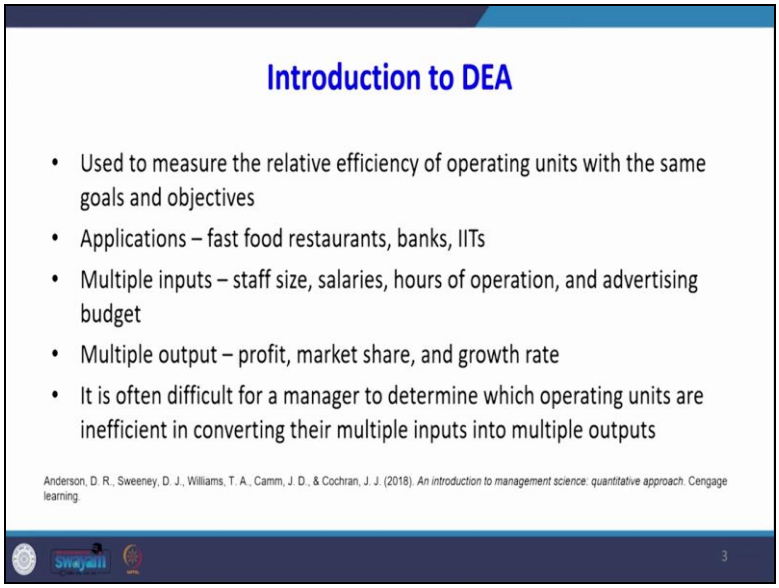
Decision Making With Spreadsheet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology-Roorkee

Lecture-14
Advanced LP Applications – Data Envelopment Analysis

Dear students, in the previous lectures and before that also we have studied the application of linear programming problems with the help of excel in marketing, finance and operations. In this area, we have had some problems, and then you solved them. Now we are going to use some advanced linear programming applications in business. So, today we are going to teach data envelopment analysis with the help of linear programming problems.

So, the agenda for this lecture is data enveloped analysis that we will use LP for solving this data environment analysis.

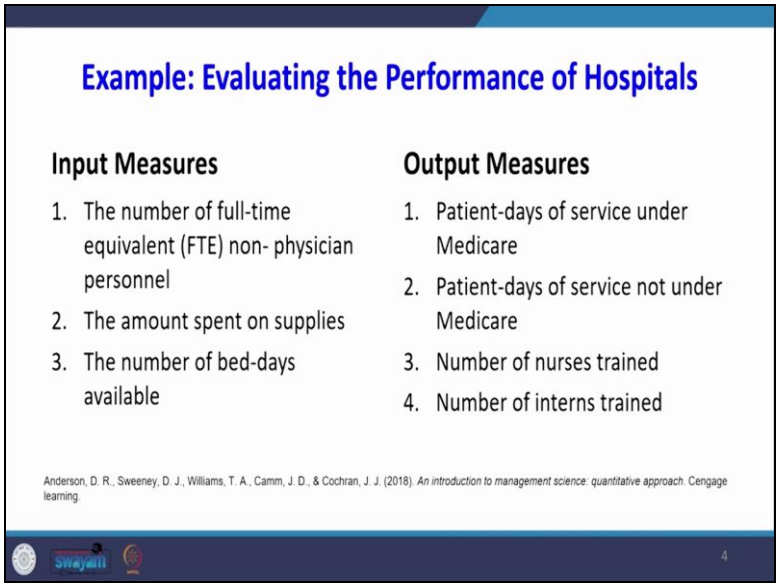


First, you should know what this DEA is. Where is it used? How does this DEA method data envelopment analysis have a connection with the LP? So, this DEA is used to measure the relative efficiency of operating units with the same goals and objectives. Assume that in a city, there are, say, different restaurants and the same company name banks say even can be used for comparing the performance of different IITs. So, where the NIRF is used like this, this DEA analysis can be used for comparing the performance of IITs, IIM, or the same within IIT different departments performance can be compared this is relative performance.

Here is another advantage of this DEA is that we can consider multiple inputs and multiple outputs. For example, if you take a restaurant. So, in a city there are five restaurants, say a restaurant chain for maybe restaurant one in different parts of the city: 1, 2, 3, 4, 5. The operation manager, so, he is willing to compare the relative performance. So, that relative performance can be done with the help of this data environment analysis.

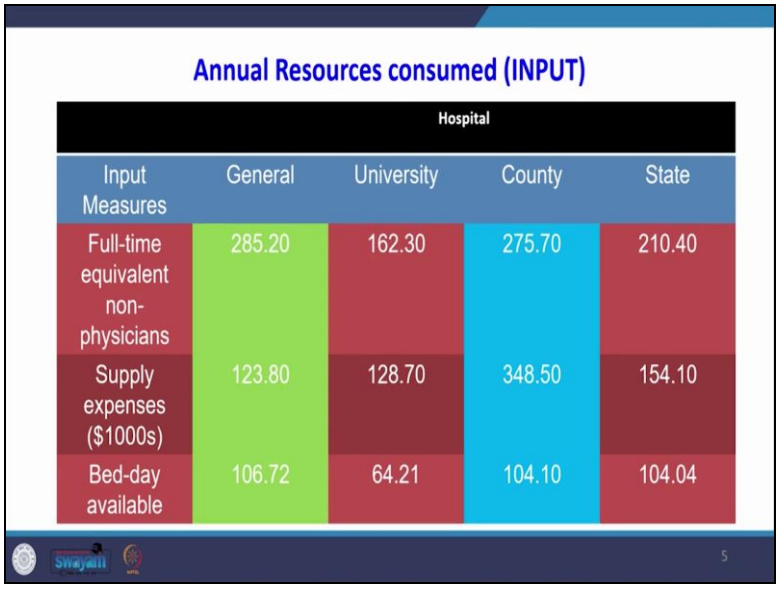
There are many research papers that will use this DEA. For example, if it is a restaurant, whatever the multiple inputs are possible, the staff size is one of the input salaries, hours of operations, and advertising budget. Obviously, when the staff size is larger, the outcome output will be more salary, which is the indirect way of representing the inputs. So, in a restaurant context, what are the multiple output profit market share growth rates? These are the multiple outputs.

So, it is often difficult for a manager to determine which operating units are inefficient in converting their multiple inputs into multiple outputs. So, whenever this kind of problem arises, the DEA, which is a data environment analysis, is the most suitable method.



Now, I have a sample problem. This example is taken from the book by Anderson et al. So, evaluating the performance of hospitals, we are going to consider four hospitals. Out of these four hospitals, we are going to measure relative efficiency. So, which hospital is efficient and which hospital is inefficient? So, we are going to consider three inputs: one is the number of full-time equivalent physician personnel, the amount spent on supplies, and the number of bed days available.

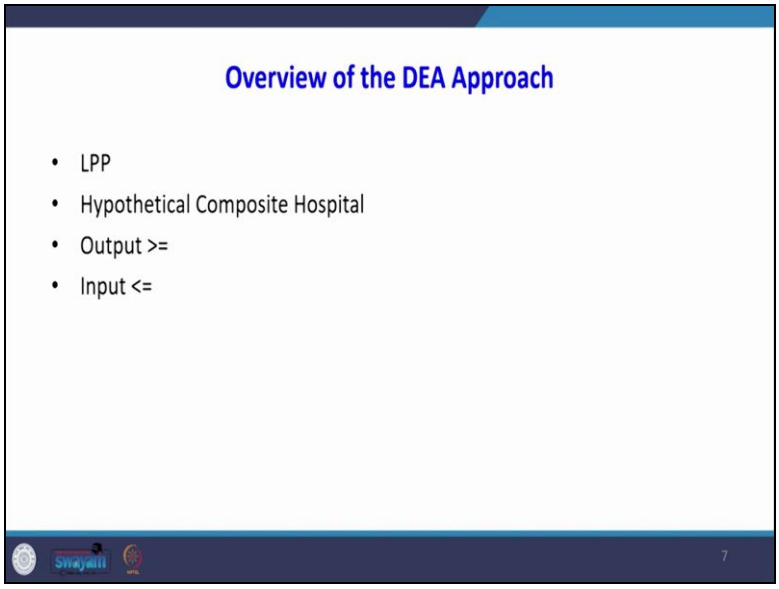
These are inputs output. There are four outputs: patient days of service under Medicare, patient days of service not under Medicare, number of nurses trained, and number of interns trained, so three inputs and four objectives.



This table shows the input. So, there are four hospitals general hospital, university hospital, county hospital, state hospital. You see, in the general hospital so, 285.20 is the full-time equivalent for non-physicians, 123.80. The values in terms of thousands are the supply expenses. Bed days available okay when there are bid days available means when there are a greater number of beds there is a greater number of bid days available 106.72.

Like that, there are four hospitals. So, when you look at this county hospital, for example, here this is the highest value, yes, 348.5010. So, now we are going to measure the efficiency of the county hospitals when compared to the other three hospitals. So, these are the inputs. Similarly, we can consider the output of these four hospitals. What are the outputs? Medicare patient days in terms of thousands is the annual input and annual output. You should be very careful with the units because these are the annual amount of input resources used. This is the annual amount of output produced.

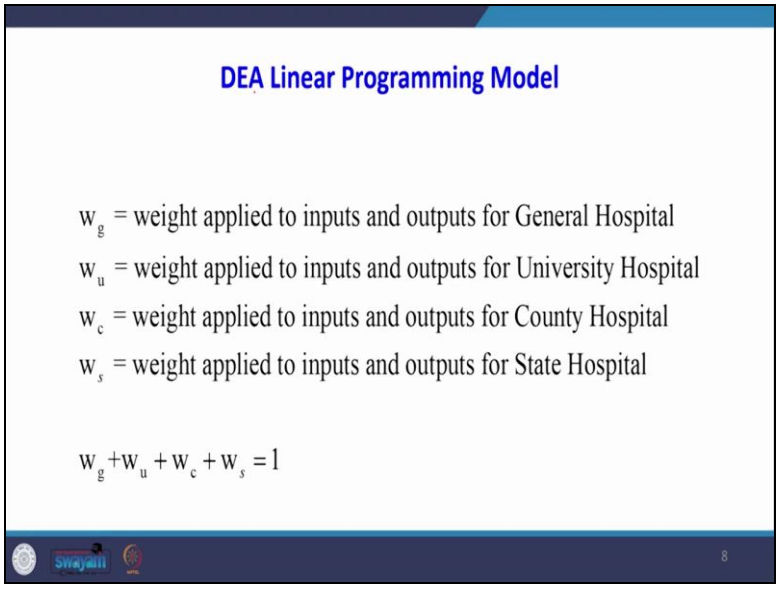
So, non-Medicare patient days 43.10 in terms of thousands, nurses trained 25.3, and interns trained 41 like that for there are four hospitals.



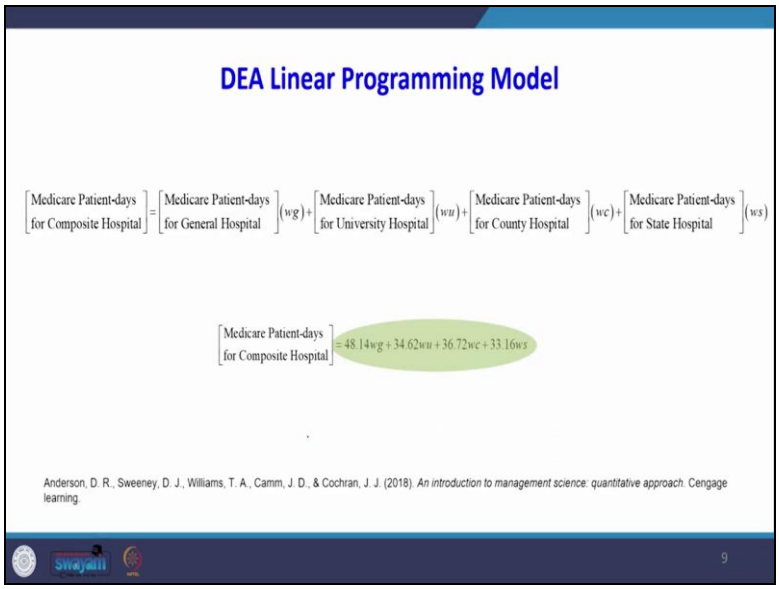
Now, we will go to the overview of the data envelopment analysis approach. So, this can be solved with the help of linear programming problems. So, what we are going to consider is we are going to consider a hypothetical composite hospital. This hospital has we are going to make some assumptions. What are the assumptions? The output will always be greater than the individual hospital's.

So, this is a composite hospital. So, what we are assuming about this composite hospital is that it is going to always produce more output than the individual hospitals, which is why this output is greater than or equal to. The second assumption is that it is going to take fewer inputs. So, inputs are less numerous than in individual hospitals. So, we are going to formulate a linear programming problem for only composite hospitals.

By looking at the efficiency or the performance of composite hospitals, we are going to compare the efficiency of the other four hospitals.

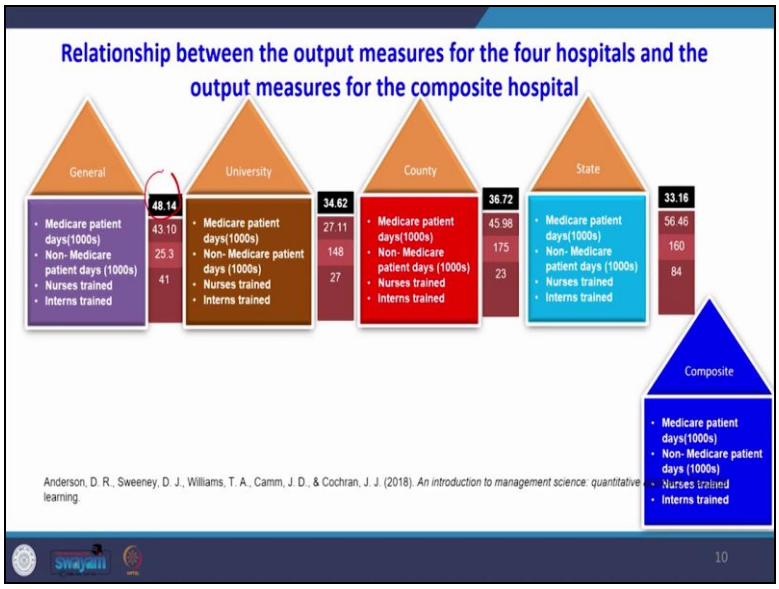


So, linear programming models are going to consider different weights. So, Wg is the weight applied to inputs and outputs for a general hospital, Wu is the weight applied to inputs and outputs for a university hospital, Wc is the weight applied to the inputs and output for a county hospital, W s weight is applied to inputs and output for a state hospital. Since its weight the sum should be one. This is one of the constraints.

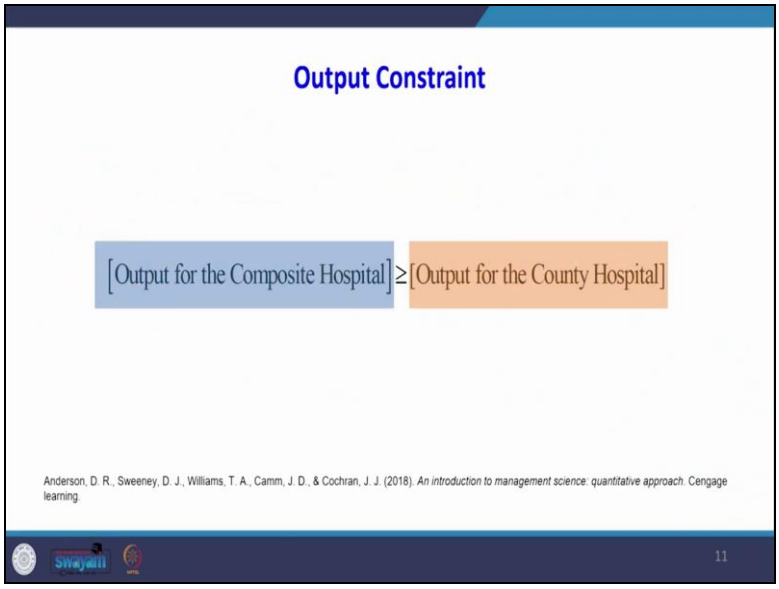


Now, we are going to take only one output, for example, Medicare patient days for composite hospitals. So, I am going to find out the Medicare patient days for composite hospitals and how I am calculating them. So, Medicare patient days for a general hospital multiplied by its weight, Medicare patient days for a university hospital multiplied by its weight, Medicare patient days for a county hospital multiplied by its weight, and Medicare patient for a state hospital multiplied by its weight.

So, the Medicare patient days for the composite hospital is 41.14 Wg + 34.62 Wu + 36.72 Wc + 33.16 Ws where I got this I got from this one.

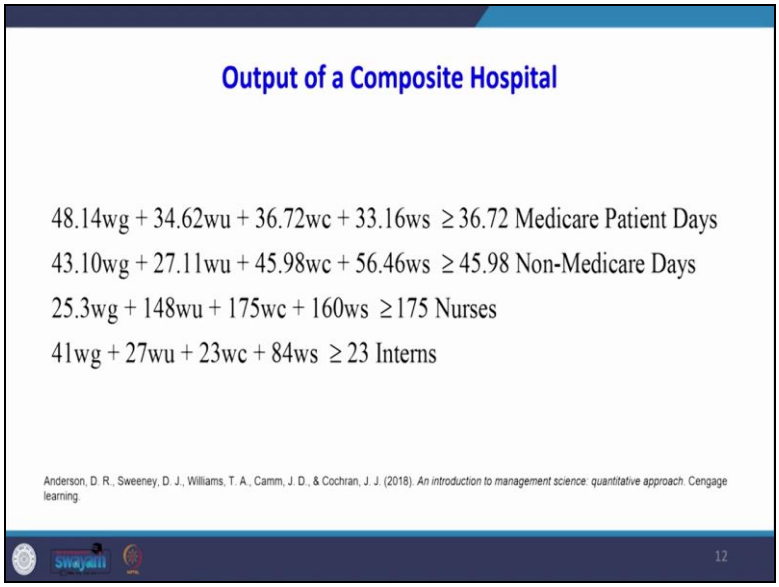


You see, this is output. There are four individual hospitals. So, here for the composite hospital, we brought it here. So, the output output of Medicare patient days for the composite hospital is this one, 48.14 Wg + 34.62 Wu + 36.72 Wc + 33.16 Ws. Like this, there are four outputs we can write constraint. We can write four outputs for the composite hospital in a similar way.



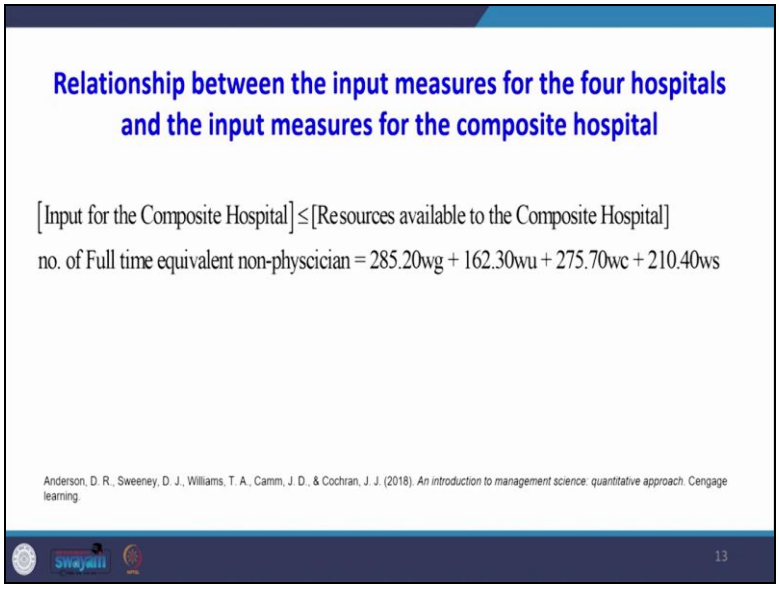
So, we know that the output of the composite hospital is greater than the output of the county hospital. So, what are we going to do? We are going to compare and go back to the previous slides. We are going to compare the efficiency of county hospitals with respect to the efficiency of the composite hospitals. Now, the problem is we are going to compare two entities: one is county hospital versus composite hospital.

So, we are assuming the output of the composite hospital is greater than that because the composite hospital is more efficient is greater than the output for the county hospital.



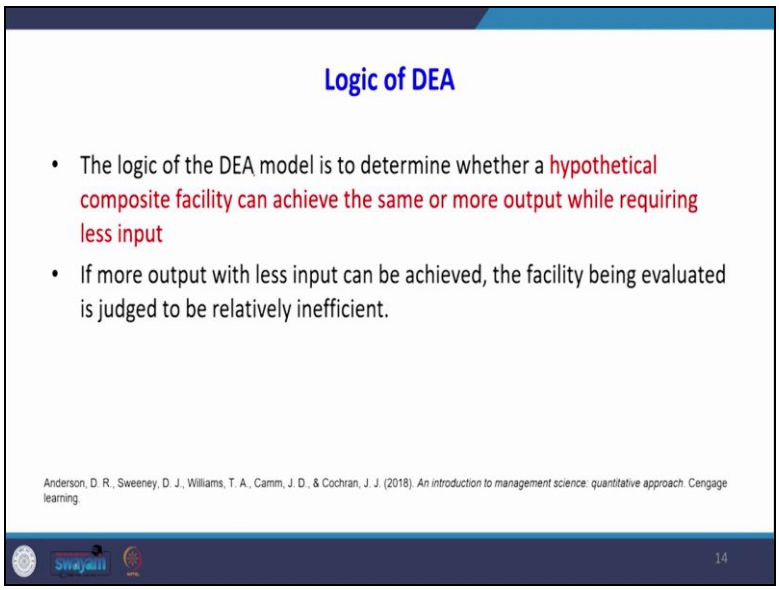
So, I have written 4 outputs for the composite hospital. So, this is 1, 2, 3, 4 most important here is this is greater than equal to. So, how did I get this 36.72? This is the output of Medicare patient days for county hospitals. So, I will go back to this one, 36.42. So, I will go back and see this is 36.42. So, as I am writing this, the output of Medicare patient days is greater than the output of the county hospital for one variable, which is Medicare patient days. That is why I have written like this.

For the second output the values are 36.72, 45.98, 175, and 23. These are only for the county hospitals. The left-hand side value is the output for the composite hospital.



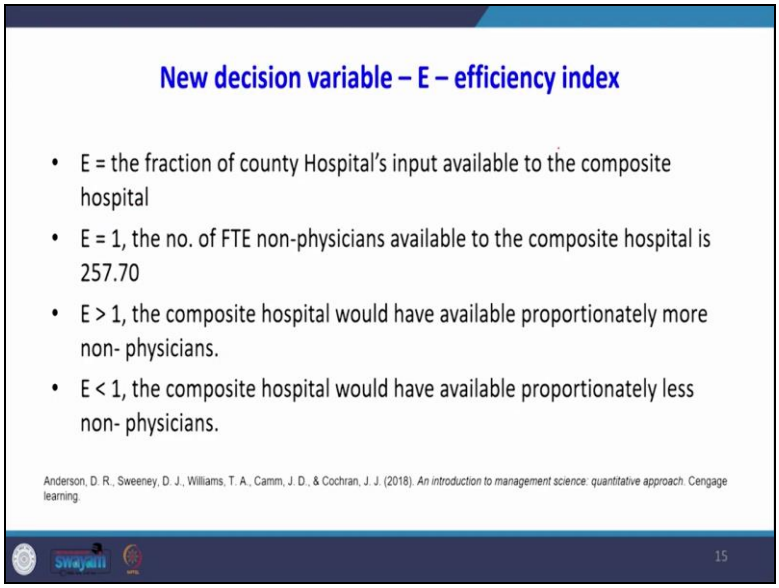
Now I am going to create another set of constraints for the inputs. So, the relationship between input measures for the four hospitals and the input measures for the composite hospital. As I told you, the input for the composite hospital is less than the resources available to the composite hospital. So, for example, the number of full-time equivalent nonphysicians for the composite hospital.

So, 285.20 Wg 162.30 + Wu + 275.70 Wc + 210.40 Ws this is the resources. This is the composite okay input.



So, the logic of DEA is to determine whether a hypothetical composite facility can achieve the same or more output while requiring less input. There are two things we are comparing to; on one side, this is a composite hospital. On the other hand, we have the county hospital. So, what we are saying is whether this hypothetical composite facility can achieve the same or more output while requiring less input. So, if this fellow composite hospital is achieving more output than the county hospital by consuming fewer resources, we can say this composite hospital is efficient.

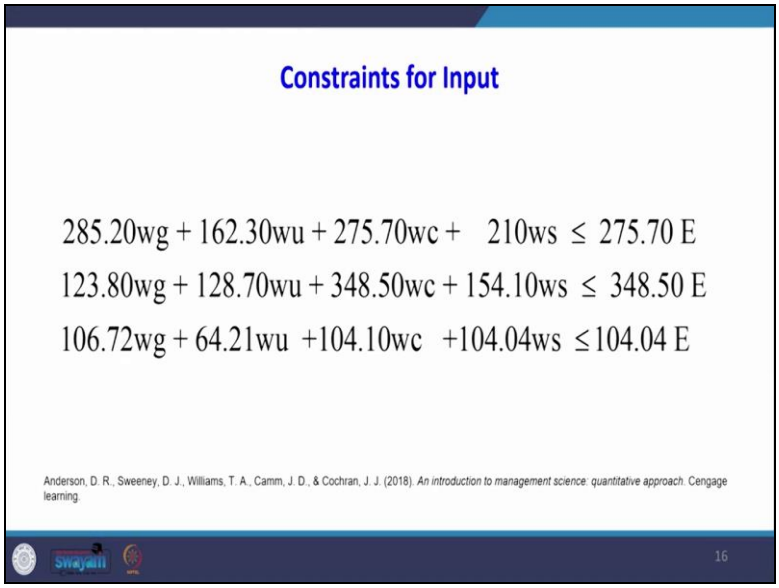
What is this? The composite hospital is going to produce more output by using fewer resources than the county hospital. Then, the composite person is more efficient. So, if more output with less input can be achieved, the facility being evaluated is judged to be relatively inefficient. So, county is inefficient but the composite is efficient because this fellow is achieving more output with a lesser input.



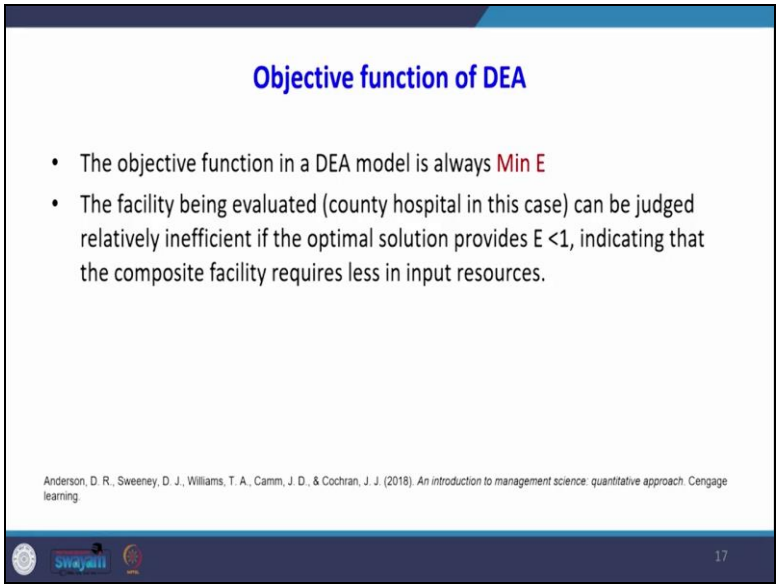
To consider that composite hospitals are going to consume more and produce more output by consuming fewer resources, we are going to reduce a new decision variable, E. The E represents the efficiency index. Value of E there are three possibilities. If E is equal to 1, what is the meaning of the fraction of county hospital input available to the composite hospital? That means the inputs utilized by the composite hospital and county hospital is the same when E is equal to 1.

So, when E is equal to 1, the total number of full-time employees and non-physicians available to the composite hospital is 257.70, which will go back to where we get 257.70. We are talking about inputting this one, which means that the composite hospital is going to consume fewer resources than the resources of the county hospital. That is why the return to 273.70 will go back again. Yes, both are consuming an equal amount of resources when E equals 1.

When E is greater than 1, the composite hospital would have available proportionately more non-physicians, which means that the composite hospital is consuming more resources than the county hospital when E is greater than 1. When E is less than one, the compositional hospital would have available proportionately fewer non-physicians, but we have assumed that previously, that composite less hospital is going to consume fewer resources than the county hospital. So, we have to minimize this E.

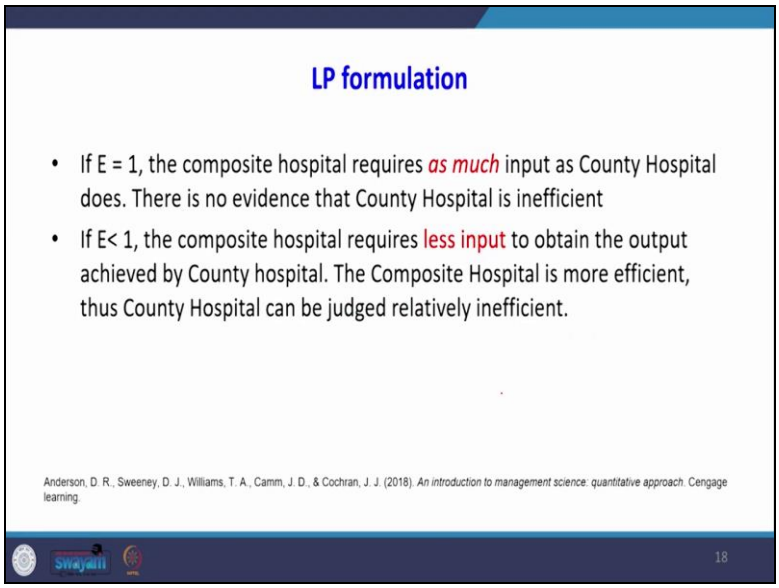


Now look at the constraints for input. So, this is the composite constraint. So, the E represents the percentage of resources utilized by the composite hospitals if the value E is equal to 1. So, the resources utilized by composite hospitals and county hospitals are the same when E is greater than one. So, the composite hospital consumes more resources than the county hospital when E is less than 1, and the composite hospital consumes fewer resources than the county hospital. We know that in the beginning I was saying that composite hospital is more efficient.

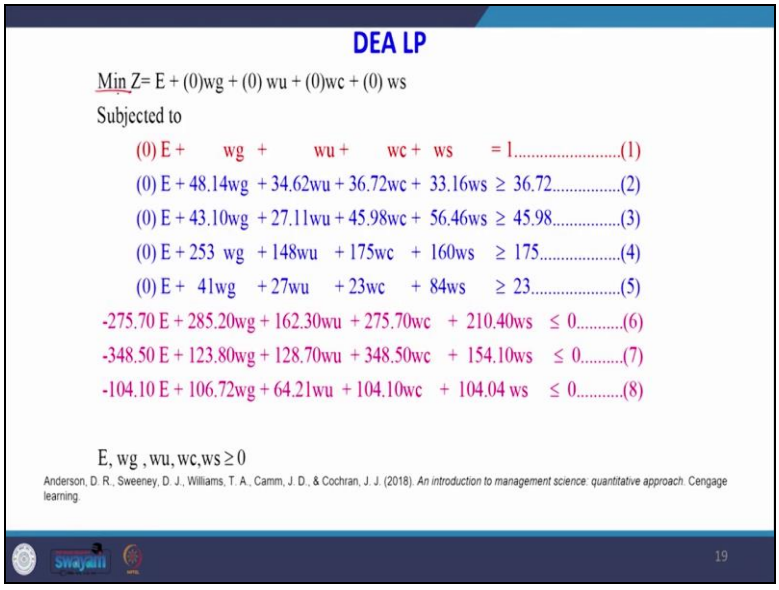


So, here, the objective function in a DEA model always minimizes E. What is the meaning of this minimizing E? The facility being evaluated, for example, a county hospital in this case, can be judged relatively inefficient if the optimal solution provides E less than 1 meaning of E less than one indicating that the composite facility requires fewer input resources. So, what

we have to do is minimize the E if the value of E is less than 1, which implies that the composite hospital consumes fewer resources than the county hospital.



Now we will go for LP formulation. If E is equal to 1, the composite hospital requires as much input as the county hospital does. So, there is no evidence that the county hospital is inefficient, but if E is less than one, the composite hospital requires less input to obtain the output achieved by the county hospital. So, the composite hospital is more efficient; thus, county hospitals can be judged relatively inefficient. So, in the data environment analysis, we are going to minimize the value of E.



Yes, that is minimizing the E. What is the constraint? This is the weightage constraint. These four constraints for output are because we are solving only for composite hospitals. You remember this LP formulation is for composite hospitals. So, we know that a composite

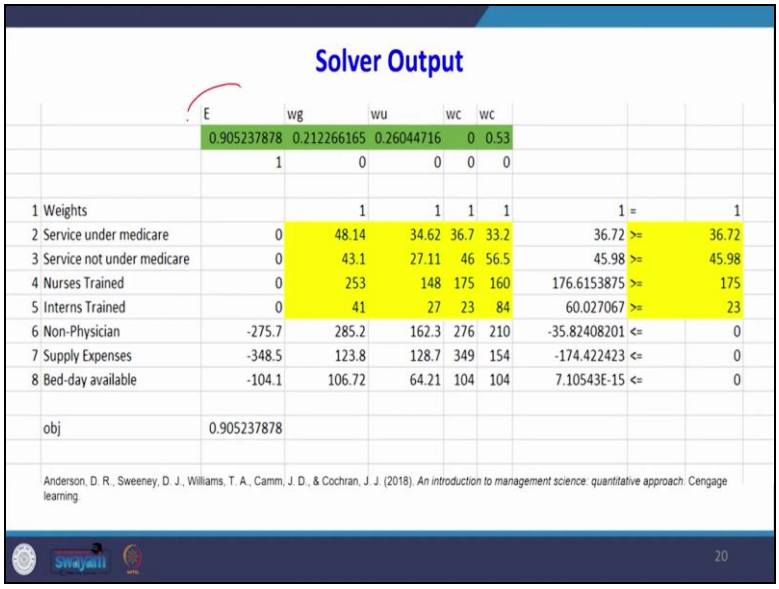
hospital is more efficient. So, they are going to produce more output than the county hospital, which is why it is greater than the output constraint for the input constraints.

So, only a fraction of the input resources of the county hospital is going to be considered by the composite hospital. So, that fraction is written as E that E we know that it is going to be less than 1 or equal to 1. So, I readjusted it, and I brought this E on the left-hand side. So, here, it is less than or equal to type. So, this is our complete linear programming problem for a composite hospital.

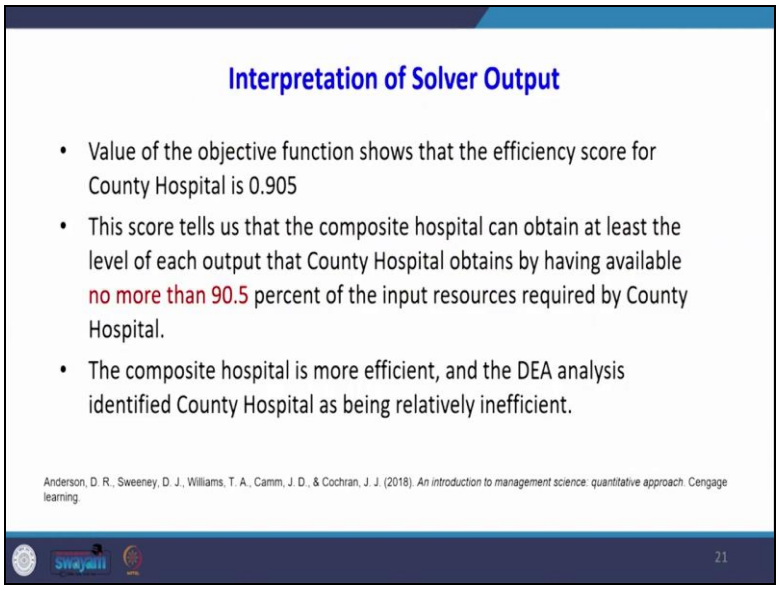
So, now I am going to solve this problem and find out the value of E. Then I go to interpret, and now I will go to excel.
Now I am going to use a solver to get the output for our data envelopment analysis. So, here, for example, in D4 to H4 is our value of decision variables in our objective function, only E is there. So, only then one should be there. Then I have written that there are 8 constraints in I7 to I14, and I have written the resources utilized. So, from K7 to K14, I have written the right-hand side. So, the value of the objective function is D16, as usual as per the constraint.

So, here, this problem is a minimization problem. Now I will go to data, then go to the solver. Then you will see that it is a minimization problem changing cell is, as I mentioned, there is one equal to type constraint. There are three less than or equal to type constraints and four greater than equal to type constraints. So, now I am going to solve it. I need an answer and am sensitive to analysis limits; click it. Now I have the answer report, and their sensitivity report is there. Now, I will go to sheet 1 where sheet 1 where I can see the value of E. The value of E is 0.905.

Now, I am going to take the output of this Excel sheet into the presentations, and from there, I am going to interpret the output.



The value of E is 0.90 when you look at here; this is both this and this is equal, but you see that there is it is a greater than or equal to types. So, it is a surplus variable here. So, this is the output.

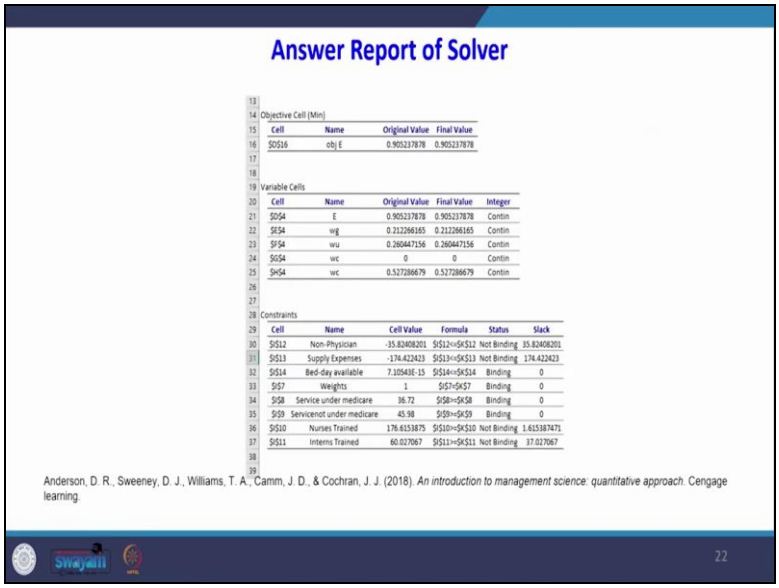


Now, we will interpret the solver output. So, the value of the objective function shows that the efficiency score for the county hospital is 0.905. Remember, we have solved a hypothetical aspect hypothetical composite hospital, but this is the efficiency of the county hospital. What is the meaning of this 0.905? This score tells us that the composite hospital can obtain at least the level of each output that the county hospital obtains by having available no more than 90.5% of the input resources.

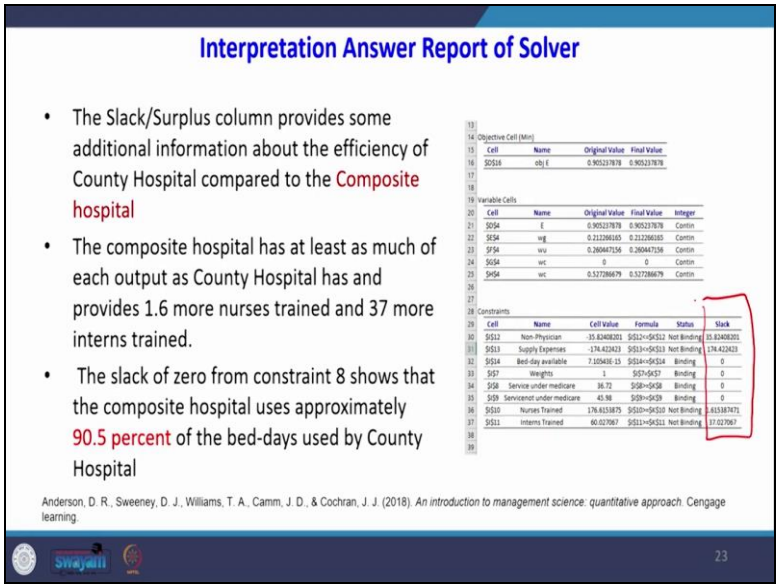
So, this fellow composite hospital right, is going to use less than 90.5% of resources consumed by the county hospital, which means this fellow composite hospital is consuming

fewer resources. So, the composite hospital is more efficient if this fellow is composite more efficient. Obviously, the county hospital is inefficient. So, what we are inferring is that the composite hospital is more efficient because it consumes fewer resources, only 90.5% of the county hospitals.

The DEA analysis identified county hospitals as being relatively inefficient. So, now we have done only one hospital like this, the same procedure has to be continued for all other four hospitals, then we can see the relative performance of all four hospitals that you should remember.



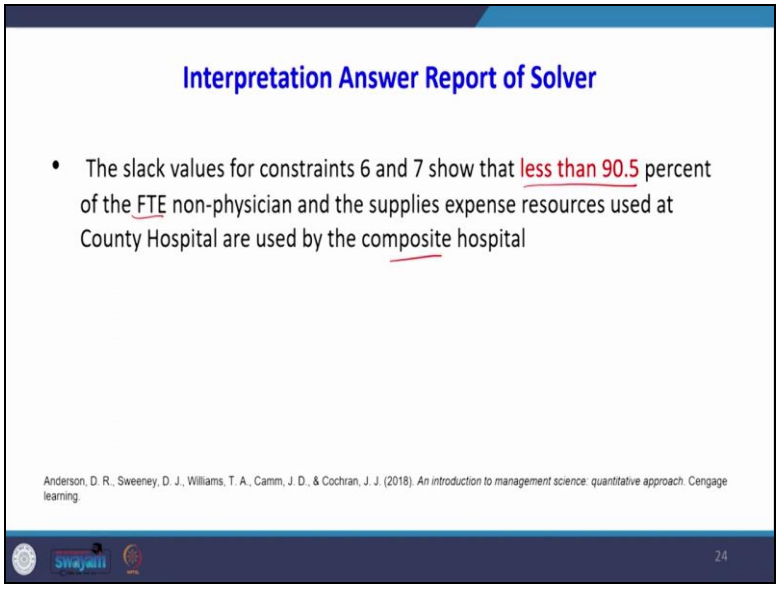
Now, there are some Slack variables. Oh, the Slack variable is there. There are some surplus variables there. The value of E is 0.9. So, now I will interpret this. So, the slack and surplus column provides some additional information about the efficiency of the county hospital compared to the composite hospital in this column.



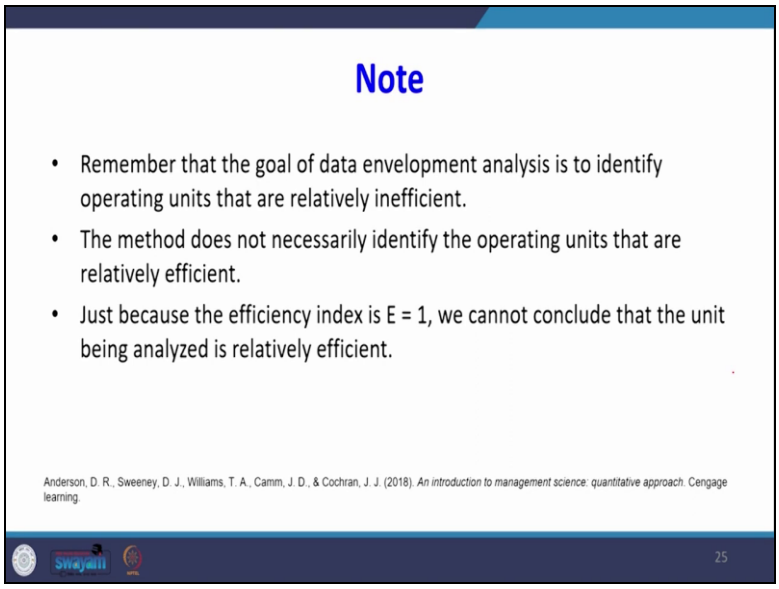
The composite hospital has at least as much as sorry. The composite hospital has at least as much of each output county hospital and provides 1.6 more nurses trained and 37 more interns trained because this is a surplus variable. Look at these two. So, it is surplus variables surplus means the constraint is greater than equal to type. So, the composite hospital is training more nurses, 1.6 more nurses, and 37 more, in turn, more interns, but by consuming only 90.5% of the resources of the county hospital.

So, the slack of 0 for constraint 8 shows that the composite hospital uses approximately 90.5% of bed days used by the county hospital. So, the bed days available here are here. So, what does slack zero mean? So, the composite hospital utilizes only 90.5% of the bed days available for the county hospital, and it achieves equal output to the county hospital.

So, we say composite is more efficient, and that is why county become inefficient. That is the meaning of this 0 slack.



Now, the slack variable for constraints 6 and 7 shows less than 90.5% of full-time employees of non-physicians, and supplies expenses resources used at the county hospital are used by the composite hospital. So, what is the meaning here that there is a slack variable? So, it says that the composite hospital has utilized less than 90.5% of the resources of the county hospital, which is the meaning of this slack. Slack means that it is unutilized resources. The county hospital did not utilize this many resources.



The very important point you should remember is that the goal of data envelopment analysis is to identify the operating units that are relatively inefficient. This method does not necessarily identify the operating units that are relatively efficient. So, what is the meaning of just because the efficiency index E equals 1, we cannot conclude that the unit being analyzed is relatively efficient? So, the focus here is more on identifying which unit unit means decision-making units they call DMU decision-making units are inefficient.

So, in this lecture, I have explained the application of data-enveloped analysis. So, I have taken on the application that compares the relative efficiency of four hospitals. So, that problem I formulated with the help of a linear programming problem then I have solved it then I found the relative efficiency of that hospital; thank you very much.