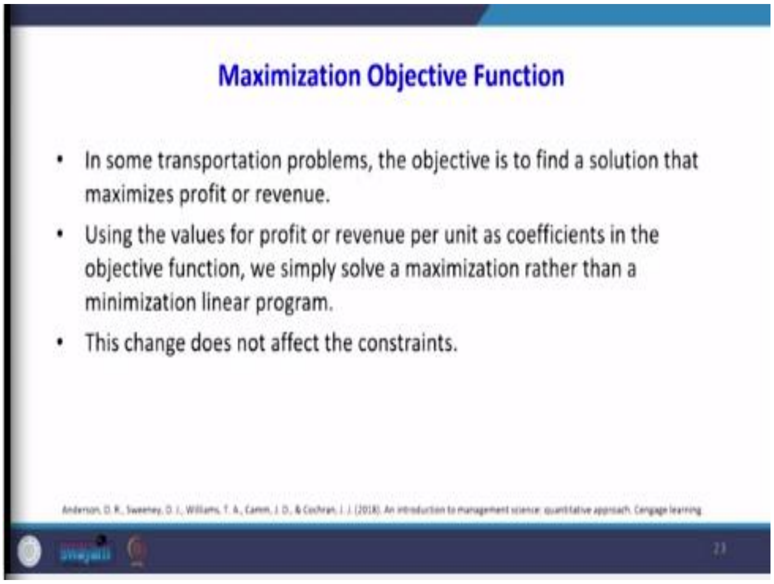
We must add a dummy origin with a supply equal to the difference between total demand and total supply. So, here we must add a dummy origin so 13500 - 13000, so this capacity is 500. When you add a dummy variable, then the LP model will have a feasible solution, so when you add a dummy variable, this must relate to all the destination nodes with zero-unit transportation cost.

So, a zero cost per unit is assigned to each arc leaving from the dummy origin so that the value of the optimal solution for the revised problem will represent the shipping cost for the unit shipped. In reality, no shipments will actually be made from the dummy origin; we have added that to balance the problem. So, suppose you are getting some solution here, so that is the shortage for this problem that is this.

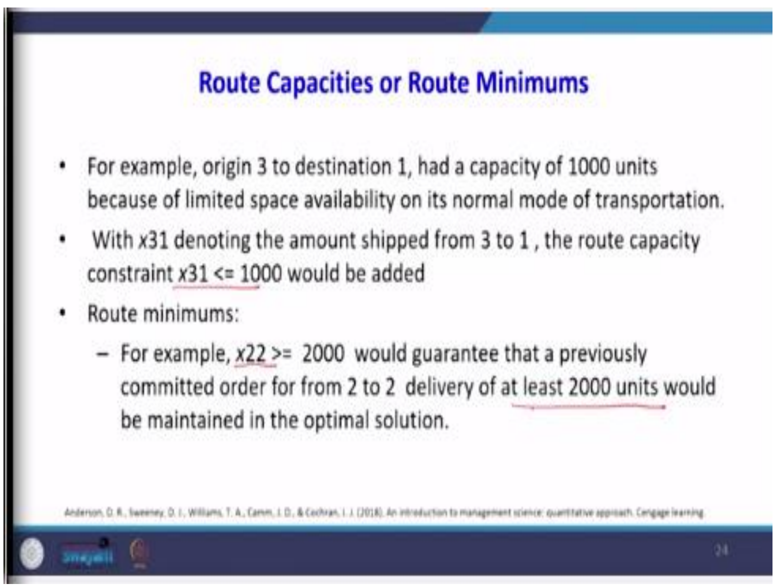


When the optimal solution is implemented, the destination showing shipments being received from the dummy origin will be the destination experiencing a shortfall or unsatisfied demand. Suppose you are getting, say, x, say, 41, say, 20. That means this is a unit of dissatisfaction.

(Refer Slide Time: 19:23)

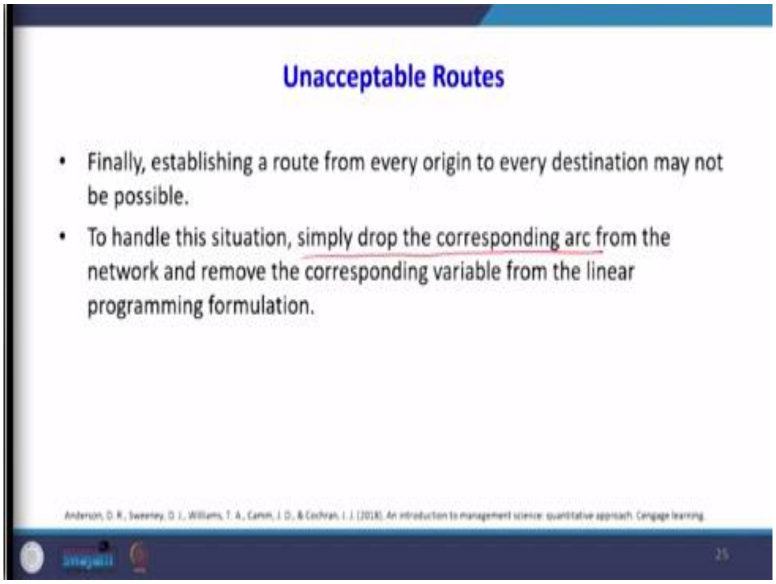


Another situation is maximization. In some transportation problems, the objective function is to find a solution that maximizes the profit or revenue. So, using the value of the profit or revenue per unit cost as a coefficient in the objective function. We simply solve maximization rather than minimization, so this change does not affect the constraint.

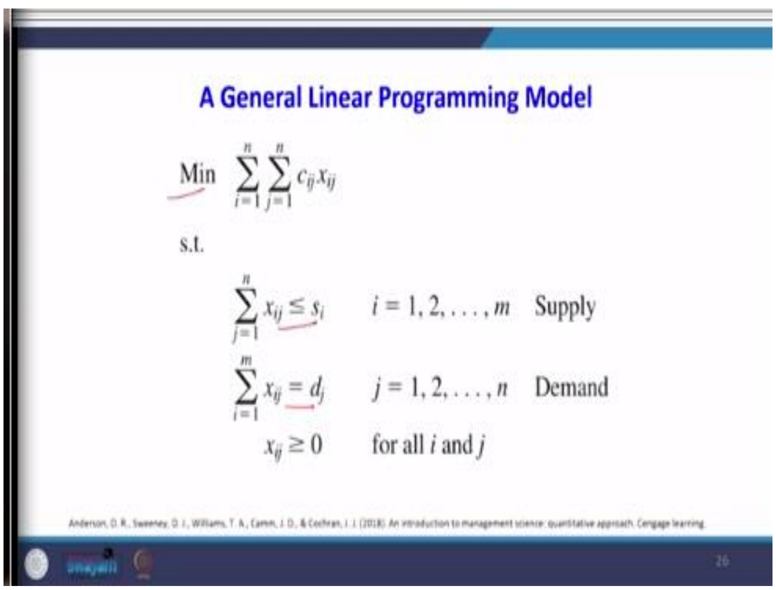


Route capacity. For example, say origin 3 to 1 had a capacity of only 1000 units because of limited space availability, which is a normal mode of transportation. So, what do you have to do? We have to add a constraint that X31 should be less than or equal to 1000. Sometimes, there may be a committed capacity, so that time should be written. For example, X22 should be greater than 2000, which would guarantee that previously committed orders from 2 to 2 deliveries of at least

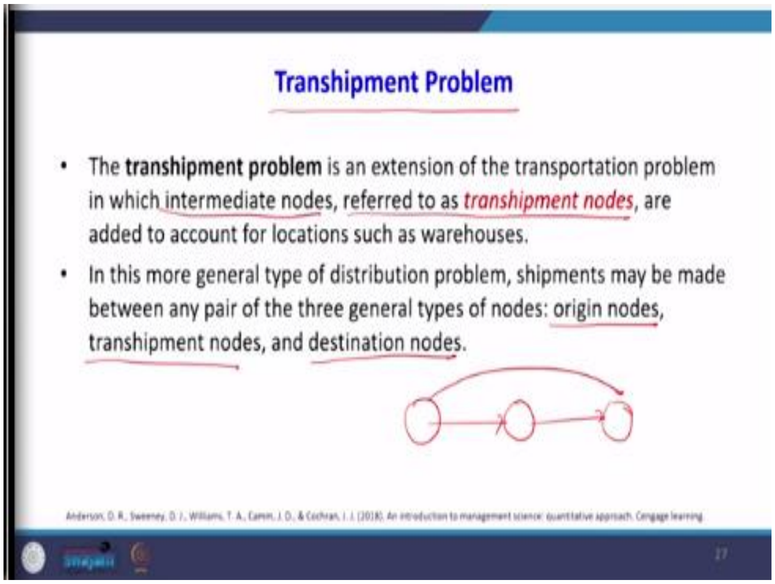
2000 units would be maintained in the optimal solution. So, this is the way to handle route capacity and root minimum.



Sometimes, there may be unacceptable routes. Finally, establishing a route from every origin to every destination may not be possible. To handle the situation, simply drop the corresponding arc, which means you drop the decision variables from the network and solve by LP. That is a way to handle an unacceptable route.

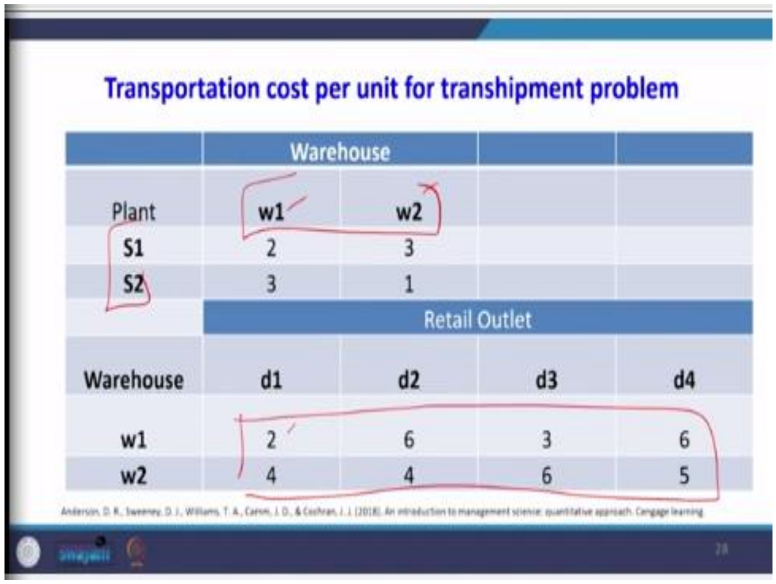


So, this is a generic form of our LP, transportation problem; what is it? It is a minimization problem; here, the supply constraints are less than or equal to type, and the demand constraints are equal to demand. This is the general LP model for transportation.



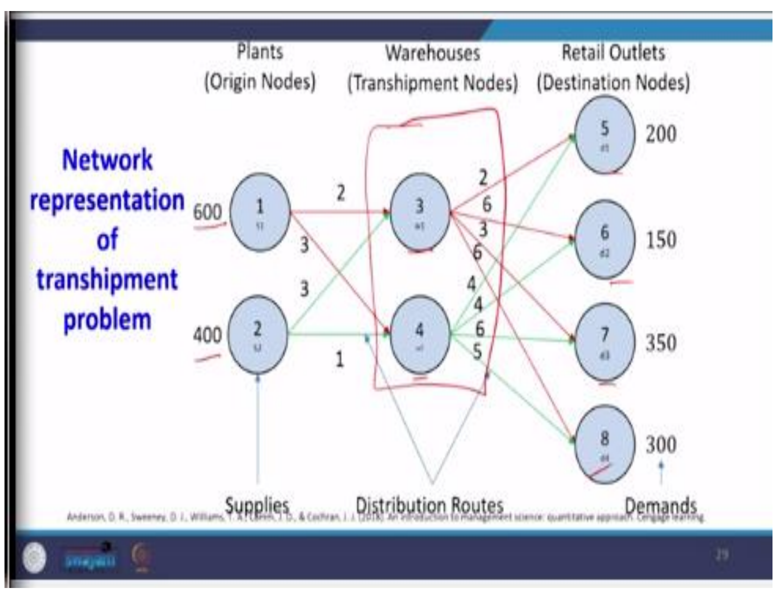
Now we will be discussing the extension of your transportation problem called a transshipment problem. What is the meaning of transshipment? Previously, there was an origin, there is a destination, and now, in between, there may be a temporary location, for example, a warehouse. So, instead of sending goods from this point to this point, first, it will reach from here, and from there, it will be sent to the demand node.

So this kind of problem is called a transmit problem. So, the transshipment problem is an extension of the transportation problem in which intermediate nodes, referred to as transshipment nodes, are added to account for locations such as warehouses. In this a more general type of distribution problem, shipments may be made between any pair of 3 general types of nodes. It can go directly from origin nodes, or shipments can be made from the transshipment node, or the shipment can be made in the destination node, 3 possibility of distribution is possible.

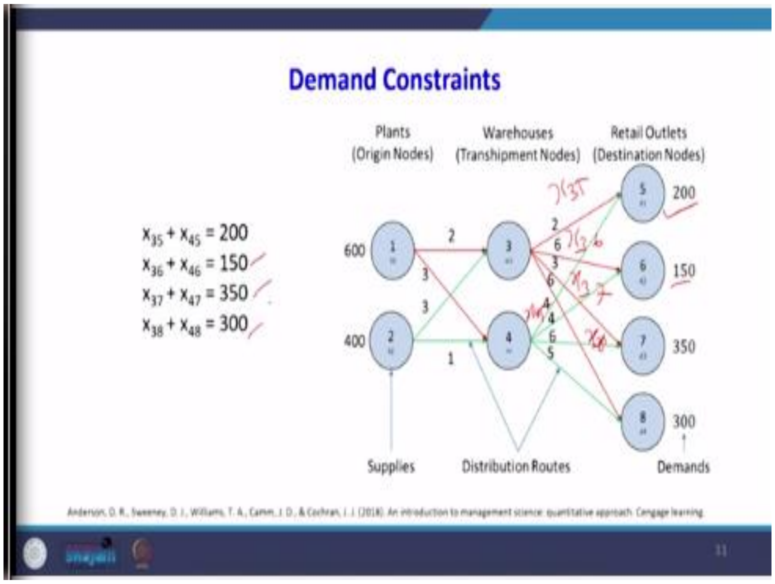


	Warehouse			
Plant	w1	w2		
S1	2	3		
S2	3	1		
	Retail Outlet			
Warehouse	d1	d2	d3	d4
w1	2	6	3	6
w2	4	4	6	5

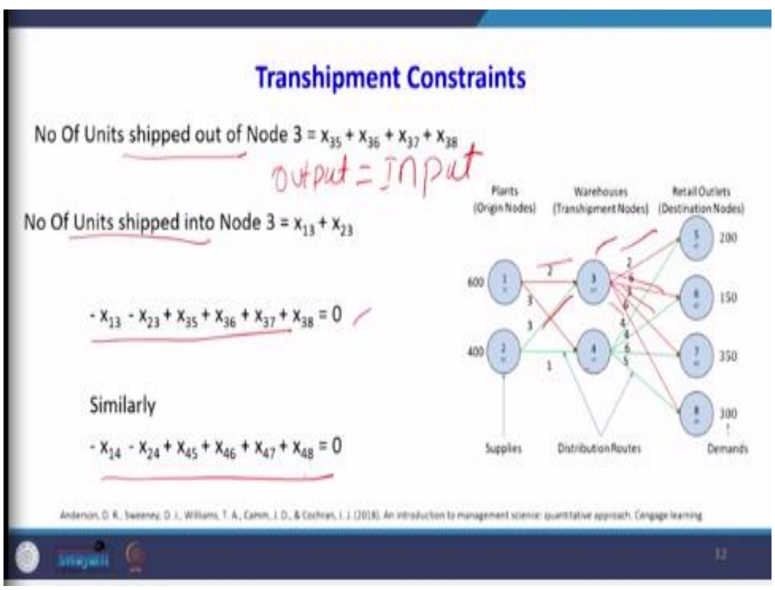
Now we will take 1 example. There is a warehouse; there are 2 warehouses there that are going to act as a transshipment node. So, for the retail outlet, the unit transportation is given, from w1 to d1, 2 unit is their transportation cost, no. The cost is 2 dollars per unit w1 to d2, d2 represents the destination nodes, 6 dollars per unit, and so on. So, this is the unit transportation cost. Between the warehouse, which is the transshipment node to the demand, and between the supply node and the transshipment node, there is another transshipment cost. So, we are going to use this transshipment node while writing the objective function.



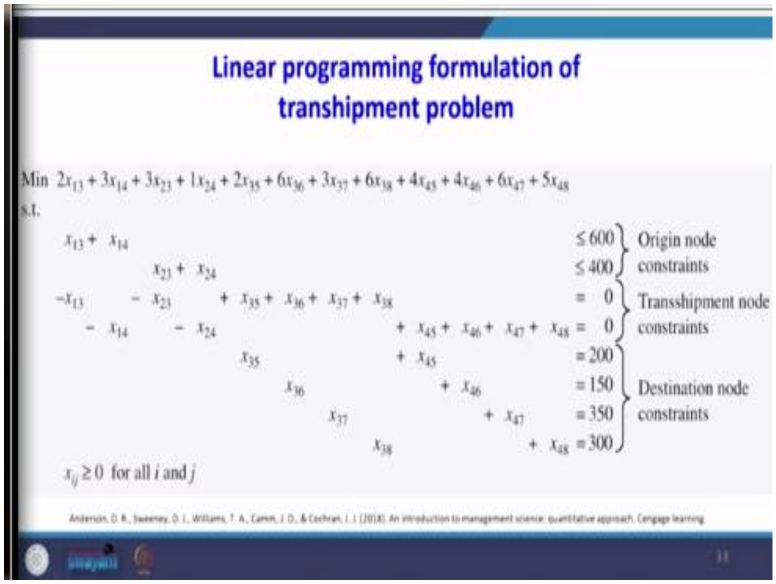
Now, this is a full picture; what is this? So, there is a supply node S1, S2, S2 there is a warehouse 1, warehouse 2, there is a d1, d2, d3, d4. So, this is your transshipment node, as usual, what we have done in the transportation problem.



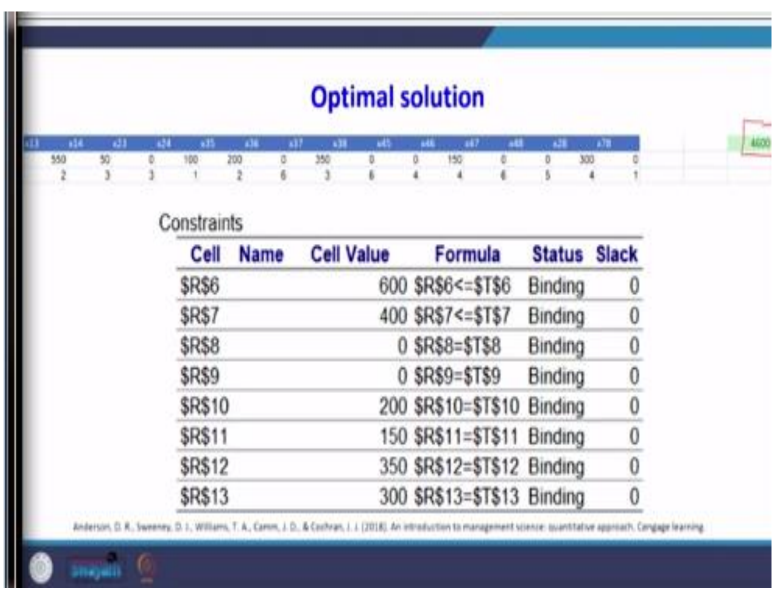
First, we will write the supply constraint; what is the supply constraint? So, X13, X14, this is 13 less than or equal to 600, X23, X24 less than or equal to 400. Then the demand constraint X 35, see this 35, 36, 37, 38. Now, when you look at here this point, what are the ways to receive the products X35? X45, here this is X45, this one X36 + X46 = 150, X37+ X47 = 350, so similarly other one, it is a demand constraint.



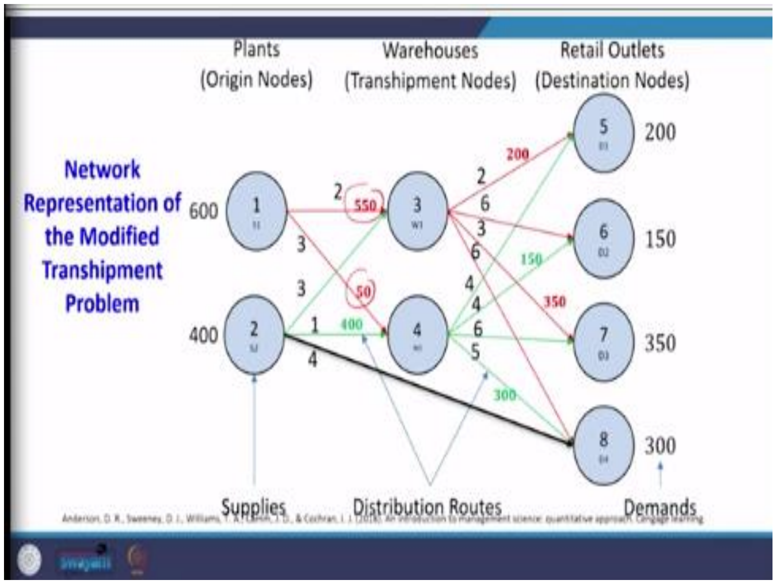
Now, this is an important additional constraint that you are adding. What is this? The number of units shipped out should be equal to the number of units shipped in. So, as we can say, the output should be equal to the input, and the number of units that are shipped out should be equal to the number of units shipped in. So, when you look at this node number of units shipped out X35, 36, 37, 38, shipped in 13, 23, so that should be equal. That is what everything brought on the lefthand side. Similarly, for node 4 also, this is written. So, the transshipment node is similar to the transportation problem. Only some additional constraint is added, which is called a transshipment constraint.



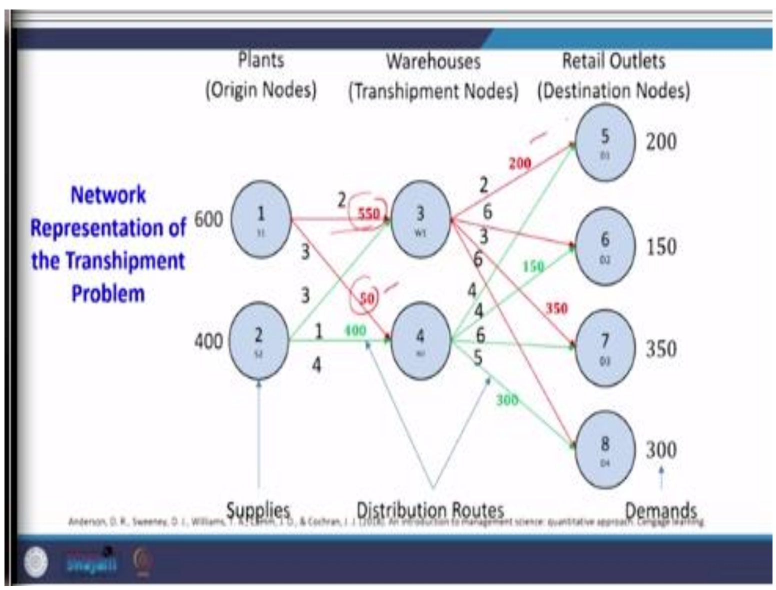
This is your complete transshipment problem. Now, we will solve this transshipment problem with the help of Excel. So, I have formulated an LP model where I have written the decision variables and objective functions, so I am going to solve it. Go to data, solver, and look at this. This is a minimization problem, and there are 2 constraints that are equal to the type in nature; those constraints are called transshipment constraints. Then, when I solved it, now I got the total transportation cost of 4600, and I got the value of how many units have to be transported from destination 1 to 3, 550, 1 to 4, 50, and so on. Now we will see the solution for our LP model.



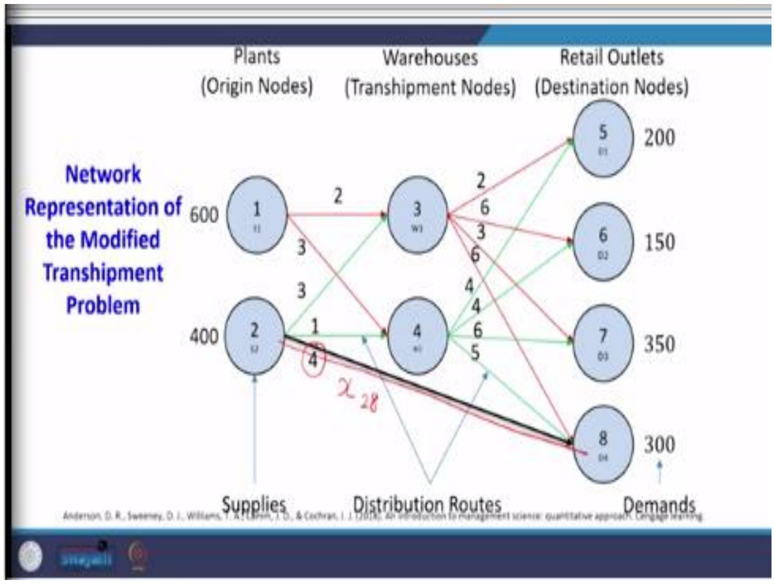
So, this was our solution; in this transshipment method, the total transportation cost is 4600. Here we are getting how much unit has to be shipped from each node to another node.



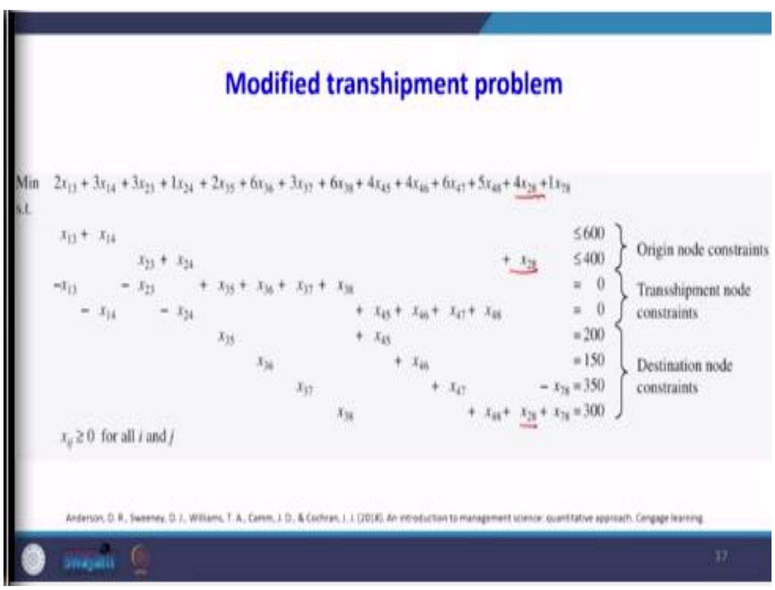
Now, here I have brought the solution, so this is 550 units have to be transported from 1 to 3, so 1 to 4 50 and so on, all the values are given.



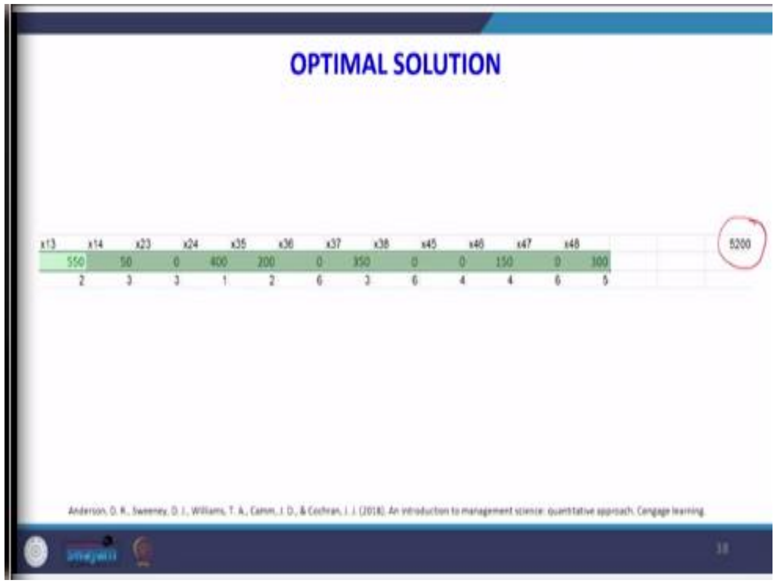
Now I have brought the solution for this transshipment problem; what does this solution say? From 1 to 3, you have to ship 550 units, 1 to 4, 50 units, 3 to 5, 200 units, and so on.



Now we look at the small modification in our transshipment problem. Suppose the company says that they want to ship certain products from source to destination 8, so this line. So, I have added a new line; there is transportation cost also there. So, when you go for a direct shipment instead of going via this transshipment node what will happen to our total transportation cost? So, what we must do in our existing problem is add a new constraint, a new variable that is x28; this x28 has to be added in every constraint and in the objective function.



So, when you resolve it, this is our modified transshipment problem. See that this X28 is added here, 28 is added here, 28 is added here, 28 is added here.



So, when you resolve it, now what is happening? Your objective function is increasing. So, when you go for direct shipment, there is an increase in transportation costs. So, in this lecture, I have explained the transportation problem, I have formulated the problem, then I have solved it with the help of a solver, then I have interpreted the result. Then we saw special cases of transportation problems. After that, I introduced an extension of transportation problems that is called a transshipment problem. That problem has also been solved with the help of Excel. In the next class we will discuss the new topic called assignment problem; thank you very much.