Decision Making With Spreadsheet

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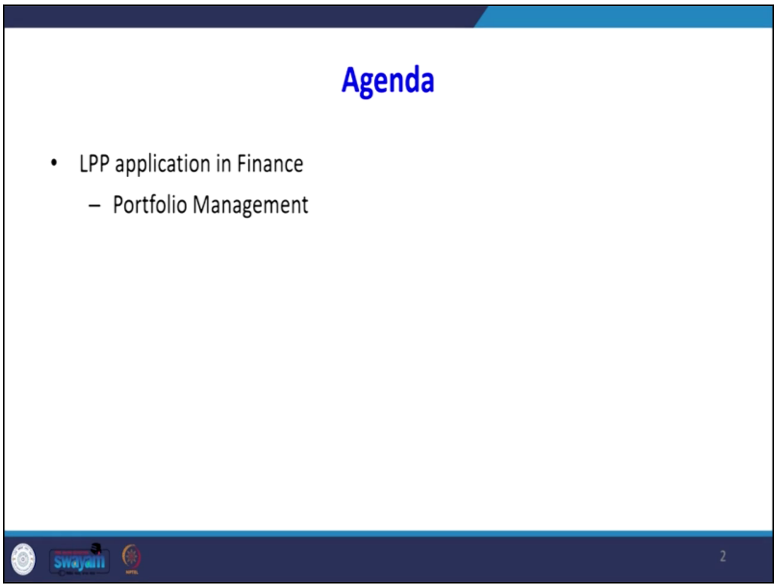
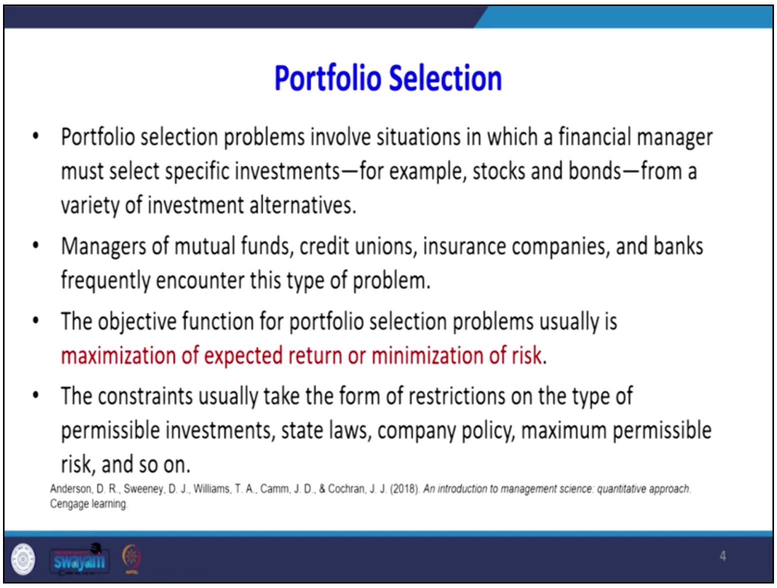
Lecture-10
LPP Applications in Finance

Dear students, in the previous lecture, I explained the application of linear programming in the marketing area. Now we will enter into another interesting year in finance. Now, I am going to explain the application of LP in the finance area. An important problem is called portfolio management.

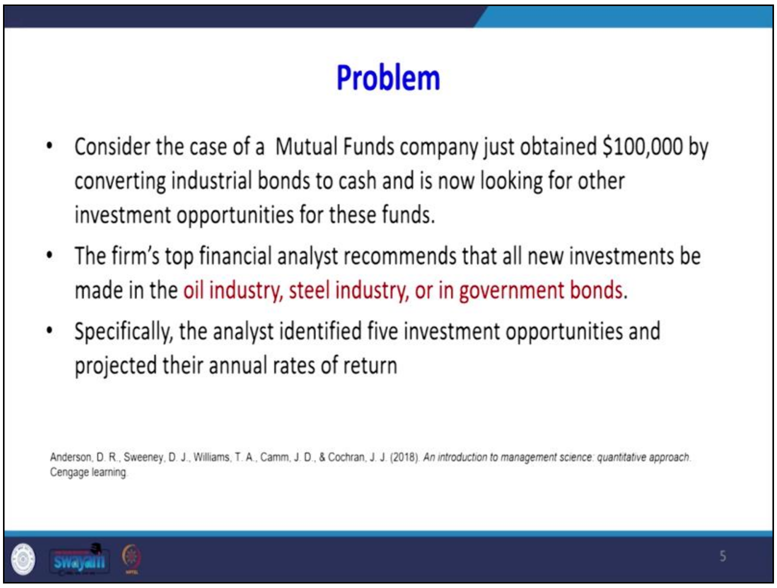


The agenda for this lecture is the application of LPP in finance, and we will take on an important problem called portfolio management.



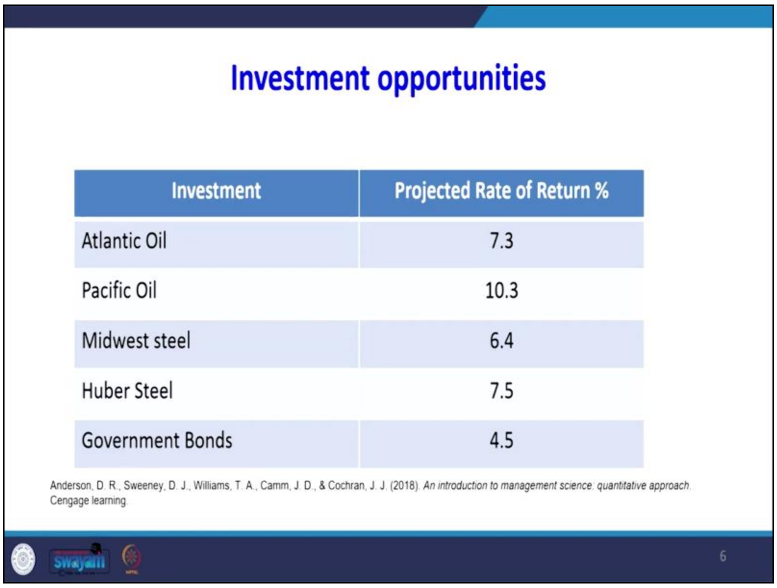
This problem is taken from the Anderson et al. book. So, what is the portfolio management? Portfolio selection problems involve situations in which a financial manager must select specific investments, for example, stocks and bonds, from a variety of investment alternatives. So, for a finance manager, there are a variety of alternatives. He has to select only certain specific investments that will maximize his return or minimize the risk that is your portfolio selection problem.

So, the managers of mutual funds, credit unions, insurance companies, and banks frequently encounter this type of problem. The objective function for portfolio selection problems is usually the maximization of expected returns and minimization of risk. The constraint usually takes the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk, and so on. These will become our constraints.



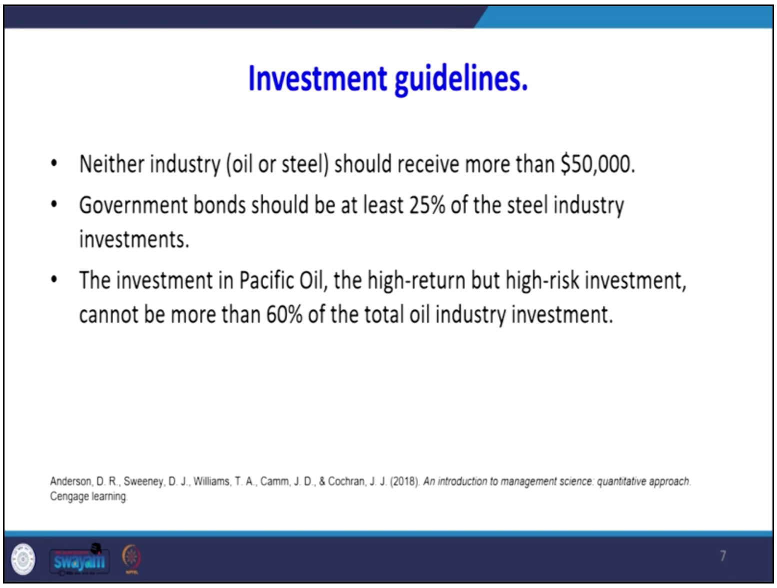
Now, what is the problem? We will take an illustrative problem. I will explain how to use linear programming problems for portfolio management. Consider the case of a mutual fund company that just obtained 100000 dollars by converting industrial bonds to cash, and he is now looking for other investment opportunities for these funds. The firm's top financial analyst recommends that all new investments be made in the oil industry, steel industry, or government bonds.

Specifically the analyst identified five investment opportunities and also given projected annual rate of return.

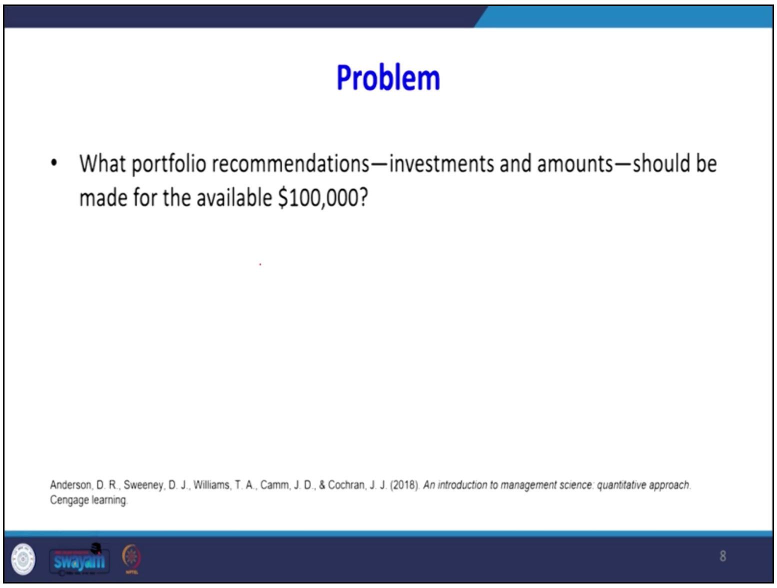


Investment	Projected Rate of Return %
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest steel	6.4
Huber Steel	7.5
Government Bonds	4.5

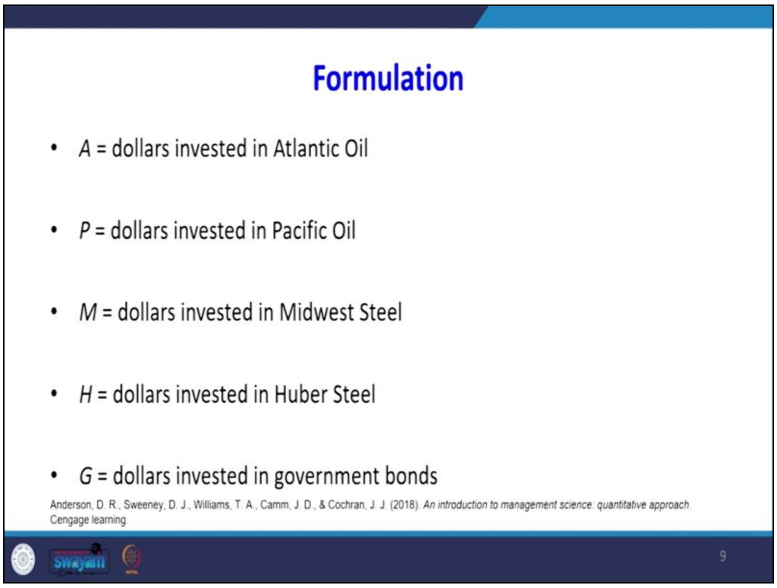
So, what are the five opportunities they can go for Atlantic oil in the oil sector? In the steel sector, they can go for Midwest Steel, Huber Steel, and government bonds. This is the projected rate of return. So, in this problem, we are maximizing the rate of return. If you want to maximize the rate of return, how many combinations or how many combinations of investment have to be made? There are some guidelines which are given by the company.



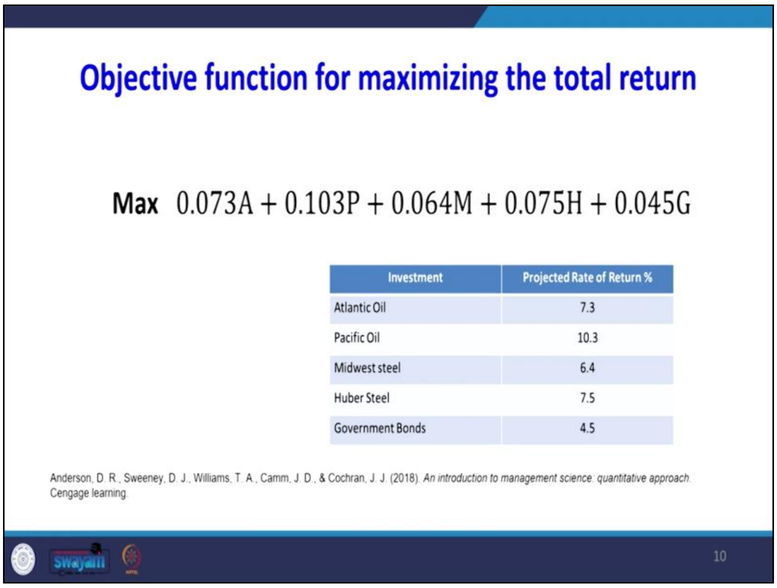
What are the guidelines? Neither industry nor steel should receive more than 50000 dollars. Should government bonds be at least 25 percent of steel industry investments? The investment in Pacific oil is a high return, but high-risk investment cannot be more than 60 percent of total oil industry investment.



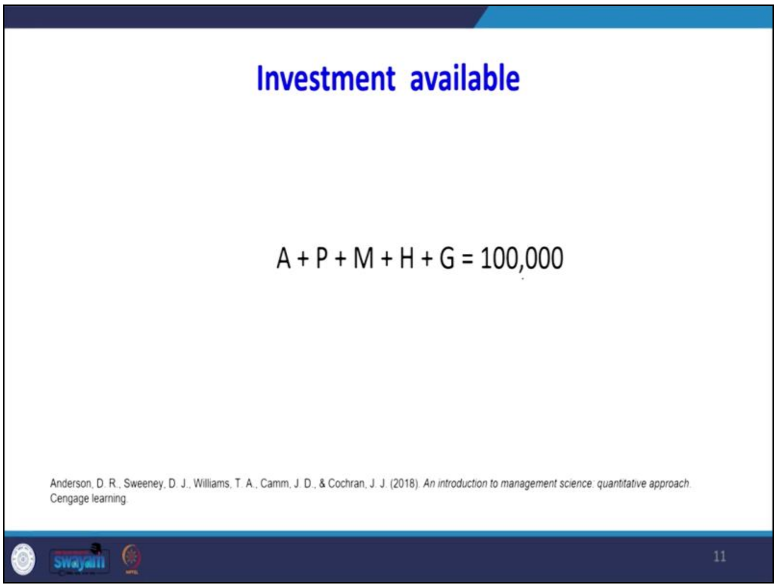
What portfolio recommendation, that is, investments and amounts, should be made available for this 100000 dollar? We have 100000 dollars with us, what portfolio recommendations, where to invest, and how much amount has to be invested that has to be given by the finance manager.



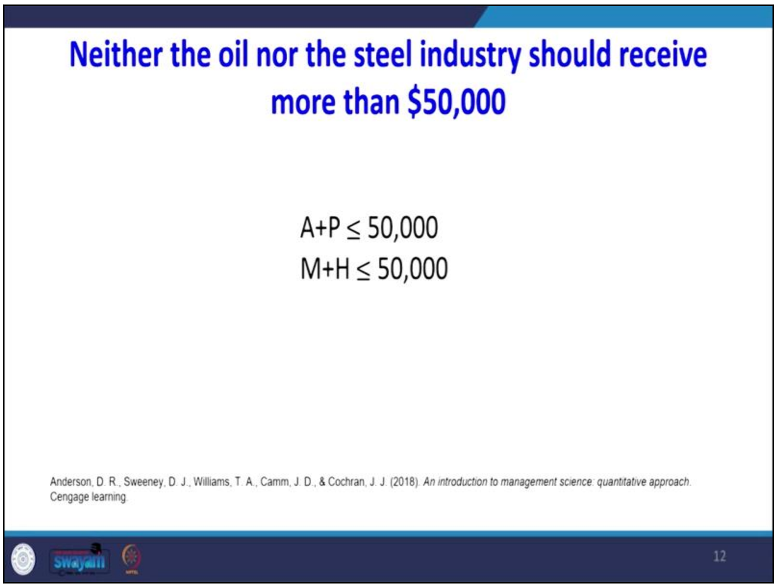
So, we have four five alternatives: A dollars invested in Atlantic oil, P dollars invested in Pacific oil, M dollars invested in Midwest steel, H dollars invested in Huber steel, and G dollars invested in government bonds. These are the decision variables.



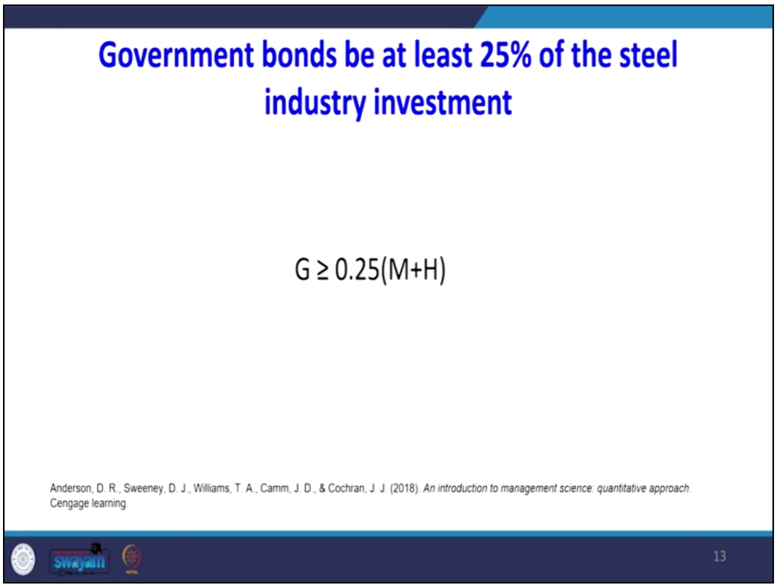
Our objective function is the maximization of our return. So, these are the returns which I have given previously that I brought again. So, 6.3 percent 10.3 I have taken in terms of decimal value.



So, this is 100000 dollars in the total amount available. So, this is A + P Atlantic Pacific Midwest Huber and Government fund. That amount should not exceed our available budget. It should be equal too.

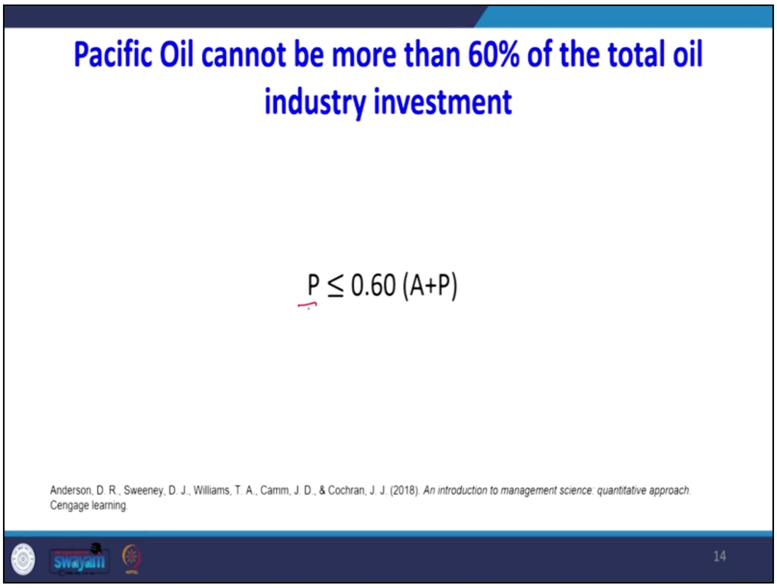


The second guideline is that neither the oil nor the steel industry should receive more than 50000. So, the total investment in the oil and steel industry should not go beyond 50000. So, there that means it should be less than 50000 dollars.

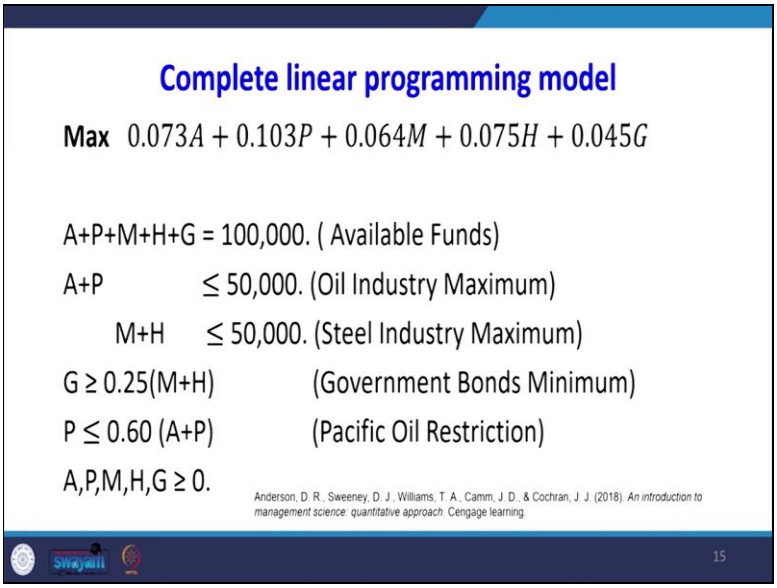


Government bonds are at least 25 percent of steel industry investment. So, the government wants G to be greater than or equal to 0.25. There are 2 steel investments.

G>=0.25(M+H)



Then, the Pacific oil cannot be more than 60 percent of the total oil industry investment. So, the P should not be more than that, which means it should be less than 60 percent of both our industry investments.

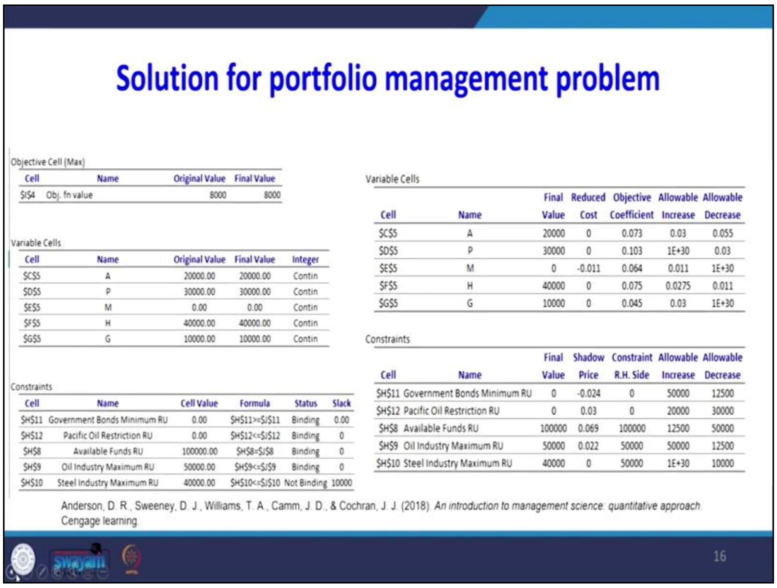


Now, I have a complete linear programming problem. So, this is a maximization problem. We have one equal-to-type constraint. There are two lesser equal to type constraints, one greater than or equal to types, one more another less than or equal to types, and three are three less than or equal to type constraints.

Now, I have brought this portfolio management problem to solve in Excel. This is the right-hand side. What you see is all the constraints and objective functions that I have formulated in Excel. So, here we know that, as usual, these are the decision values for which we will be getting the answer. The resources utilized here are objective functions. A sign is there, and this is a maximization type. So, the right-hand side value is given. So, I go to data solver.

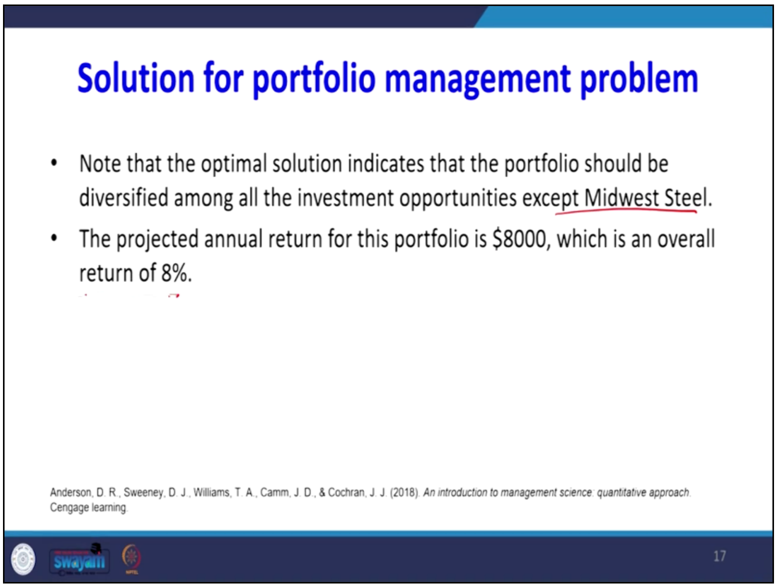
So, when you solve it, we need to answer sensitivity analysis and limits. Click it. So, what we are getting there are 5 investment opportunities. In A, we should invest 20000; in P, we should invest 30000, M you need not invest; in H, we should invest 40000. In G, we should invest 10000. So, when you go for this kind of investment opportunity, your objective function will be 8000.

Now, we will explain the sensitivity of this answer report. Now, I will explain the answer to the sensitivity analysis report in detail.

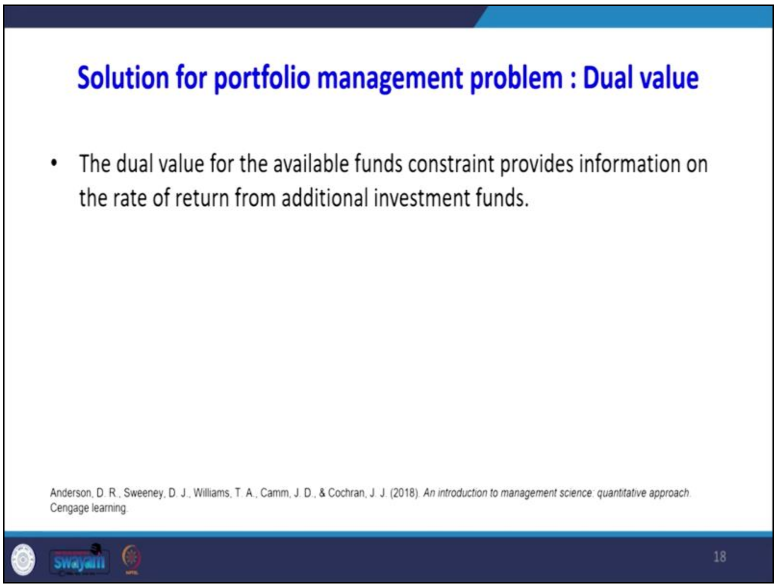


Now, I have brought the output of the solver. Now, we will interpret the answer in detail. First, we look at the objective function. So, the total return is 8000 dollars. What are the different 5, 1, 2, 3, 4, and 5 investment opportunities? See that you should go for 20000 in A, 30000 in B, 40000 in H, and 10000 in G. On The right-hand side, you can see the reduced cost. For example, here, the M does not appear in the solutions, which means there is a negative reduction cost.

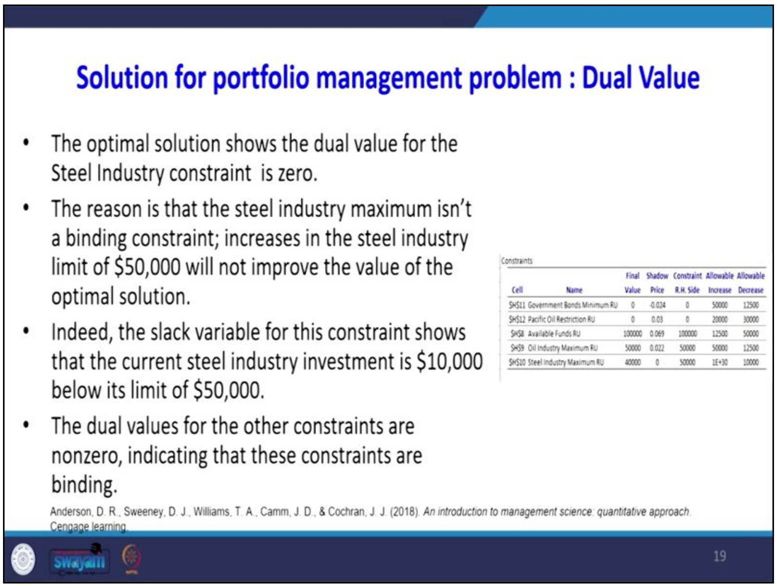
And there are slack variables there, so this is our surplus. Here, there is are slack values is, and then there also shadow prices. I will explain this output in detail.



Note that the optimal solution indicates that the portfolio should be diversified among all investment opportunities except Midwest steel. So, we should go for diversification, but we should not invest in Midwest steel. The projected annual return for this portfolio is 8000 dollars, which is an overall return of 8%.



The dual value for the available funds constraint provides information on the rate of return from additional investment funds.



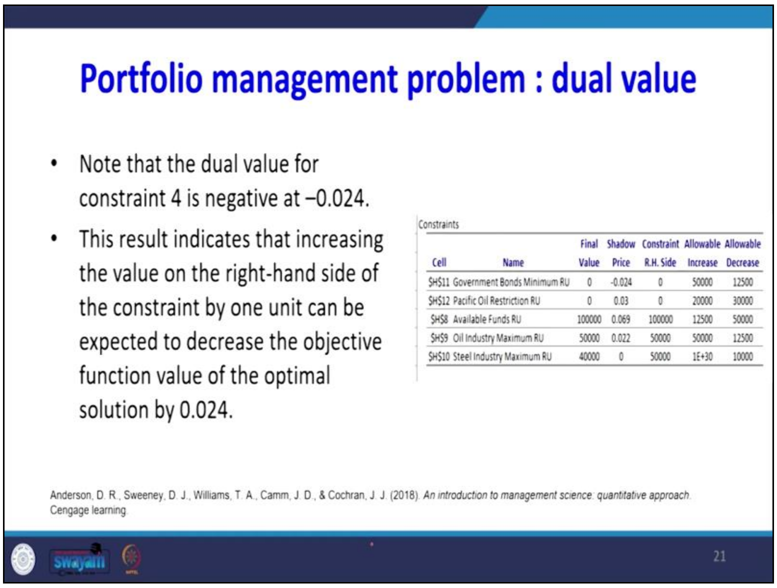
Now, I will explain the dual value of the problem. The optimal solution shows that the dual value for the steel industry constraint is zero. This is the reason that the steel industry maximum is not a binding constraint. So, increases in the steel industry limit of 50000 dollars will not improve the value of the optimal solution because it is not a binding constraint. So, that will not help to increase your objective function value.

Indeed, the slack variable for this constraint shows that the current steel industry investment is 10000 dollars below its limit of 50000 because, you see, the hand side is 50000, but the final value is forty thousand. So, 10000 dollars is the slack variable that has not been utilized yet. There are unutilized resources there. So, adding any extra resources will not help in improving your objective function, which is why the shadow price is zero.

The dual values for the other constraints are nonzero, for example, here, indicating that these constraints are binding constraints. The dual value of 0.0694 funds available constraint shows that the value of the optimal solution can be increased by 0.069 if one more dollar can be made available for portfolio investment. We are discussing the following: funds available constraint this constraint.

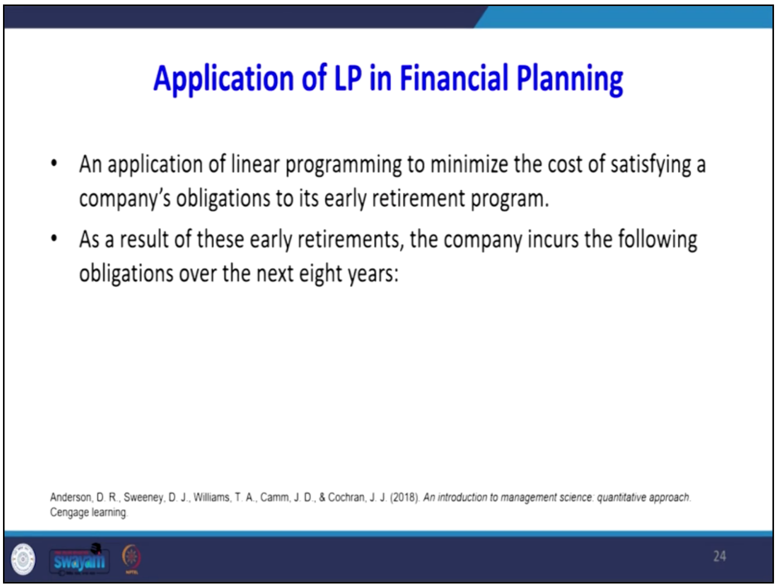
Because we have a positive shadow price and a positive dual value, if the funds available are increased by one dollar, our objective function will increase by 0.069. If more funds can be obtained at the cost of less than 6.9 percent, the management should consider obtaining them because we will be earning more value here; however, if a return of more than 6.9 percent can be obtained by investing funds elsewhere other than these five securities.

So, management should question the wisdom of investing the entire 100000 dollar in this portfolio what is the meaning of this one. If you can get your fund at the interest rate of 6.9 percent, the management should go for that. If you can get the fund, which is more than more than 6.9 percent, we need not go for these five options. We can go for other options to get more return than this portfolio than these investment opportunities.

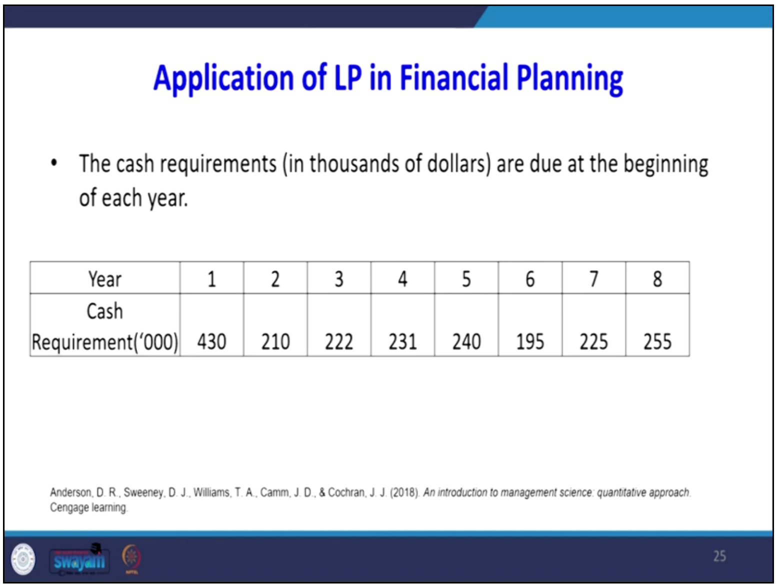


Note that the dual value for constraint 4 is negative here for government bonds here, it is negatives ok. The result indicates that increasing the dual value and the right-hand side of the constraint by one unit can be expected to decrease the objective function because the shadow price is negative. If you increase on the right-hand side by one unit, the objective function will decrease by 0.024.

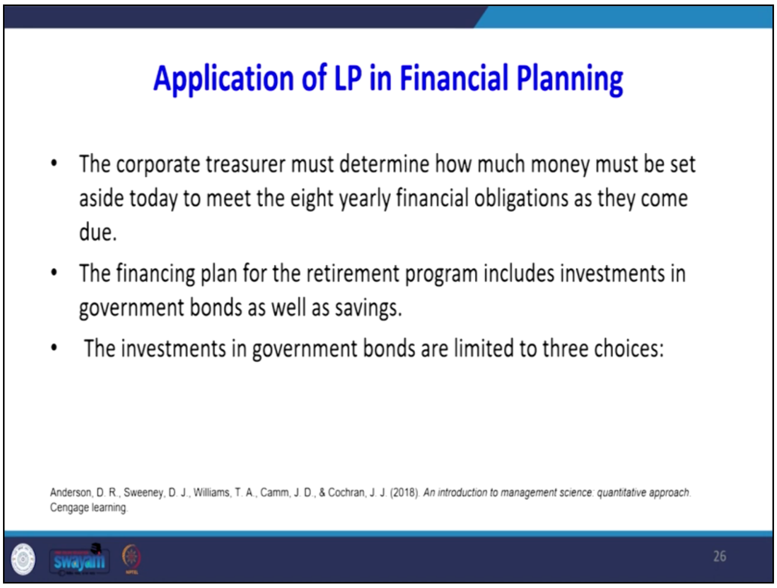
Now, we will go for the application of linear programming problems in another application called financial planning.



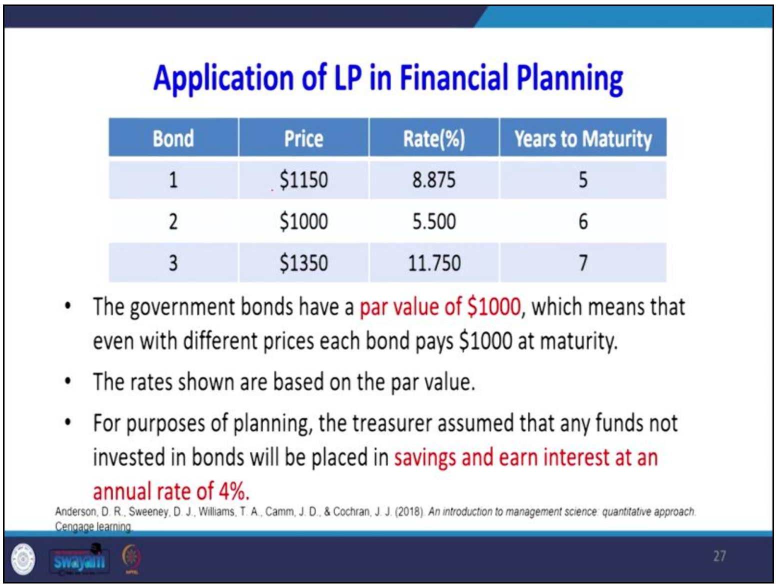
The application of linear programming to minimize the cost of satisfying a company's obligations to its early retirement program is the next application of our LP in the finance area. As a result of this early retirement, the company incurs the following obligations over the next eight years: what are the obligations?



The cash requirement in terms of thousands of dollars is due at the beginning of each year. So, because of this yearly retirement plan, in year one, you need 430000-dollar cash requirement in years 2, year 3, up to 8 years.

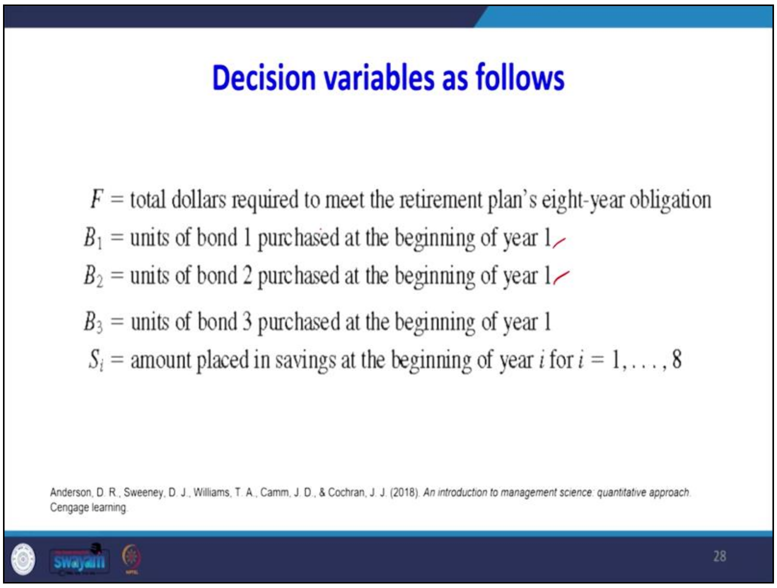


The corporate treasurer must determine how much money must be set aside today to meet the eight-year financial obligations as they come due. The financial plan for the retirement program includes investment in government bonds as well as savings. The investment in government bonds is limited to 3 choices: what are they?

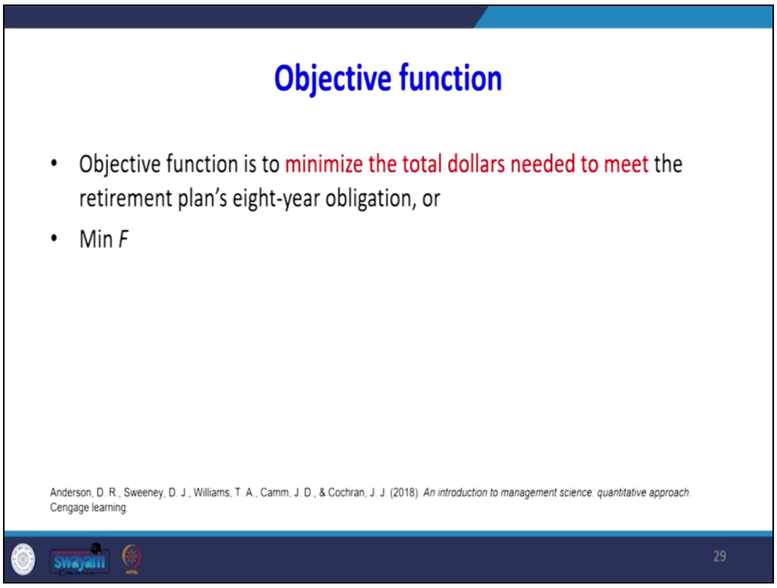


Bond 1, the price is 1150 dollars, the return rate is 8.87 percent, but the year to maturity is 5 years. The second bond is priced at a thousand dollars at a rate of 5.5 percentage years, with a maturity of six. The third one is the price of 1350, a rate of 11.75 percentage years to mature with a maturity of 7. The government bonds have a par value of 1000 dollars, which means that even with the different prices, each bond pays 1000 dollars at maturity.

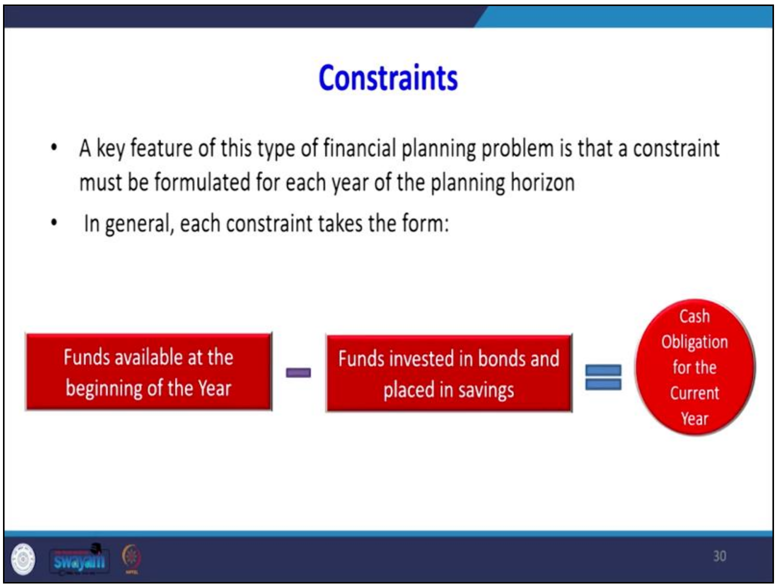
The par value is 1000 dollars, and the rates shown are based on the power value. For the purpose of planning, the treasurer assumed that any funds not invested in bonds would be placed in savings and earn interest at an annual rate of 4 percent. So, the problem is that the company has some financial obligations.



For that financial obligation, decision variable F is equal to the total dollars required to meet the retirement plan's eight-year obligation. B_1 units of bond 1 were purchased at the beginning of year one, and B_2 units of bond 2 were purchased at the beginning of year one. B_3 units of bond 3 were purchased at the beginning of year one. Si amount is placed in savings at the beginning of year i for i equal to 1 to 8.

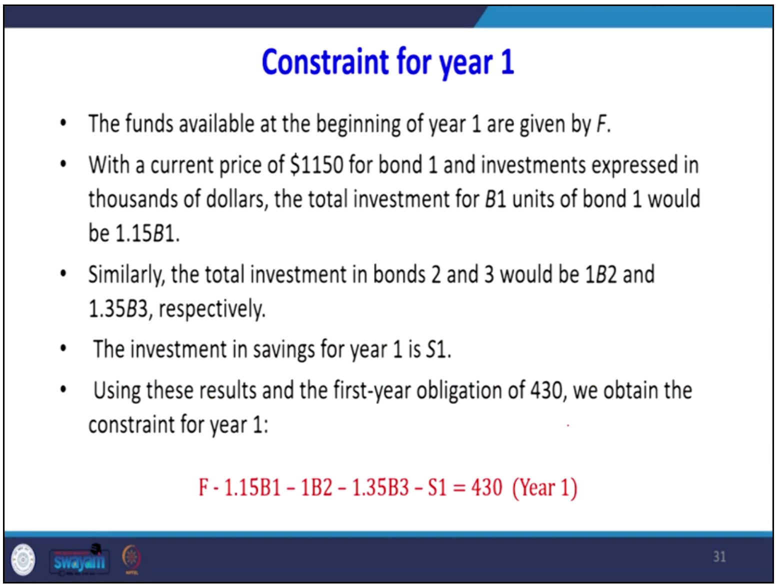


The objective function is to minimize the total dollars needed to meet the retirement plan year obligations. That is, this F has to minimize what is the F says total dollar needed for meeting the eight years obligations.



Constraints A key feature of this type of financial planning problem is that a constraint must be formulated for each year of the planning horizon. In general, each constraint takes the form of like this. So, the funds available at the beginning of the year from this point are invested in bonds and placed in savings, which is equal to cash obligation for the current year.

So, the amount initially available is minus that year how much fund is invested in bonds and savings that should be equal to the cash obligation for the current year this way the constraint needs to be formulated.

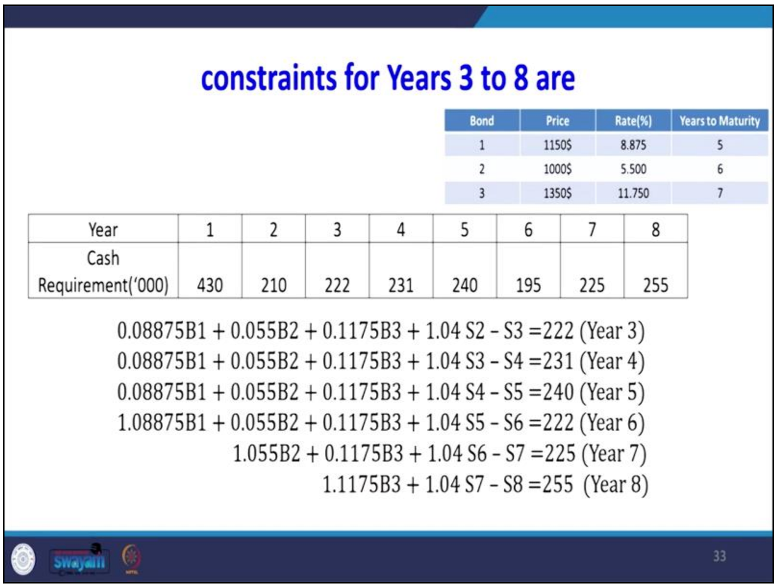


So, the constraint for year one is that the funds available at the beginning of year one is given by F with a current price of 1150 dollars for bond 1, and the investment expressed in 1000 dollars, the total investment for B_1 unit of bond 1 would be $1.15B_1$. So, that much will be invested in bond 1. Similarly, the total investment in bonds 2 and 3 would be one B_2 because of the 1000 we have taken in terms of thousand one B_2 and 1.35 B_3, respectively.
The investment in saving for year one is S_1. Using this result, the first-year obligation of 430, we obtain constraints for year one. So, the F is the beginning of the year, and how much is required? Then, how much return from bond 1, bond 2, and bond 3 must be subtracted if any bond any uninvested money is there that will go for the savings? So, that will fulfill the 430. So, that means the first-year obligation is 430000 dollars. I have the initial money, right? This will be invested in bonds.

Still, there are monies there that will be invested in savings, but that should meet my first-year obligation of 430.

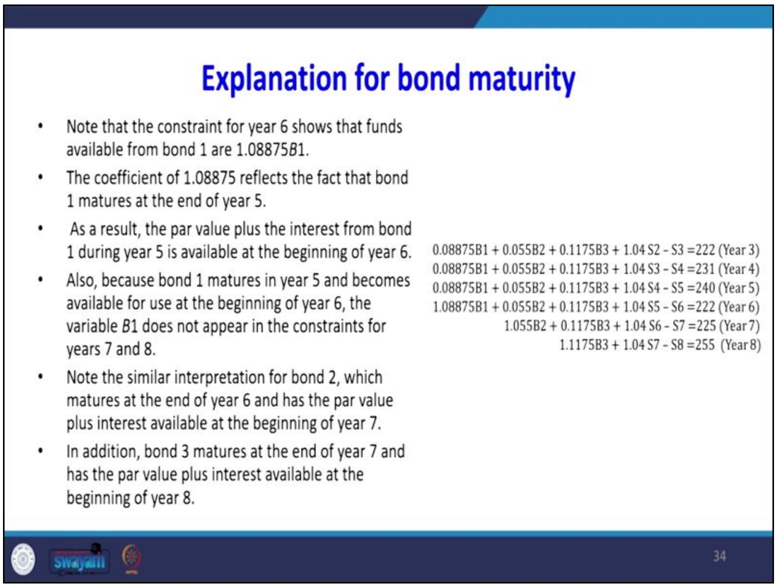
Constraint year 2 investment in bonds can take place only in the first year and the bonds will be held until maturity. The funds available at the beginning of year 2 include an investment return of 8.87 percent on the power value of bond 1 because in year one already, we have invested. So, we will get 8.875 percent on the power value of bond 1, 5.5 percent on the power value of bond 2, 11.75 percent on the power value of bond 3, and 4 percent on saving.

The new amount to be invested in saving for year 2 is S_2 ok. So, with the obligation of 210, the constraint for year 2 is like this. So, on the right-hand side, you can directly write 210, which is an obligation. So, this amount of the bond has earned some interest, and the investments made in year 1 also earn some interest. So, this money will be used to fulfill the second-year obligation even after fulfilling any money left that will be invested in S_2. That is why I brought it to the left-hand side; it will be $(S - S_2)$.



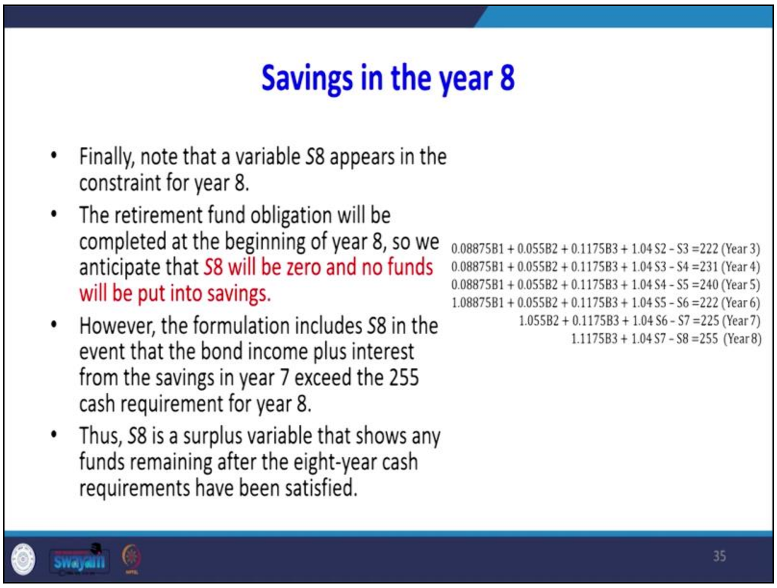
Now, constraints for years 3 to eight are the same way we have to form the constraint for year 3, year 4, year 5, and year 6. There is a difference; I will explain what is why it is 1.08. Similarly, for year 7, it is 1.055. For year eight, it is 1.17 because the bond 1 will take 5 years for maturity. So, in the first year, we have matured. So, in the sixth year, it is coming along with the principle, principle, and interest.

So, bond 2 is in the seventh year because it is maturity is six years. In the seventh year, we are coming along with the principal interest, which is why it is 1.05. Bond 3 will take seven years. So, in the eighth year, it will come along with the principle and interest, which is why it is 1.17.



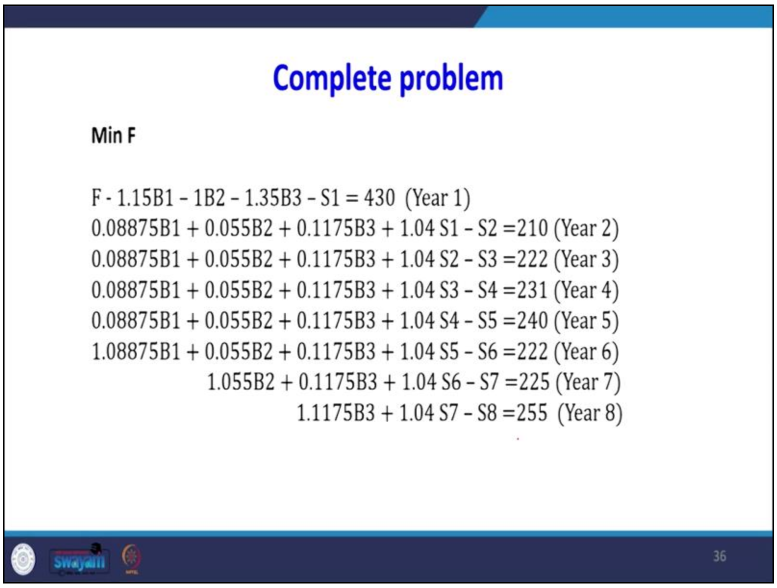
Note that the constraint for year 6 here shows that the funds available for bond 1 is 1.08875B 1. The coefficient of 1.08875 reflects the fact that bond 1 matures at the end of year five. As a result, the par value plus the interest from the bond 1 during the year five is available at the beginning of year six. Also, because bond 1 matures in five years and becomes available for use at the beginning of year 6, the variable B 1 does not appear in constraint for years 7 and 8.

You see that here, there is no variable B1 or B2. Note that the similar interpretation for bond 2 matures at the end of year 6 and has the par value plus interest available at the beginning of year 7. In addition, bond 3 matures at the end of year seven and has the power value plus interest available at the beginning of year eight.



Now, the savings are in year eight because year eight is the last year. How are we? What is the meaning of why we introduce this S8? Finally, note that the variable S8 appears in the constraint for year eight. The retirement fund obligation will be completed at the beginning of year 8. So, we anticipate that S8 will be 0, and no funds will be put into savings. Even though we wrote S8 because we made an equation in such a way that it fulfilled the applications, the value of S8 will become zero.

However, the formulation includes S8 if the bond income plus interest from saving in year seven exceeds our 255 cash requirement for year 8. Thus, yes, it is a surplus variable that shows any funds remaining after the eight-year cash requirements have been satisfied. That is why we have introduced variable S8.

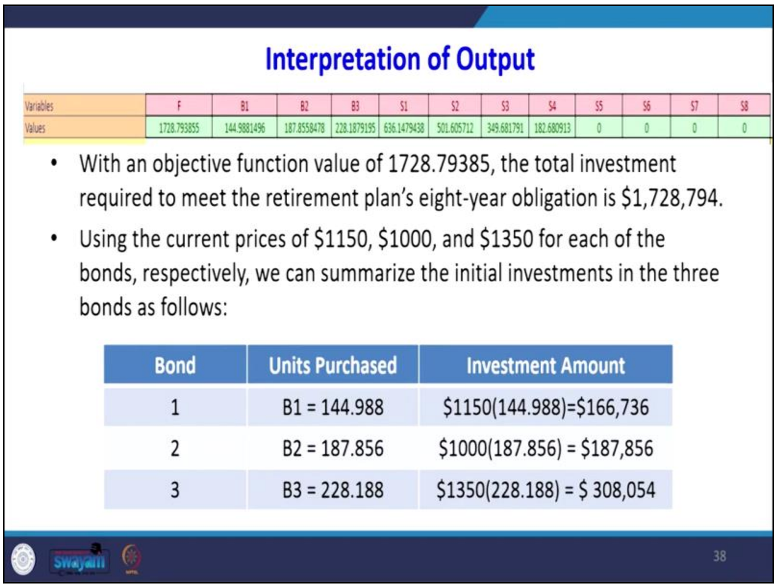


Now, this is our complete problem that I am going to solve with the help of a solver.

Now, I have brought this our formulated problem to solve in Excel. So, here this is the value as usual where we are going to get the answer, which is the changing cells. The hand side is written on this side, and the resources utilize the resources on resources written here. So, now I will go to the data solver. Look at this problem. This is a minimization problem. When you solve it, we need to answer the limit.

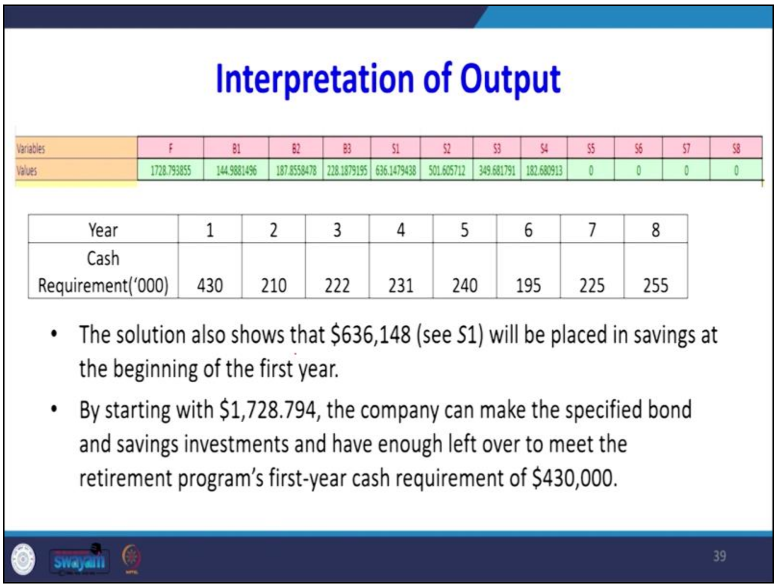
Why we are saying, it is a minimization because, at the beginning of the year, we should have the minimum amount of amount minimum requirement of amount. So, that should be invested in bonds. If it remains, it should be invested in savings to meet the first-year obligations. That is why it is a minimization problem. So this says I will interpret the answers and sensitivity report now.

Now I have brought the Excel output. So, the value of F is 1728. So, that is the amount we required at the beginning of year one. I will explain this slack variable and shadow price.

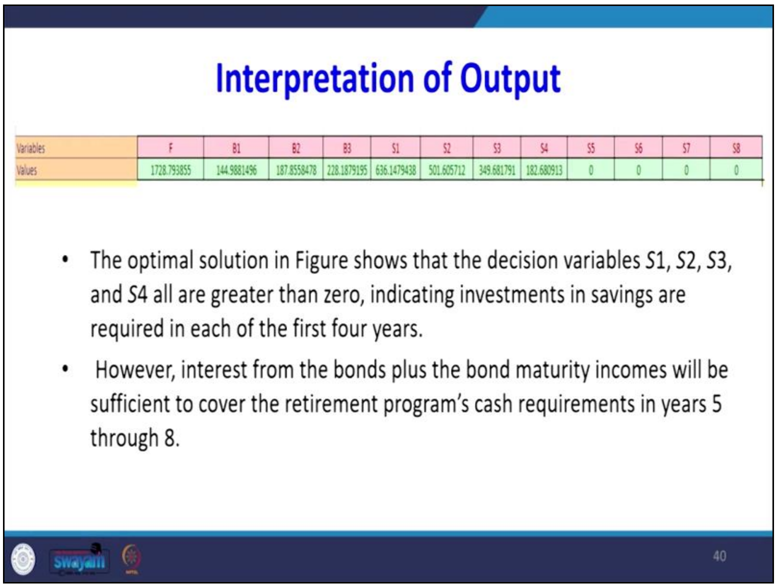


With an objective function value of this much, 1728, the total investment required to meet the requirement plan 8-year obligation that is we need because all the values in terms of 1000, we need 1728797 using the current prices of 1150, 1000, and 1350 dollars for each of the bonds respectively we can summarize the initial investment in the 3 bonds as follows. So, initially, we purchased B1= 144, B2 =187, and B3 =228.

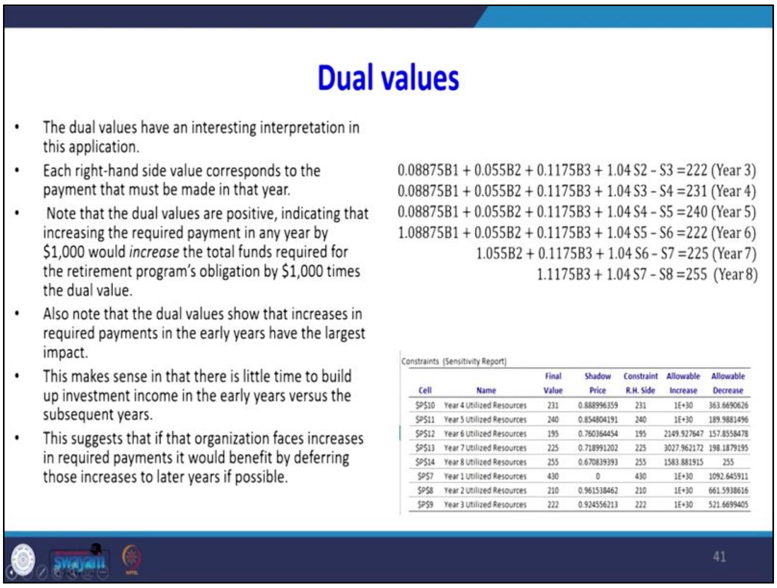
So, the actual amount is this quantity amount when you multiply by their power value. When you multiply this, you will get this many answers.



The solution also shows that 636148, that is, S1 where is this one S 1 will be placed in savings at the beginning of the first year. By starting with 1728.797, the company can make a specific bond and save investment and have enough left to meet the retirement programs for the first year cash requirement of 430000 dollars. So, after meeting the first requirement this much so the remaining amount invested is $ 636.17.



The optimal solution figure shows that the decision variables S1, S2, S3, and S4 this one S1 S2, S3, S4 are all greater than zero, indicating that investment in savings is required in each of the first four years. However, the interest from the bond plus the bond maturity income will be sufficient to cover the retirement program cash requirement in years five through eight, which is why no saving is required in years five, six, seven, or eight.



Now, we will interpret the dual value. The dual values have an interesting interpretation in this application, where the dual value is where we have this dual value as our dual value. Each right-hand side value corresponds to the payment that must be made in that year. So, this is the value of this amount we might be paying in that year. Note that the dual values are positive, indicating that increasing the required payment in any year by one unit we have in terms of thousands.

So, one unit would increase the total funds required for the retirement program application by one times the dual value, that is 1000 times. Also, note that the dual values show that increases in the required payments in the early years have the largest impact on early years. Look at years 1, 2, and 3. For example, in years 2 and 3, the dual values are 0.96, 0.92. This makes sense because there is little time to build up investment income in the early years versus subsequent years.

But in the subsequent years, the shadow price has become less. So, what we are inferring from this is if that organization faces increases in required payments, it would benefit by differing those increases to a later year if possible; if they defer that payment in later years, that is good for that organization because they can have the lesser initial amount requirement. In this lecture, I explained the application of linear programming problems in finance.

We have taken on 2 problems: one problem is portfolio management; another problem is financial planning. I formulated the problem and solved it with the help of Excel, and then we interpreted the results; thank you very much.