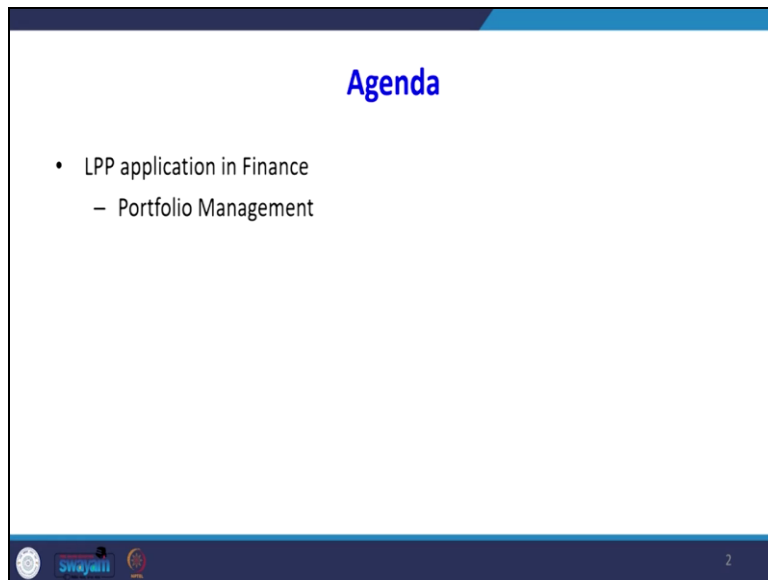


Decision Making With Spreadsheet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology-Roorkee




Lecture-10
LPP Applications in Finance

Dear students, in the previous lecture, I explained the application of linear programming in the marketing area. Now we will enter into another interesting year in finance. Now, I am going to explain the application of LP in the finance area. An important problem is called portfolio management.



Agenda

- LPP application in Finance
 - Portfolio Management

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The agenda for this lecture is the application of LPP in finance, and we will take on an important problem called portfolio management.

Portfolio Selection

- Portfolio selection problems involve situations in which a financial manager must select specific investments—for example, stocks and bonds—from a variety of investment alternatives.
- Managers of mutual funds, credit unions, insurance companies, and banks frequently encounter this type of problem.
- The objective function for portfolio selection problems usually is **maximization of expected return or minimization of risk**.
- The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk, and so on.

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning



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This problem is taken from the Anderson et al. book. So, what is the portfolio management? Portfolio selection problems involve situations in which a financial manager must select specific investments, for example, stocks and bonds, from a variety of investment alternatives. So, for a finance manager, there are a variety of alternatives. He has to select only certain specific investments that will maximize his return or minimize the risk that is your portfolio selection problem.

So, the managers of mutual funds, credit unions, insurance companies, and banks frequently encounter this type of problem. The objective function for portfolio selection problems is usually the maximization of expected returns and minimization of risk. The constraint usually takes the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk, and so on. These will become our constraints.

Problem

- Consider the case of a Mutual Funds company just obtained \$100,000 by converting industrial bonds to cash and is now looking for other investment opportunities for these funds.
- The firm's top financial analyst recommends that all new investments be made in the **oil industry, steel industry, or in government bonds**.
- Specifically, the analyst identified five investment opportunities and projected their annual rates of return

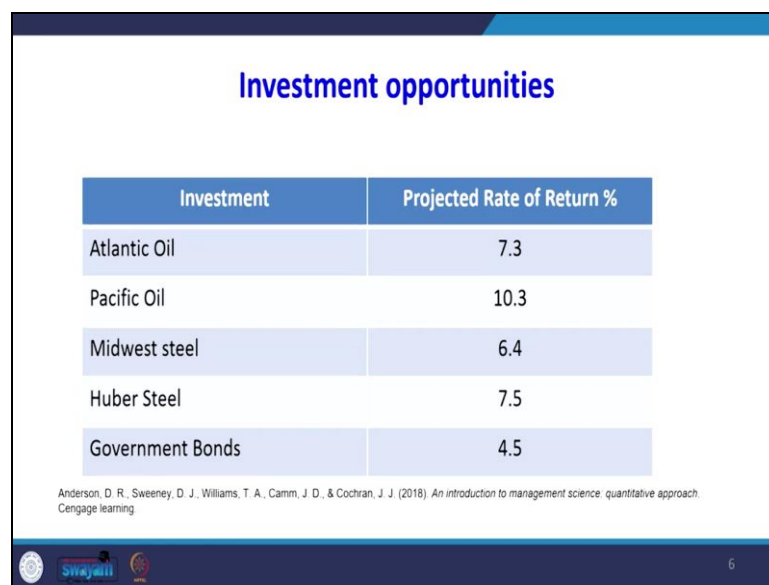
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Now, what is the problem? We will take an illustrative problem. I will explain how to use linear programming problems for portfolio management. Consider the case of a mutual fund company that just obtained 100000 dollars by converting industrial bonds to cash, and he is now looking for other investment opportunities for these funds. The firm's top financial analyst recommends that all new investments be made in the oil industry, steel industry, or government bonds.

Specifically the analyst identified five investment opportunities and also given projected annual rate of return.



The image shows a slide titled "Investment opportunities" with a table listing five investment options and their projected rates of return. The table is as follows:

Investment	Projected Rate of Return %
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest steel	6.4
Huber Steel	7.5
Government Bonds	4.5

Below the table, there is a citation: Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.


The slide also features a footer with logos for Swayam and a page number 6.

So, what are the five opportunities they can go for Atlantic oil in the oil sector? In the steel sector, they can go for Midwest Steel, Huber Steel, and government bonds. This is the projected rate of return. So, in this problem, we are maximizing the rate of return. If you want to maximize the rate of return, how many combinations or how many combinations of investment have to be made? There are some guidelines which are given by the company.

Investment guidelines.

- Neither industry (oil or steel) should receive more than \$50,000.
- Government bonds should be at least 25% of the steel industry investments.
- The investment in Pacific Oil, the high-return but high-risk investment, cannot be more than 60% of the total oil industry investment.

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What are the guidelines? Neither industry nor steel should receive more than 50000 dollars. Should government bonds be at least 25 percent of steel industry investments? The investment in Pacific oil is a high return, but high-risk investment cannot be more than 60 percent of total oil industry investment.

Problem

- What portfolio recommendations—investments and amounts—should be made for the available \$100,000?

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What portfolio recommendation, that is, investments and amounts, should be made available for this 100000 dollar? We have 100000 dollars with us, what portfolio recommendations, where to invest, and how much amount has to be invested that has to be given by the finance manager.

Formulation

- A = dollars invested in Atlantic Oil
- P = dollars invested in Pacific Oil
- M = dollars invested in Midwest Steel
- H = dollars invested in Huber Steel
- G = dollars invested in government bonds

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So, we have four five alternatives: A dollars invested in Atlantic oil, P dollars invested in Pacific oil, M dollars invested in Midwest steel, H dollars invested in Huber steel, and G dollars invested in government bonds. These are the decision variables.

Objective function for maximizing the total return

$$\text{Max } 0.073A + 0.103P + 0.064M + 0.075H + 0.045G$$

Investment	Projected Rate of Return %
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest steel	6.4
Huber Steel	7.5
Government Bonds	4.5

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Our objective function is the maximization of our return. So, these are the returns which I have given previously that I brought again. So, 6.3 percent 10.3 I have taken in terms of decimal value.

Investment available

$$A + P + M + H + G = 100,000$$

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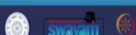
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So, this is 100000 dollars in the total amount available. So, this is A + P Atlantic Pacific Midwest Huber and Government fund. That amount should not exceed our available budget. It should be equal too.

Neither the oil nor the steel industry should receive more than \$50,000

$$A+P \leq 50,000$$
$$M+H \leq 50,000$$

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The second guideline is that neither the oil nor the steel industry should receive more than 50000. So, the total investment in the oil and steel industry should not go beyond 50000. So, there that means it should be less than 50000 dollars.

Government bonds be at least 25% of the steel industry investment

$$G \geq 0.25(M+H)$$

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Government bonds are at least 25 percent of steel industry investment. So, the government wants G to be greater than or equal to 0.25. There are 2 steel investments.
 $G \geq 0.25(M+H)$

Pacific Oil cannot be more than 60% of the total oil industry investment

$$P \leq 0.60(A+P)$$

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Then, the Pacific oil cannot be more than 60 percent of the total oil industry investment. So, the P should not be more than that, which means it should be less than 60 percent of both our industry investments.

Complete linear programming model

Max $0.073A + 0.103P + 0.064M + 0.075H + 0.045G$

$A+P+M+H+G = 100,000.$ (Available Funds)

$A+P \leq 50,000.$ (Oil Industry Maximum)


$M+H \leq 50,000.$ (Steel Industry Maximum)

$G \geq 0.25(M+H)$ (Government Bonds Minimum)

$P \leq 0.60 (A+P)$ (Pacific Oil Restriction)

$A,P,M,H,G \geq 0.$

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Now, I have a complete linear programming problem. So, this is a maximization problem. We have one equal-to-type constraint. There are two lesser equal to type constraints, one greater than or equal to types, one more another less than or equal to types, and three are three less than or equal to type constraints.

Now, I have brought this portfolio management problem to solve in Excel. This is the right-hand side. What you see is all the constraints and objective functions that I have formulated in Excel. So, here we know that, as usual, these are the decision values for which we will be getting the answer. The resources utilized here are objective functions. A sign is there, and this is a maximization type. So, the right-hand side value is given. So, I go to data solver.

So, when you solve it, we need to answer sensitivity analysis and limits. Click it. So, what we are getting there are 5 investment opportunities. In A, we should invest 20000; in P, we should invest 30000, M you need not invest; in H, we should invest 40000. In G, we should invest 10000. So, when you go for this kind of investment opportunity, your objective function will be 8000.

Now, we will explain the sensitivity of this answer report. Now, I will explain the answer to the sensitivity analysis report in detail.

Solution for portfolio management problem

Objective Cell (Max)			
Cell	Name	Original Value	Final Value
\$S\$4	Obj. fn value	8000	8000

Variable Cells			
Cell	Name	Original Value	Final Value
\$C\$5	A	20000.00	20000.00
\$D\$5	P	30000.00	30000.00
\$E\$5	M	0.00	0.00
\$F\$5	H	40000.00	40000.00
\$G\$5	G	10000.00	10000.00

Constraints			
Cell	Name	Cell Value	Formula
\$H\$11	Government Bonds Minimum RU	0.00	\$H\$11>=\$I\$11
\$H\$12	Pacific Oil Restriction RU	0.00	\$H\$12<=\$I\$12
\$H\$8	Available Funds RU	100000.00	\$H\$8<=\$I\$8
\$H\$9	Oil Industry Maximum RU	50000.00	\$H\$9<=\$I\$9
\$H\$10	Steel Industry Maximum RU	40000.00	\$H\$10<=\$I\$10

Variable Cells			
Cell	Name	Final Value	Reduced Cost
\$C\$5	A	20000	0
\$D\$5	P	30000	0
\$E\$5	M	0	-0.011
\$F\$5	H	40000	0
\$G\$5	G	10000	0

Constraints			
Cell	Name	Final Value	Shadow Price
\$H\$11	Government Bonds Minimum RU	0	-0.024
\$H\$12	Pacific Oil Restriction RU	0	0.03
\$H\$8	Available Funds RU	100000	0.069
\$H\$9	Oil Industry Maximum RU	50000	0.022
\$H\$10	Steel Industry Maximum RU	40000	0

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Now, I have brought the output of the solver. Now, we will interpret the answer in detail. First, we look at the objective function. So, the total return is 8000 dollars. What are the different 5, 1, 2, 3, 4, and 5 investment opportunities? See that you should go for 20000 in A, 30000 in B, 40000 in H, and 10000 in G. On The right-hand side, you can see the reduced cost. For example, here, the M does not appear in the solutions, which means there is a negative reduction cost.

And there are slack variables there, so this is our surplus. Here, there is are slack values is, and then there also shadow prices. I will explain this output in detail.

Solution for portfolio management problem

- Note that the optimal solution indicates that the portfolio should be diversified among all the investment opportunities except Midwest Steel.
- The projected annual return for this portfolio is \$8000, which is an overall return of 8%.

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Note that the optimal solution indicates that the portfolio should be diversified among all investment opportunities except Midwest steel. So, we should go for diversification, but we

should not invest in Midwest steel. The projected annual return for this portfolio is 8000 dollars, which is an overall return of 8%.

Solution for portfolio management problem : Dual value

- The dual value for the available funds constraint provides information on the rate of return from additional investment funds.

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The dual value for the available funds constraint provides information on the rate of return from additional investment funds.

Solution for portfolio management problem : Dual Value

- The optimal solution shows the dual value for the Steel Industry constraint is zero.
- The reason is that the steel industry maximum isn't a binding constraint; increases in the steel industry limit of \$50,000 will not improve the value of the optimal solution.
- Indeed, the slack variable for this constraint shows that the current steel industry investment is \$10,000 below its limit of \$50,000.
- The dual values for the other constraints are nonzero, indicating that these constraints are binding.

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$11	Government Bonds Minimum Ru	0	0.0214	0	50000	12500
\$B\$12	Specific Oil Refraction Ru	0	0.03	0	20000	30000
\$B\$8	Available Funds Ru	100000	0.069	100000	12500	50000
\$B\$9	Oil Industry Maximum Ru	50000	0.022	50000	50000	12500
\$B\$10	Steel Industry Maximum Ru	40000	0	50000	1E+30	10000

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Now, I will explain the dual value of the problem. The optimal solution shows that the dual value for the steel industry constraint is zero. This is the reason that the steel industry maximum is not a binding constraint. So, increases in the steel industry limit of 50000 dollars

will not improve the value of the optimal solution because it is not a binding constraint. So, that will not help to increase your objective function value.

Indeed, the slack variable for this constraint shows that the current steel industry investment is 10000 dollars below its limit of 50000 because, you see, the hand side is 50000, but the final value is forty thousand. So, 10000 dollars is the slack variable that has not been utilized yet. There are unutilized resources there. So, adding any extra resources will not help in improving your objective function, which is why the shadow price is zero.

The dual values for the other constraints are nonzero, for example, here, indicating that these constraints are binding constraints. The dual value of 0.0694 funds available constraint shows that the value of the optimal solution can be increased by 0.069 if one more dollar can be made available for portfolio investment. We are discussing the following: funds available constraint this constraint.

Because we have a positive shadow price and a positive dual value, if the funds available are increased by one dollar, our objective function will increase by 0.069. If more funds can be obtained at the cost of less than 6.9 percent, the management should consider obtaining them because we will be earning more value here; however, if a return of more than 6.9 percent can be obtained by investing funds elsewhere other than these five securities.

So, management should question the wisdom of investing the entire 100000 dollar in this portfolio what is the meaning of this one. If you can get your fund at the interest rate of 6.9 percent, the management should go for that. If you can get the fund, which is more than more than 6.9 percent, we need not go for these five options. We can go for other options to get more return than this portfolio than these investment opportunities.

Portfolio management problem : dual value

- Note that the dual value for constraint 4 is negative at -0.024 .
- This result indicates that increasing the value on the right-hand side of the constraint by one unit can be expected to decrease the objective function value of the optimal solution by 0.024 .

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$11	Government Bonds Minimum RU	0	-0.024	0	50000	12500
\$H\$12	Pacific Oil Restriction RU	0	0.03	0	20000	30000
\$H\$8	Available Funds RU	100000	0.069	100000	12500	50000
\$H\$9	Oil Industry Maximum RU	50000	0.022	50000	50000	12500
\$H\$10	Steel Industry Maximum RU	40000	0	50000	1E+30	10000

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Note that the dual value for constraint 4 is negative here for government bonds here, it is negatives ok. The result indicates that increasing the dual value and the right-hand side of the constraint by one unit can be expected to decrease the objective function because the shadow price is negative. If you increase on the right-hand side by one unit, the objective function will decrease by 0.024 .

Financial Planning



<https://www.rowling.com/2019/08/27/why-should-you-want-professional-help-with-your-financial-planning/>



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Now, we will go for the application of linear programming problems in another application called financial planning.

Application of LP in Financial Planning

- An application of linear programming to minimize the cost of satisfying a company's obligations to its early retirement program.
- As a result of these early retirements, the company incurs the following obligations over the next eight years:

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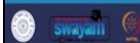
The application of linear programming to minimize the cost of satisfying a company's obligations to its early retirement program is the next application of our LP in the finance area. As a result of this early retirement, the company incurs the following obligations over the next eight years: what are the obligations?

Application of LP in Financial Planning

- The cash requirements (in thousands of dollars) are due at the beginning of each year.

Year	1	2	3	4	5	6	7	8
Cash Requirement('000)	430	210	222	231	240	195	225	255

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The cash requirement in terms of thousands of dollars is due at the beginning of each year. So, because of this yearly retirement plan, in year one, you need 430000-dollar cash requirement in years 2, year 3, up to 8 years.

Application of LP in Financial Planning

- The corporate treasurer must determine how much money must be set aside today to meet the eight yearly financial obligations as they come due.
- The financing plan for the retirement program includes investments in government bonds as well as savings.
- The investments in government bonds are limited to three choices:

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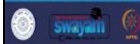
The corporate treasurer must determine how much money must be set aside today to meet the eight-year financial obligations as they come due. The financial plan for the retirement program includes investment in government bonds as well as savings. The investment in government bonds is limited to 3 choices: what are they?

Application of LP in Financial Planning

Bond	Price	Rate(%)	Years to Maturity
1	\$1150	8.875	5
2	\$1000	5.500	6
3	\$1350	11.750	7

- The government bonds have a **par value of \$1000**, which means that even with different prices each bond pays \$1000 at maturity.
- The rates shown are based on the par value.
- For purposes of planning, the treasurer assumed that any funds not invested in bonds will be placed in **savings and earn interest at an annual rate of 4%**.

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Bond 1, the price is 1150 dollars, the return rate is 8.87 percent, but the year to maturity is 5 years. The second bond is priced at a thousand dollars at a rate of 5.5 percentage years, with a maturity of six. The third one is the price of 1350, a rate of 11.75 percentage years to mature with a maturity of 7. The government bonds have a par value of 1000 dollars, which means that even with the different prices, each bond pays 1000 dollars at maturity.

The par value is 1000 dollars, and the rates shown are based on the power value. For the purpose of planning, the treasurer assumed that any funds not invested in bonds would be placed in savings and earn interest at an annual rate of 4 percent. So, the problem is that the company has some financial obligations.

Decision variables as follows

F = total dollars required to meet the retirement plan's eight-year obligation
 B_1 = units of bond 1 purchased at the beginning of year 1 ✓
 B_2 = units of bond 2 purchased at the beginning of year 1 ✓
 B_3 = units of bond 3 purchased at the beginning of year 1
 S_i = amount placed in savings at the beginning of year i for $i = 1, \dots, 8$

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For that financial obligation, decision variable F is equal to the total dollars required to meet the retirement plan's eight-year obligation. B_1 units of bond 1 were purchased at the beginning of year one, and B_2 units of bond 2 were purchased at the beginning of year one. B_3 units of bond 3 were purchased at the beginning of year one. S_i amount is placed in savings at the beginning of year i for i equal to 1 to 8.

Objective function

- Objective function is to **minimize the total dollars needed to meet** the retirement plan's eight-year obligation, or
- Min F

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The objective function is to minimize the total dollars needed to meet the retirement plan year obligations. That is, this F has to minimize what is the F says total dollar needed for meeting the eight years obligations.

Constraints

- A key feature of this type of financial planning problem is that a constraint must be formulated for each year of the planning horizon
- In general, each constraint takes the form:



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Constraints A key feature of this type of financial planning problem is that a constraint must be formulated for each year of the planning horizon. In general, each constraint takes the form of like this. So, the funds available at the beginning of the year from this point are invested in bonds and placed in savings, which is equal to cash obligation for the current year.

So, the amount initially available is minus that year how much fund is invested in bonds and savings that should be equal to the cash obligation for the current year this way the constraint needs to be formulated.

Constraint for year 1

- The funds available at the beginning of year 1 are given by F .
- With a current price of \$1150 for bond 1 and investments expressed in thousands of dollars, the total investment for B_1 units of bond 1 would be $1.15B_1$.
- Similarly, the total investment in bonds 2 and 3 would be $1B_2$ and $1.35B_3$, respectively.
- The investment in savings for year 1 is S_1 .
- Using these results and the first-year obligation of 430, we obtain the constraint for year 1:

$$F - 1.15B_1 - 1B_2 - 1.35B_3 - S_1 = 430 \text{ (Year 1)}$$




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So, the constraint for year one is that the funds available at the beginning of year one is given by F with a current price of 1150 dollars for bond 1, and the investment expressed in 1000 dollars, the total investment for B_1 unit of bond 1 would be $1.15B_1$. So, that much will be invested in bond 1. Similarly, the total investment in bonds 2 and 3 would be one B_2 because of the 1000 we have taken in terms of thousand one B_2 and $1.35 B_3$, respectively.

The investment in saving for year one is S_1 . Using this result, the first-year obligation of 430, we obtain constraints for year one. So, the F is the beginning of the year, and how much is required? Then, how much return from bond 1, bond 2, and bond 3 must be subtracted if any bond any uninvested money is there that will go for the savings? So, that will fulfill the 430. So, that means the first-year obligation is 430000 dollars. I have the initial money, right? This will be invested in bonds.

Still, there are monies there that will be invested in savings, but that should meet my first-year obligation of 430.

Constraint year 2 investment in bonds can take place only in the first year and the bonds will be held until maturity. The funds available at the beginning of year 2 include an investment return of 8.87 percent on the power value of bond 1 because in year one already, we have

invested. So, we will get 8.875 percent on the power value of bond 1, 5.5 percent on the power value of bond 2, 11.75 percent on the power value of bond 3, and 4 percent on saving.

The new amount to be invested in saving for year 2 is S_2 ok. So, with the obligation of 210, the constraint for year 2 is like this. So, on the right-hand side, you can directly write 210, which is an obligation. So, this amount of the bond has earned some interest, and the investments made in year 1 also earn some interest. So, this money will be used to fulfill the second-year obligation even after fulfilling any money left that will be invested in S_2 . That is why I brought it to the left-hand side; it will be $(S - S_2)$.

constraints for Years 3 to 8 are

Bond	Price	Rate(%)	Years to Maturity
1	1150\$	8.875	5
2	1000\$	5.500	6
3	1350\$	11.750	7

Year	1	2	3	4	5	6	7	8
Cash Requirement('000)	430	210	222	231	240	195	225	255

$$0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_2 - S_3 = 222 \text{ (Year 3)}$$

$$0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_3 - S_4 = 231 \text{ (Year 4)}$$

$$0.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_4 - S_5 = 240 \text{ (Year 5)}$$

$$1.08875B_1 + 0.055B_2 + 0.1175B_3 + 1.04S_5 - S_6 = 222 \text{ (Year 6)}$$

$$1.055B_2 + 0.1175B_3 + 1.04S_6 - S_7 = 225 \text{ (Year 7)}$$

$$1.1175B_3 + 1.04S_7 - S_8 = 255 \text{ (Year 8)}$$

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Now, constraints for years 3 to eight are the same way we have to form the constraint for year 3, year 4, year 5, and year 6. There is a difference; I will explain what is why it is 1.08. Similarly, for year 7, it is 1.055. For year eight, it is 1.17 because the bond 1 will take 5 years for maturity. So, in the first year, we have matured. So, in the sixth year, it is coming along with the principle, principle, and interest.

So, bond 2 is in the seventh year because its maturity is six years. In the seventh year, we are coming along with the principal interest, which is why it is 1.05. Bond 3 will take seven years. So, in the eighth year, it will come along with the principle and interest, which is why it is 1.17.

Explanation for bond maturity

- Note that the constraint for year 6 shows that funds available from bond 1 are 1.08875B1.
- The coefficient of 1.08875 reflects the fact that bond 1 matures at the end of year 5.
- As a result, the par value plus the interest from bond 1 during year 5 is available at the beginning of year 6.
- Also, because bond 1 matures in year 5 and becomes available for use at the beginning of year 6, the variable B1 does not appear in the constraints for years 7 and 8.
- Note the similar interpretation for bond 2, which matures at the end of year 6 and has the par value plus interest available at the beginning of year 7.
- In addition, bond 3 matures at the end of year 7 and has the par value plus interest available at the beginning of year 8.

$$\begin{aligned}
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S2 - S3 = 222 \text{ (Year 3)} \\
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S3 - S4 = 231 \text{ (Year 4)} \\
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S4 - S5 = 240 \text{ (Year 5)} \\
 &1.08875B1 + 0.055B2 + 0.1175B3 + 1.04S5 - S6 = 222 \text{ (Year 6)} \\
 &\quad 1.055B2 + 0.1175B3 + 1.04S6 - S7 = 225 \text{ (Year 7)} \\
 &\quad\quad 1.1175B3 + 1.04S7 - S8 = 255 \text{ (Year 8)}
 \end{aligned}$$

Note that the constraint for year 6 here shows that the funds available for bond 1 is 1.08875B1. The coefficient of 1.08875 reflects the fact that bond 1 matures at the end of year five. As a result, the par value plus the interest from the bond 1 during the year five is available at the beginning of year six. Also, because bond 1 matures in five years and becomes available for use at the beginning of year 6, the variable B 1 does not appear in constraint for years 7 and 8.

You see that here, there is no variable B1 or B2. Note that the similar interpretation for bond 2 matures at the end of year 6 and has the par value plus interest available at the beginning of year 7. In addition, bond 3 matures at the end of year seven and has the power value plus interest available at the beginning of year eight.

Savings in the year 8

- Finally, note that a variable S8 appears in the constraint for year 8.
- The retirement fund obligation will be completed at the beginning of year 8, so we anticipate that **S8 will be zero and no funds will be put into savings.**
- However, the formulation includes S8 in the event that the bond income plus interest from the savings in year 7 exceed the 255 cash requirement for year 8.
- Thus, S8 is a surplus variable that shows any funds remaining after the eight-year cash requirements have been satisfied.

$$\begin{aligned}
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S2 - S3 = 222 \text{ (Year 3)} \\
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S3 - S4 = 231 \text{ (Year 4)} \\
 &0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S4 - S5 = 240 \text{ (Year 5)} \\
 &1.08875B1 + 0.055B2 + 0.1175B3 + 1.04S5 - S6 = 222 \text{ (Year 6)} \\
 &\quad 1.055B2 + 0.1175B3 + 1.04S6 - S7 = 225 \text{ (Year 7)} \\
 &\quad\quad 1.1175B3 + 1.04S7 - S8 = 255 \text{ (Year 8)}
 \end{aligned}$$

Now, the savings are in year eight because year eight is the last year. How are we? What is the meaning of why we introduce this S8? Finally, note that the variable S8 appears in the constraint for year eight. The retirement fund obligation will be completed at the beginning of year 8. So, we anticipate that S8 will be 0, and no funds will be put into savings. Even though we wrote S8 because we made an equation in such a way that it fulfilled the applications, the value of S8 will become zero.

However, the formulation includes S8 if the bond income plus interest from saving in year seven exceeds our 255 cash requirement for year 8. Thus, yes, it is a surplus variable that shows any funds remaining after the eight-year cash requirements have been satisfied. That is why we have introduced variable S8.

Complete problem

Min F

F - 1.15B1 - 1B2 - 1.35B3 - S1 = 430 (Year 1)

0.08875B1 + 0.055B2 + 0.1175B3 + 1.04 S1 - S2 = 210 (Year 2)

0.08875B1 + 0.055B2 + 0.1175B3 + 1.04 S2 - S3 = 222 (Year 3)


0.08875B1 + 0.055B2 + 0.1175B3 + 1.04 S3 - S4 = 231 (Year 4)

0.08875B1 + 0.055B2 + 0.1175B3 + 1.04 S4 - S5 = 240 (Year 5)

1.08875B1 + 0.055B2 + 0.1175B3 + 1.04 S5 - S6 = 222 (Year 6)

1.055B2 + 0.1175B3 + 1.04 S6 - S7 = 225 (Year 7)

1.1175B3 + 1.04 S7 - S8 = 255 (Year 8)


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Now, this is our complete problem that I am going to solve with the help of a solver.

Now, I have brought this our formulated problem to solve in Excel. So, here this is the value as usual where we are going to get the answer, which is the changing cells. The hand side is written on this side, and the resources utilize the resources on resources written here. So, now I will go to the data solver. Look at this problem. This is a minimization problem. When you solve it, we need to answer the limit.

Why we are saying, it is a minimization because, at the beginning of the year, we should have the minimum amount of amount minimum requirement of amount. So, that should be invested in bonds. If it remains, it should be invested in savings to meet the first-year

obligations. That is why it is a minimization problem. So this says I will interpret the answers and sensitivity report now.

Now I have brought the Excel output. So, the value of F is 1728. So, that is the amount we required at the beginning of year one. I will explain this slack variable and shadow price.

Interpretation of Output

Variables	F	B1	B2	B3	S1	S2	S3	S4	S5	S6	S7	S8
Values	1728.793855	144.9881496	187.8558478	228.1879195	636.1479438	501.605712	349.681791	182.680913	0	0	0	0

- With an objective function value of 1728.79385, the total investment required to meet the retirement plan's eight-year obligation is \$1,728,794.
- Using the current prices of \$1150, \$1000, and \$1350 for each of the bonds, respectively, we can summarize the initial investments in the three bonds as follows:

Bond	Units Purchased	Investment Amount
1	B1 = 144.988	\$1150(144.988)=\$166,736
2	B2 = 187.856	\$1000(187.856) = \$187,856
3	B3 = 228.188	\$1350(228.188) = \$ 308,054

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With an objective function value of this much, 1728, the total investment required to meet the requirement plan 8-year obligation that is we need because all the values in terms of 1000, we need 1728797 using the current prices of 1150, 1000, and 1350 dollars for each of the bonds respectively we can summarize the initial investment in the 3 bonds as follows. So, initially, we purchased B1= 144, B2 =187, and B3 =228.

So, the actual amount is this quantity amount when you multiply by their power value. When you multiply this, you will get this many answers.

Interpretation of Output

Variables	F	B1	B2	B3	S1	S2	S3	S4	S5	S6	S7	S8
Values	1728.793855	144.9881496	187.8558478	228.1879195	636.1479438	501.605712	349.681791	182.680913	0	0	0	0

Year	1	2	3	4	5	6	7	8
Cash Requirement('000)	430	210	222	231	240	195	225	255

- The solution also shows that \$636,148 (see S1) will be placed in savings at the beginning of the first year.
- By starting with \$1,728,794, the company can make the specified bond and savings investments and have enough left over to meet the retirement program's first-year cash requirement of \$430,000.

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The solution also shows that 636148, that is, S1 where is this one S 1 will be placed in savings at the beginning of the first year. By starting with 1728.797, the company can make a specific bond and save investment and have enough left to meet the retirement programs for the first year cash requirement of 430000 dollars. So, after meeting the first requirement this much so the remaining amount invested is \$ 636.17.

Interpretation of Output

Variables	F	B1	B2	B3	S1	S2	S3	S4	S5	S6	S7	S8
Values	1728.793855	144.9881496	187.8558478	228.1879195	636.1479438	501.605712	349.681791	182.680913	0	0	0	0

- The optimal solution in Figure shows that the decision variables S1, S2, S3, and S4 all are greater than zero, indicating investments in savings are required in each of the first four years.
- However, interest from the bonds plus the bond maturity incomes will be sufficient to cover the retirement program's cash requirements in years 5 through 8.

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The optimal solution figure shows that the decision variables S1, S2, S3, and S4 this one S1 S2, S3, S4 are all greater than zero, indicating that investment in savings is required in each of the first four years. However, the interest from the bond plus the bond maturity income

will be sufficient to cover the retirement program cash requirement in years five through eight, which is why no saving is required in years five, six, seven, or eight.

Dual values

- The dual values have an interesting interpretation in this application.
- Each right-hand side value corresponds to the payment that must be made in that year.
- Note that the dual values are positive, indicating that increasing the required payment in any year by \$1,000 would increase the total funds required for the retirement program's obligation by \$1,000 times the dual value.
- Also note that the dual values show that increases in required payments in the early years have the largest impact.
- This makes sense in that there is little time to build up investment income in the early years versus the subsequent years.
- This suggests that if that organization faces increases in required payments it would benefit by deferring those increases to later years if possible.

$0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S2 - S3 = 222$ (Year 3)
 $0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S3 - S4 = 231$ (Year 4)
 $0.08875B1 + 0.055B2 + 0.1175B3 + 1.04S4 - S5 = 240$ (Year 5)
 $1.08875B1 + 0.055B2 + 0.1175B3 + 1.04S5 - S6 = 222$ (Year 6)
 $1.055B2 + 0.1175B3 + 1.04S6 - S7 = 225$ (Year 7)
 $1.1175B3 + 1.04S7 - S8 = 255$ (Year 8)

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
SP10	Year 4 Utilized Resources	231	0.888996339	231	1E+30	363.6690626
SP11	Year 5 Utilized Resources	240	0.854804291	240	1E+30	189.9881496
SP12	Year 6 Utilized Resources	195	0.760304654	195	2148.927647	157.8558476
SP13	Year 7 Utilized Resources	225	0.718991202	225	3027.962172	198.1879195
SP14	Year 8 Utilized Resources	255	0.670819393	255	1583.881915	255
SP7	Year 1 Utilized Resources	430	0	430	1E+30	1093.0459311
SP8	Year 2 Utilized Resources	210	0.961538462	210	1E+30	661.5938616
SP9	Year 3 Utilized Resources	222	0.924556213	222	1E+30	521.6699405

Now, we will interpret the dual value. The dual values have an interesting interpretation in this application, where the dual value is where we have this dual value as our dual value. Each right-hand side value corresponds to the payment that must be made in that year. So, this is the value of this amount we might be paying in that year. Note that the dual values are positive, indicating that increasing the required payment in any year by one unit we have in terms of thousands.

So, one unit would increase the total funds required for the retirement program application by one times the dual value, that is 1000 times. Also, note that the dual values show that increases in the required payments in the early years have the largest impact on early years. Look at years 1, 2, and 3. For example, in years 2 and 3, the dual values are 0.96, 0.92. This makes sense because there is little time to build up investment income in the early years versus subsequent years.

But in the subsequent years, the shadow price has become less. So, what we are inferring from this is if that organization faces increases in required payments, it would benefit by differing those increases to a later year if possible; if they defer that payment in later years, that is good for that organization because they can have the lesser initial amount requirement. In this lecture, I explained the application of linear programming problems in finance.

We have taken on 2 problems: one problem is portfolio management; another problem is financial planning. I formulated the problem and solved it with the help of Excel, and then we interpreted the results; thank you very much.