Decision Making With Spreadsheet
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Lecture-11
LPP Applications in Operations - 1

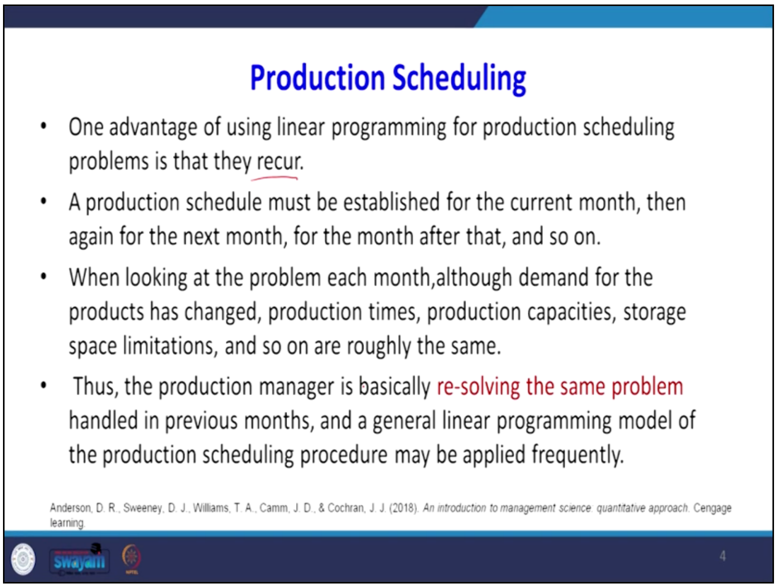
Dear students, in our previous lecture, I explained how to use linear programming problems for solving problems related to finance with the help of a solver. In this lecture, I am going to explain how to use linear programming problems for solving problems related to operations. One of the very well-known, very reputed examples is production scheduling.

The reference for this problem is from the book Anderson et al. So, first, we will see what production scheduling is. So, one of the most important applications of linear programming deals with multi-period planning, such as production scheduling. The solution to a production scheduling problem enables the manager to establish an efficient, low-cost production schedule. So, the objective of any scheduling problem is to give a schedule that will minimize the low cost to the production schedule.

For one or more products over several time periods, it may be months or weeks. Essentially, a production scheduling problem can be viewed as a product mix problem for each of several periods for each of several periods in the future. This is similar to our product mix problem.

What is the product mix problem? Are there different products, say x1, say x2, and so x3? How many quantities in each product x1, x2, x3 have to be manufactured?

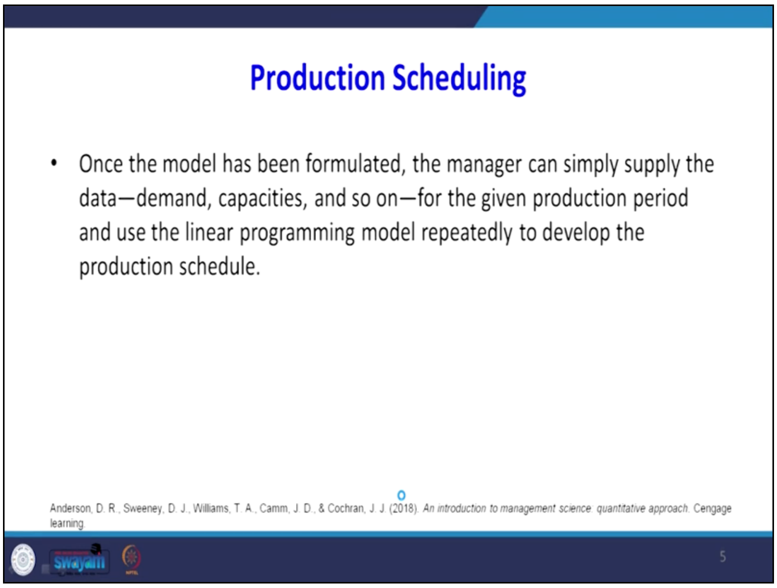
This scheduling is also similar to that problem. The manager must determine the production levels that will allow the company to meet product demand requirements given the limitation on production capacity, capacity of the plant, and storage space while minimizing the total production cost. So, what is the objective of your production scheduling problem? We have to prepare a schedule that will minimize our production costs.



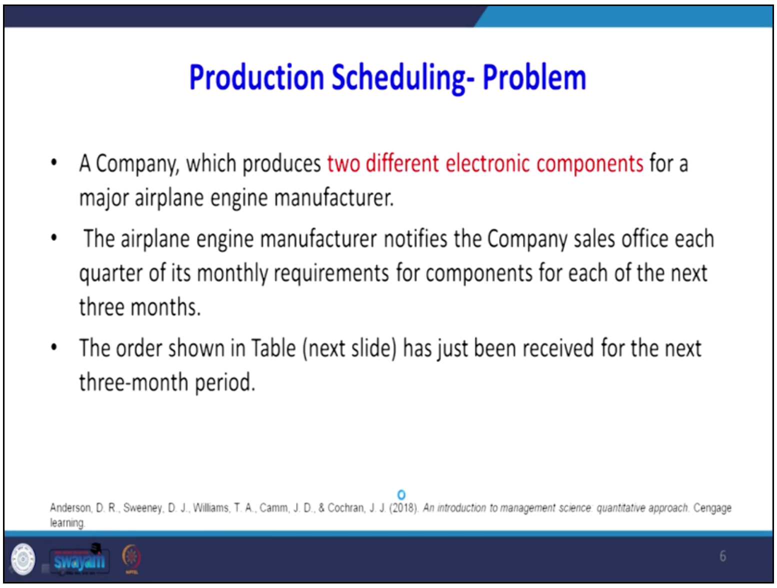
One advantage of using linear programming for production scheduling problems is that they recur because the scheduling is a very operational operation problem every month or every year that scheduling needs to be done. So, once a mathematical model or an Excel model is prepared that can be reused, that is the advantage of using Excel. A production schedule must be established for the current month, then again for the next month, for the month after that, and so on.

When looking at the problem each month, although demand for the product has changed, the remaining factors, like production times, production capacities, storage space limitations, and so on, are roughly the same. Only the demand will be changed, the same model can be reused for different periods. Thus, the production manager is basically resolving the same problem handled in the previous month, and a general linear programming model of the production scheduling procedure may be applied frequently.

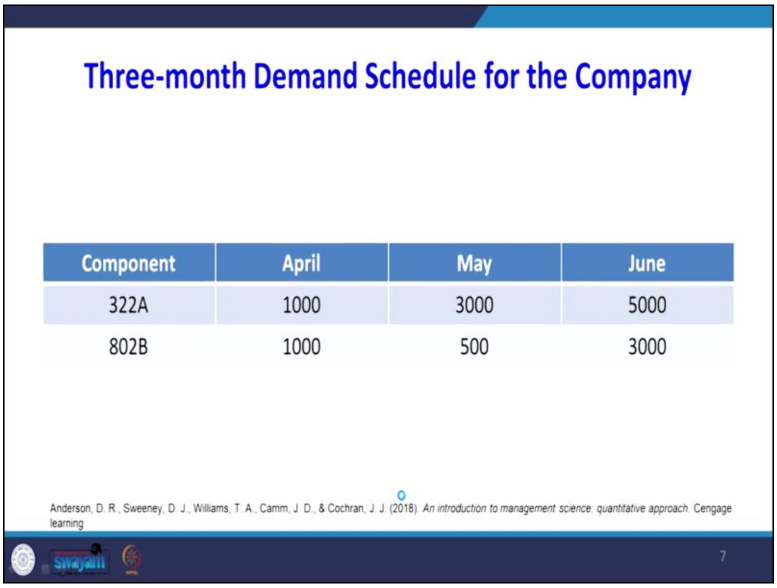
So, once you have a spreadsheet modeling for the scheduling, that model can be used for the coming months, which is advantageous.



Once the model has been formulated, the manager can simply supply the data, for example, demand capacities and so on, for the given production period and use the linear programming model repeatedly to develop the production schedule.



We will take one example. I will explain how to formulate a production scheduling problem, then solve it with the help of a solver, and then we will interpret the result. A company that produces 2 different electronic components for a major airplane engine manufacturer. The airplane engine manufacturer notifies the company sales office each quarter of its monthly component requirement for each of the next 3 months. So, the company is receiving the order for the next 3 months. The order is shown in the next slide.



Three-month Demand Schedule for the Company

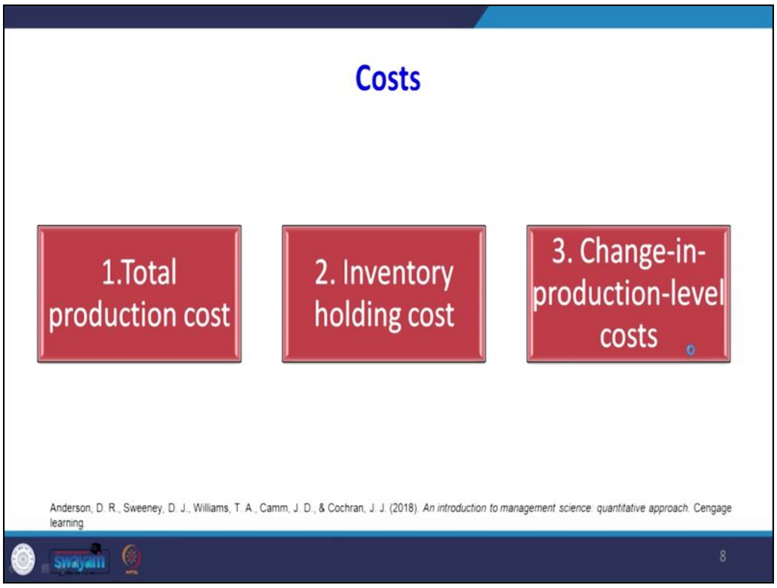
Component	April	May	June
322A	1000	3000	5000
802B	1000	500	3000

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning

7

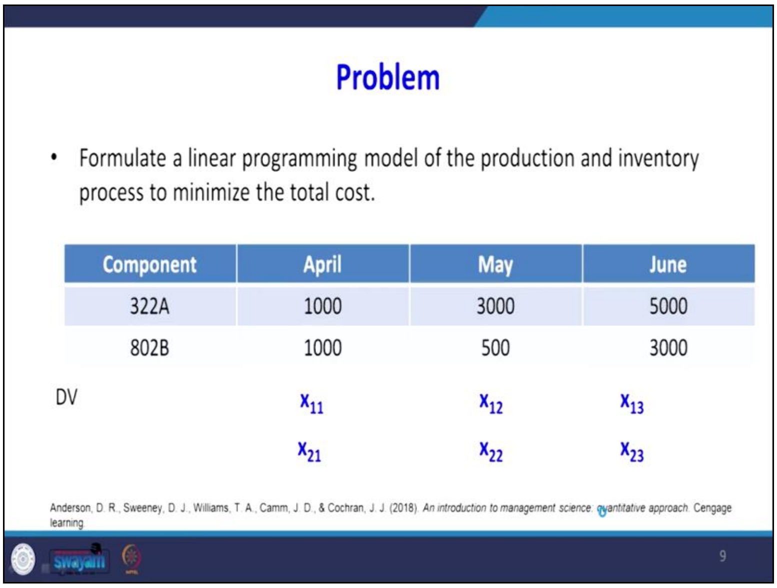
For example, there are 2 products: 322A and 802B. So, April month, May month, June month. This is the demand that we need to fulfill. So, for this, we must give a production schedule. What is the meaning of the production schedule and how many products? For example, here, how many products are using x11? That is for a product. This first one represents the product type, and the second one represents the month. Say here it is x12 products one month two.

Suppose I write x13 product one month 3 similarly for a product 2; x21, x22, x23. So, we have to suggest each month for each product how many units have to be produced, which is an objective of this production scheduling problem.

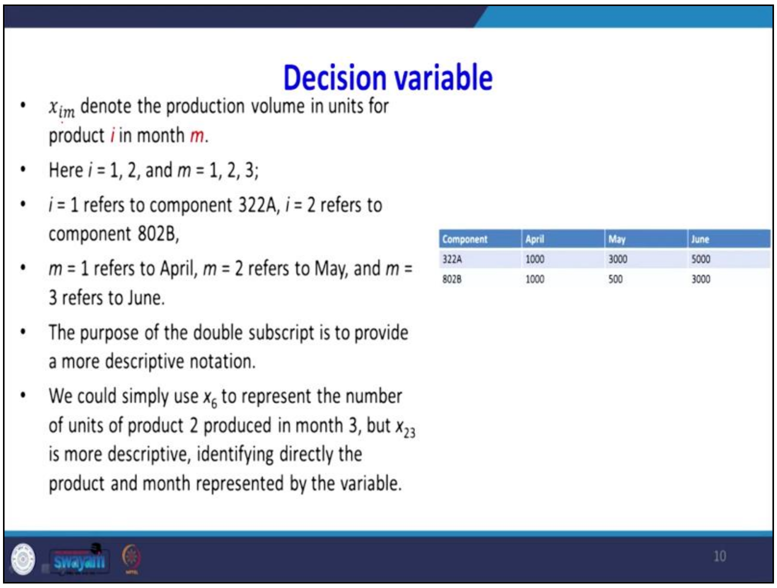


When we go for a production share, there are 3 elements of cost: one is actual production cost, then inventory holding cost. Sometimes, we may produce more than the demand that has to be stored for the next year and the next period's demand is inventory holding cost. Then the other is the change in production level cost. Suppose this is an example. We can say the setup cost. So, currently, my company says, man, suppose I manufacture with 5000 units.

So, my demand for next month is, say, 6000 units. Now I must increase my capacity from 5000 to 6000. So, that cost is called setup cost or a change in production level cost. It is not only increasing sometimes, but it may also be 4000. Now we must decrease my production level. So, that cost is a change in production level cost.



So, here, the problem is to formulate a linear programming model of the production and inventory process to minimize the total cost. As I discussed in the previous slides, this is my decision variable x11, x12, x13 for product 1, for product 2, x21. So, this first number represents the product's second number. For example, 1, 2, and 3 that number represent the month. So, if it is one, it is a month: April 2 it is May 3, it is June.



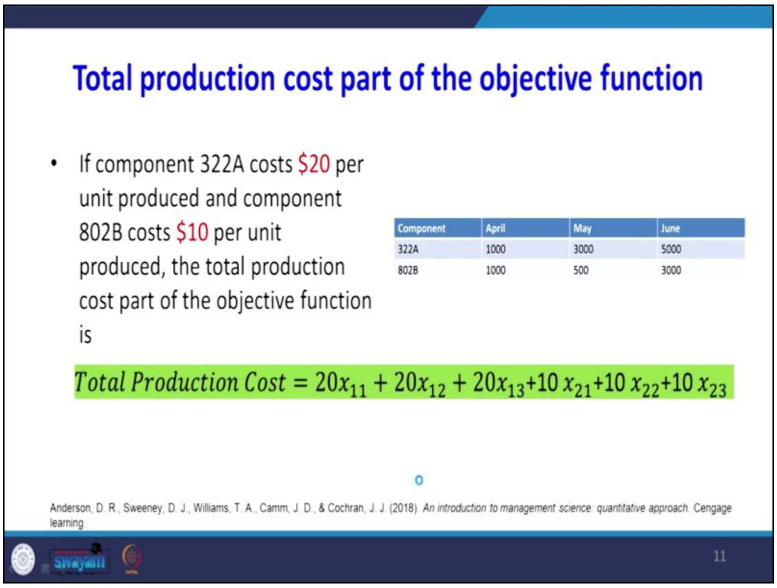
x_{im} denote the production volume in units for product *i* in month *m*.

Here *i* = 1, 2, and *m* = 1, 2, 3;

i = 1 refers to component 322A, *i* = 2 refers to component 802B,

m = 1 refers to April, *m* = 2 refers to May, and *m* = 3 refers to June.

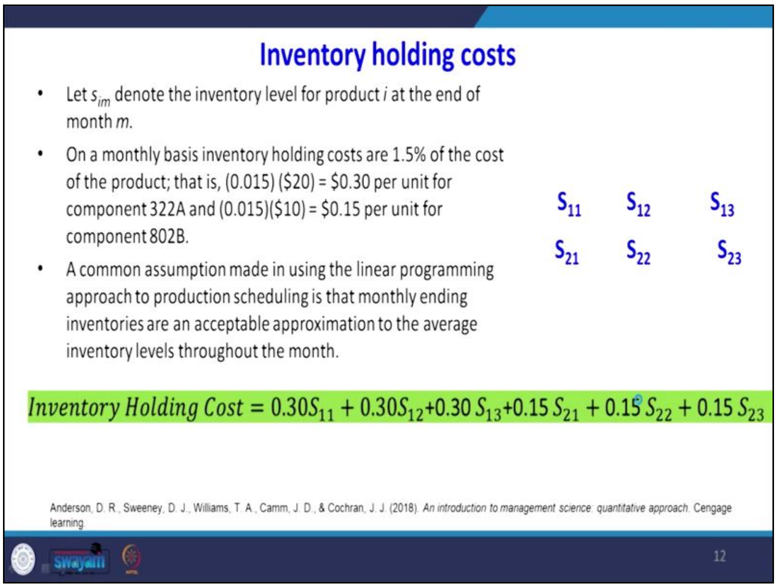
So, the purpose of the double subscript is to provide a more descriptive notation. We could simply use x6. For example, you can call it if this is x1, x2, x3, x4, x5, x6. We can use product 2. The June month requirement is x 6. But if you provide x23 instead of x6 when you provide x23, it is more descriptive and identifies directly the product and month represented by the variable. That is why we should go for double subscript, which will provide more clarity on the decision variables.



Now we will formulate our objective function. As I discussed earlier, the objective function has 3 components: production cost, inventory cost, and setup cost. Now we will go for the total production cost as part of the objective function. If component 322A costs 20 dollars, 20 dollars per unit produced, and component 802B costs 10 per unit produced, the total production cost part of our objective function is 20x11.

Total Production Cost = $20x_{11} + 20x_{12} + 20x_{13} + 10x_{21} + 10x_{22} + 10x_{23}$

For example, say this is twenty, this is 10. See what you see. This is our production cost. So, what you have to do you have to multiply by corresponding to decision variables.



The next component is inventory holding cost. So, for inventory holding cost also, we are going to use this decision variable S. Say when I say S_{11}, for example, this is product one product 2, this is month one month 2 month 3, but here, S represents the inventory cost. So, if I use Sim to denote the inventory level for production I at the end of month m. On a monthly basis, inventory holding costs are 1.5 percent of the total production product. For example, 1.5 percent is 0.015, and the product cost for product one is 20.

On a monthly basis inventory holding costs are 1.5% of the cost of the product;

that is, (0.015) ($20) = $0.30 per unit for component 322A

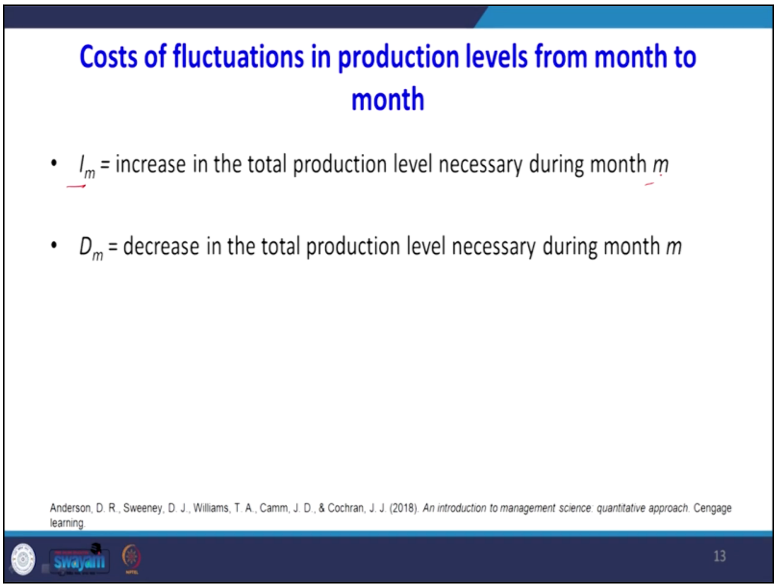
and (0.015)($10) = $0.15 per unit for component 802B.

So, there is a 0.30 dollar per unit for component A, and if you multiplied by when you multiply 0.015 multiplied by 10 for the second product. So, for the second product inventory holding cost is 0.15 per unit per component. Here, the common assumption made in using this LP approach to production scheduling is that the monthly ending inventory is an acceptable approximation to the average inventory level throughout the month.

So, generally, how it is generally done, inventory suppose inventory like this assumes that I have ordered 500 units. So, this is week 1, week 2, week 3. So, my inventory level will keep on decreasing. For example, it is zero. Generally, we will do the average of this, but in this problem. So, the average inventory is equivalent to that month ending inventory that is the meaning. So, monthly ending inventory is an acceptable approximation to the average inventory level. That is what the meaning of this sentence is.

So, now we will go for the inventory holding cost. We know that for the first product, the inventory holding cost is 30 0.15. So, when you multiply by corresponding decision variables for inventory,

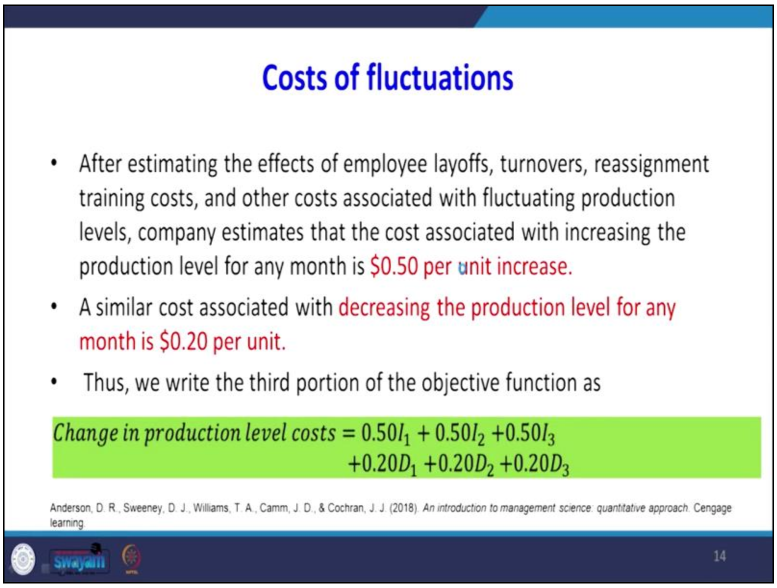
Inventory Holding Cost = $0.30S_{11} + 0.30S_{12} + 0.30S_{13} + 0.15S_{21} + 0.15S_{22} + 0.15S_{23}$



Next, we will go for the setup cost. The cost of fluctuation in production level from month to month. So, we are going to use this notation

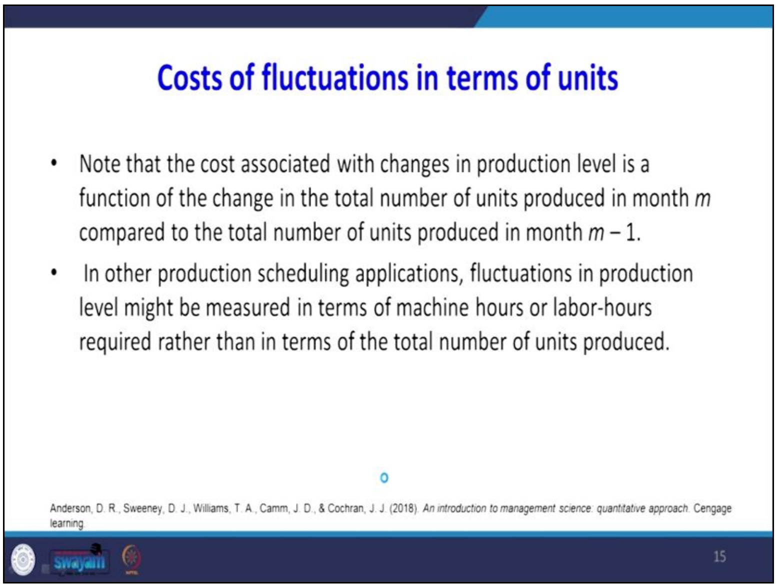
I_m = increase in the total production level necessary during month m

D_m = decrease in the total production level necessary during month m



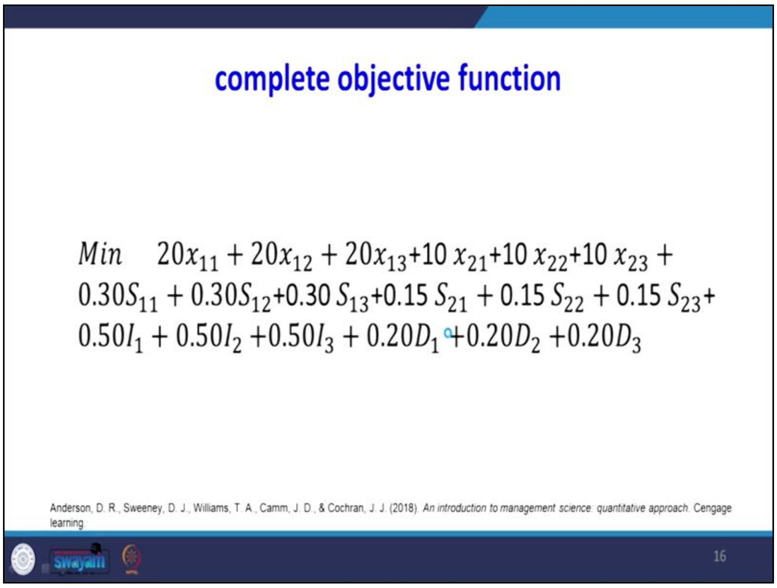
We discuss about the cost fluctuations. After estimating the effect of employee layoff, turnovers, reassignment training costs, and other costs associated with fluctuating production levels, the company estimates that the cost associated with increasing the production level for any month is 0.5 dollars per unit increase. A similar cost associated with a decrease in production level for any month is 0.20 dollars per unit.

So, we write the third portion of our objective function. So, there are we have 3 months 3 months there is a possibility of increasing, there is a possibility of decreasing also. For increasing cost is 0.501 per month 1, month 2, month 3. In the same month, there is a possibility of decreasing D1, D2, and D3. But the increment and decrease decrement will not take place simultaneously. In the next slide, I will explain.

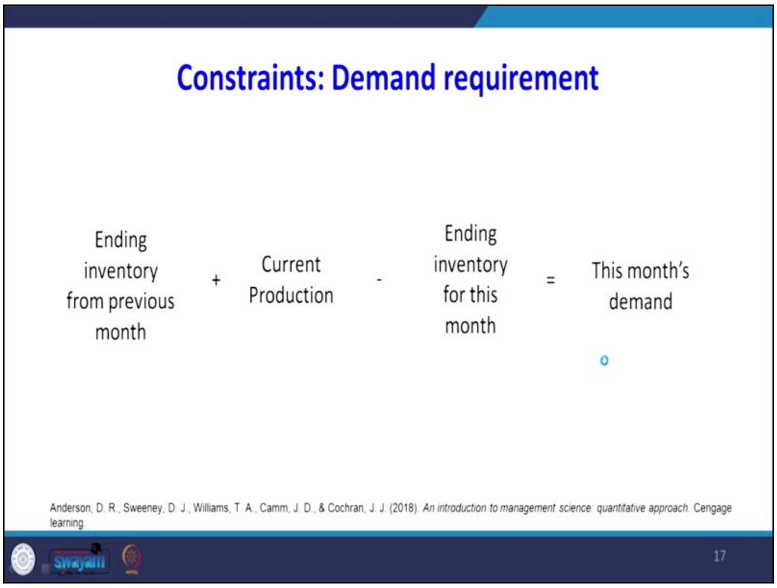


Note that the cost associated with the changes in production level is a function of change in total number of units produced in month m compared to the total number of units produced in month m - 1. Suppose you say 1, 2, 3 we start from zero months of say suppose here this is m - 1 this is say m. So, the difference in units is called your change in production level. In other production scheduling applications, fluctuation in production level might be measured in terms of machine hours or labor hours required rather than in terms of a total number of units produced.

Now, we are considering the change in production level only in terms of numbers. Okay, it is not necessary sometimes, we can consider a change in machine hours change in labor hours.

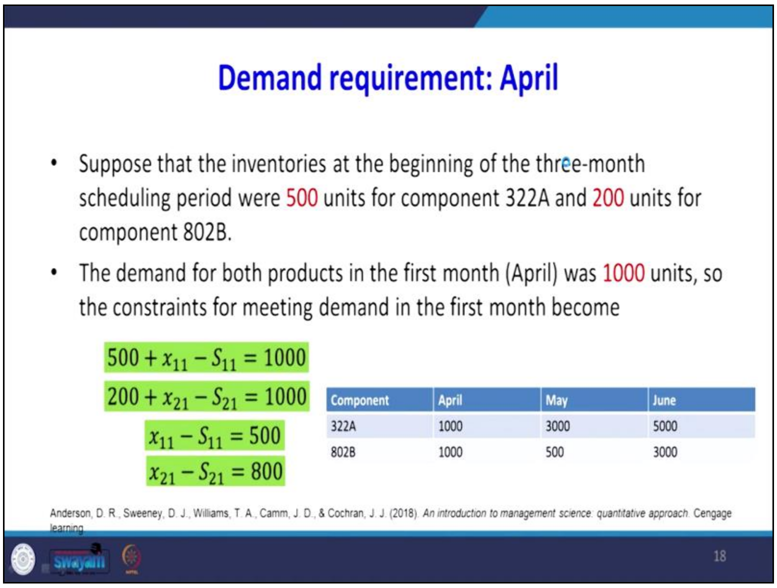


Now, I am going to write the function. So, this is my portion x 1 up to this is my production cost S this is S up to this, this is my inventory cost this up to this, this is my setup cost. Now I will go for writing the constraint.



So, here we will write the constraints. First, we must fulfill the demand. So, here you see the extreme right of this one. So, this month's demand ok we must fulfill this demand for that ending inventory from the previous month. So, the current month's production has to be added. So, your ending inventory plus current production will directly satisfy your monthly demand. If any unused units are stored, I have brought in the left-hand side that will stored as an inventory for this month.

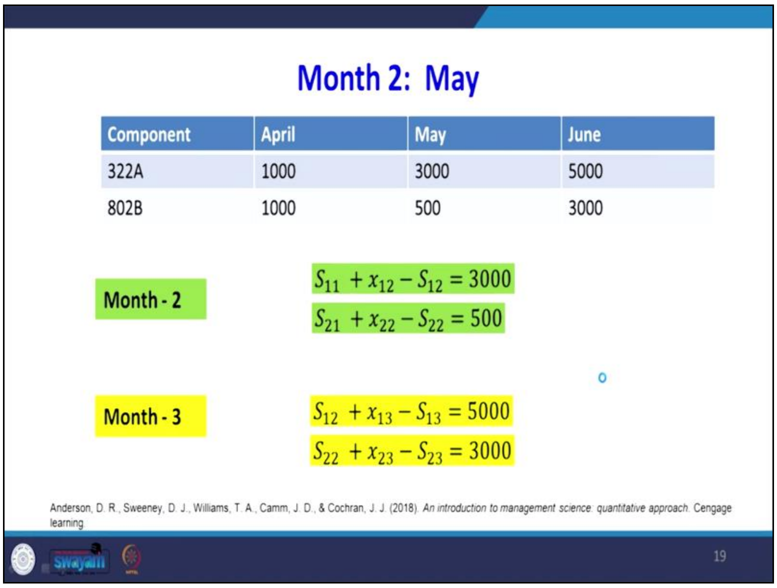
So, the general equation is ending inventory this month plus current production that will satisfy that month's obligations after satisfying that month's demand. If any excess unit is there that will be stored as actually, it should be this side. So, I brought it on the left-hand side. So, it will be stored as an inventory in that month. So, that can be used for the next month; it will just be carried out for the next month.



Now, we will write about the constraints for the month of April. So, what will happen in April? There may be some initial inventory in the month of March. Suppose that the inventory at the beginning of the 3-month scheduling period was 500 units for product A and 200 units for product B. So, the demand for both products in the first month is 1000, 1000. So, the constraint for meeting the demand in the first month is that we use the same idea.

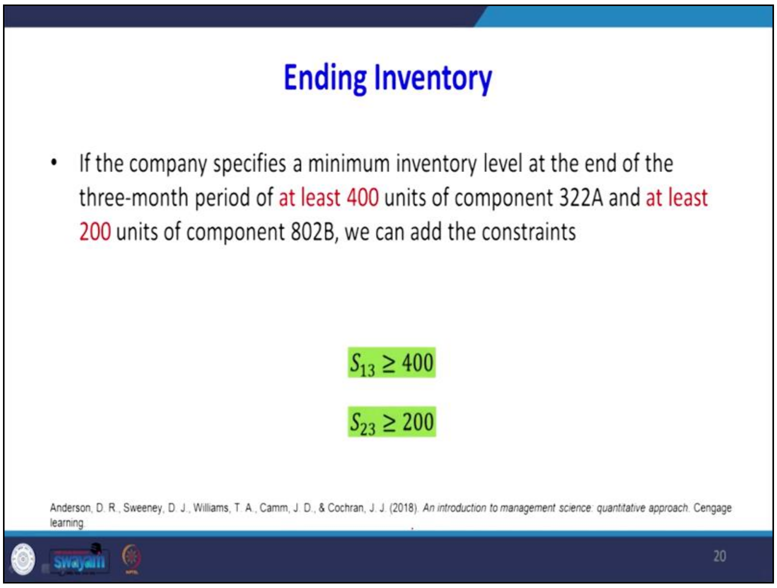
So, the beginning inventory and then that month's production for product 1 will be fulfilling the 1000. If there is any extra that will be stored for the inventory, this is for product 1. For product 2, we have a beginning inventory of 200 units plus April month production that will used to fulfill that month's requirement if any extra units are stored. So, I have brought on the left-hand side as inventory.

After simplifying this are the final equations we are going to write for May month and month June month.



Say, for the month of May, see this S11, right? We carried from this is 1, 2, and 3, which were carried from April. This is my beginning inventory, plus the second month of production. When I add this, I should fulfill my 3000 units. Yes, after fulfilling any additional unit is there that will be stored as inventory S12. So, product 1 is for product 2, the third month. You see this; this S12 will be my beginning inventory from the second period, then that month's third-month production.

This will help you to fulfill 5000 for any additional unit that will be added as an S13, similarly for product 2.



Now, there may be a requirement for the company that this much-ending inventory has to be maintained. If the company specifies a minimum inventory level at the end of the 3-month period, say at least 400 units for component A. So, S13 should be greater than or equal to 400 and at least 200 units of component B. So, we can add this constraint S23 because this is in the third period of the inventory available.

Machine, Labour, and storage capacities for the company

Month	Machine Capacity (hours)	Labour Capacity (hours)	Storage Capacity (square feet)
April	400	300	10,000
May	500	300	10,000
June	600	300	10,000

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

21

Now, we will go for our capacity constraints capacity for machine, labor, and storage capacities. So, we have machine capacity. This is resources available for 400 hours in April and 500 hours in May and June. Similarly, for labour hours and storage capacity, we have 2 products.

Machine, labour, and storage requirements for components 322A and 802B

Component	Machine (hours/unit)	Labour (hours/unit)	Storage (square feet/unit)
322A	0.10	0.05	2
802B	0.08	0.07	3

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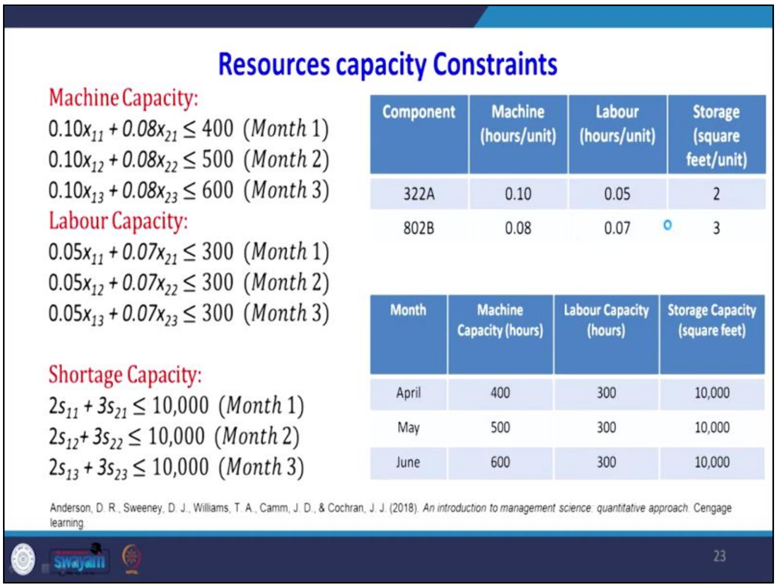
22

So, this says how many hours per unit are consumed by product A in the machine, say 0.10 this 10.02. This many hours per unit is consumed by-product, this is for machine, labor, and storage. So, we have to write constraint for machine hours, we have to write constraint for labor hours, and we should write the constraint for storage capacity always; this should not exceed. So, it should be less than or equal to type. So, we will be writing that constraint.



So, I have included the labor hours per unit and the total available resources. So, what will happen? We know that this will consume 0.10 x11 for product one, and then this will be 0.08 x21. So, that should not exceed over 400 400 in month one. That is why we wrote this 400. Similarly, for the May month 500 and June month 600, then we will go to labour capacity. So, this is x1. Ok, now we will write the constraint for the labor capacity.

Machine Capacity:

$0.10x11 + 0.08x21 \leq 400\ \ (Month\ 1)$

$0.10x12 + 0.08x22 \leq 500\ \ (Month\ 2)$

$0.10x13 + 0.08x23 \leq 600\ \ (Month\ 3)$

Labour Capacity:

$0.05x11 + 0.07x21 \leq 300\ \ (Month\ 1)$

$0.05x12 + 0.07x22 \leq 300\ \ (Month\ 2)$

$0.05x13 + 0.07x23 \leq 300\ \ (Month\ 3)$

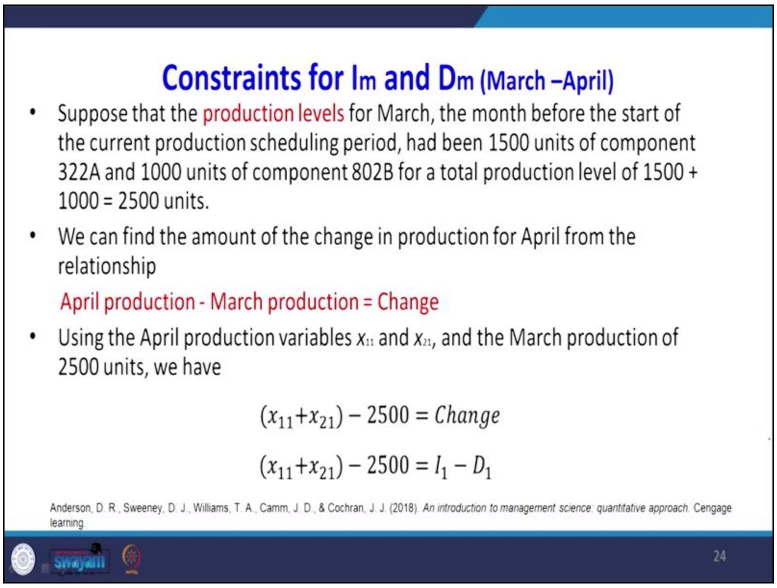
Shortage Capacity:

$2s11 + 3s21 \leq 10{,}000\ \ (Month\ 1)$

$2s12 + 3s22 \leq 10{,}000 \ \ (Month\ 2)$

$2s13 + 3s23 \leq 10{,}000 \ \ (Month\ 3)$

So, for product one, it consumes 0.05 hours per unit. So, 0.05 x11 for product one for product to 0.07 x21. So, that should not exceed your 300 months of April, then May and June. Now, we will go for writing constraint for storage, so product A occupies 2 square feet. So, 2S 11, because you see that this is our inventory variable, will be stored at 2 S 11 and 3 S 21, and should not exceed 10000 in the month of April. Similarly, that should not exceed 10000 for the month of May and then 10000 for the month of June.



Now we will write the constraint for our setup cost. For example, if you consider March and April see, we have problems only from April onwards, April, May, and June, but I am going to consider what is the setup cost between March and April. Suppose that the production level for March, which is a month before the start of the current production scheduling period, had been 1500 units of component and component A and thousand units of component B.

So, the total production level is 2500 units 1500 + 1000, 2500 units. We can find the amount of change in production for April from the following relationship. The same thing. The first one is April month production, and the previous month's production says the previous month's production, the differences change. Using this April production variable x11 and x21 and the March production of 2500 units because this is given for the March.

So, April production is this one x11, x21, and the March production is 2500.

$$(x_{11} + x_{21}) - 2500 = Change$$

$$(x_{11} + x_{21}) - 2500 = I_1 - D_1$$

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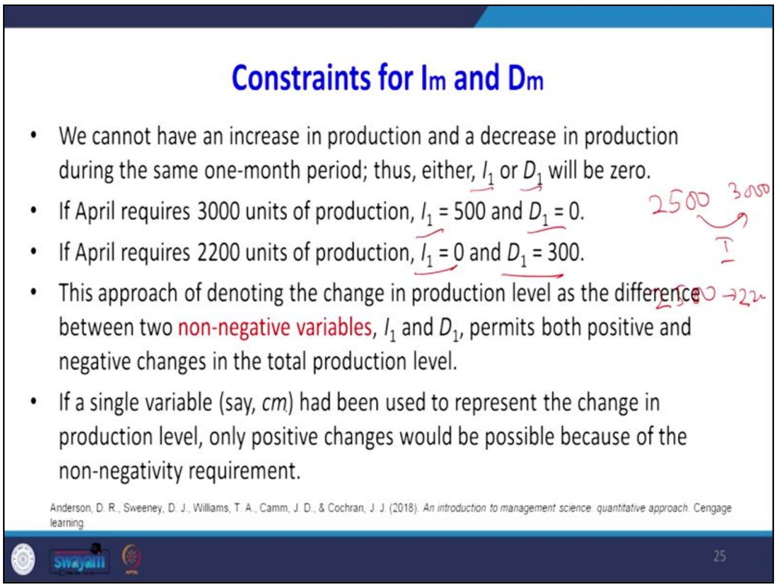


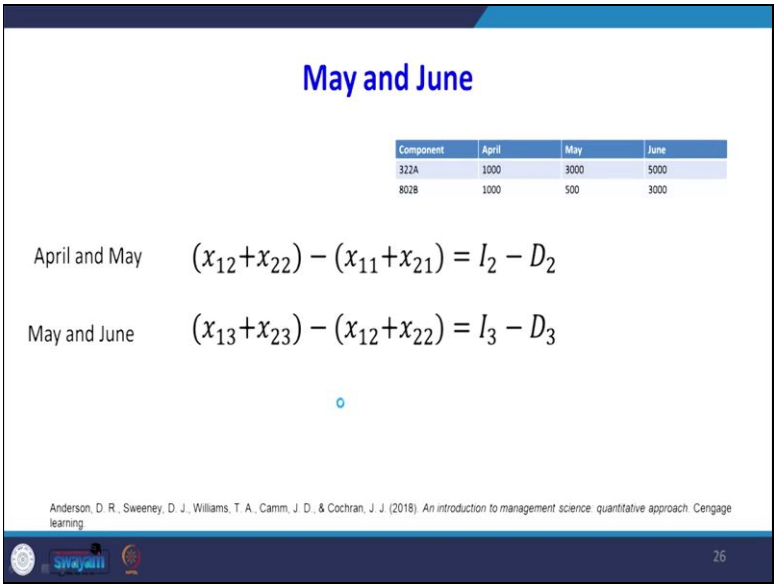
We cannot have an increase in production and a decrease in production during the same one-month period. That is, it can be either I_1 or D_1. It cannot be both.

If April requires 3000 units of production, $I_1 = 500$ and $D_1 = 0$.

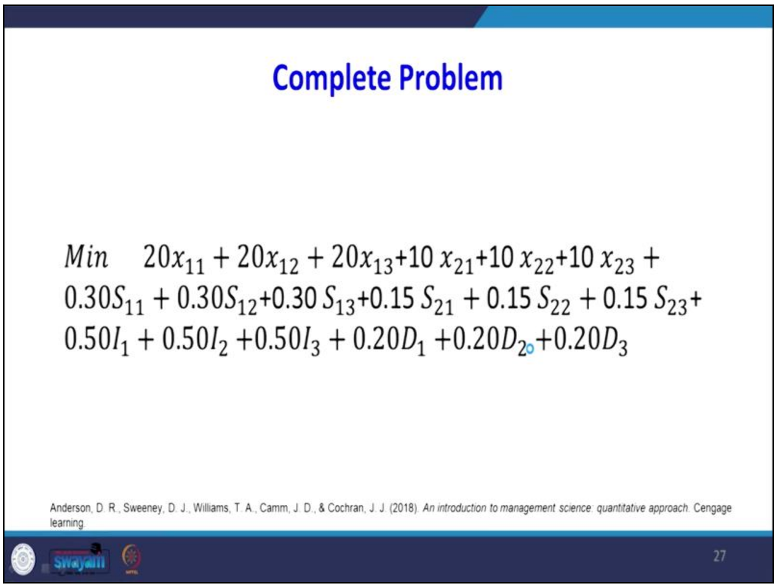
If April requires 2200 units of production, $I_1 = 0$ and $D_1 = 300$..

This approach of denoting the change in production level as the difference between two non-negative variables, I_1 and D_1, permits both positive and negative changes in the total production level. If a single variable (say, *cm*) had been used to represent the change in production level, only positive changes would be possible because of the non-negativity requirement.

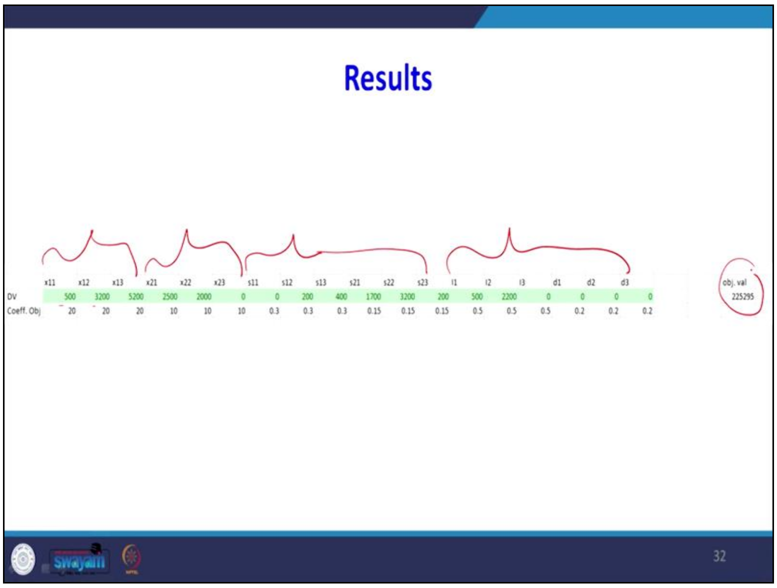
The negative changes cannot be represented that is why the change is represented by 2 variable I 1 D 1 advantages we can represent positive increment value and decrement value.



I have already explained it in April. Now we will write in the month suppose if you are jumping from the if you are producing from April to May. So, we have to go for a month's production that is $(x_{12}+x_{22}) - (x_{11}+x_{21}) = I_2 - D_2$ so, if you want to determine the variability in May and June. So, June month production $(x_{13}+x_{23}) - (x_{12}+x_{22}) = I_3 - D_3$



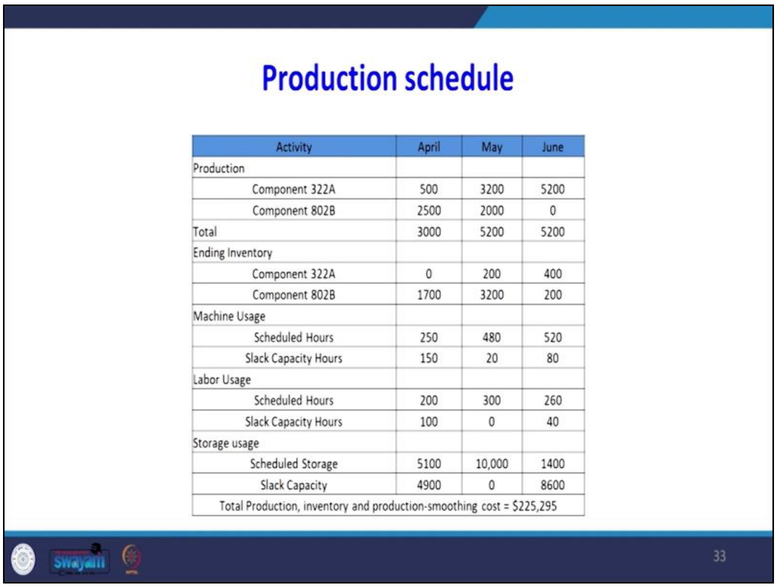
Now we will bring all our complete problems. This is a minimization problem because we are minimizing the cost. I have written my objective function, then I brought the I have brought the demand constraint for each month, then brought the ending inventory constraint, then I brought the capacity constraint machine capacity labor capacity the production scheduling problem.



I have solved it with the help of a solver. I brought the output of the solver. How do I read this output? You see, there is one product, one product, one month, one this one, this is product type: 1 month, 1 month, and 2 months. Today, this is month 3, and this is a production schedule. Product 2 month 1 month 2 month 3: This is inventory. How much inventory is this? This is our inventory cost. This is our increment or decrement cost. So, this is we have to minimize the cost.

So, the value of the objective function is 225295 dollars. Now I will explain with the help of the solver how I got this result.

Now I have brought this Excel sheet already, and I have solved it. So, I am going to change these values here. So, I will solve this problem here so that you will have a better understanding just ok. So, these are decision variable cells. This is a coefficient of the objective function. These are the various constraints, so go to data. So, go to solver. So this is a minimization type, so when you solve it.



Activity	April	May	June
Production			
Component 322A	500	3200	5200
Component 802B	2500	2000	0
Total	3000	5200	5200
Ending Inventory			
Component 322A	0	200	400
Component 802B	1700	3200	200
Machine Usage			
Scheduled Hours	250	480	520
Slack Capacity Hours	150	20	80
Labor Usage			
Scheduled Hours	200	300	260
Slack Capacity Hours	100	0	40
Storage usage			
Scheduled Storage	5100	10,000	1400
Slack Capacity	4900	0	8600
Total Production, inventory and production-smoothing cost = $225,295			

Now, I have brought the production schedule in the form of a table. So, this table gives the, for example, say, product A in the month of April. This is a production schedule for the month of May 3200 and the month of June 5200. The most important thing is when you are providing the output, you should give this output in such a way that the manager is able to easily understand instead of simply showing the Excel you have to bring in a form.

So that the manager can easily interpret what is more important. So, the total cost for this production schedule is 225295; otherwise, you can say 225,295 dollars. So, in this lecture, I have explained the application of linear programming problems in the operations area. The problem I have is production scheduling. So, we have taken a problem and formulated the problem, and we have been given a production schedule for the company. So that it will minimize the total production scheduling cost; thank you very much.