Decision Making With Spreadsheet
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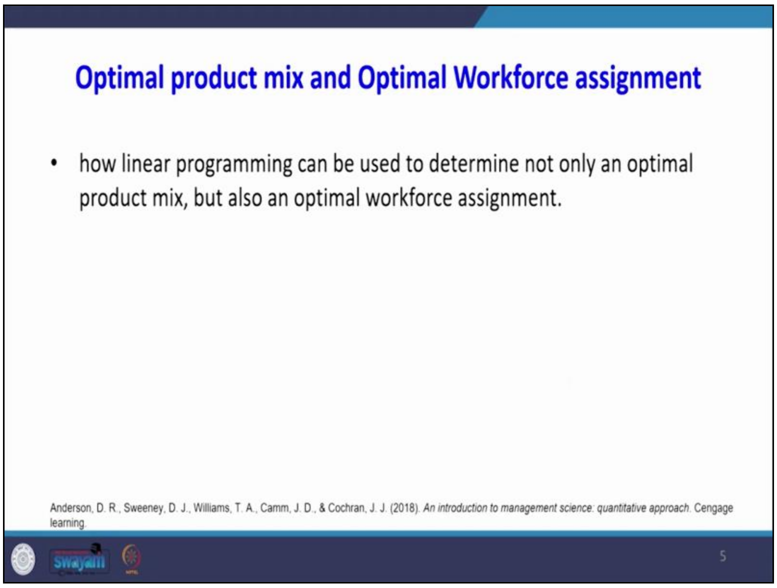
Lecture-13
LPP Applications in Operations - 3

Dear students, in the previous lecture, I explained how to use linear programming problems to make decisions in operations management. In this lecture, I will take another application of linear programming in workforce allocation.

So, the agenda for this lecture is linear programming problems in workforce allocation.

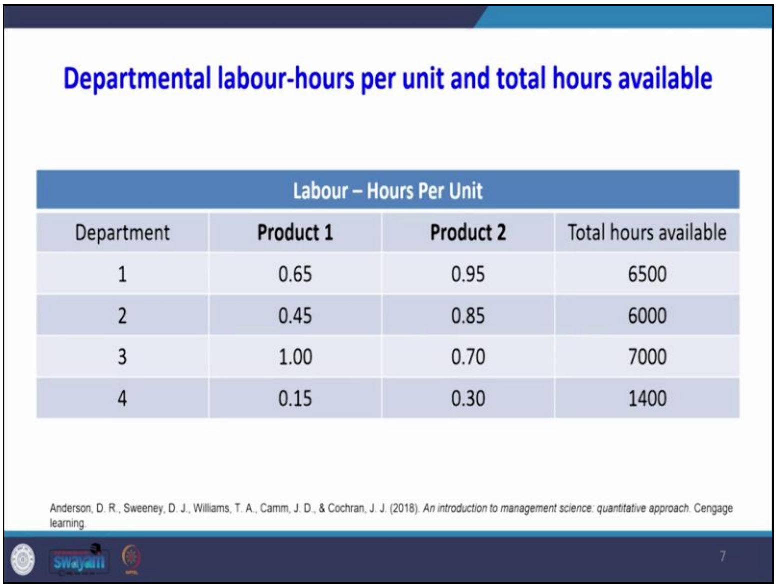
The reference for this problem is from the book Anderson et al. So, first, what is workforce assignment? So, workforce assignment problems frequently occur when the production managers must make the decision involving staffing requirements for a given planning period. What is the meaning of workforce how to allocate the workforce into various jobs that is the meaning of workforce allocation.

So, workforce assignments often have some flexibility, and at least some personnel can be assigned to more than one department or work center. Such is the case when employees have been cross trained in 2 or more jobs, for instance, when sales personnel can be transferred between stores.

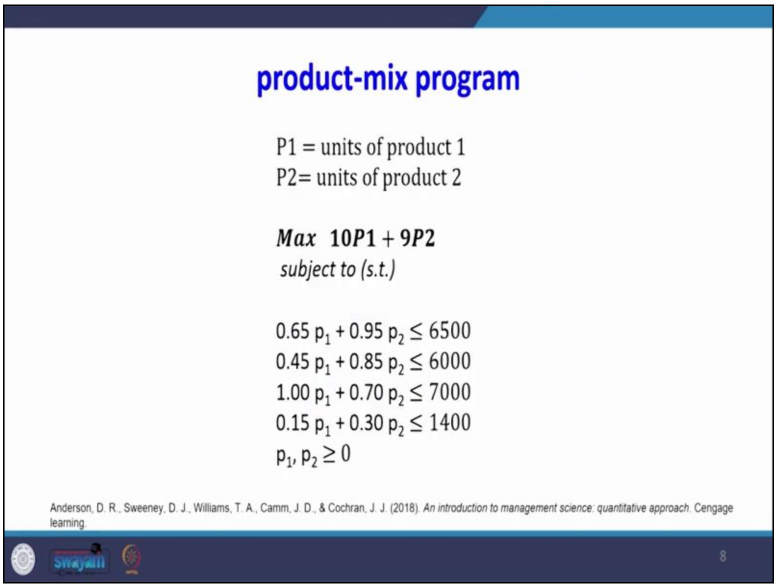


I am going to take one problem: how an optimal product mix problem can be used for assigning workforce allocation. So, the problem that I have taken is how linear programming problems can be used to determine not only the optimal product mix but also an optimal workforce assignment. This is a hypothetical problem: a company produces 2 products with a contribution to profit per unit of 10 dollars and 9 dollars, respectively.

The labor requirement per unit produced and the total hours of labor available from personnel assigned to each of the 4 departments are shown in this table.



So, we are going to manufacture 2 products, product one and product 2 we. We are going to use 4 departments. There is a restriction on the total number of hours available for each department, and this is going to be constrained.



So, if you formulate that problem, so, we are going to take decision variables P 1 and P 2.

$0.65\ p1 + 0.95\ p2 \leq 6500$

$0.45\ p1 + 0.85\ p2 \leq 6000$

$1.00\ p1 + 0.70\ p2 \leq 7000$

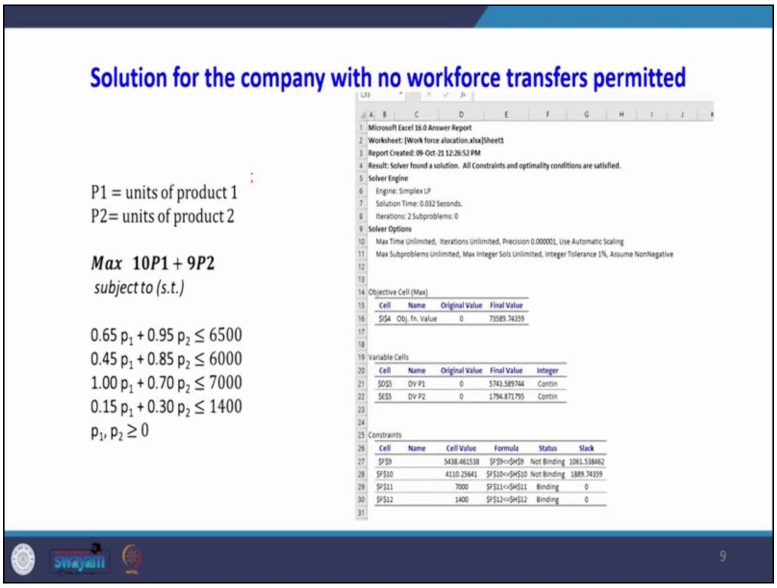
$0.15\ p1 + 0.30\ p2 \leq 1400$

$p1, p2 \geq 0$

So, I have formulated this constraint from the table given in the previous slide. I am going to solve this problem on the left-hand side. I am going to get the result.

Now I have brought the problem into the Excel document. I am going to solve it with the help of a solver. So, on the right-hand side, I have the problem on the left-hand side. If you look at that, I have formulated the problem as you show our usual procedure. So, P, what is the P that is the D5 is the value of detection variable one that is a P1E5 that is the changing cells ok there is a coefficient objective function there is a constraint less than or equal to, and there is a right-hand side also there are the limitations.

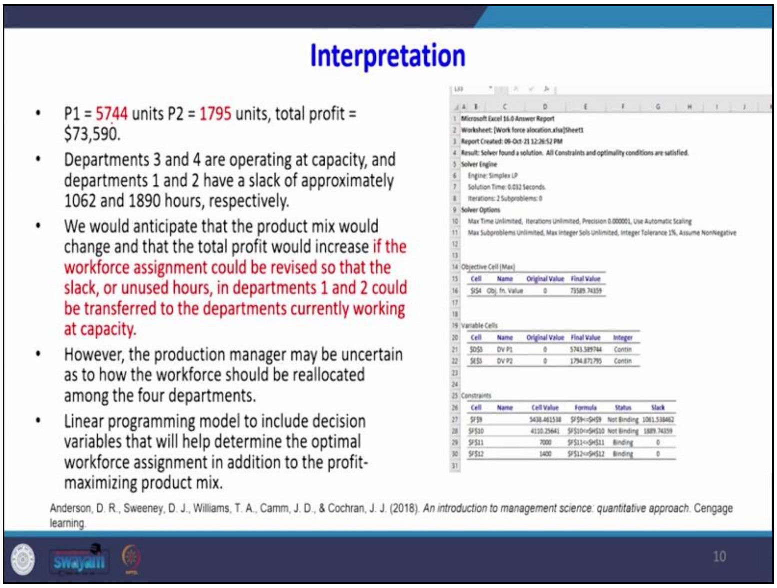
Now, I am going to solve our data and go to the solver. So, already I have formulated. So, this is a maximization type please solve. So, select this answer sent sensitivity limits and press ok answer report. So, the answer says that the value of the objective function is 73589.74 dollars. We got the value for P1, which is 5743.58. For P2, it is 1794. So, I have taken this screenshot of this solution, which is our answer report and sensitivity report, in the PPT. So, I will explain the interpretation of this output in our PPT in the presentation.



So, on the left-hand side, there is a maximization of 10P1 + 9P2.

So, here on the right-hand side, I have brought the screenshot of our solver solution. So, this is our final value. Here, we are getting the values of P1 and P2. Here we can look at the problem here how we are going to use you see that there are 2 constraints there is these are less than or equal to type. So, it is a slack variable. So, this Slack represents unutilized resources.

So, in the coming slides, I am going to explain how to use these unutilized resources and how we interpret this.

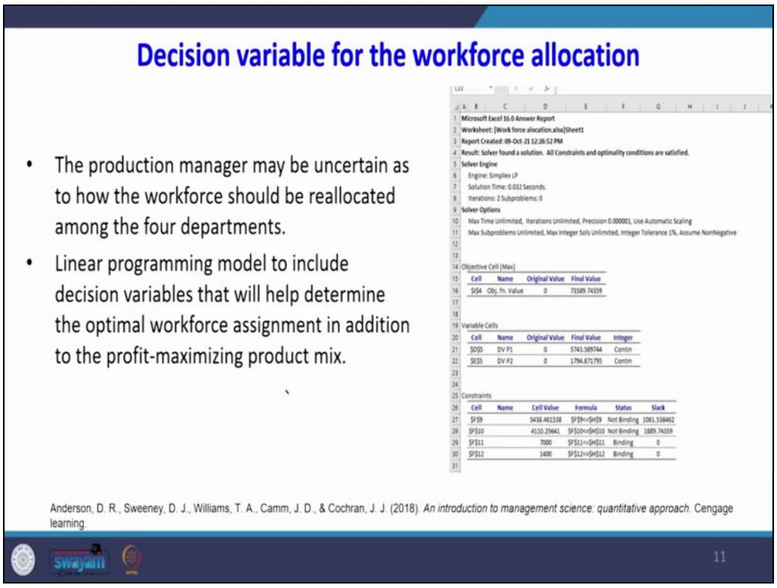


We know the value of P1 is 5744. I have rounded it, but we can not round it off because there may be an issue in sensitivity. at present, for only our convenience, I have rounded it. But technically, rounding off will lead to an infeasible solution. Later, we will be studying when we are studying integer programming. So, rounding off is not possible, but here, only for convenience, I am only, for example, rounded.

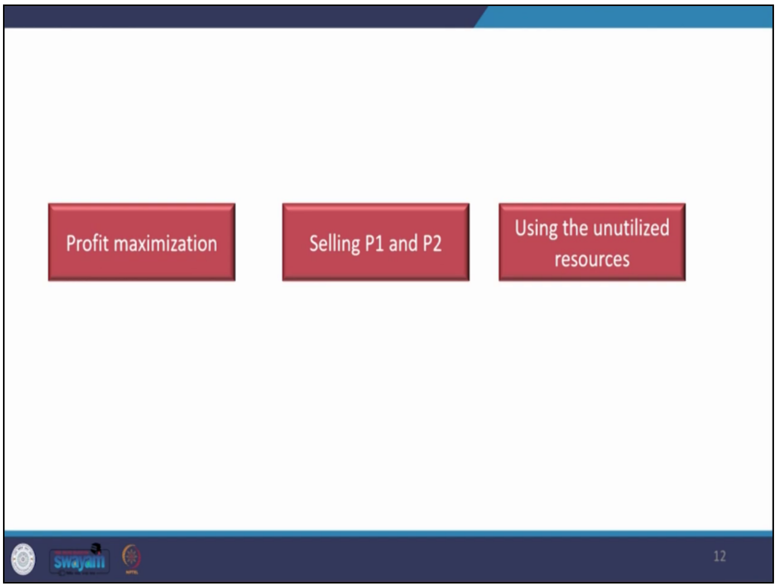
So, the value of the total profit is 73590 dollars. When you look at this output, department 3 operates at full capacity, and see this because the slack is zero value, and departments 1 and 2 have a slack of approximately 1062 and 1890 hours. So, we would anticipate that the product mix would change and the total profit would increase if the workforce assignment could be revised.

So, the slack that is unused resources and unused hours in departments 1 and 2 could be transferred to the departments currently working at capacity. So, what we are going to do here, there are some unused resources, we are going to train the people who are there who are working in this department, and we are going to transfer to the where the resources are fully utilized. However, the production manager may be uncertain as to how the workforce should be reallocated among the 4 departments, which is our agenda for this lecture.

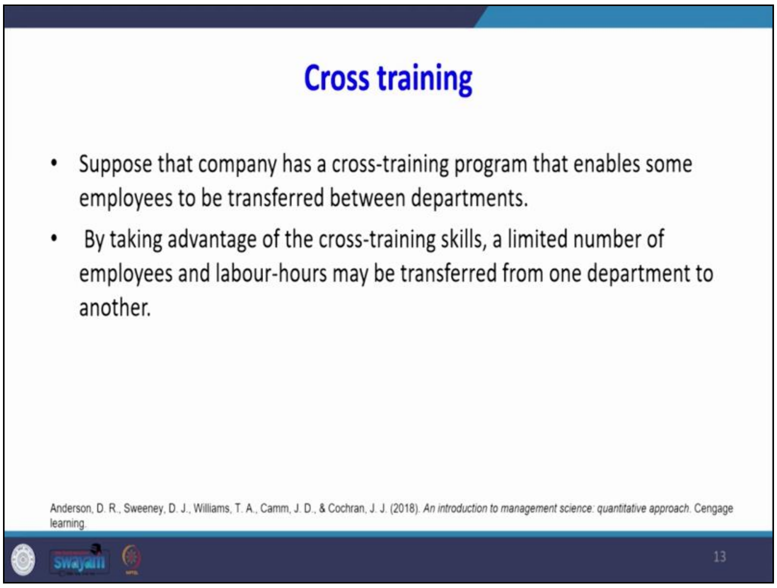
So, what we are going to do is to take some unused resources, and we are going to train those employees, and we are going to utilize the departments where it is of working at full capacity. So, the linear programming model includes decision variables that will help determine the optimal workforce assignment in addition to the profit maximization product mix. So, this problem is a simple profit maximization product mix. Apart from this, when we reuse these unutilized resources, that also will increase your objective function.



Now, we are going to include decision variables for the workforce allocation. The production manager may be uncertain as to how the workforce should be reallocated among the 4 departments. So, the linear programming model includes decision variables that will help determine the optimal workforce assignment in addition to the profit maximization product mix.

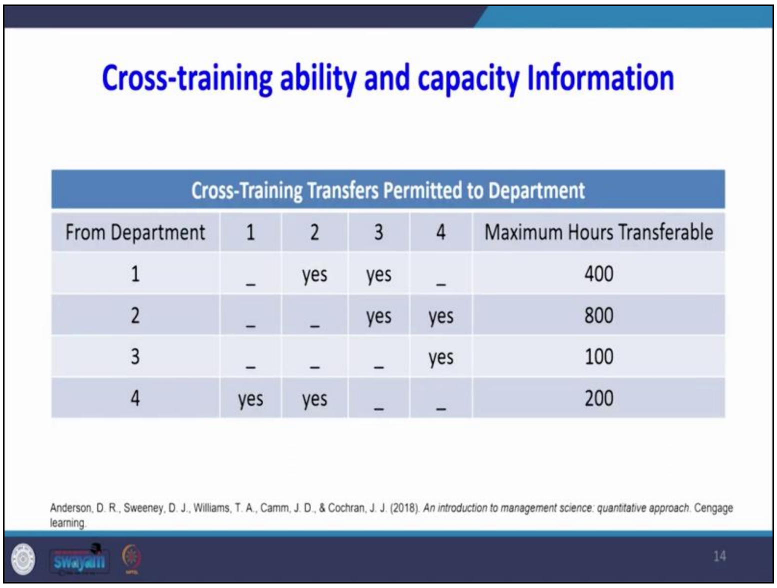


So, we are already going for the profit maximization type. So, we can maximize the profit by knowing how many units of P1 and P2 have to be produced. So, apart from this, profit maximization can be done by selling P1 and P2 units plus using the unutilized resources. The whole agenda for this lecture is, yes, we are making a profit by selling P 1 and P 2, but there are some unutilized resources in some departments. How can unutilized resources be utilized and the profit how can the profit be maximized? That is the objective.



So, how are we going to do cross-training? What is cross-training? Suppose that a company has a cross-training program that enables some employees to be transferred between departments. Cross-training means that the person is specialized in one department. So, he can be his skills can be increased, his skills may be updated, and he may be eligible for working in other departments. Also, that is the meaning of cross-training.

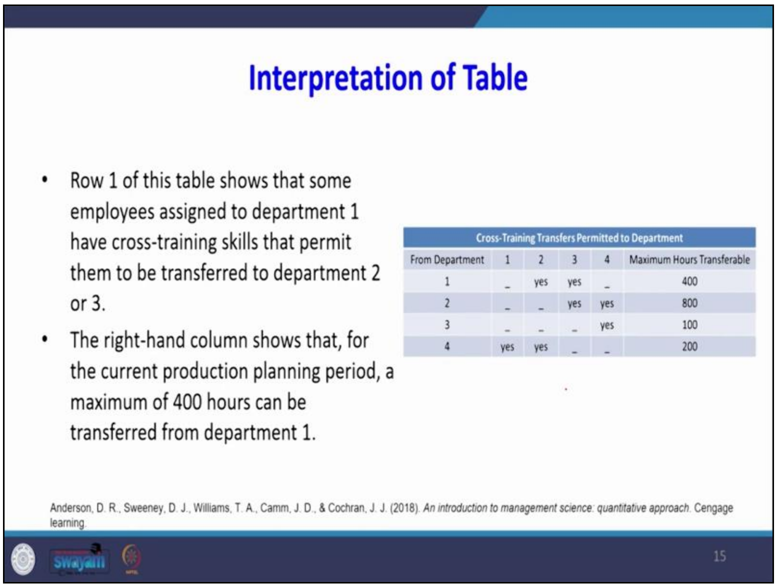
So, by taking advantage of the cross-training skills, a limited number of employees and labor hours may be transferred from one department to another department. That is what we are going to do.


Cross-training ability and capacity Information

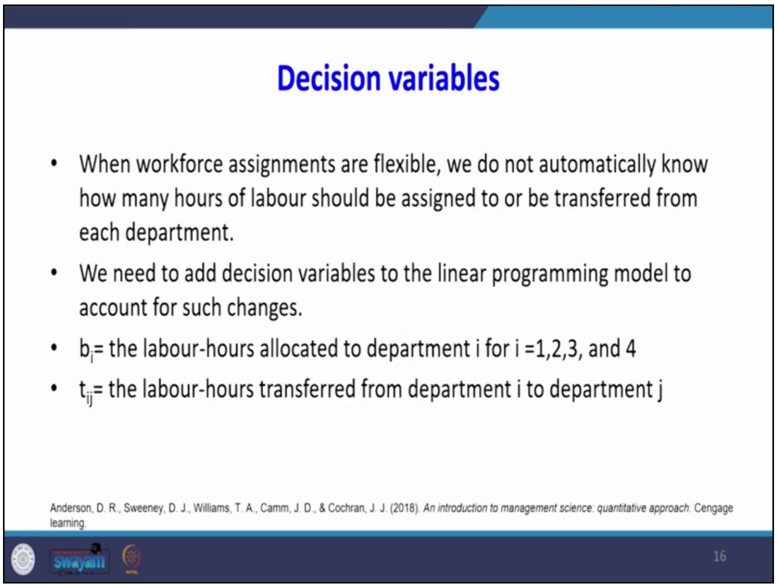
Cross-Training Transfers Permitted to Department					
From Department	1	2	3	4	Maximum Hours Transferable
1	_	yes	yes	_	400
2	_	_	yes	yes	800
3	_	_	_	yes	100
4	yes	yes	_	_	200

This is cross-training ability and capacity information. So, this table says that there are 4 departments from department 1 to this one from departments 1 and 2. Yes, there is a possibility of sending a person who is working in department 1 to department 2 after re-skilling. So, 1, 2, 3 is also possible, but 1, 2, 4 is not possible. So, from one from the department, one maximum of 4 hundred hours is transferable.

Similarly, from, say, department 2, you see we can transfer some. There is a possibility of cross-training and setting the workforce to department 3. So, 2, 3 is possible 2, 4 is possible. From the third department, only one option, only 3 to 4, is possible. From the 4th department, 4 to 1 is possible 4 to 2 is possible.



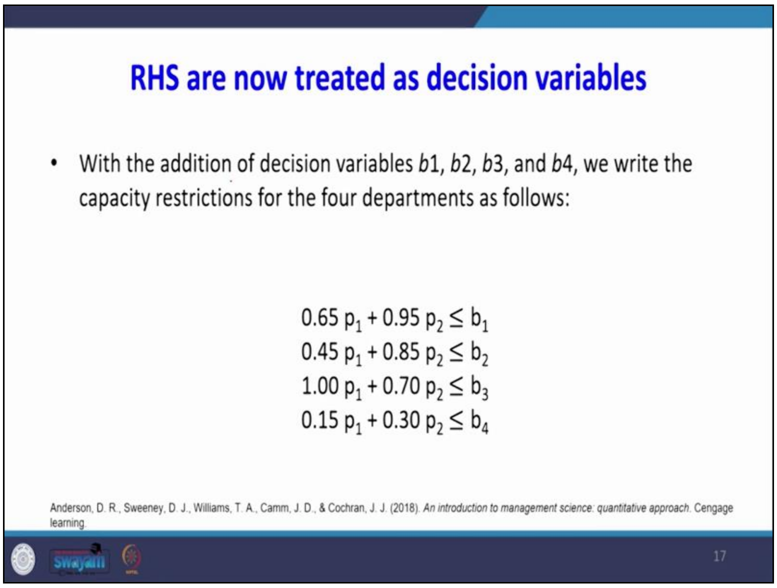
So, here is how to interpret this table. So, row one of this table shows that some employees assigned to department one has cross-training skills that permit them to be transferred to departments 2 and 3. That is why these 2 and 3 are yes. The right-hand column shows that for the current production planning period a maximum of 400 hours can be transferred from department 1.



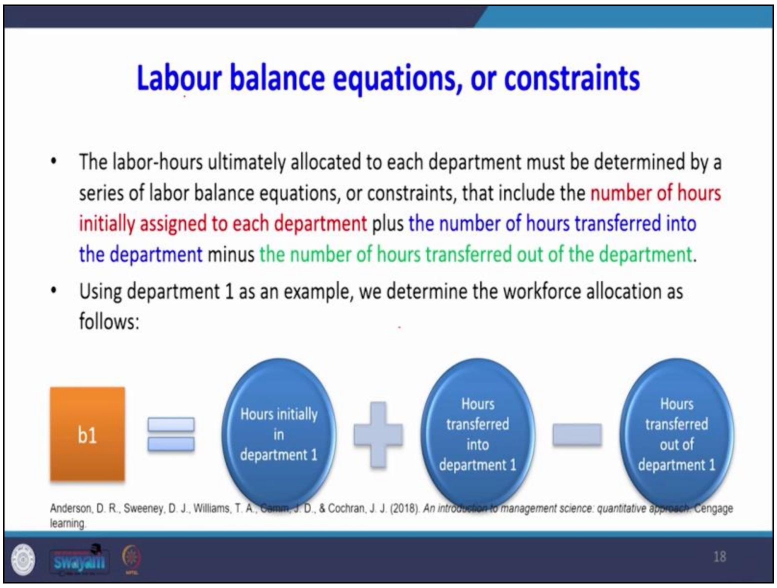
So, now we are going to introduce some more decision variables. We already have the 2 digital variables, P1 and P2, ok in the product mix problem. Apart from this, we are going to introduce 2 more types of decision-making. So, when workforce assignments are flexible, we do not automatically know how many hours of labor should be assigned to or transferred from each department. So, we need to add decision variables to the linear programming model to account for such changes.

So, there are 2 types of decision variables. b_i= the labour hours allocated to department i for i =1,2,3, and 4

t_{ij}= the labour hours transferred from department i to department j

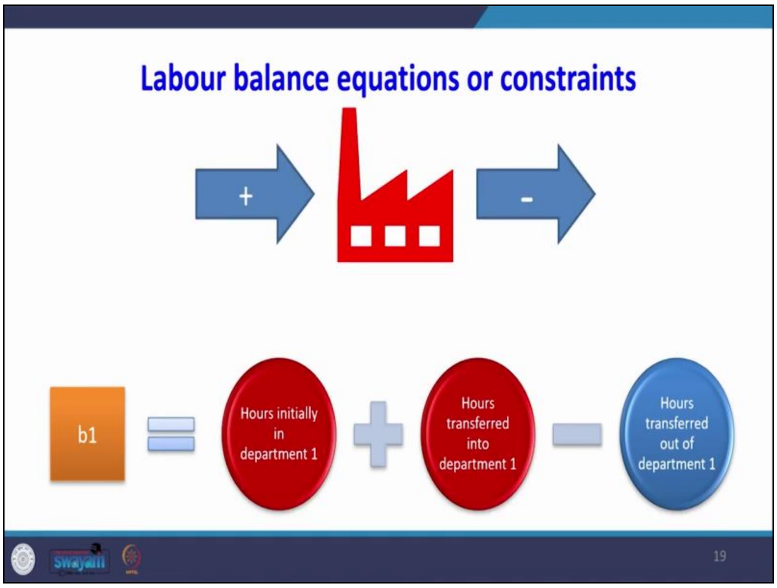


Very important point here: the right-hand side is now treated as the decision variables. how with the addition of decision variables b1, b2, b3, b4, we write the capacity restriction for 4 departments as shown as follows: b1, b2, b3, b4, which represents the maximum how many hours we are going to transfer.



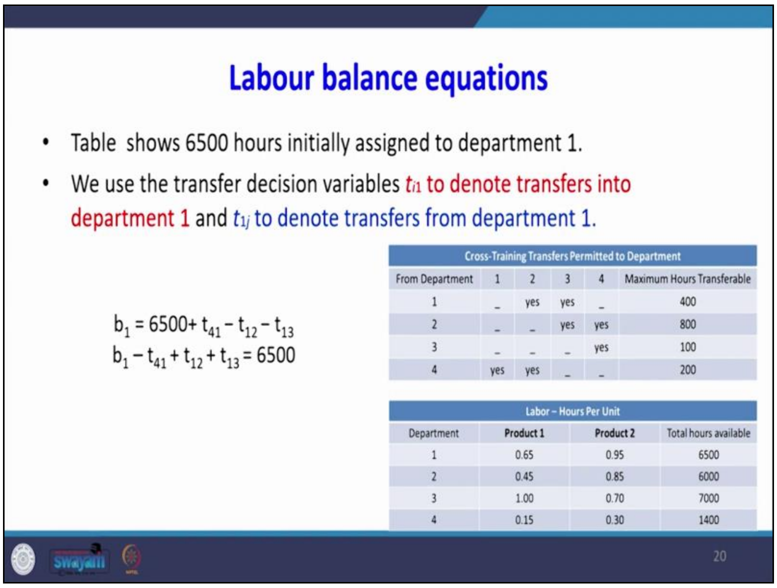
So, here, we are going to use the labor balance equations as constraints. The labor hours ultimately allocated to each department must be determined by a series of labor balance equations or constraints that include the number of hours initially assigned to each department plus the number of hours transferred into the department minus the number of hours transferred out of the department.

So, this one, so if any labor hours are transferred to a particular department, we are adding plus any labor versus moving away from that department, we are going to add minus that is the meaning of this b 1.



So, when you look at this picture, what does this labor balance equation say? So, this is a department if any additional hours are coming into the department. So, we are adding plus from this department any hours transferred to another department that is written as minus. So, the b 1 is equal to the initial however hours initially in department 1 plus this one. This plus, however, is transferred into the department.

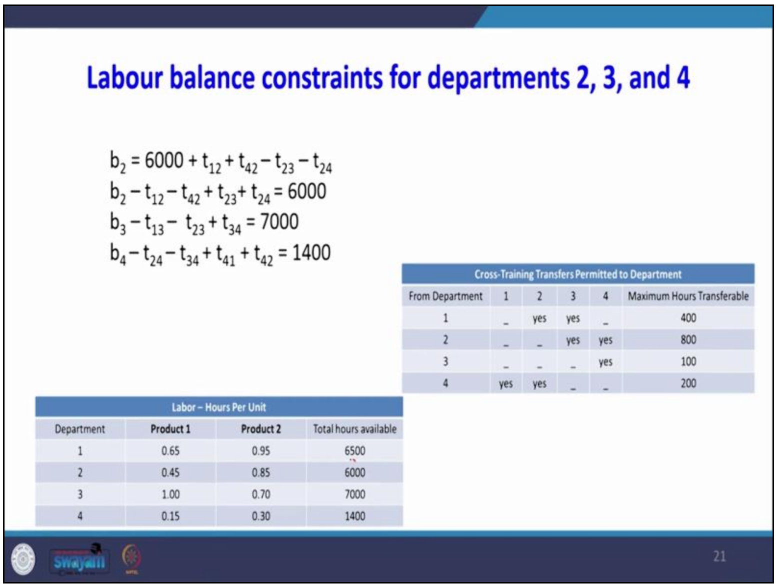
So, if any hour is going away from the department, the hours transferred out of the department have to be put minus. I will explain this concept with the help of a numerical value.



$$b_1 = 6500 + t_{41} - t_{12} - t_{13}$$

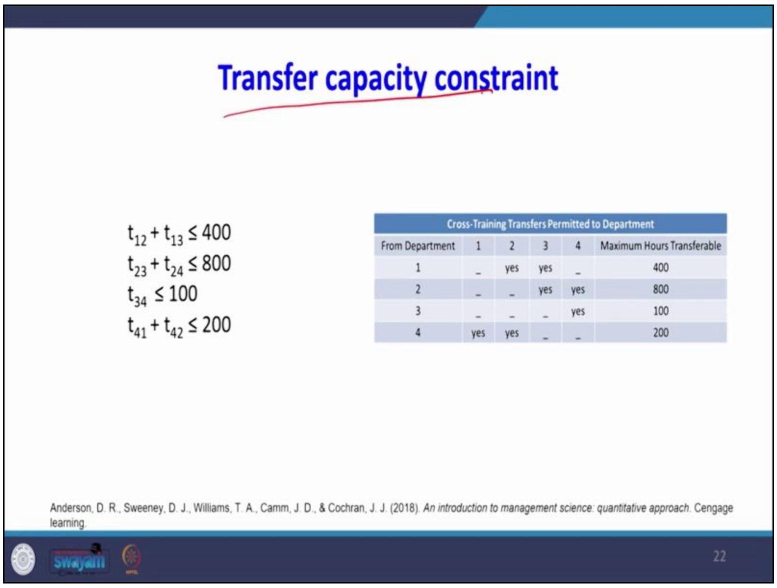
.

$$b_1 - t_{41} + t_{12} + t_{13} = 6500$$



For the second department, we initially allocated 6000 hours, which is why it is the initial hours allocated. In Department 2, you see that from Department 1, it is possible So, now, when you simplify, this is $b_2 - t_{12} - t_{42} + t_{23} + t_{24} = 6000$

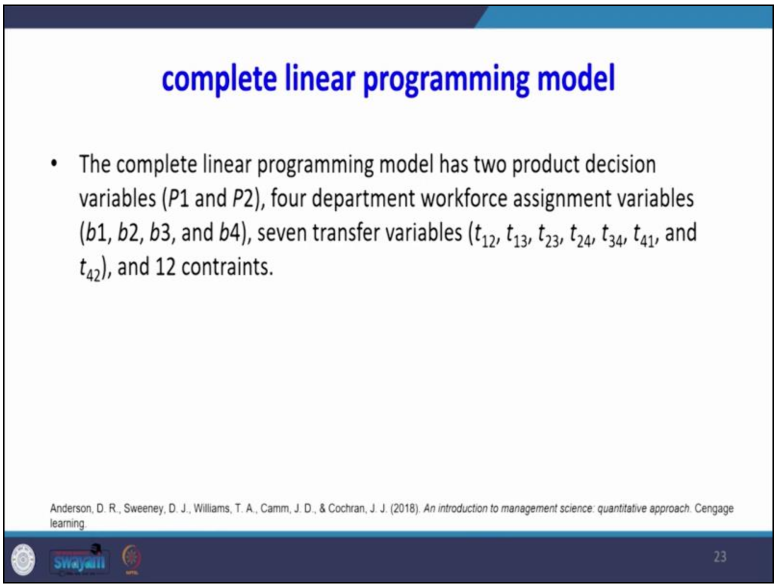
Similarly, by looking at the total hours available in department 3, we can write the equations for departments 3 and 4.



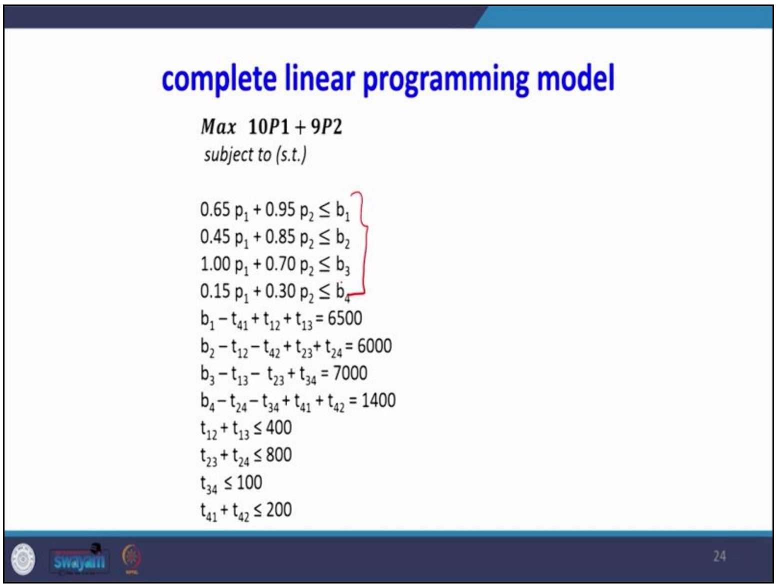
Now we have to add the transfer capacity constraint, which is the transfer capacity from departments 1 to 12 so that when you transfer, it cannot exceed 400 hours from department 1. Similarly from department 2 so $t_{12} + t_{13} \leq 400$

from department 3 ; $t_{34} \leq 100$

from department 4 $t_{41} + t_{42} \leq 200$



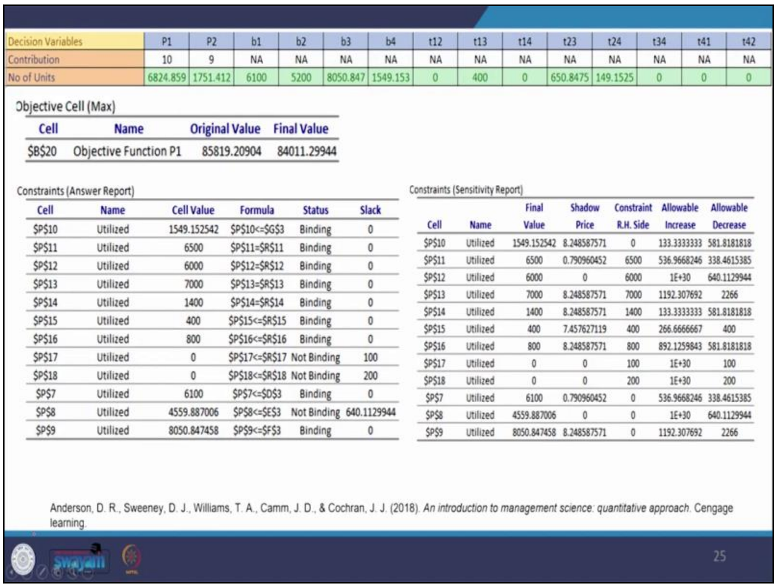
So, now I am going to write the complete linear programming problem considering all decision variables. The complete linear programming model has two product decision variables (*P*1 and *P*2), four department workforce assignment variables (*b*1, *b*2, *b*3, and *b*4), seven transfer variables (t_{12}, t_{13}, t_{23}, t_{24}, t_{34}, t_{41}, and t_{42}), and 12 contraints.



So, the complete problem is like this transfer constraint. These are capacity constraints, and these capacity constraints are maximum transferable. Now, we are going to solve this problem with the help of Excel.

Now, I have brought the problem into Excel. So, there are P1 and P2 product mix variables b1, b2, b3, and b4, the transferable variable, and other variables, there are contributions, and there are a number of units. So, I have written all the constraints. So, the resources utilized in the sign are mentioned on the right-hand side. So, my objective function is in B21. So, now I am going to solve this one and go to a data solver to solve it. Yes, now I have the solution values of P1, P2, D3, E4, and so on.

Now I will go to answer the report. So, I have brought this answer report. So, our objective function value is 84011.299. similarly, I am going to use a sensitivity report. So, I have brought the screenshot of these reports into my presentations, and from there, I am going to explain the interpretation of this answer in detail.



Decision Variables	P1	P2	b1	b2	b3	b4	t12	t13	t14	t23	t24	t34	t41	t42
Contribution	10	9	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
No of Units	6824.859	1751.412	6100	5200	8050.847	1549.153	0	400	0	650.8475	149.1525	0	0	0

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$20	Objective Function P1	85819.20904	84011.29944

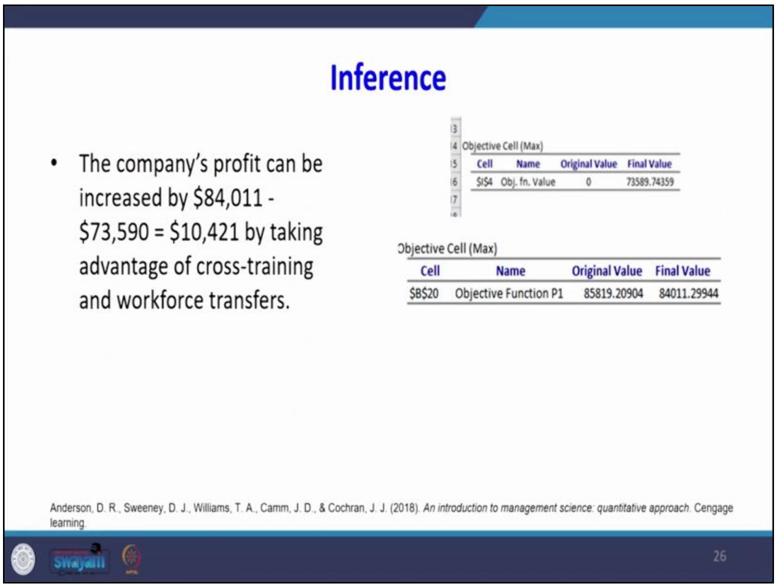
Constraints (Answer Report)

Cell	Name	Cell Value	Formula	Status	Slack
$P$10	Utilized	1549.152542	$P$10<=$G$3	Binding	0
$P$11	Utilized	6500	$P$11=$R$11	Binding	0
$P$12	Utilized	6000	$P$12=$R$12	Binding	0
$P$13	Utilized	7000	$P$13=$R$13	Binding	0
$P$14	Utilized	1400	$P$14=$R$14	Binding	0
$P$15	Utilized	400	$P$15<=$R$15	Binding	0
$P$16	Utilized	800	$P$16<=$R$16	Binding	0
$P$17	Utilized	0	$P$17<=$R$17	Not Binding	100
$P$18	Utilized	0	$P$18<=$R$18	Not Binding	200
$P$7	Utilized	6100	$P$7<=$D$3	Binding	0
$P$8	Utilized	4559.887006	$P$8<=$E$3	Not Binding	640.1129944
$P$9	Utilized	8050.847458	$P$9<=$F$3	Binding	0

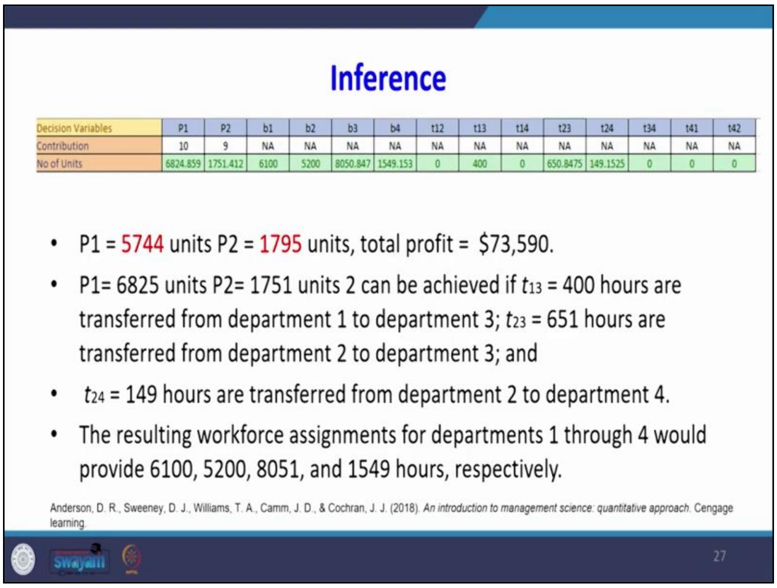
Constraints (Sensitivity Report)

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$P$10	Utilized	1549.152542	8.248587571	0	133.3333333	581.8181818
$P$11	Utilized	6500	0.790960452	6500	536.9668246	338.4615385
$P$12	Utilized	6000	0	6000	1E+30	640.1129944
$P$13	Utilized	7000	8.248587571	7000	1192.307692	2266
$P$14	Utilized	1400	8.248587571	1400	133.3333333	581.8181818
$P$15	Utilized	400	7.457627119	400	266.6666667	400
$P$16	Utilized	800	8.248587571	800	892.1259843	581.8181818
$P$17	Utilized	0	0	100	1E+30	100
$P$18	Utilized	0	0	200	1E+30	200
$P$7	Utilized	6100	0.790960452	0	536.9668246	338.4615385
$P$8	Utilized	4559.887006	0	0	1E+30	640.1129944
$P$9	Utilized	8050.847458	8.248587571	0	1192.307692	2266

Now I have brought this screenshot of our answer. Here we have the values of P1 P2 and b1, b2 b3 and so on. Now this is the value of our objective function right 84011.299 dollars.



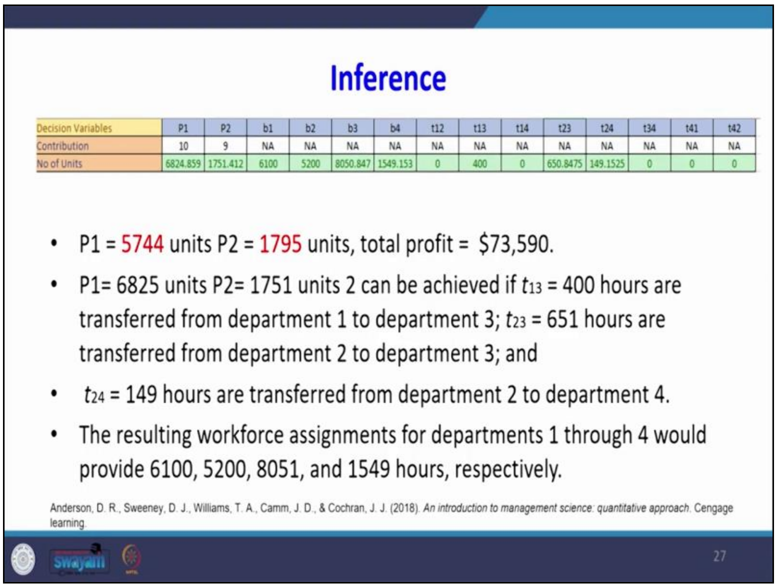
The first inference is that the company's profit can be increased by 84011 - 73590, equal to 10421, by taking advantage of cross-training and workforce transfers. So, what is the meaning without cross-training? This was 73589.74 the value of our objective function. After cross-training, you see that the value of our objective function is now 84011.29. So, the difference is increase in the value of objective function due to cross training how much 10421 dollar is the increase in the value of objective function due to cross training that is the first inference.



In the past, without cross-training, the value of P 1 was this much 5744. Now you see that we are producing more products because of the cross training, there is an increase in product one. Similarly, without cross-training, the number of units produced is 1795. Now it is reduced to 1751, and this can be achieved by transferring t 13 equal to 400, whereas t 13, we got this answer that means from department one, the person can be transferred to 3 how much our how many hours 400 hours.

Similarly, t23 from Department 2, the people can be the workforce can be transferred to Department 3 how many hours, maximum of 651. So, from department 2 to the 4th department, 149 hours can be transferred. So, what we are inferring here when we cross training when we do the cross training is that the workforce can be re-employed in some other department that will increase the value of our objective function.

So, the resulting workforce assignment for departments 1 to 4 is 6100, 5200 this one. Value of b1, b2, b3 and 4 up to b 4 this is the first inference.



If the manager has the flexibility to assign personnel to different departments, reduced workforce idle time, improved workforce utilization, and improved profit should result. So, the linear programming model in this section automatically assigns the employees and labor hours to the departments in the most profitable manner. In this lecture, we took a simple product mix problem and found the value of our objective function.

Then, we saw the possibility of transferring the work from one department to another department, where we transfer wherever unutilized resources are from that department. After cross-training the employees, they are transferred to other departments where the requirement is needed. So, by transferring, we realize that the value of an objective function can be increased. That is the outcome of this lecture; thank you.